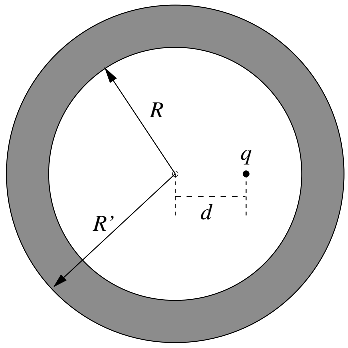
A charge in a conducting shell

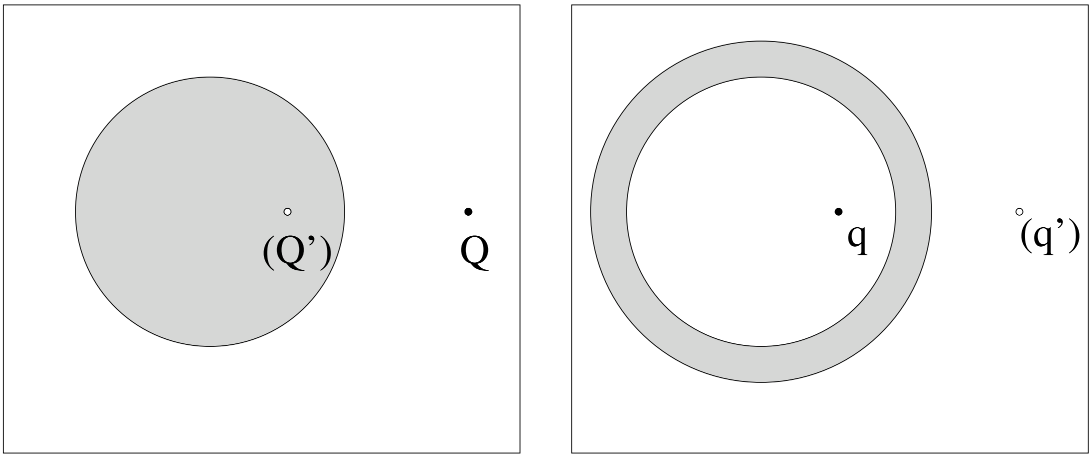
A point charge q is located inside a spherical conducting shell of internal radius R and external radius R, at a distance d from the center. The shell is grounded.



a) Find the electric potential and the electric field in the whole space.
b) Find the force on the charge.
c) Show that the total charge induced on the sphere is $-q$.
d) How does the answer to **a)** change if the shell is not grounded, but electrically isolated?

Solution

a) Let us search for a system of image charges which potential, summed up with the point charge potential, would yield a zero potential on the internal surface of the shell. Inaside the shell the field and the potential will be those produced by q and the image charge(s). Outside the shell the field and the potential will be zero.



It is known that if a charge Q is located outside a conducting sphere of radius R, at a distance a from the center O of the sphere, placing on the axis from O the the position of Q an image charge $Q' = -QR/a$ at a distance $a' = R^2/a$ from O, the sum of the potentials of Q and Q' is zero at the surface of the sphere.

From such a case we can find a solution for our present problem by exchaning the roles of the image and the real charge. We place an image charge q' outside the inner surface of the shell, on the axis trough the position of q and O and at a distance d' from O. Using the above mentioned result with the substitutions $Q \leftrightarrow q'$, $Q' \leftrightarrow q$, $a \leftrightarrow d'$, $a' \leftrightarrow d$, if $q = -q'R/d'$ and $d = R^2/d'$ the potential on the sphere of radius R is zero. Thus we obtain $d' = R^2/d$ and $q' = -qd'/R = -qR/d$.

b) The force is the Coulomb force between q and the image charge, and it is attractive:

$$F = k_0 \frac{qq'}{(d'-d)^2} = -k_0 \frac{q^2 Rd}{R^2 - d^2} .$$

c) Since the field outside the inner cavity of the shell is zero, due to Gauss theorem the charge inside any sphere that contains the inner cavity is zero. Thus, the charge induced on the inner surface is $-q$, as may be verified directly calculating the surface charge and integrating over the whole surface.

d) If the shell is isolated, the total charge on it is zero. An amount of charge $-q$ is distributed on the inner surface in order to reproduce the field of the image charge; the remaining amount of charge $+q$ is distributed uniformly on the outer surface, in order for the shell to be at a constant potential. For the superposition principle, outside the shell the potential and the field are those produced by the charge q uniformly distributed on the outer surface, i.e. those of a point charge located in the center of the shell (thus, we cannot determine the position of the charge inside the shell only by measuring the field outside the shell). In the inner cavity the field is the same both in the case of a grounded or of an isolated shell.