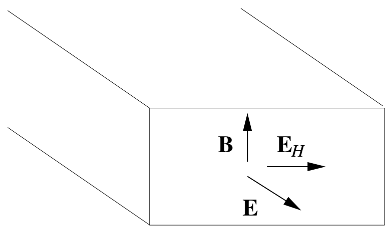
Hall effect for two charged species

Consider a conducting medium containing *two* species of charge carriers, having opposite charges $q_+ = q > 0$ and $q_- = -q$, equal densities n, but different masses m_+ and m_+. For both species we assume the validity of Drude's classical model, thus the motion of charge carriers is described by the two equations

$$m_\pm \frac{d\mathbf{v}_\pm}{dt} = \mathbf{F} - \nu_\pm m_\pm \mathbf{v}_\pm, \qquad (1)$$

where the damping frequencies ν_+ and ν_- are in general not the same.
Consider the effect of an uniform and constant electric field $\mathbf{E}$ on the medium.
a) Assuming a steady state, find the drift velocities of both species and write the conductivity σ as a function of the known parameters (q, n, $m_\pm$, $\nu_\pm$).



Now consider the medium in the presence of an electric field $\mathbf{E}$ and a magnetic field $\mathbf{B}$ (both constant and uniform), with $\mathbf{E}$ perpendicular to $\mathbf{B}$. Assume than an electric field $\mathbf{E}_H$, perpendicular to both $\mathbf{E}$ and $\mathbf{B}$, is generated similarly for what happens for the common classical Hall effect.
b) Show that, contrary to what happens for a single charged species, it is in general *not* possible to determine $\mathbf{E}_H$ from the condition that the velocities of charge carriers along the direction of $\mathbf{E}_H$ must vanish.
c) If the medium has boundaries in the direction of $\mathbf{E}_H$, a necessary condition for a steady state is that the component J_H of the current density in the direction of $\mathbf{E}_H$ must vanish ($J_H = 0$); explain why it is so, and use such condition to determine in steady state the expression of $\mathbf{E}_H$ as a function of E, B and other parameters (neglect the magnetic force in the direction of $\mathbf{E}$).

Solution

a) In a steady state, the drift velocity is found posing $d\mathbf{v}_\pm/dt = 0$ in Eq.(1), obtaining

$$\mathbf{v}_\pm = \frac{q}{m_\pm \nu_\pm}\mathbf{E}. \tag{2}$$

From the definition of the current density

$$\mathbf{J} = n_+ q_+ \mathbf{v}_+ + n_- q_- \mathbf{v}_- = nq^2 \left(\frac{1}{m_+\nu_+} + \frac{1}{m_-\nu_-} \right) \mathbf{E}, \equiv \sigma \mathbf{E}, \tag{3}$$

we obtain

$$\sigma = nq^2 \left(\frac{1}{m_+\nu_+} + \frac{1}{m_-\nu_-} \right). \tag{4}$$

b) We take $\mathbf{E} = E\hat{\mathbf{x}}$ and $\mathbf{B} = B\hat{\mathbf{z}}$. The magnetic field deflects the charge carriers along $\hat{\mathbf{y}}$, and $\mathbf{E}_H = E_H\hat{\mathbf{y}}$. The component along $\hat{\mathbf{y}}$ of the force on the carries is given by

$$F_{+,y} = q(E_H + v_{+,x}B), \qquad F_{-,y} = -q(E_H + v_{-,x}B), \tag{5}$$

where $v_{\pm,x}$ are the drift velocities found at point **a)**. Because in general $v_{+,x} \neq v_{-,x}$ holds, it is in general impossible that $F_{+,y} = 0$ and $F_{-,y} = 0$ at the same time.

c) If a current density J_y is present, a net charge will accumulate on the boundaries delimiting the medium along y. Such a charge is the source of E_H. If the medium is confined by plane walls, the surface charge density Σ on such walls will vary according to

$$\partial_t \Sigma = -J_y. \tag{6}$$

In a steady state $\partial_t \Sigma = 0$ and thus $J_y = 0$ must hold.

The condition of zero current imposes

$$0 = J_y = qn(v_{+,y} - v_{-,y}), \tag{7}$$

which implies $v_{+,y} = v_{-,y}$: the drift velocity along y is the same for both species. On the other hand, posing $dv_{\pm,y}/dt = 0$ in Eq.(1), we obtain

$$v_{\pm,y} = \pm\frac{q}{m_\pm\nu_\pm}(E_H + v_{\pm,x}B) = \pm\frac{q}{m_\pm\nu_\pm}\left(E_H \pm \frac{q}{m_\pm\nu_\pm}BE\right). \tag{8}$$

Posing $v_{+,y} = v_{-,y}$ we obtain after some simple algebra

$$E_H = -qEB\frac{m_+\nu_+ - m_-\nu_-}{m_+\nu_+ m_-\nu_-}. \tag{9}$$

We observe that while the current density, i.e. the flux of electric charge, along y is zero, there is a net flux *matter* in the same direcion, given by

$$f = m_+ n v_{+,y} + m_- n v_{-,y} = (m_+ + m_-) n v_{\pm,y}. \tag{10}$$

In a fully ionized plasma, whose constituents are electrons and ions only, in regimes where Eq.(1) holds there must be a net drift of the plasma as a whole in the direction of $\mathbf{E} \times \mathbf{B}$.

Note that this problem is also given by *Kittel*[1], but no discussion is given about of what should happen, e.g., in a semiconductor slab to account for the net flow of carriers in the direction of $\mathbf{E} \times \mathbf{B}$.

[1] C. Kittel, *Introduction to solid state physics.*