Intense Laser-Matter Interaction

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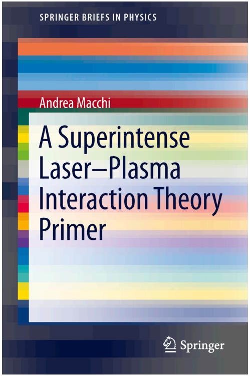
Outline

- Some Numbers & Facts on (Super-)Intense Laser-Matter Interaction
- (Ultrafast) Plasma Production by Laser
- Single Electron Motion: Relativistic & Nonlinear Effects
- Nonlinear Laser-Plasma Optics
  - Self-induced transparency
  - Self-focusing
  - Moving Mirrors: High Harmonics & Attosecond Pulses

Compact References

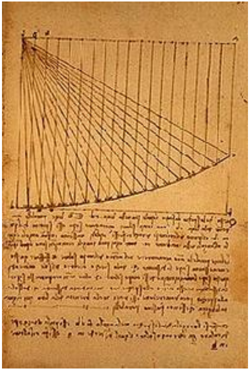
A. Macchi,
- *A Superintense Laser-Plasma Interaction Theory Primer* (Springer, 2013)

- *Basics of Laser-Plasma Interaction: a Selection of Topics*, in:
Laser-Driven Sources of High Energy Particles and Radiation,
Springer Proceedings in Physics **231**, 25-49 (2019)
arXiv:1806.06014



Focused Light-Matter Interaction: an Old Story . . .

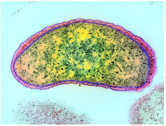
Archimedes' use of "Burning Mirrors" in the Siege of Syracuse; Giulio Parigi (~ 1600)
Uffizi Museum, Florence, Italy
Sunlight intensity: $I \simeq 10^{-1}\ \mathrm{Wcm}^{-2}$
tight focusing $\longrightarrow \simeq 10^{3}\ \mathrm{Wcm}^{-2}$

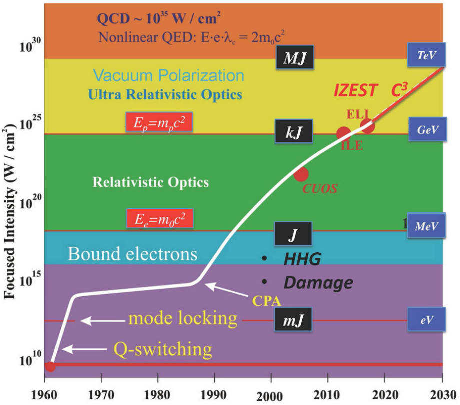


Leonardo da Vinci,
Codex Arundel (1480-1518),
British Library, London

Present Day: Extreme Intensities

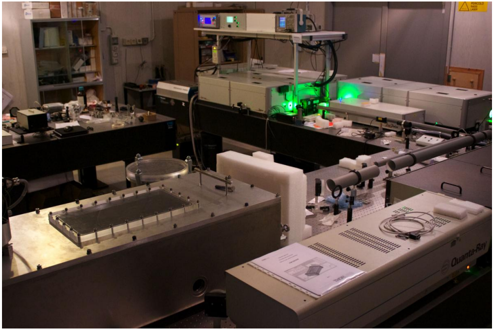
Chirped Pulse Amplification
D.Strickland & G.Mourou
(Nobel Prize 2018)
"λ^3" pulses: extreme focusing of laser pulses in space and time (few femtoseconds $= 10^{-15}$ s):
Highest intensity so far $I = 10^{23}$ W cm^{-2}
Yoon et al, Optica **8** (2021) 630





Mourou & Tajima, "Exploring fundamental physics at the highest-intensity-laser frontier", SPIE news, August 3, 2012

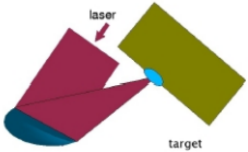
“Extreme” Physics on a Table Top



The 200 Terawatt laser system at Intense Laser Irradiation Laboratory, CNR/INO, Pisa

The Dawn of Laser-Plasma Physics

"*The laser is a solution looking for a problem*"
(I. D'Haenens to T. Maiman, 1960)
Q-switched lasers (1962):
$I = 10^{13}\ \mathrm{Wcm}^{-2}$
$\longrightarrow$ matter is ionized and heated up to $\sim 10^9$ K: hot, dense plasma state
"*It should be possible to do many interesting experiments on such plasmas*"
J.Dawson, "On the Production of Plasma by Giant Pulse Lasers"
Phys. Fluids **7** (1964) 981



Plasma generated by blasting droplets with a laser (ETH-Zurich/B.Newton)

Route to Plasma Production by Laser

Multiphoton ionization frees electrons which absorb energy via collisions with ions (*inverse Bremsstrahlung*) causing heating and further ionization

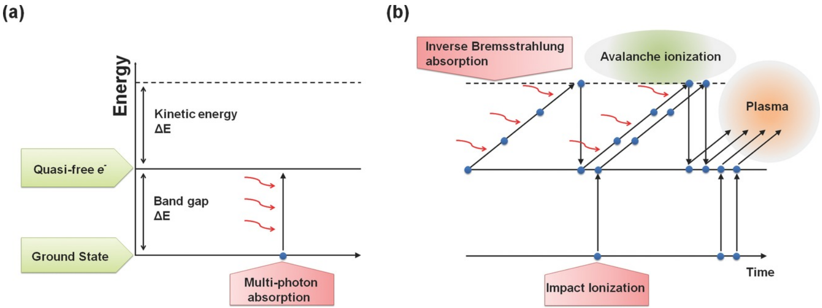


Figure from: J. Yoon et al, Annalen der Physik **525** (2013) 205

Ultrafast Field Ionization

Laser intensity
$I_{18} \equiv I/(10^{18}\ \mathrm{W\ cm}^{-2})$
Laser electric field
$E_L = 2.7 \times 10^{10} I_{18}^{1/2}\ \mathrm{V\ cm}^{-1}$
Binding electric field in H
$E_{\mathrm{H}} \approx \frac{e}{r_{\mathrm{B}}^2} = 5.1 \times 10^9\ \mathrm{Vcm}^{-1}$
Barrier lowering $\longrightarrow$
ionization via tunnel effect
Barrier suppression for
$I > 7.5 \times 10^{14}\ \mathrm{W\ cm}^{-2} Z^3$

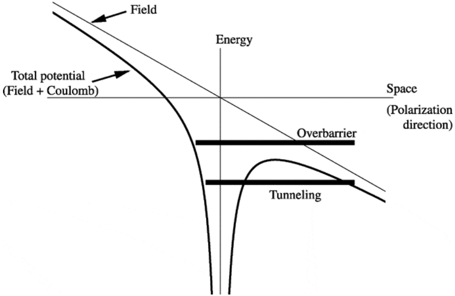


Figure from D. Bauer "Plasma formation through field ionization in intense laser–matter interaction"
Laser & Particle Beams **21** (2003) 489

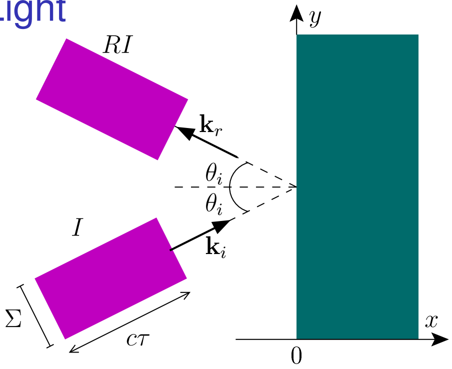
Tunneling & barrier suppression ionization are "instantaneous" (faster than the laser period)

The Heavy Pressure of Light

EM field contains linear momentum with density $\mathbf{g} = \mathbf{S}/c^2$ ($I = |\mathbf{S}|$ for a propagating wave)

$\longrightarrow$ reflection on a mirror generates a pressure:

$$P_L = (1+R)\frac{I}{c}\cos^2\theta_i$$



At normal incidence ($\theta_i = 0$) on a perfect mirror ($R = 1$)

$$P_L = \frac{2I}{c} = 6 \times 10^8 I_{18}\ \mathrm{atm}$$

strongest pressure generated on Earth

Single electron in a plane wave

An EM plane wave can be described by the vector potential:

$$\mathbf{A}(x,t) = \mathbf{A}(x-ct) \quad \longrightarrow \quad \mathbf{E} = -\frac{1}{c}\partial_t \mathbf{A} \quad , \quad \mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}$$

Equations of Motion (EoM):

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \mathbf{v} = \frac{\mathbf{p}}{m_e\gamma} \, , \qquad \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = -e\left[\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right]$$

$$\mathbf{r} = \mathbf{r}(t) \qquad \mathbf{p} = \mathbf{p}(t) \qquad \gamma = (\mathbf{p}^2 + m_e^2 c^2)^{1/2} = (1 - \mathbf{v}^2/c^2)^{-1/2}$$

The EoM are nonlinear because of the $\mathbf{v} \times \mathbf{B}$ term and the dependence of the fields on the instantaneous position:

$$\mathbf{E} = \mathbf{E}(\mathbf{r}(t), t) \qquad \mathbf{B} = \mathbf{B}(\mathbf{r}(t), t)$$

When is the motion relativistic?

(Quasi-)Monochromatic wave $\mathbf{A}(x,t) = \mathrm{Re}\left[\hat{\mathbf{A}}(x,t)\mathrm{e}^{ikx-i\omega t}\right]$

(with $\hat{A}(x,t)$ a slowly varying envelope, i.e. the wavepacket profile)

$$\text{Assume} \quad |\mathbf{v}| \ll c \quad \Rightarrow \quad |\mathbf{r}| \ll \lambda = \frac{2\pi c}{\omega} \quad \Rightarrow \quad k|\mathbf{r}| = 2\pi\frac{|\mathbf{r}|}{\lambda} \simeq 0$$

$$\Rightarrow \mathbf{E}(\mathbf{r}(t),t) = \mathbf{E}(kx(t),t) \simeq \mathbf{E}(x=0,t) \quad \text{and} \quad \frac{\mathbf{v}}{c} \times \mathbf{B} \simeq 0$$

$$\text{Solution} \quad \mathbf{p}(t) \simeq \frac{e}{c}\mathbf{A}(0,t) \propto \mathrm{e}^{-i\omega t} \qquad \frac{|\mathbf{v}|}{c} = \frac{p}{m_e c} = \frac{eA_0}{m_e c^2} \equiv a_0$$

The motion becomes relativistic and nonlinear when $a_0 \gtrsim 1$

$$a_0 = 0.85\left(\frac{I\lambda^2}{10^{18}\ \mathrm{W\ cm^{-2}}}\right)^{1/2} \quad \text{where} \quad I \equiv \langle|\mathbf{S}|\rangle = \left\langle\frac{c}{4\pi}|\mathbf{E}\times\mathbf{B}|\right\rangle$$

Constants of motion in a plane wave

Symmetry properties of the EoM $\longrightarrow$ conserved quantities:

$$\mathbf{p}_\perp - \frac{e}{c}\mathbf{A} = \mathbf{C}_1 \qquad p_x - m_e\gamma c = C_2$$

("$\perp$" denotes the transverse direction, i.e. yz plane)
Initial conditions $\mathbf{p} = 0$, $\mathbf{A} = 0 \longrightarrow \mathbf{C}_1 = 0$, $C_2 = -m_e c$

$$p_x = \frac{\mathbf{p}_\perp^2}{2m_e c} = \frac{1}{2m_e c}\left(\frac{e}{c}\mathbf{A}\right)^2$$

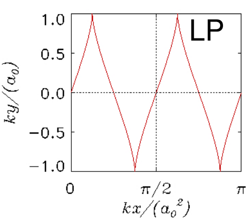
After the EM pulse is gone $\mathbf{A} = 0$ again $\Rightarrow p_x = 0$
$\Rightarrow$ no net acceleration by EM plane wave in vacuum

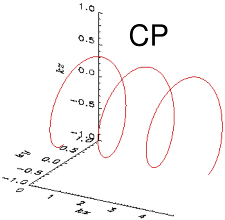
Solutions for a plane monochromatic wave

$$\mathbf{A}(x,t) = A_0 \left[\hat{\mathbf{y}} \cos\theta \cos(kx - \omega t) - \hat{\mathbf{z}} \sin\theta \sin(kx - \omega t)\right]$$

with $\mathbf{C}_1 = \mathbf{0}$, $C_2 = -m_e c$ (*adiabatic* field rising in an infinite time)

$\theta = 0\,,\ \pm\frac{\pi}{2}$
linear polarization (LP)
$\gamma = \gamma(t)$





$\theta = \pm\frac{\pi}{4}$
circular polarization (CP)
$\gamma = \left(1 + \frac{a_0^2}{2}\right)^{1/2}$
=<u>constant</u>

Constant longitudinal drift: $\langle p_x \rangle = m_e c a_0^2/4$, $\langle v_x \rangle = c a_0^2/(a_0^2 + 4)$
(origin: absorption of EM energy $\propto$ absorption of EM momentum)

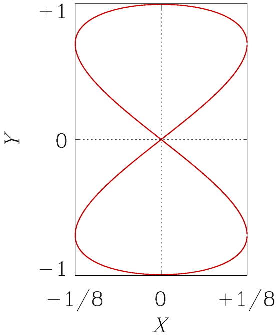
Figure of Eight

LP in the frame where $\langle v_x \rangle = 0$
i.e. $\mathbf{C}_1 = \mathbf{0}$, $C_2 = m_e \gamma_0 c$

Closed self-similar trajectory

$$16X^2 = Y^2(1 - Y^2)$$

$$X \equiv \frac{\gamma_0}{a_0^2} kx \qquad Y \equiv \frac{\gamma_0}{a_0} ky$$



Messages learnt:

- initial conditions are crucial
- polarization matters
- EM field properties constrain the dynamics

Ponderomotive approximation

Aim: describe the motion in a *quasi-periodic* field ($T = 2\pi/\omega$)

$$\mathbf{A}(\mathbf{r},t) = \mathrm{Re}\left[\tilde{\mathbf{A}}(\mathbf{r},t)\mathrm{e}^{-i\omega t}\right]$$

for which the average over a period $\left(\langle f\rangle \equiv T^{-1}\int_0^T f(t')\mathrm{d}t'\right)$

$$\langle \mathbf{A}(\mathbf{r},t)\rangle \simeq 0 \qquad \left\langle \tilde{\mathbf{A}}(\mathbf{r},t)\right\rangle \simeq \tilde{\mathbf{A}}(\mathbf{r},t)$$

Idea: find an EoM for the "slow" (period-averaged) motion

$$\mathbf{r}(t) \equiv \mathbf{r}_s(t) + \mathbf{r}_o(t) \qquad \langle \mathbf{r}_o(t)\rangle \simeq 0 \qquad \langle \mathbf{r}_s(t)\rangle \simeq \mathbf{r}_s(t)$$

(analogy: *guiding center* in a non-uniform magnetic field)

Non-Relativistic Ponderomotive force

Newton-like EoM for $\mathbf{v}_s(t) = \langle \mathbf{v}(t) \rangle$ and $\mathbf{r}_s(t) = \langle \mathbf{r}(t) \rangle$ from iterative (*perturbative*) approach including lowest order contributions from the $\mathbf{v} \times \mathbf{B}$ term and the spatial variation of $\mathbf{E} = \mathbf{E}(\mathbf{r}, t)$:

$$m_e \frac{\mathrm{d}\mathbf{v}_s}{\mathrm{d}t} \equiv \mathbf{f}_p = -\boldsymbol{\nabla} U_p(\mathbf{r}_s(t), t \qquad \frac{\mathrm{d}\mathbf{r}_s}{\mathrm{d}t} = \mathbf{v}_s$$

U_p: "ponderomotive potential" i.e. local "oscillation energy"

$$U_p = \frac{m_e}{2} v_{\mathrm{osc}}^2 = \frac{e^2}{2 m_e \omega^2} \left\langle \mathbf{E}^2(\mathbf{r}_s(t), t) \right\rangle$$

Remark: $\mathbf{f}_p$ is a *slowly-varying* force (no high frequency components!)

$$m_{\mathrm{eff}} \equiv m_e (1 + \left\langle \mathbf{a}^2 \right\rangle)^{1/2} \qquad \left(\mathbf{a} \equiv e\mathbf{A}/m_e c^2\right)$$

Relativistic Ponderomotive force

The relativistic extension of the PF theory is not straightforward (all assumptions need to be discussed carefully when $v_s \lesssim c$) and has generated controversies and different interpretations. Proposed equation for near-plane wave fields:

$$\frac{\mathrm{d}}{\mathrm{d}t}(m_{\text{eff}}\mathbf{v}_s) = -\boldsymbol{\nabla}(m_{\text{eff}}c^2) \equiv \mathbf{f}_p$$

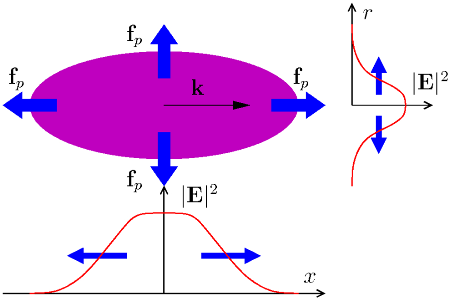
$$m_{\text{eff}} \equiv m_e(1 + \langle \mathbf{a}^2 \rangle)^{1/2} \qquad (\mathbf{a} \equiv e\mathbf{A}/m_e c^2)$$

The (time- and space-dependent) *effective mass* m_{eff} accounts for relativistic inertia due to the oscillatory motion

Ponderomotive Effects

$$\mathbf{f}_p \propto -\boldsymbol{\nabla}|\mathbf{E}|^2$$

$\Rightarrow$ electrons are pushed out of higher field regions
A laser pulse (of finite length and width) pushes electrons in both longitudinal (x) and radial (r) directions



Effects in a plasma: steepening of the density profile at the laser pulse front, channel formation, hole boring, wake generation, ...

Linear waves in a plasma

General wave equation for $\mathbf{E}$ from Maxwell's equations:

$$\left(\nabla^2 - \frac{1}{c^2}\partial_t^2\right)\mathbf{E} - \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{E}) = \frac{4\pi}{c^2}\partial_t\mathbf{J}$$

Assume monochromatic fields i.e. $\mathbf{E}(\mathbf{r},t) = \tilde{\mathbf{E}}(\mathbf{r})\mathrm{e}^{-i\omega t}$
Using linearized, non-relativistic fluid equations ($|\mathbf{u}_e| \ll c$)

$$\partial_t\mathbf{u}_e = -\frac{e}{m_e}\mathbf{E} \qquad \mathbf{u}_e = \mathbf{u}_e(\mathbf{r},t) \qquad \text{(ions taken at rest)}$$

$$\tilde{\mathbf{J}} = -en_e\tilde{\mathbf{u}}_e = -\frac{i}{4\pi}\frac{\omega_p^2}{\omega}\tilde{\mathbf{E}}, \qquad \omega_p \equiv \left(\frac{4\pi e^2 n_e}{m_e}\right)^{1/2}$$

$$\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\tilde{\mathbf{E}} - \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\tilde{\mathbf{E}}) = \frac{\omega_p^2}{c^2}\tilde{\mathbf{E}} \qquad \text{Helmoltz equation}$$

Linear transverse (EM) waves

Taking $\boldsymbol{\nabla}\cdot\mathbf{E}=0$ and introducing $\varepsilon(\omega)=\mathsf{n}^2(\omega)=1-\frac{\omega_p^2}{\omega^2}$

$$\left(\nabla^2+\varepsilon(\omega)\frac{\omega^2}{c^2}\right)\tilde{\mathbf{E}}=\left(\nabla^2+\mathsf{n}^2(\omega)\frac{\omega^2}{c^2}\right)\tilde{\mathbf{E}}=0$$

$\varepsilon(\omega)$ dielectric function, $\mathsf{n}(\omega)$ refractive index
Plane waves $\tilde{\mathbf{E}}(\mathbf{r})=E_0\boldsymbol{\epsilon}\mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}}$, $\mathbf{k}\cdot\boldsymbol{\epsilon}=0$, $\mathbf{B}=\mathbf{k}\times\mathbf{E}/k$

dispersion relation $\quad k^2c^2=\varepsilon(\omega)\omega^2=\omega^2-\omega_p^2$

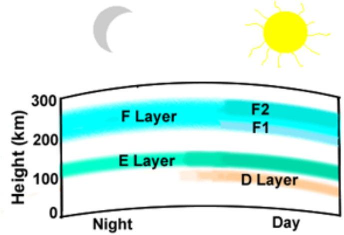
Propagation requires a real value of k i.e.

$$k^2>0 \quad\leftrightarrow\quad \varepsilon(\omega)>0 \quad\leftrightarrow\quad \omega>\omega_p$$

The Cut-off or “Critical” Density

$$\omega > \omega_p \quad \leftrightarrow \quad n_e < n_c \equiv \frac{m_e\omega^2}{4\pi e^2} = 1.1 \times 10^{21}\ \text{cm}^{-3}(\lambda/1\ \mu\text{m})^{-2}$$

$n_e < n_c$: “underdense” transparent plasma
$n_e > n_c$: “overdense” opaque plasma
Metals ($n_e \simeq 10^{23}\ \text{cm}^{-3}$) reflect visible light
Ionosphere ($n_e \lesssim\simeq 10^{6}\ \text{cm}^{-3}$) reflects radio waves



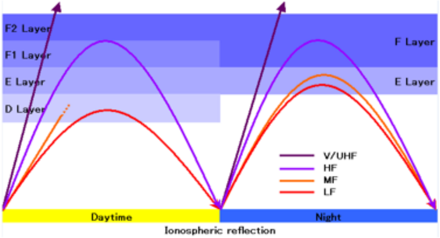


Figure from engineerstutor.com

A Nonlinear Relativistic Wave

Nonlinear terms for $a_0 \gtrsim 1$:
$\mathbf{p}_e = \mathbf{p}_e(\mathbf{r}, t)$

$$\partial_t \mathbf{p}_e + \mathbf{u}_e \cdot \nabla \mathbf{p}_e = -e\mathbf{E} - \frac{e}{c}\mathbf{u}_e \times \mathbf{B}$$

$$\mathbf{J} = -en_e\mathbf{u}_e = -en_e \frac{\mathbf{p}_e/m_e c}{(1 + \mathbf{p}_e^2/m_e^2c^2)^{1/2}}$$

In general plane wave solutions are neither monochromatic nor transverse ($\mathbf{u}_e \times \mathbf{B} \parallel \mathbf{k}$)
Particular monochromatic solution for *circular* polarization:
$\mathbf{p}_e \cdot \mathbf{k} = 0$, $\mathbf{u}_e \cdot \nabla \mathbf{p}_e = 0$, $\mathbf{u}_e \times \mathbf{B} = 0$, and

$$\gamma = (1 + \mathbf{p}_e^2/m_e^2c^2)^{1/2} = \text{const.} = \left(1 + a_0^2/2\right)^{1/2}$$

$$\partial_t \mathbf{p}_e = m_e\gamma\partial_t \mathbf{u}_e = -e\mathbf{E} \qquad \mathbf{J} = -en_e\mathbf{u}_e$$

Identical to the non-relativistic equations but for $m_e \to m_e\gamma$

“Relativistic” Self-Induced Transparency

For the *particular* solution (CP, plane wave, monochromatic) the replacement $m_e \to m_e\gamma$ yields

$$\omega_p \longrightarrow \frac{\omega_p}{\gamma^{1/2}} \qquad k^2c^2 = \omega^2 - \frac{\omega_p^2}{\gamma}$$

The cut-off density $n_c \longrightarrow n_c\gamma = n_c\left(1 + a_0^2/2\right)^{1/2}$
the more intense the wave, the higher the cut-off density

Warning: $n_e = n_c\gamma$ is not a “sharp” transparency threshold because of nonlinear pulse dispersion and distortion, effect of boundary conditions, ...
Message: use with care the “relativistically corrected critical density” $n_c^{(\mathrm{rel})} = n_c\gamma$ concept

Transparency of Semi-Infinite Plasma

The ponderomotive force pushes and piles up electrons
$\longrightarrow$ increase of density which keeps the plasma opaque

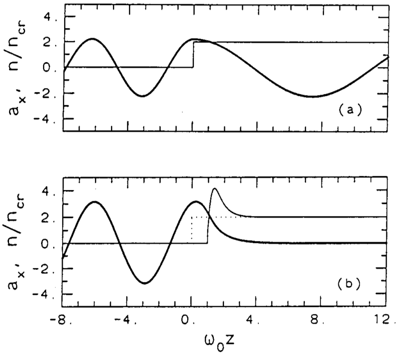
Both evanescent and propagating solutions for same laser & plasma parameters (hysteresis, instability)
Evanescent solution exists for

$$a_0 \lesssim \frac{3^{3/2}}{2^3}\left(\frac{n_e}{n_c}\right)^2 \simeq 0.65\left(\frac{n_e}{n_c}\right)^2$$

instead of

$n_e > n_c\gamma \leftrightarrow a_0 \lesssim \sqrt{2}n_e/n_c$

[F. Cattani et al,
Phys. Rev. E **62** (2000) 1234]



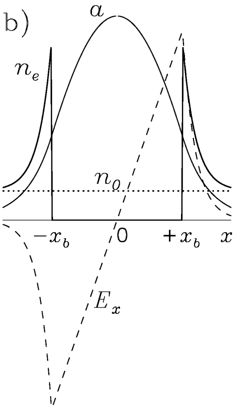
Goloviznin & Schep,
Phys. Plasmas **7** (2000) 1564

Electromagnetic Caviton

For certain values of a_0 and n, at the plasma (ion) boundary $x = 0$

$$\frac{\mathrm{d}\tilde{a}}{\mathrm{d}x}(x = 0) = 0$$

$\rightarrow$ we can build a continuous symmetrical solution between two plasma layers: resonant "optomechanical" EM cavity sustained by the ponderomotive force (*caviton*, improperly aka soliton)

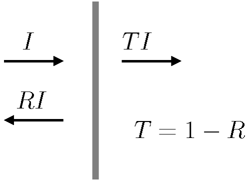


On the time scale of ion motion the caviton expands because of the electrostatic force (model for "post-solitons" which have been observed in experiments)

Transparency of ultrathin plasma foil

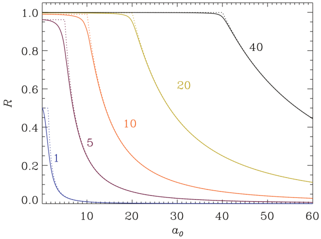
$n_e(x) \simeq n_0 \ell \delta(x)$ (ℓ: foil thickness)

[V.A.Vshivkov et al, Phys. Plasmas **5** (1996) 2727]



Nonlinear reflectivity:

$$R \simeq \begin{cases} 1 & (a_0 < \zeta) \\ \dfrac{\zeta^2}{a_0^2} & (a_0 < \zeta) \end{cases} \qquad \zeta \equiv \pi \frac{n_0 \ell}{n_c \lambda}$$

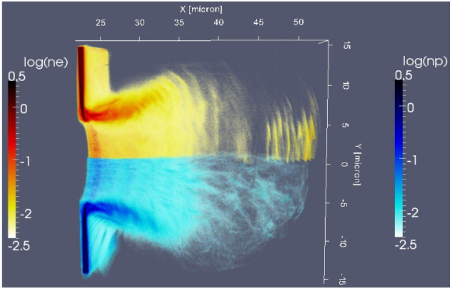


The transparency threshold $a_0 \simeq \zeta$ depends on areal density $n_0 \ell$

Self-induced transparency is a complex process

Several effects contribute to SIT: target heating & expansion, 3D bending & rarefaction, instabilities . . .
Only kinetic simulations can take most effects simultaneously into account

3D PIC simulation of laser interaction with a thin target showing breakup to transparency
[A. Sgattoni, AlaDyn code]

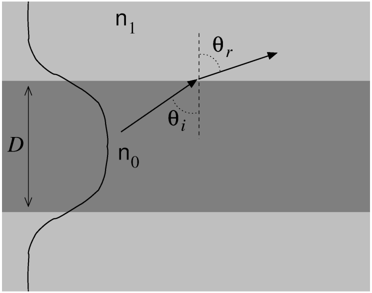


Relativistic Self-Focusing

Nonlinear refractive index (to be used with care!)

$$\mathrm{n}_{\mathrm{NL}} = \left(1 - \frac{\omega_p^2}{\gamma\omega^2}\right)^{1/2} = \mathrm{n}_{\mathrm{NL}}(|\mathbf{a}|^2) \qquad \gamma = (1 + |\mathbf{a}|^2/2)^{1/2}$$

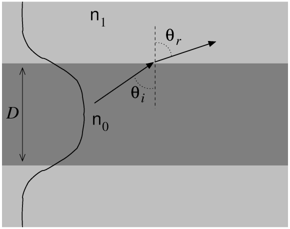
For a laser beam with ordinary intensity profile n_{NL} is higher on the axis than at the edge: $\mathrm{n}_0 = \mathrm{n}_{\mathrm{NL}}(a_0) > \mathrm{n}_{\mathrm{NL}}(0) = \mathrm{n}_1$
$\longrightarrow$ pulse guiding effect
as in an optical fiber



Self-Focusing threshold: simple model

Assumptions: $a_0 \ll 1$, $\omega_p \ll \omega$, $\lambda/D \ll 1$

Impose total reflection in Snell's law of refraction



$$\sin\theta_r = \frac{n_0}{n_1}\sin\theta_i = \frac{n_{NL}(a_0)}{n_{NL}(0)}\sin\theta_i \doteq 1$$

$\cos\theta_i \simeq \lambda/D$ diffraction angle

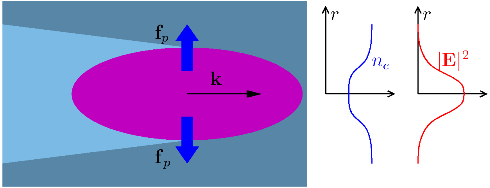
$$\longrightarrow \quad \pi\left(\frac{D}{2}\right)^2 a_0^2 \simeq \pi\lambda^2\frac{\omega^2}{\omega_p^2}$$

Threshold *power* for self-focusing

$$P_c \simeq \frac{\pi^2}{2}\frac{m_e c^3}{r_c}\left(\frac{\omega}{\omega_p}\right)^2 = 43\ \mathrm{GW}\frac{n_c}{n_e}$$

Advanced modeling of self-focusing

The radial ponderomotive force creates a low-density channel $\longrightarrow$ further "optical fiber" effect (*self-channeling*)



A non-perturbative, multiple-scale modeling for Gaussian beam characterizes the propagation modes

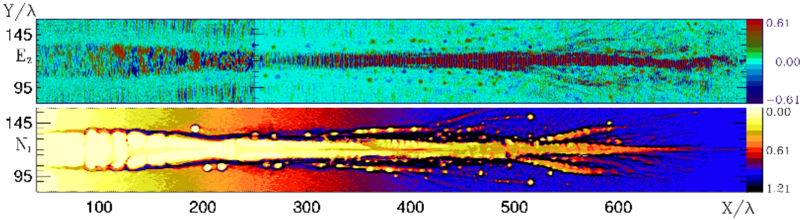
[Sun et al Phys. Fluids **30** (1987) 526]

"Minimal" threshold power $P_c = 17.5\ \mathrm{GW}\dfrac{n_c}{n_e}$

Warning: it applies only to not-so-short, not-so-tightly focused pulses

Nonlinear propagation is a complex process . . .

2D simulation of the propagation of a laser pulse ($a_0 = 2.5$, $\tau_p = 1$ ps) in an inhomogeneous plasma with peak density $n_e = 0.1 n_c$
Self-focusing and channeling followed by beam breakup, caviton formation, ion acceleration, steady magnetic field generation, . . .



T. V. Liseykina & A. Macchi, IEEE Trans. Plasma Science **36** (2008) 1136, special issue on Images in Plasma Science

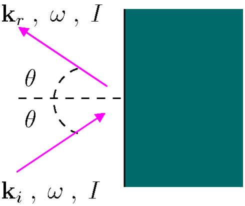
Moving mirrors

A step-boundary plasma described by n $= (1 - n_e/n_c)^{1/2}$ with $n_e \gg n_c$ is a perfect mirror (100% reflection)

Linear theory assumptions:

the interface ($x = 0$) is immobile

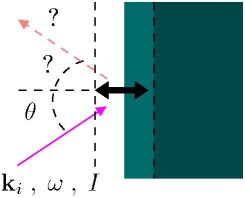
electrons are confined in the $x > 0$ region)



At high intensities the surface is:

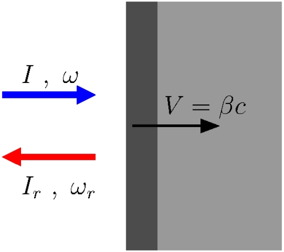
- pushed/pulled by oscillating components of the Lorentz force
- pushed by the steady ponderomotive force

$\rightarrow$ pulse is reflected from a “moving” mirror



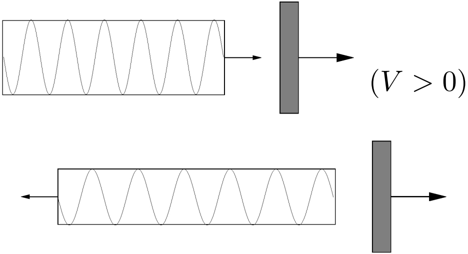
Reflection from a moving mirror

Reflection kinematics can be studied via Lorentz transformations
(the mirror is “perfect” in its rest frame; normal incidence for simplicity)



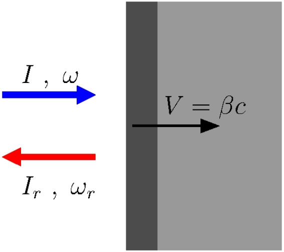
$$\omega_r = \omega \frac{1-\beta}{1+\beta} \qquad \beta = \frac{V}{c}$$

red shift for $V > 0$
blue shift for $V < 0$
The number of cycles is a Lorentz invariant $\longrightarrow$
$V > 0$: pulse stretching
$V < 0$: pulse shortening



Force on/by the moving mirror

The force on the mirror can be derived from Lorentz transformations of fields and forces or also by the conservation of photon number N



$$I = \frac{N\hbar\omega}{\tau} \quad \text{intensity } (\tau\text{: pulse duration})$$

$$\Delta\mathbf{p} = N\hbar(\mathbf{k}_i - \mathbf{k}_r) = N\frac{\hbar}{c}(\omega + \omega_r)\hat{\mathbf{x}} \quad \text{exchanged momentum}$$

$$\omega_r = \omega\frac{1-\beta}{1+\beta} \qquad \Delta t = \frac{\tau}{1-\beta} \qquad \Delta t\text{: reflection time}$$

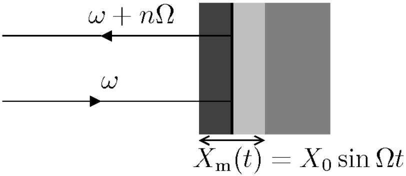
$$F \equiv \frac{\Delta p}{\Delta t} = \frac{2I}{c}\frac{1-\beta}{1+\beta} = \begin{cases} > 0 & \text{for} \quad \beta > 0 \quad \text{(work done on the mirror)} \\ < 0 & \text{for} \quad \beta < 0 \quad \text{(work done on the pulse)} \end{cases}$$

$\longrightarrow$ a moving mirror may amplify the reflected pulse!

Oscillating mirror and high harmonics

$X_{\rm m}(t) = X_0 \sin \Omega t$

Boundary condition
in instantaneous rest frame



$$E'_{\|}(x = X'_m) = 0$$

$\rightarrow \quad A_{\|}(x = X_m(t)) = 0$ in lab frame

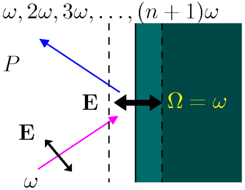
$A_{\|}(x, t) = A_i(x - ct) + A_r(x + ct)$ with $A_i(t) = A_0 \cos(\omega t)$

$$\longrightarrow A_r(t) \sim \sin\left(\omega t + \frac{2\omega X_0}{c}\sin \Omega t\right) \sim \sum_{n=0}^{\infty} J_n\left(\frac{2\omega X_0}{c}\right)\sin(\omega + n\Omega)t$$

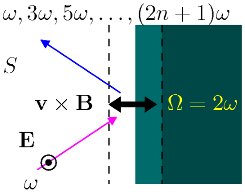
The reflected spectrum contains sums of wave frequency and mirror harmonics $\omega_r, n = \omega + n\Omega$

Self-generated high harmonics

The laser pulse drives surface oscillations with either ω or 2ω frequency depending on the polarization

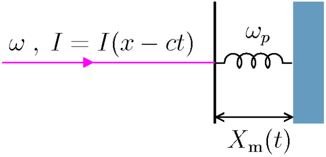


P-polarization: $\mathbf{E}$-driven, $\Omega = \omega$
$\longrightarrow$ even & odd HH, P-polarized



S-polarization: $\mathbf{v} \times \mathbf{B}$-driven, $\Omega = 2\omega$
$\longrightarrow$ odd HH only, S-polarized

Toy model for moving mirror HH



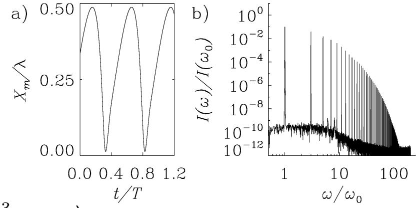
Perfect mirror with position X_m, velocity $V_m = \beta_m c$ and recoil force at the plasma frequency

$$\frac{\mathrm{d}}{\mathrm{d}t}(\gamma_m \beta_m) = \frac{2I}{\sigma c^2}\left(1 + 2\cos(2\omega t_r)\right)\frac{1-\beta_m}{1+\beta_m} - \omega_p^2 X_m \qquad \frac{\mathrm{d}X_m}{\mathrm{d}t} = \beta_m c$$

$t_r = t - X_m/c$: retarded time $\qquad \sigma$: mass per unit area

Cut-off frequency depends on $\beta_\mathrm{max} = \max(\beta_m)$

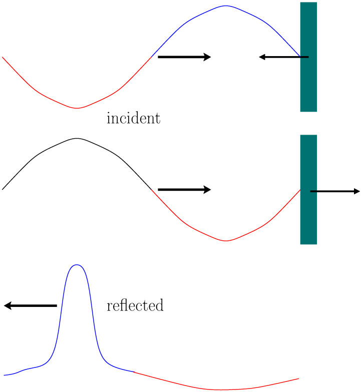
$$\omega_\mathrm{co} = \omega \frac{1-\beta_\mathrm{max}}{1-\beta_\mathrm{max}} \simeq 4\omega\gamma_\mathrm{max}^2$$

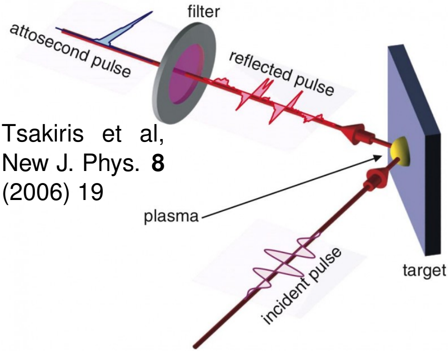


(other scalings proposed e.g. $\omega_\mathrm{co} \sim \gamma_\mathrm{max}^3 \ldots$)

Attosecond pulse train

HH are phase-locked
Reflected light is an
attosecond pulse train





Tsakiris et al, New J. Phys. **8** (2006) 19

Simple picture:
successive half-cycles are alternately compressed-enhanced and stretched-quenched

Achieving extreme intensities via harmonic focusing

Intensity ehnancement of attosecond pulses plus focusing by the self-consistently curved target surface may yield $I \simeq 6 \times 10^{27}$ W cm^{-2} sufficient to investigate strong field QED effects

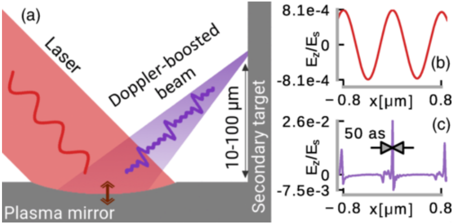


figure: L. Fedeli et al, Phys. Rev. Lett. **127** (2021) 114801

Earlier similar studies:
V. A. Vshivkov et al, Phys. Plasmas **5** (1998) 2727
S. Gordienko et al, PRL **94** (2005) 103903
Alternate approach based on reflection from plasma wake waves:
S. V. Bulanov et al, PRL **91** (2003) 085001

Summary

- (Super-)intense lasers (instantaneously) turn matter into plasma
- Relativistic electron dynamics is strongly nonlinear
- New optical phenomena arise in the relativistic regime

Tomorrow: exploiting laser-plasma acceleration for new particle accelerators