The Tolman-Stewart experiment

The experiment of Tolman and Stewart (1916)[1], designed to show that the conduction in metals is due to electrons, can be schematically described as follows. A metallic ring of radius a is kept in rotation around its axis with angular velocity ω. We assume that the ring has a section of surface S and that its thickness is much smaller than the radius, in order that any radial motion of charge carriers can be neglected.

At the time $t = 0$ the rotation of the ring is suddenly stopped. For $t > 0$ a current $I = I(t)$ flowing in the ring and decaying in time is observed.

a) Using Drude's model for conduction in metals, find $I = I(t)$ and its characteristic decay time τ for a ring of Copper (having electrical conductivity $\sigma \simeq 10^7\ \Omega^{-1}\mathrm{m}^{-1}$ and electron density $n_e = 8.5 \times 10^{28}\ \mathrm{m}^{-3}$).

b) Find as a function of σ the total value of the charge that flows in the ring from $t = 0$ to $t = \infty$.

c) To detect the current signal, suppose that a circular loop of radius $b \ll a$ is placed at the centre of the ring and in the same plane. Find the induced electromotive force in the loop.

[1] R. C. Tolman and T. D. Stewart, Phys. Rev. **8** (1916) 97–116.

Solution

a) In Drude's model the equation of motion of electrons is

$$m\frac{d\mathbf{v}}{dt} = \mathbf{F} - m\nu\mathbf{v}\,, \tag{1}$$

where $\mathbf{F}$ is the external force on electrons, and $m\nu\mathbf{v}$ is a phenomenological friction force. In steady state ($d\mathbf{v}/dt = 0$), under the action of an electric force $\mathbf{F} = -e\mathbf{E}$ the electrons acquire a steady velocity

$$\mathbf{v} = -\frac{e}{m\nu}\mathbf{E}\,.$$

The current density is given by $\mathbf{J} = -en_e\mathbf{v}$. From this we obtain the microscopic form of Ohm's law

$$\mathbf{J} = \frac{n_e e^2}{m\nu}\mathbf{E} \equiv \sigma\mathbf{E}\,.$$

The value of the damping frequency ν for Copper is

$$\nu = \frac{n_e e^2}{m\sigma} = \frac{8.5\times10^{28}(1.6\times10^{-19})^2}{9.1\times10^{-31}\times10^{7}} \simeq 2.4\times10^{14}\ \mathrm{s}^{-1}$$

($m = m_e = 9.1\times10^{-31}$ kg).

At $t = 0$ the electron tangential velocity is $v_0 = a\omega$. For $t > 0$, due to the absence of external forces the solution of Eq.(1) is

$$\mathbf{v} = \mathbf{v}_0 e^{-\nu t}\,.$$

The total current is thus given by

$$I = I_0 e^{-\nu t}\,, \qquad I_0 = -(en_e v_0)S\,,$$

and the decay time is $\tau = 1/\nu \simeq 4\times10^{-15}$ s.

b) The total charge flown in the ring is

$$Q = \int_0^\infty I(t)dt = \frac{I_0}{\nu} = -\frac{m}{e}\sigma S v_0\,.$$

Thus, measuring σ, S, v_0 and Q the value of e/m can be obtained. In the original experiment, Tolman and Stewart were able to measure Q using a ballistic galvanometer in a circuit coupled with a rotating coil.

c) The field at the center of the ring is

$$B = \frac{\mu_0 I}{2a}\,.$$

The flux on the loop is $\Phi \simeq \pi b^2 B$, thus the induced EMF is

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{\mu_0\nu I_0\pi b^2}{2a}e^{-\nu t}\,.$$