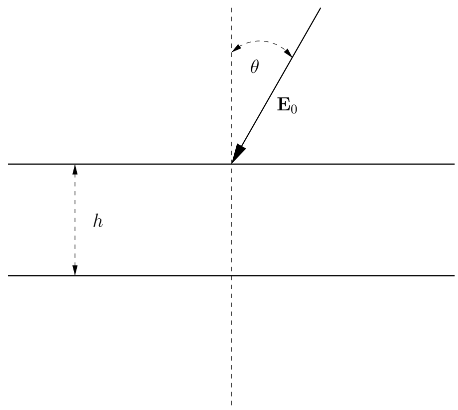
Refraction of field lines in a dielectric



A dielectric slab of thickness h, length $L \gg h$, and dielectric permittivity ϵ_r is placed in an uniform electric field $\mathbf{E}_0$. Let θ be the angle between $\mathbf{E}_0$ and the direction perpendicular to the slab surface.

a) Find the electric field inside the slab and the angle θ' between the field and the direction perpendicular to the slab surface.

b) Find the polarization charge densities.

c) Find whether the external field exerts a moment on the slab.

Boundary effects may be neglected.

Solution

a) The component of the electric field parallel to the surface, $E_\parallel = E_0 \sin\theta$, is continous. In vacuum the perpendicular component is given by $E_{v,\perp} = E_0 \cos\theta$. We use the continuity of $D_\perp = \epsilon E_\perp$ to determine $E_{v,\perp} = \epsilon_r E_{d,\perp}$ where $\mathbf{E}_d$ is the field inside the dielectric slab. Thus $E_{d,\perp} = E_0 \cos\theta/\epsilon_r$ and

$$\tan\theta' = \frac{E_{d,\parallel}}{E_{d,\perp}} = \frac{E_0 \sin\theta}{E_0 \cos\theta/\epsilon_r} = \epsilon_r \tan\theta\,.$$

Since $\epsilon_r > 1$, $\theta' > \theta$ holds.
b) From Gauss's theorem we obtain

$$\sigma_p = \epsilon_0 (E_{v,\perp} - E_{d,\perp}) = \epsilon_0 E_0 \cos\theta \left(1 - \frac{1}{\epsilon_r}\right)\,.$$

c) The electrostatic energy density inside the slab is

$$u_E = \frac{\epsilon}{2}\mathbf{E}^2 = \frac{\epsilon}{2} E_0^2 \left(\epsilon_r^{-2}\cos^2\theta + \sin^2\theta\right)\,.$$

Hence u_E grows with increasing θ and we expect electrostatic force to act to try to rotate the slab in order to reach the angle for which the energy has a minimum, i.e. $\theta = 0$.

Assuming the energy to be uniformly distributed, the total energy is $U_E = V u_E$, where V is the volume of the slab, and the momentum exerted by the electrostatic force is

$$M = -\frac{\partial U_E}{\partial\theta} = \frac{\epsilon}{2} E_0^2 V \sin 2\theta \left(\epsilon_r^{-2} - 1\right) < 0\,.$$