Basics of Laser-Plasma Interaction (lecture 2/3)

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Outline of Lecture 2

Nonlinear “*relativistic*” optics

- Review of linear EM waves in a plasma
- Self-induced transparency
- Self-focusing

Moving mirrors

- Basic formulas
- High harmonics
- “Flying mirrors” from plasma wakes
- Light sails

Linear transverse (EM) waves

Taking $\tilde{\mathbf{E}}(\mathbf{r}) = E_0 \boldsymbol{\epsilon} \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}}$, $\quad \nabla\cdot\mathbf{E} = 0,\quad \mathbf{k}\cdot\boldsymbol{\epsilon} = 0 \quad \mathbf{B} = \mathbf{k}\times\mathbf{E}/k$

$$\left(\nabla^2 + \varepsilon(\omega)\frac{\omega^2}{c^2}\right)\tilde{\mathbf{E}} = \left(\nabla^2 + \mathrm{n}^2(\omega)\frac{\omega^2}{c^2}\right)\tilde{\mathbf{E}} = 0$$

$\varepsilon(\omega) = \mathrm{n}^2(\omega) = 1 - \dfrac{\omega_p^2}{\omega^2}$ $\quad \varepsilon(\omega)$ dielectric function, $\mathrm{n}(\omega)$ refractive index

dispersion relation $\quad k^2c^2 = \varepsilon(\omega)\omega^2 = \omega^2 - \omega_p^2$

Propagation requires a real value of k i.e.

$$k^2 > 0 \quad \leftrightarrow \quad \varepsilon(\omega) > 0 \quad \leftrightarrow \quad \omega > \omega_p \quad \leftrightarrow \quad n_e < n_c \equiv m_e\omega^2/4\pi e^2$$

$n_c = \dfrac{1.1\times 10^{21}\ \mathrm{cm}^{-3}}{(\lambda/1\ \mu\mathrm{m})^2}$: cut-off or “critical” density

A nonlinear relativistic wave

As $a_0 \longrightarrow 1$ nonlinear terms complicate the picture:

$$\partial_t \mathbf{p}_e + \mathbf{u}_e \cdot \nabla \mathbf{p}_e = -e\mathbf{E} - \frac{e}{c}\mathbf{u}_e \times \mathbf{B} \qquad \mathbf{J} = -en_e\mathbf{u}_e = -en_e \frac{\mathbf{p}_e c}{(\mathbf{p}_e^2 + m_e^2 c^2)^{1/2}}$$

In general a plane wave solution to is neither monochromatic nor transverse ($\mathbf{u}_e \times \mathbf{B} \parallel \mathbf{k}$)
However for *circular* polarization there is a monochromatic solution for which $\mathbf{p}_e \cdot \mathbf{k} = 0 \quad , \quad \mathbf{u}_e \cdot \nabla \mathbf{p}_e = 0 \quad , \quad \mathbf{u}_e \times \mathbf{B} = 0 \quad ,$ and

$$\gamma = (1 + \mathbf{p}_e^2 / m_e^2 c^2)^{1/2} = \text{cost.} = \left(1 + a_0^2/2\right)^{1/2}$$

$$\partial_t \mathbf{p}_e = m_e \gamma \partial_t \mathbf{u}_e == -e\mathbf{E} \qquad \mathbf{J} = -en_e\mathbf{u}_e$$

The equations are *identical* to the non-relativistic case but for

$$m_e \rightarrow m_e \gamma$$

Self-induced transparency (with words of caution ...)

For the *particular* solution (CP, plane wave, monochromatic) the replacement $m_e \to m_e\gamma$ yields

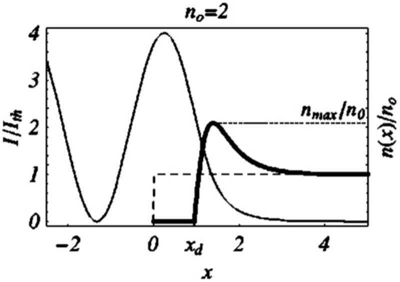
$$\omega_p \longrightarrow \frac{\omega_p}{\gamma^{1/2}} \qquad k^2c^2 = \omega^2 - \frac{\omega_p^2}{\gamma}$$

The cut-off density $n_c \longrightarrow n_c\gamma = n_c\left(1 + a_0^2/2\right)^{1/2}$
the more intense the wave, the higher the cut-off density

However one cannot define $n_e = n_c\gamma$ as a transparency threshold because of nonlinear pulse dispersion and distortion, effect of boundary conditions, ...
Message: distrust the "relativistically corrected critical density" $n_c^{(\mathrm{rel})} = n_c\gamma$ concept

Transparency of semi-infinite plasma

The ponderomotive force pushes and piles up electrons
$\longrightarrow$ increase of density
$\longrightarrow$ change of the transparency threshold
[F. Cattani et al,
Phys. Rev. E **62** (2000) 1234]



Evanescent solution (assuming steady state, circular polarization, immobile ions ...) exists up to a threshold (for $n_e \gg n_c$)

$$a_0 \simeq \frac{3^{3/2}}{2^3}\left(\frac{n_e}{n_c}\right)^2 \simeq 0.65\left(\frac{n_e}{n_c}\right)^2$$

instead of $n_e = n_c\gamma \leftrightarrow a_0 \simeq \sqrt{2}n_e/n_c$

The evanescent solution - I

Assumptions in the cold fluid plasma equations

- steady state
- balance between ponderomotive and electrostatic forces

→ ODE for $\tilde{a}(x)$ (that may be put in Hamiltonian form)

$$\frac{\mathrm{d}^2\tilde{a}}{\mathrm{d}x^2} - \frac{\tilde{a}}{1+\tilde{a}}\left(\frac{\mathrm{d}\tilde{a}}{\mathrm{d}x}\right)^2 + \left(1+\tilde{a}^2 - n(1+\tilde{a}^2)^{1/2}\right) = 0$$

Evanescent solution in the plasma

$$\tilde{a}(x) = \frac{2n^{1/2}\kappa\cosh\left(\kappa(x-x_0)\right)}{n\cosh^2\left(\kappa(x-x_0)\right) - n + 1}$$

$n = n_0/n_c$, $\kappa = (n-1)^{1/2}$

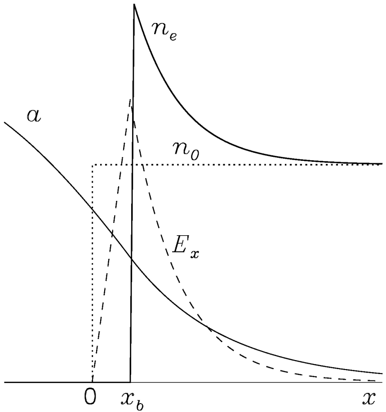
The evanescent solution - II

The parameter x_0 is determined by matching with the vacuum solution (standing wave) at the electron density boundary $x = x_b$ (to be determined self-consistently)

Condition of "monotonic evanescence" $\mathrm{d}\tilde{a}/\mathrm{d}x < 0$ determine existence condition

→ transparency threshold

[F. Cattani et al PRE **62** (2000) 1234]



Note: for given n and a_0 *both* evanescent and propagating solution may exist (hysteresis) [Goloviznin & Schep, Phys. Plasmas **7** (2000) 1564]

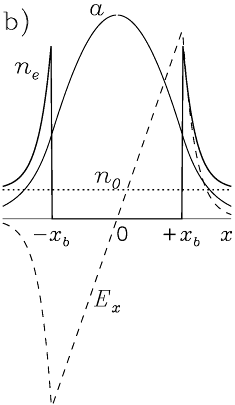
Electromagnetic caviton

For certain values of a_0 and n we obtain at the plasma (ion) boundary $x = 0$

$$\frac{\mathrm{d}\tilde{a}}{\mathrm{d}x}(x = 0) = 0$$

→ one can “build” a continous, symmetrical solution between two plasma layers: resonant EM cavity sustained by the ponderomotive force (*caviton*, improperly aka soliton)

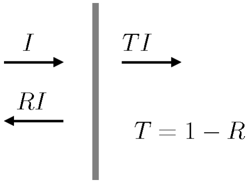
On the time scale of ion motion the caviton expands because of the electrostatic force (“post-solitons” also observed experimentally)



Transparency of ultrathin plasma foil

$n_e(x) \simeq n_0 \ell \delta(x)$ (ℓ: foil thickness)

[V.A.Vshivkov et al, Phys. Plasmas **5** (1996) 2727]



Nonlinear reflectivity can be calculated

$$R \simeq \begin{cases} 1 & (a_0 < \zeta) \\ \dfrac{\zeta^2}{a_0^2} & (a_0 < \zeta) \end{cases} \qquad \zeta \equiv \pi \frac{n_0 \ell}{n_c \lambda}$$

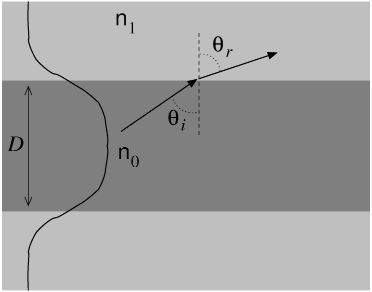
The transparency threshold $a_0 \simeq \zeta$ depends on areal density $n_0 \ell$

Relativistic Self-Focusing

Nonlinear refractive index (to be used with care!)

$$n_{\rm NL} = \left(1 - \frac{\omega_p^2}{\gamma\omega^2}\right)^{1/2} = n_{\rm NL}(|\mathbf{a}|^2) \qquad \gamma = (1 + |\mathbf{a}|^2/2)^{1/2}$$

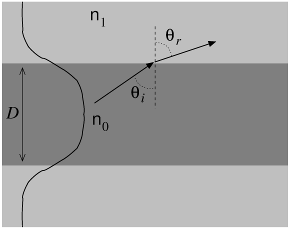
For a laser beam with ordinary intensity profile $n_{\rm NL}$ is higher on the axis than at the edge: $n_0 = n_{\rm NL}(a_0) > n_{\rm NL}(0) = n_1$
$\longrightarrow$ pulse guiding effect as in an optical fiber



Self-Focusing threshold: simple model

Assumptions: $a_0 \ll 1$, $\omega_p \ll \omega$, $\lambda/D \ll 1$

Impose total reflection in Snell's law of refraction



$$\sin\theta_r = \frac{\mathrm{n}_0}{\mathrm{n}_1}\sin\theta_i = \frac{\mathrm{n}_{\mathrm{NL}}(a_0)}{\mathrm{n}_{\mathrm{NL}}(0)}\sin\theta_i \doteq 1$$

with $\theta_i \simeq \arccos(\lambda/D)$ the diffraction angle

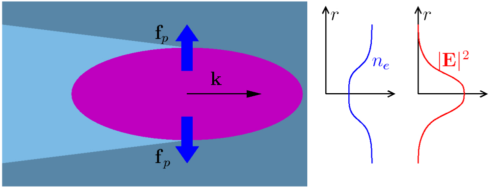
$$\longrightarrow \pi\left(\frac{D}{2}\right)^2 a_0^2 \simeq \pi\lambda^2\frac{\omega^2}{\omega_p^2}$$

Threshold *power* for self-focusing

$$P_c \simeq \frac{\pi^2}{2}\frac{m_e c^3}{r_c}\left(\frac{\omega}{\omega_p}\right)^2 = 43\ \mathrm{GW}\frac{n_c}{n_e}$$

Advanced modeling of self-focusing

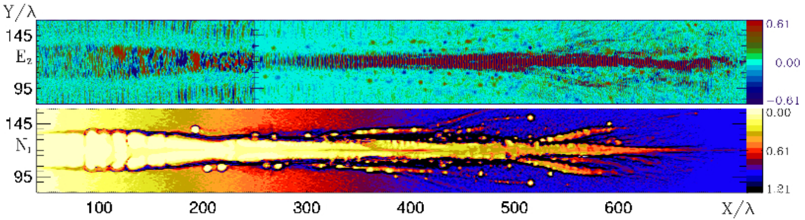
The radial ponderomotive force creates a low-density channel
$\longrightarrow$ further "optical fiber" effect (*self-channeling*)



A non-perturbative, multiple-scale modeling for Gaussian beam characterizes the propagation modes
[Sun et al Phys. Fluids **30** (1987) 526]
"Minimal" threshold power $P_c = 17.5\ \mathrm{GW}\dfrac{n_c}{n_e}$
Warning: it applies only to not-so-short, not-so-tightly focused pulses

Nonlinear propagation is a complex process . . .

2D simulation of the propagation of a laser pulse ($a_0 = 2.5$, $\tau_p = 1$ ps) in an inhomogeneous plasma with peak density $n_e = 0.1 n_c$
Self-focusing and channeling followed by beam breakup, caviton formation, ion acceleration, steady magnetic field generation, . . .



T. V. Liseykina & A. Macchi, IEEE Trans. Plasma Science **36** (2008) 1136, special issue on Images in Plasma Science

Moving mirrors

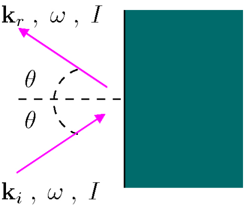
A step-boundary plasma described by n $= (1 - n_e/n_c)^{1/2}$ with $n_e \gg n_c$ is a perfect mirror (100% reflection)

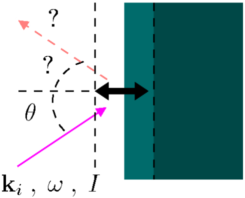
Linear theory assumes the interface $(x = 0)$ to be immobile and electrons to be confined in the mirror $(x > 0$ region)

At very high intensities the interface is

- pushed/pulled by oscillating components of the Lorentz force
- pushed by the steady ponderomotive force

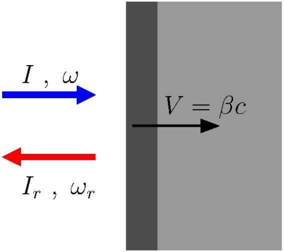
$\rightarrow$ pulse is reflected from a "moving" mirror





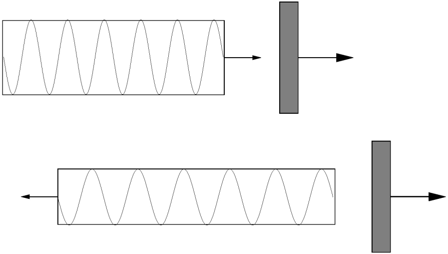
Reflection from a moving mirror

Reflection kinematics can be studied via Lorentz transformations (the mirror is “perfect” in its rest frame; normal incidence for simplicity)



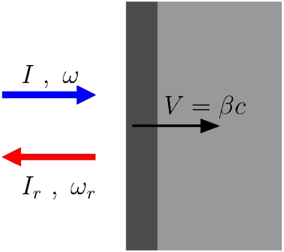
$$\omega_r = \omega \frac{1-\beta}{1+\beta} \qquad \beta = \frac{V}{c}$$

red shift for $V > 0$
blue shift for $V < 0$
The number of cycles is a Lorentz invariant $\longrightarrow$
$V > 0$: pulse stretching
$V < 0$: pulse shortening



Force on/by the moving mirror

The force on the mirror can be derived from Lorentz transformations of fields and forces or also by the conservation of photon number N



$$I = \frac{N\hbar\omega}{\tau}$$ intensity (τ: pulse duration)

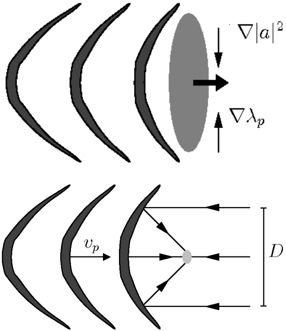
$$\Delta\mathbf{p} = N\hbar(\mathbf{k}_i - \mathbf{k}_r) = N\frac{\hbar}{c}(\omega + \omega_r)\hat{\mathbf{x}}$$ exchanged momentum

$$\omega_r = \omega\frac{1-\beta}{1+\beta} \qquad \Delta t = \frac{\tau}{1-\beta}$$ Δt: reflection time

$$F \equiv \frac{\Delta p}{\Delta t} = \frac{2I}{c}\frac{1-\beta}{1+\beta} = \begin{cases} >0 & \text{for} \quad \beta > 0 \quad \text{(work done on the mirror)} \\ <0 & \text{for} \quad \beta < 0 \quad \text{(work done on the pulse)} \end{cases}$$

$\longrightarrow$ a moving mirror may amplify the reflected pulse!

Flying mirror for intensity amplification

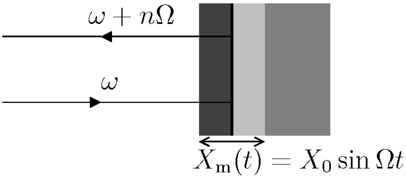


In 3D the wake wave has concave wave-fronts because of relativistic effects
$\longrightarrow$ moving mirror with focusing!
Decrease of ω and $\lambda = 2\pi c/\omega$ causes compression of reflected pulse in space as well as in time with intensity gain at focus

$$\mathscr{G} \simeq 64\gamma_p^6 (D/\lambda)^2 \qquad \gamma_p = (1 - v_p^2/c^2)^{-1/2}$$

The low reflectivity of the wake keeps the amplification factor to $\mathscr{A} \simeq 32\gamma_p^3 (D/\lambda)^2$ (quite substantial anyway)
S.V.Bulanov et al, Phys. Rev. Lett. **91**(2003) 085001

Oscillating mirror and high harmonics



Oscillatory motion

$X_\mathrm{m}(t) = X_0 \sin \Omega t$

Boundary condition

in instantaneous rest frame

$$E'_\parallel(x = X'_m) = 0$$

$\rightarrow \quad A_\parallel(x = X_m(t)) = 0$ in lab frame (note that $E_\parallel(x = X_m(t)) \neq 0$)

$A_\parallel(x, t) = A_i(x - ct) + A_r(x + ct)$ with $A_i(t) = A_0 \cos(\omega t)$

$$\longrightarrow \quad A_r(t) \sim \sin\left(\omega t + \frac{2\omega}{c} X_0 \sin \Omega t\right) \sim \sum_{n=0}^{\infty} J_n\left(\frac{2\omega X_0}{c}\right) \sin(\omega + n\Omega) t$$

(J_n: Bessel functions)

The reflected spectrum contains sums of wave frequency and mirror harmonics

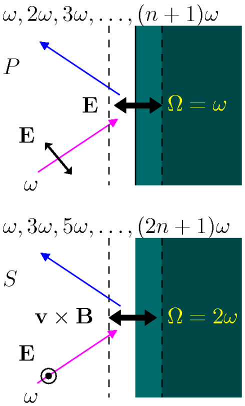
Self-generated high harmonics

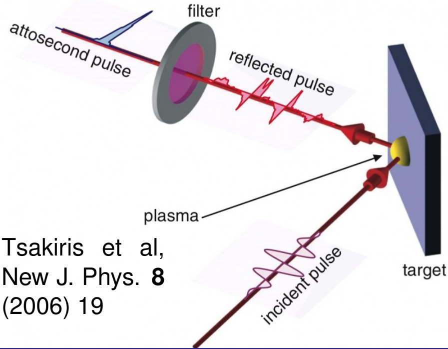
The laser pulse drives surface oscillations depending on the polarization

P-polarization: $\mathbf{E}$-driven, $\Omega = \omega$

S-polarization: $\mathbf{v} \times \mathbf{B}$-driven, $\Omega = 2\omega$

$\longrightarrow$ selection rules for high harmonics (HH)



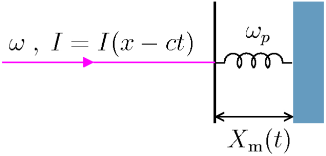


Tsakiris et al, New J. Phys. **8** (2006) 19

HH are phase-locked
Reflected light is an attosecond pulse train

Toy model for moving mirror HH

Perfect mirror with position X_m, velocity $V_m = \beta_m c$ and recoil force at the plasma frequency



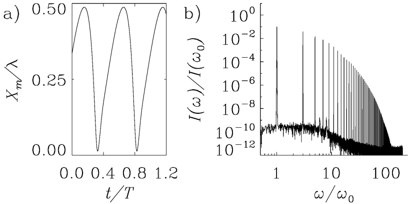
$$\frac{\mathrm{d}}{\mathrm{d}t}(\gamma_m \beta_m) = \frac{2I}{\sigma c^2}(1 + 2\cos(2\omega t_r))\frac{1-\beta_m}{1+\beta_m} - \omega_p^2 X_m \qquad \frac{\mathrm{d}X_m}{\mathrm{d}t} = \beta_m c$$

$t_r = t - X_m/c$: retarded time $\qquad \sigma$: mass per unit area

Cut-off frequency depends on $\beta_{\max} = \max(\beta_m)$

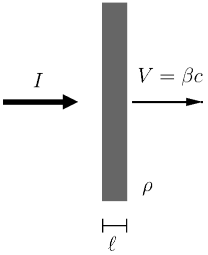
$$\omega_{\mathrm{co}} = \omega \frac{1-\beta_{\max}}{1-\beta_{\max}} \simeq 4\omega\gamma_{\max}^2$$

(other scalings proposed e.g. $\omega_{\mathrm{co}} \sim \gamma_{\max}^3 \ldots$)



Light Sail

EoM for a mirror of finite mass pushed by *steady* radiation pressure



$$\frac{\mathrm{d}(\gamma\beta)}{\mathrm{d}t} = \frac{2}{\rho\ell c^2} I\left(t - \frac{X}{c}\right)\frac{1-\beta}{1+\beta} \qquad \frac{\mathrm{d}X}{\mathrm{d}t} = \beta c$$

Analytical solution yields for final gamma-factor

$$\gamma(\infty) - 1 = \frac{\mathscr{F}^2}{(2(\mathscr{F}+1))} \qquad \mathscr{F} = \frac{2}{(\rho\ell)}\int_0^\infty I(t')dt' \simeq \frac{2I\tau_p}{\rho\ell}$$

Mechanical efficiency η can be estimated using photon number conservation + frequency shift

$$\eta \equiv \frac{\Delta\mathscr{E}}{I\tau_p} = \frac{N\hbar(\omega+\omega_r)}{N\hbar} = \frac{2\beta}{1+\beta} \xrightarrow{\beta\to 1} 1$$ a "perfect" engine!

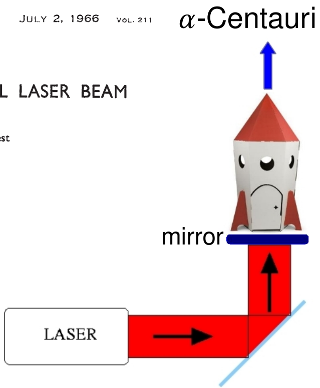
Early vision of radiation pressure acceleration (1966)

22 NATURE JULY 2, 1966 VOL. 211

INTERSTELLAR VEHICLE PROPELLED BY TERRESTRIAL LASER BEAM

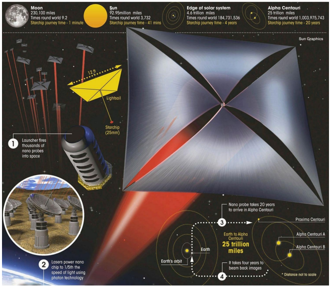
By PROF. G. MARX

Institute of Theoretical Physics, Roland Eötvös University, Budapest



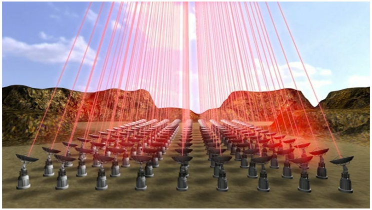
A solution to “Fermi’s paradox”: “*Laser propulsion from Earth ...would solve the problem of acceleration but not of deceleration at arrival ...no planet could be invaded by unexpected visitors from outer space*”

Starshot: laser-boosted light sails for space travel



(credit: Breakthrough Starshot, breakthroughinitiatives.org)

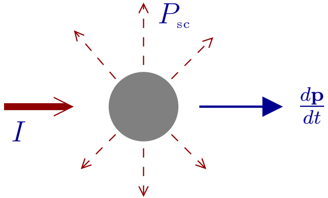
Critical analysis: H. Milchberg, "Challenges abound for propelling interstellar probes", Physics Today, 26 April 2016



Analogy with Thomson scattering acceleration

Light Sail equations of motion have the same form as those of a particle undergoing Thomson Scattering

[Landau & Lifshitz, *The Classical Theory of Fields*, ch.78 p.250 (1962)]



$$\frac{\mathrm{d}p}{\mathrm{d}t} = \sigma_T I \propto P_{\mathrm{sc}} \text{ in rest frame}$$

Veksler's 1957 proposal: coherent scattering by a cluster of radius $a \ll \lambda$ with N $(\gg 1)$ particles

$$P_{\mathrm{sc}} \rightarrow N^2 P_{\mathrm{sc}} \quad \Rightarrow \quad \sigma_T \rightarrow N^2 \sigma_T$$

$\Rightarrow$ N-fold increase in acceleration

V. I. Veksler, "The principle of coherent acceleration", At. Energ. **2** (1957) 525