Induced electric current in the ocean

A fluid flows with uniform velocity $\mathbf{v}$ in the presence of a constant and uniform magnetic field $\mathbf{B}$ perpendicular to $\mathbf{v}$. The fluid has an electrical conductivity σ.
a) Find the electric current density $\mathbf{J}$ induced in the fluid.
b) Give a numerical estimate of $|\mathbf{J}|$ for the terrestrial oceans, knowing that the Earth's magnetic field has a typical value $B \simeq 0.5$ Gauss $= 5 \times 10^{-5}$ Tesla, the conductivity of sea water is $\sigma \simeq 4\ \Omega^{-1}\text{m}^{-1}$ an a typical value of the flow velocity is $v = 1$ m/s.
c) Due to the appearance of the induced current the magnetic force tends to slow the flow. By considering the force on a fluid element, estimate the time it would take for this effect to stop the flow, if the magnetic force only was in action.

Solution

a) Due to the flow of the fluid, the charge carriers feel a force for unit charge equale to $\mathbf{v} \times \mathbf{B}$, that is equivalent to an electric field $\mathbf{E}_{eq} \equiv \mathbf{v} \times \mathbf{B}$. The induced current density is

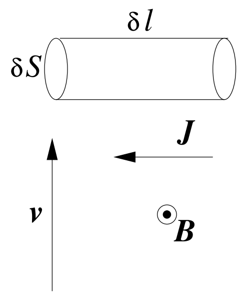
$$\mathbf{J} = \sigma \mathbf{E}_{eq} = \sigma \mathbf{v} \times \mathbf{B}. \tag{1}$$

b) Inserting the typical values given in the text we obtain

$$J \simeq 4 \times 1 \times 5 \times 10^{-5} \text{ A/m}^2 = 2 \times 10^{-4} \text{ A/m}^2. \tag{2}$$

c) For the fluid element we take a small cylinder with base surface δS and height $|\delta \mathbf{l}|$, with $\delta \mathbf{l} \parallel \mathbf{J}$. The current intensity in the cylinder is $I = J\delta S$ an the force is thus given by $\mathbf{F} = I\delta\mathbf{l} \times \mathbf{B} = -BJ\delta\mathcal{V}\mathbf{v}$, where $\delta\mathcal{V}$ is the volume of the cylinder, which in turn has a mass $m = \rho\delta\mathcal{V}$. The equation of motion, eliminating $\delta\mathcal{V}$ and inserting $J = \sigma v B$, is

$$\rho \frac{d\mathbf{v}}{dt} = -\sigma B^2 \mathbf{v}, \tag{3}$$



whose solution is a decreasing exponential with a time constant

$$\tau = \frac{\rho}{\sigma B^2} \simeq 10^{11} \text{ s} \simeq 3.5 \times 10^3 \text{ yr}, \tag{4}$$

being $\rho = 10^3$ kg/m^3.