A test of the neutrality of matter

It is known that the electric field at the Earth surface is of the order of 100 V/m. This data provides information about the degree of neutrality of matter. Let us consider the hypothesis that such field is due to a small unbalance between the charge of a proton and the charge of an electron, which would result in a net charge per atom. From the knowledge of the Earth's gravitational field g derive an upper limit to $\delta q = q_p - |q_e|$ where q_p and q_e are the proton and electron charges, respectively. Notice that the result will depend on universal constants but *not* on the radius and mass of the Earth.

Solution

If the Earth has a total net charge Q, assuming that its distribution has a spherical symmetry the electrostatic field at the surface is

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{R_T^2}\,,$$

where R_T is the Earth radius. The gravitational field is given by the similar formula

$$g = G\frac{M_T}{R_T^2}\,,$$

where M_T is the mass of the Earth and $g \simeq 9.8$ m/s^2.

If the Earth contains N_p protons, its total mass is approximately

$$M_T \simeq 2N_p m_p$$

assuming that the number of protons is almost equal to that of neutrons and the mass of the neutron is close to the proton mass m_p. If the number of electrons is the same as the number of protons, but a net charge difference δq exists, the total charge of Earth is

$$Q = \delta q N_p\,.$$

The ratio between the total charge and mass is given by

$$\frac{Q}{M_T} \simeq \frac{\delta q}{2m_p}\,,$$

and the ratio between the electric and gravitational fields by

$$\frac{E}{g} = \frac{(4\pi\epsilon_0)^{-1}Q/R_T^2}{GM_T/R_T^2} = \frac{1}{4\pi\epsilon_0 G}\frac{\delta q}{2m_p}\,.$$

Since

$$\frac{E}{g} \leq \frac{100 \text{ V/m}}{9.8 \text{ m/s}^2} \simeq 10 \qquad \text{S.I. units,}$$

assuming that the measured value of E gives an upper limit to the possible effect of $q_p \neq |q_e|$, we obtain

$$\delta q \leq 10 \times 2m_p \times 4\pi\epsilon_0 G\,.$$

By inserting the values (in S.I. units) $m_p = 1.6 \times 10^{-27}$, $G = 6.67 \times 10^{-11}$, and $4\pi\epsilon_0 = 10^7 c^{-2} = (9 \times 10^9)^{-1}$, we obtain

$$\delta q \leq 2 \times 10^{-46} \text{ C} \sim 10^{-27} e\,.$$

This stringent limit is consistent with the absence of any indication for an intrinsic charge non-neutrality of matter.