Basic Phenomena of Superintense Laser-Plasma Optics

Andrea Macchi

CNR/INO, Adriano Gozzini laboratory, Pisa, Italy

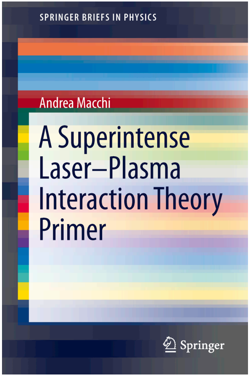
Enrico Fermi Department of Physics, University of Pisa, Italy

47th International Nathiagali Summer College
"High Power Laser Systems & Applications"
Islamabad, Pakistan, June 20, 2017

Compact References

A. Macchi,
- *A Superintense Laser-Plasma Interaction Theory Primer* (Springer, 2013)

- *Basics of Laser-Plasma Interaction: a Selection of Topics*, in:
Laser-Driven Sources of High Energy Particles and Radiation,
Springer Proceedings in Physics **231**, 25-49 (2019)
arXiv:1806.06014



Outline

Single electron dynamics

- Relativistic motion in a plane wave
- Ponderomotive force
- Radiation friction

Nonlinear "*relativistic*" propagation

- Review of linear EM waves in a plasma
- Self-induced transparency
- EM cavitons
- Self-focusing

Moving mirrors

- Basic formulas
- High harmonics
- Light sails

Single electron in a plane wave

An EM plane wave can be described by the vector potential:

$$\mathbf{A}(x,t) = \mathbf{A}(x-ct) \quad \longrightarrow \quad \mathbf{E} = -\frac{1}{c}\partial_t \mathbf{A} \quad , \quad \mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}$$

Equations of Motion (EoM):

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \mathbf{v} = \frac{\mathbf{p}}{m_e \gamma} \, , \qquad \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = -e\left[\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right]$$

$$\mathbf{r} = \mathbf{r}(t) \qquad \mathbf{p} = \mathbf{p}(t) \qquad \gamma = (\mathbf{p}^2 + m_e^2 c^2)^{1/2} = (1 - \mathbf{v}^2/c^2)^{-1/2}$$

The EoM are nonlinear because of the $\mathbf{v} \times \mathbf{B}$ term and the dependence of the fields on the instantaneous position:

$$\mathbf{E} = \mathbf{E}(\mathbf{r}(t), t) \qquad \mathbf{B} = \mathbf{B}(\mathbf{r}(t), t)$$

When is the motion relativistic?

(Quasi-)Monochromatic wave $\mathbf{A}(x,t) = \mathrm{Re}\left[\hat{\mathbf{A}}(x,t)\mathrm{e}^{ikx-i\omega t}\right]$

(with $\hat{A}(x,t)$ a slowly varying envelope, i.e. the wavepacket profile)

Assume $|\mathbf{v}| \ll c \quad \Rightarrow \quad |\mathbf{r}| \ll \lambda = \frac{2\pi c}{\omega} \quad \Rightarrow \quad k|\mathbf{r}| = 2\pi\frac{|\mathbf{r}|}{\lambda} \simeq 0$

$\Rightarrow \mathbf{E}(\mathbf{r}(t),t) = \mathbf{E}(kx(t),t) \simeq \mathbf{E}(x=0,t)$ and $\frac{\mathbf{v}}{c} \times \mathbf{B} \simeq 0$

Solution $\mathbf{p}(t) \simeq \frac{e}{c}\mathbf{A}(0,t) \propto \mathrm{e}^{-i\omega t} \qquad \frac{|\mathbf{v}|}{c} = \frac{p}{m_e c} = \frac{eA_0}{m_e c^2} \equiv a_0$

The motion becomes relativistic and nonlinear when $a_0 \gtrsim 1$

$a_0 = 0.85\left(\frac{I\lambda^2}{10^{18}\ \mathrm{W\ cm^{-2}}}\right)^{1/2}$ where $I \equiv \langle|\mathbf{S}|\rangle = \left\langle\frac{c}{4\pi}|\mathbf{E}\times\mathbf{B}|\right\rangle$

Constants of motion in a plane wave

Symmetry properties of the EoM $\longrightarrow$ conserved quantities:

$$\mathbf{p}_\perp - \frac{e}{c}\mathbf{A} = \mathbf{C}_1 \qquad p_x - m_e\gamma c = C_2$$

("$\perp$" denotes the transverse direction, i.e. yz plane)
Initial conditions $\mathbf{p} = 0$, $\mathbf{A} = 0 \longrightarrow \mathbf{C}_1 = 0$, $C_2 = -m_e c$

$$p_x = \frac{\mathbf{p}_\perp^2}{2m_e c} = \frac{1}{2m_e c}\left(\frac{e}{c}\mathbf{A}\right)^2$$

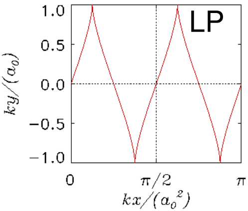
After the EM pulse is gone $\mathbf{A} = 0$ again $\Rightarrow p_x = 0$
$\Rightarrow$ no net acceleration by EM plane wave in vacuum

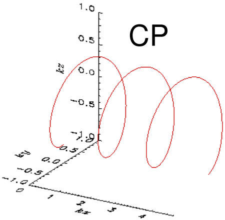
Solutions for a plane monochromatic wave

$$\mathbf{A}(x,t) = A_0\left[\hat{\mathbf{y}}\cos\theta\cos(kx-\omega t) - \hat{\mathbf{z}}\sin\theta\sin(kx-\omega t)\right]$$

with $\mathbf{C}_1 = \mathbf{0}$, $C_2 = -m_e c$ (*adiabatic* field rising in an infinite time)

$\theta = 0\,,\ \pm\frac{\pi}{2}$
linear polarization (LP)
$\gamma = \gamma(t)$





$\theta = \pm\frac{\pi}{4}$
circular polarization (CP)
$\gamma = \left(1+\frac{a_0^2}{2}\right)^{1/2}$
=<u>constant</u>

Constant longitudinal drift: $\langle p_x\rangle = m_e c a_0^2/4$, $\langle v_x\rangle = c a_0^2/(a_0^2+4)$
(origin: absorption of EM energy $\propto$ absorption of EM momentum)

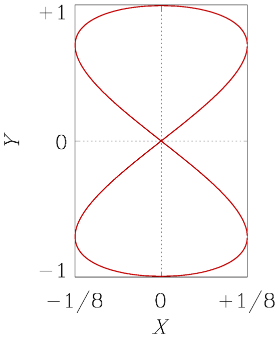
Figure of Eight

LP in the frame where $\langle v_x \rangle = 0$
i.e. $\mathbf{C}_1 = \mathbf{0}$, $C_2 = m_e \gamma_0 c$

Closed self-similar trajectory

$$16X^2 = Y^2(1 - Y^2)$$

$$X \equiv \frac{\gamma_0}{a_0^2} kx \qquad Y \equiv \frac{\gamma_0}{a_0} ky$$



Messages learnt:

- initial conditions are crucial
- polarization matters
- EM field properties constrain the dynamics

Ponderomotive approximation

Aim: describe the motion in a *quasi-periodic* field ($T = 2\pi/\omega$)

$$\mathbf{A}(\mathbf{r},t) = \mathrm{Re}\left[\tilde{\mathbf{A}}(\mathbf{r},t)\mathrm{e}^{-i\omega t}\right]$$

for which the average over a period $\left(\langle f\rangle \equiv T^{-1}\int_0^T f(t')\mathrm{d}t'\right)$

$$\langle \mathbf{A}(\mathbf{r},t)\rangle \simeq 0 \qquad \left\langle \tilde{\mathbf{A}}(\mathbf{r},t)\right\rangle \simeq \tilde{\mathbf{A}}(\mathbf{r},t)$$

Idea: find an EoM for the "slow" (period-averaged) motion

$$\mathbf{r}(t) \equiv \mathbf{r}_s(t) + \mathbf{r}_o(t) \qquad \langle \mathbf{r}_o(t)\rangle \simeq 0 \qquad \langle \mathbf{r}_s(t)\rangle \simeq \mathbf{r}_s(t)$$

(analogy: *guiding center* in a non-uniform magnetic field)

Ponderomotive force

A *perturbative*, *non-relativistic* approach including lowest order contributions from the $\mathbf{v} \times \mathbf{B}$ term and the spatial variation of $\mathbf{E} = \mathbf{E}(\mathbf{r}, t)$ yields the EoM for $\mathbf{v}_s(t) = \langle \mathbf{v}(t) \rangle$ and $\mathbf{r}_s(t) = \langle \mathbf{r}(t) \rangle$

$$m_e \frac{\mathrm{d}\mathbf{v}_s}{\mathrm{d}t} = -\frac{e^2}{2m_e\omega^2} \boldsymbol{\nabla} \left\langle \mathbf{E}^2(\mathbf{r}_s(t), t \right\rangle \equiv \mathbf{f}_p \qquad \frac{\mathrm{d}\mathbf{r}_s}{\mathrm{d}t} = \mathbf{v}_s$$

Relativistic extension (slightly controversial):

$$\frac{\mathrm{d}}{\mathrm{d}t}(m_{\text{eff}} \mathbf{v}_s) = -\boldsymbol{\nabla}(m_{\text{eff}} c^2) \equiv \mathbf{f}_p$$

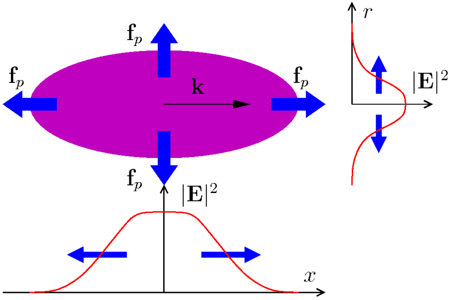
$$m_{\text{eff}} \equiv m_e(1 + \left\langle \mathbf{a}^2 \right\rangle)^{1/2} \qquad \left(\mathbf{a} \equiv e\mathbf{A}/m_e c^2\right)$$

The (time- and space-dependent) effective mass m_{eff} accounts for relativistic inertia due to the oscillatory motion

Ponderomotive effects

$$\mathbf{f}_p \propto -\nabla|\mathbf{E}|^2$$

$\Rightarrow$ electrons are pushed out of higher field regions
A laser pulse (of finite length and width) pushes electrons in both longitudinal (x) and radial (r) directions



Notice: we *define* f_p as a *secular*, "slow" force (it does *not* include oscillating nonlinear terms)
The ponderomotive force concept is tightly related to that of radiation pressure

Introducing radiation friction - I

Example:

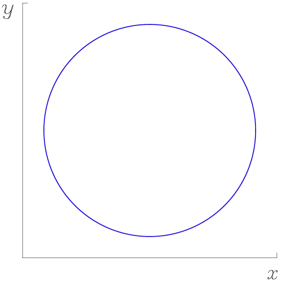
electron in a magnetic field $\mathbf{B}_0$

$$\mathbf{f}_L = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}/c) \text{ Lorentz force}$$

$$m_e \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \mathbf{f}_L = -\frac{e}{c}\mathbf{v} \times \mathbf{B}_0$$

Solution: uniform circular motion

$$|\mathbf{v}| = v = \text{cost.}$$



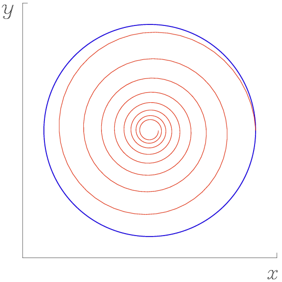
$$K = \tfrac{1}{2} m_e v^2 = \text{constant} \qquad \omega_c = \frac{eB_0}{m_e c} \qquad r = \frac{v}{\omega_c}$$

Introducing radiation friction - II

But the electron radiates:

$$P_{\text{rad}} = \frac{2e^2}{3c^3}\left|\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}\right|^2 = \frac{2e^2}{3c^3}\omega_c^2 v^2$$

(Larmor's formula for radiated power)



Energy loss due to radiation:

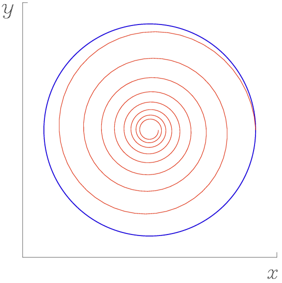
$$\frac{\mathrm{d}K}{\mathrm{d}t} = -P_{\text{rad}} \quad \longrightarrow \quad v(t) = v(0)\mathrm{e}^{-t/\tau}$$

$$\tau = \frac{3m_e c^3}{2e^2\omega_c^2} = \frac{3c}{2r_c\omega_c^2} \qquad r_c = \frac{e^2}{m_e c^2}$$

If $r(t) \simeq v(t)/\omega_c$, electron "falls" along a spiral

Introducing radiation friction - III

The Lorentz force does not describe the electron motion consistently: need to include an extra force



$$m_e \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \mathbf{f}_L + \mathbf{f}_{\text{rad}}$$

Work done by extra force = energy loss

$$\int_0^t \mathbf{f}_{\text{rad}} \cdot \mathbf{v}\mathrm{d}t = -\int_0^t P_{\text{rad}}\mathrm{d}t \longrightarrow \mathbf{f}_{\text{rad}} = -\frac{2e^2}{3c^3}\frac{\mathrm{d}^2\mathbf{v}}{\mathrm{d}t^2}$$

Physical interpretation: the electron is affected by the self-generated radiation field (radiation *reaction* or *self-force*)

Landau-Lifshitz force (non-relativistic)

$\mathbf{f}_{\text{rad}} = -\frac{2e^2}{3c^3}\frac{\mathrm{d}^2\mathbf{v}}{\mathrm{d}t^2}$ introduces unphysical ("runaway") solutions ($\dot{\mathbf{v}}(t) = \dot{\mathbf{v}}(0)\mathrm{e}^{t/\tau}$) and "extra" initial conditions

LL[1] iterative approach yields $\mathbf{f}_{\text{rad}} = \mathbf{f}_{\text{rad}}(\mathbf{E},\mathbf{B})$:

$$\mathbf{f}_{\text{rad}} = -\frac{2e^2}{3c^3}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}\right) \simeq -\frac{2e^2}{3c^3}\frac{\mathrm{d}}{\mathrm{d}t}\left(-\frac{e}{m_e}\mathbf{f}_L\right)$$

$$= \frac{2e^3}{3m_ec^3}\left(\frac{\mathrm{d}\mathbf{E}}{\mathrm{d}t} - \frac{e}{m_ec}\mathbf{E}\times\mathbf{B}\right)$$

in the "instantaneous" $\mathbf{v} = 0$ frame

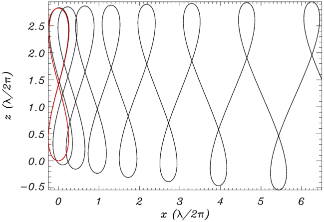
[1] Landau & Lifshitz, *The Classical Theory of Fields*, 2nd Ed., par.76

Landau-Lifshitz force (relativistic)

$$\mathbf{f}_{\text{rad}} = -\frac{2r_c^2}{3}\left\{\gamma^2\left[\left(\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B}\right)^2-\left(\frac{\mathbf{v}}{c}\cdot\mathbf{E}\right)^2\right]\frac{\mathbf{v}}{c}+\right.$$
$$\left.-\left[\left(\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B}\right)\times\mathbf{B}+\left(\frac{\mathbf{v}}{c}\cdot\mathbf{E}\right)\mathbf{E}\right]+\gamma\frac{m_e c}{e}\left(\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{E}+\frac{\mathbf{v}}{c}\times\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{B}\right)\right\}$$

Dominant term ($\sim -\gamma^2\mathbf{v}$) acts as a nonlinear friction force

Effect of including $\mathbf{f}_{\text{rad}}$ on the motion in a plane wave: accelerating drift of the figure-of-eight (constraints of "no-acceleration" are broken)



Linear waves in a plasma

General wave equation for $\mathbf{E}$ from Maxwell's equations:

$$\left(\nabla^2 - \frac{1}{c^2}\partial_t^2\right)\mathbf{E} - \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{E}) = \frac{4\pi}{c^2}\partial_t\mathbf{J}$$

Assume monochromatic fields i.e. $\mathbf{E}(\mathbf{r},t) = \tilde{\mathbf{E}}(\mathbf{r})\mathrm{e}^{-i\omega t}$
Using linearized, non-relativistic equations ($|\mathbf{u}_e| \ll c$)

$$\partial_t\mathbf{u}_e = -\frac{e}{m_e}\mathbf{E} \qquad \mathbf{J} = -en_e\mathbf{u}_e \quad \text{(ions taken at rest)}$$

$$\tilde{\mathbf{J}} = -i\frac{n_e e^2}{m_e\omega}\tilde{\mathbf{E}} = -\frac{i}{4\pi}\frac{\omega_p^2}{\omega}\tilde{\mathbf{E}}, \qquad \omega_p \equiv \left(\frac{4\pi e^2 n_e}{m_e}\right)^{1/2}$$

$$\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\tilde{\mathbf{E}} - \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\tilde{\mathbf{E}}) = \frac{\omega_p^2}{c^2}\tilde{\mathbf{E}} \qquad \text{Helmoltz equation}$$

Linear transverse (EM) waves

Taking $\nabla \cdot \mathbf{E} = 0$ and introducing $\varepsilon(\omega) = \mathsf{n}^2(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$

$$\left(\nabla^2 + \varepsilon(\omega)\frac{\omega^2}{c^2}\right)\tilde{\mathbf{E}} = \left(\nabla^2 + \mathsf{n}^2(\omega)\frac{\omega^2}{c^2}\right)\tilde{\mathbf{E}} = 0$$

$\varepsilon(\omega)$ dielectric function, $\mathsf{n}(\omega)$ refractive index

Plane waves $\tilde{\mathbf{E}}(\mathbf{r}) = E_0 \boldsymbol{\epsilon} \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}}$, $\mathbf{k}\cdot\boldsymbol{\epsilon} = 0$, $\mathbf{B} = \mathbf{k}\times\mathbf{E}/k$

dispersion relation $\qquad k^2c^2 = \varepsilon(\omega)\omega^2 = \omega^2 - \omega_p^2$

Propagation requires a real value of k i.e.

$$k^2 > 0 \quad \leftrightarrow \quad \varepsilon(\omega) > 0 \quad \leftrightarrow \quad \omega > \omega_p \quad \leftrightarrow \quad n_e < n_c \equiv m_e\omega^2/4\pi e^2$$

$n_c = 1.1 \times 10^{21}\ \mathrm{cm}^{-3}(\lambda/1\ \mu\mathrm{m})^{-2}$: cut-off or "critical" density

A nonlinear relativistic wave

Nonlinear terms for $a_0 \gtrsim 1$:

$$\partial_t \mathbf{p}_e + \mathbf{u}_e \cdot \nabla \mathbf{p}_e = -e\mathbf{E} - \frac{e}{c}\mathbf{u}_e \times \mathbf{B}$$

$$\mathbf{J} = -en_e\mathbf{u}_e = -en_e \frac{\mathbf{p}_e/m_e c}{(1 + \mathbf{p}_e^2/m_e^2c^2)^{1/2}}$$

In general plane wave solutions are neither monochromatic nor transverse ($\mathbf{u}_e \times \mathbf{B} \parallel \mathbf{k}$)
Particular monochromatic solution for *circular* polarization:
$\mathbf{p}_e \cdot \mathbf{k} = 0 \quad , \quad \mathbf{u}_e \cdot \nabla \mathbf{p}_e = 0 \quad , \quad \mathbf{u}_e \times \mathbf{B} = 0 \quad ,$ and

$$\gamma = (1 + \mathbf{p}_e^2/m_e^2c^2)^{1/2} = \text{const.} = \left(1 + a_0^2/2\right)^{1/2}$$

$$\partial_t \mathbf{p}_e = m_e\gamma\partial_t \mathbf{u}_e = -e\mathbf{E} \qquad \mathbf{J} = -en_e\mathbf{u}_e$$

Identical to the non-relativistic equations but for $m_e \to m_e\gamma$

Self-induced transparency (with words of caution . . .)

For the *particular* solution (CP, plane wave, monochromatic) the replacement $m_e \rightarrow m_e\gamma$ yields

$$\omega_p \longrightarrow \frac{\omega_p}{\gamma^{1/2}} \qquad k^2c^2 = \omega^2 - \frac{\omega_p^2}{\gamma}$$

The cut-off density $n_c \longrightarrow n_c\gamma = n_c\left(1 + a_0^2/2\right)^{1/2}$
the more intense the wave, the higher the cut-off density

However one cannot define $n_e = n_c\gamma$ as a transparency threshold because of nonlinear pulse dispersion and distortion, effect of boundary conditions, . . .
Message: distrust (or at least use with care) the "relativistically corrected critical density" $n_c^{(\mathrm{rel})} = n_c\gamma$ concept

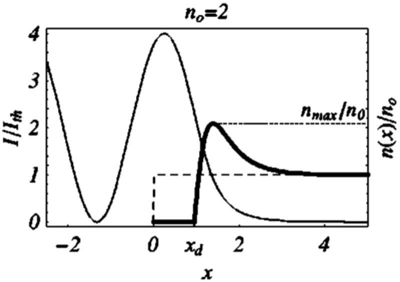
Transparency of semi-infinite plasma

The ponderomotive force pushes and piles up electrons
$\longrightarrow$ increase of density & change of the transparency threshold
[F. Cattani et al, Phys. Rev. E **62** (2000) 1234]

Evanescent solution (assuming steady state, circular polarization, immobile ions ...) exists up to a threshold (for $n_e \gg n_c$)

$$a_0 \simeq \frac{3^{3/2}}{2^3}\left(\frac{n_e}{n_c}\right)^2 \simeq 0.65\left(\frac{n_e}{n_c}\right)^2$$

instead of
$n_e = n_c\gamma \leftrightarrow a_0 \simeq \sqrt{2}n_e/n_c$



The evanescent solution - I

Assumptions in the cold fluid plasma equations

- steady state
- balance between ponderomotive and electrostatic forces

$\rightarrow$ ODE for $\tilde{a}(x)$ (that may be put in Hamiltonian form)

$$\frac{d^2\tilde{a}}{dx^2} - \frac{\tilde{a}}{1+\tilde{a}}\left(\frac{d\tilde{a}}{dx}\right)^2 + \left(1+\tilde{a}^2 - n(1+\tilde{a}^2)^{1/2}\right) = 0$$

Evanescent solution in the plasma

$$\tilde{a}(x) = \frac{2n^{1/2}\kappa\cosh\left(\kappa(x-x_0)\right)}{n\cosh^2\left(\kappa(x-x_0)\right) - n + 1}$$

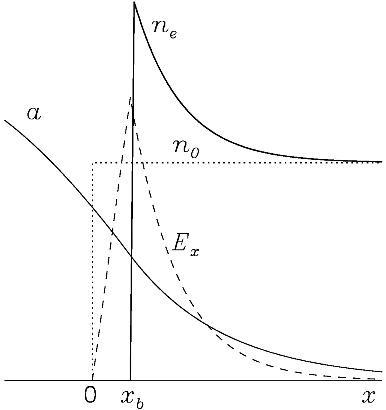
$n = n_0/n_c$, $\kappa = (n-1)^{1/2}$

The evanescent solution - II

The parameter x_0 is determined by matching with the vacuum solution (standing wave) at the electron density boundary $x = x_b$ (to be determined self-consistently)

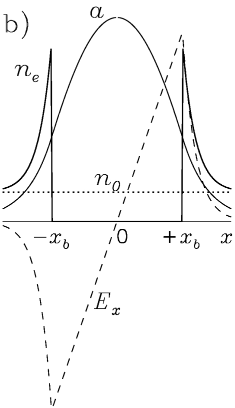
Condition of "monotonic evanescence" $\mathrm{d}\tilde{a}/\mathrm{d}x < 0$ determine existence condition $\rightarrow$ transparency threshold

[F. Cattani et al PRE **62** (2000) 1234]



Note: for given n and a_0 *both* evanescent and propagating solution may exist (hysteresis) [Goloviznin & Schep, Phys. Plasmas **7** (2000) 1564]

Electromagnetic caviton



For certain values of a_0 and n, at the plasma (ion) boundary $x = 0$

$$\frac{\mathrm{d}\tilde{a}}{\mathrm{d}x}(x = 0) = 0$$

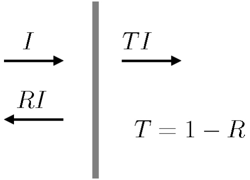
$\rightarrow$ we can build a continuous symmetrical solution between two plasma layers: resonant "optomechanical" EM cavity sustained by the ponderomotive force (*caviton*, improperly aka soliton)

On the time scale of ion motion the caviton expands because of the electrostatic force (model for "post-solitons" which have been observed in experiments)

Transparency of ultrathin plasma foil

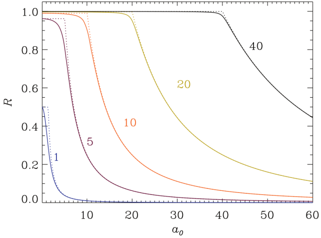
$n_e(x) \simeq n_0 \ell \delta(x)$ (ℓ: foil thickness)

[V.A.Vshivkov et al, Phys. Plasmas **5** (1996) 2727]



Nonlinear reflectivity:

$$R \simeq \begin{cases} 1 & (a_0 < \zeta) \\ \dfrac{\zeta^2}{a_0^2} & (a_0 < \zeta) \end{cases} \qquad \zeta \equiv \pi \frac{n_0 \ell}{n_c \lambda}$$

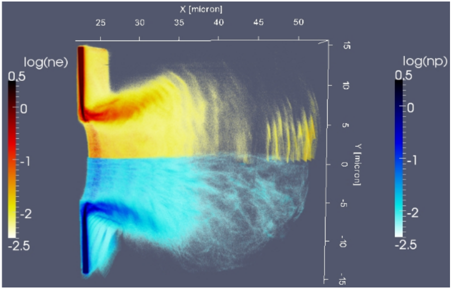


The transparency threshold $a_0 \simeq \zeta$ depends on areal density $n_0 \ell$

Self-induced transparency is a complex process . . .

Several effects contribute to SIT: target heating & expansion, 3D bending & rarefaction, instabilities . . .
Only kinetic simulations can take most effects simultaneously into account

3D PIC simulation of laser interaction with a thin target showing breakup to transparency
[A. Sgattoni, AlaDyn code]

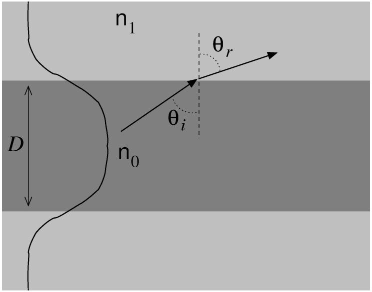


Relativistic Self-Focusing

Nonlinear refractive index (to be used with care!)

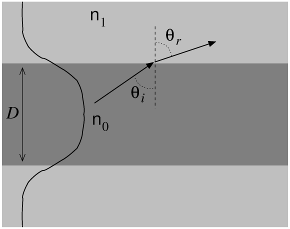
$$n_{\rm NL} = \left(1 - \frac{\omega_p^2}{\gamma\omega^2}\right)^{1/2} = n_{\rm NL}(|\mathbf{a}|^2) \qquad \gamma = (1 + |\mathbf{a}|^2/2)^{1/2}$$

For a laser beam with ordinary intensity profile $n_{\rm NL}$ is higher on the axis than at the edge: $n_0 = n_{\rm NL}(a_0) > n_{\rm NL}(0) = n_1$
$\longrightarrow$ pulse guiding effect
as in an optical fiber



Self-Focusing threshold: simple model

Assumptions: $a_0 \ll 1$, $\omega_p \ll \omega$, $\lambda/D \ll 1$
Impose total reflection in Snell's law of refraction



$$\sin\theta_r = \frac{\mathsf{n}_0}{\mathsf{n}_1}\sin\theta_i = \frac{\mathsf{n}_{\mathsf{NL}}(a_0)}{\mathsf{n}_{\mathsf{NL}}(0)}\sin\theta_i \doteq 1$$

$\cos\theta_i \simeq \lambda/D$ diffraction angle

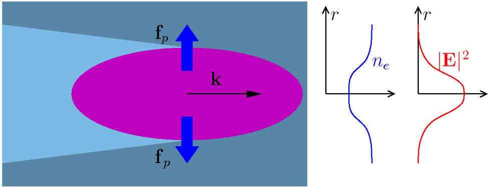
$$\longrightarrow \quad \pi\left(\frac{D}{2}\right)^2 a_0^2 \simeq \pi\lambda^2\frac{\omega^2}{\omega_p^2}$$

Threshold *power* for self-focusing

$$P_c \simeq \frac{\pi^2}{2}\frac{m_e c^3}{r_c}\left(\frac{\omega}{\omega_p}\right)^2 = 43\ \mathrm{GW}\frac{n_c}{n_e}$$

Advanced modeling of self-focusing

The radial ponderomotive force creates a low-density channel
$\longrightarrow$ further "optical fiber" effect (*self-channeling*)



A non-perturbative, multiple-scale modeling for Gaussian beam characterizes the propagation modes

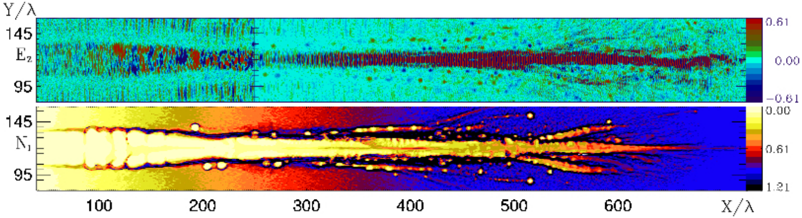
[Sun et al Phys. Fluids **30** (1987) 526]

"Minimal" threshold power $P_c = 17.5\ \mathrm{GW}\dfrac{n_c}{n_e}$

Warning: it applies only to not-so-short, not-so-tightly focused pulses

Nonlinear propagation is a complex process ...

2D simulation of the propagation of a laser pulse ($a_0 = 2.5$, $\tau_p = 1$ ps) in an inhomogeneous plasma with peak density $n_e = 0.1 n_c$
Self-focusing and channeling followed by beam breakup, caviton formation, ion acceleration, steady magnetic field generation, ...



T. V. Liseykina & A. Macchi, IEEE Trans. Plasma Science **36** (2008) 1136, special issue on Images in Plasma Science

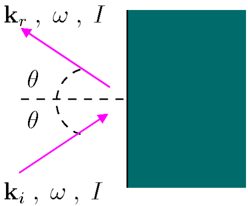
Moving mirrors

A step-boundary plasma described by n $= (1 - n_e/n_c)^{1/2}$ with $n_e \gg n_c$ is a perfect mirror (100% reflection)

Linear theory assumptions:

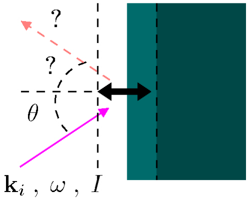
the interface ($x = 0$) is immobile

electrons are confined in the $x > 0$ region)



At high intensities the surface is:

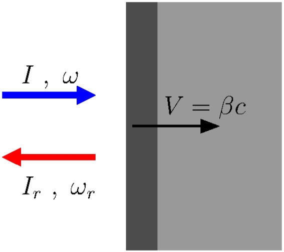
- pushed/pulled by oscillating components of the Lorentz force
- pushed by the steady ponderomotive force



$\rightarrow$ pulse is reflected from a "moving" mirror

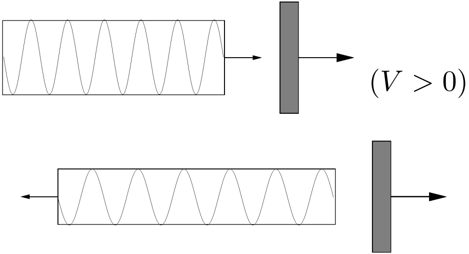
Reflection from a moving mirror

Reflection kinematics can be studied via Lorentz transformations
(the mirror is “perfect” in its rest frame; normal incidence for simplicity)



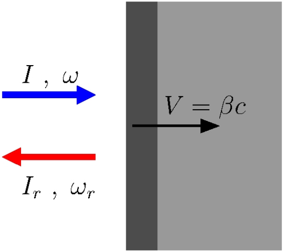
$$\omega_r = \omega\frac{1-\beta}{1+\beta} \qquad \beta = \frac{V}{c}$$

red shift for $V > 0$
blue shift for $V < 0$
The number of cycles is a Lorentz invariant $\longrightarrow$
$V > 0$: pulse stretching
$V < 0$: pulse shortening



Force on/by the moving mirror

The force on the mirror can be derived from Lorentz transformations of fields and forces or also by the conservation of photon number N



$I = \frac{N\hbar\omega}{\tau}$ intensity (τ: pulse duration)

$\Delta\mathbf{p} = N\hbar(\mathbf{k}_i - \mathbf{k}_r) = N\frac{\hbar}{c}(\omega + \omega_r)\hat{\mathbf{x}}$ exchanged momentum

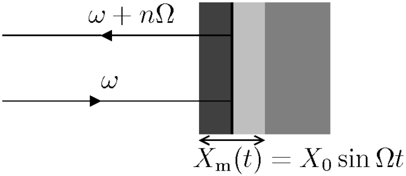
$\omega_r = \omega\frac{1-\beta}{1+\beta}$ $\quad \Delta t = \frac{\tau}{1-\beta}$ $\quad \Delta t$: reflection time

$$F \equiv \frac{\Delta p}{\Delta t} = \frac{2I}{c}\frac{1-\beta}{1+\beta} = \begin{cases} > 0 & \text{for} \quad \beta > 0 \quad \text{(work done on the mirror)} \\ < 0 & \text{for} \quad \beta < 0 \quad \text{(work done on the pulse)} \end{cases}$$

$\longrightarrow$ a moving mirror may amplify the reflected pulse!

Oscillating mirror and high harmonics

$X_{\rm m}(t) = X_0 \sin \Omega t$
Boundary condition
in instantaneous rest frame



$$E'_{\|}(x = X'_m) = 0$$

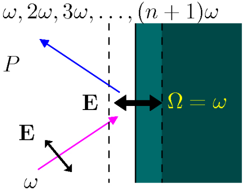
$\rightarrow \quad A_{\|}(x = X_m(t)) = 0$ in lab frame
$A_{\|}(x,t) = A_i(x - ct) + A_r(x + ct)$ with $A_i(t) = A_0 \cos(\omega t)$

$$\longrightarrow A_r(t) \sim \sin\left(\omega t + \frac{2\omega X_0}{c}\sin \Omega t\right) \sim \sum_{n=0}^{\infty} J_n\left(\frac{2\omega X_0}{c}\right)\sin(\omega + n\Omega)t$$

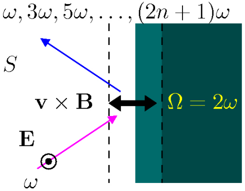
The reflected spectrum contains sums of wave frequency and mirror harmonics $\omega_r, n = \omega + n\Omega$

Self-generated high harmonics

The laser pulse drives surface oscillations with either ω or 2ω frequency depending on the polarization



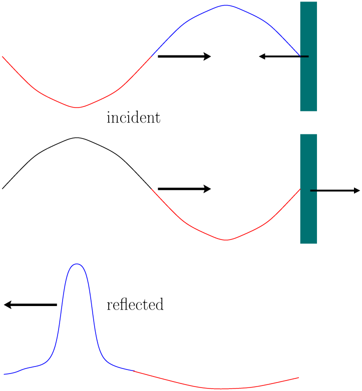
P-polarization: $\mathbf{E}$-driven, $\Omega = \omega$
$\longrightarrow$ even & odd HH, P-polarized

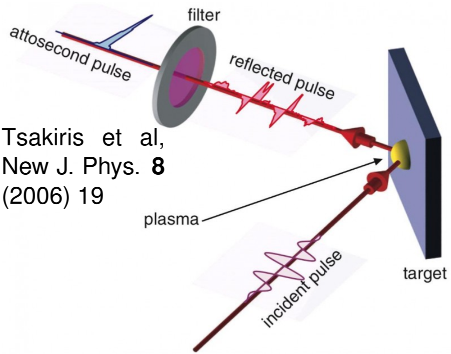


S-polarization: $\mathbf{v} \times \mathbf{B}$-driven, $\Omega = 2\omega$
$\longrightarrow$ odd HH only, S-polarized

Attosecond pulse train

HH are phase-locked
Reflected light is an attosecond pulse train





Tsakiris et al, New J. Phys. **8** (2006) 19

Simple picture:
successive half-cycles are alternately compressed-enhanced and stretched-quenched

Achieving extreme intensities via harmonic focusing

Intensity ehnancement of attosecond pulses plus focusing by the self-consistently curved target surface may yield $I \simeq 6 \times 10^{27}\ \mathrm{W\ cm^{-2}}$ sufficient to investigate strong field QED effects

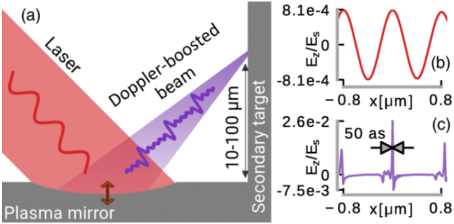


figure: L. Fedeli et al, Phys. Rev. Lett. **127** (2021) 114801

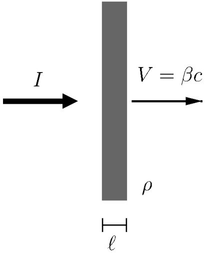
Earlier similar studies:
V. A. Vshivkov et al, Phys. Plasmas **5** (1998) 2727
S. Gordienko et al, PRL **94** (2005) 103903
Alternate approach based on reflection from plasma wake waves:
S. V. Bulanov et al, PRL **91 2003** 085001

Light Sail boosted by radiation pressure

At normal incidence the total cycle-averaged $\mathbf{J} \times \mathbf{B}$ force per unit surface is $P = 2\dfrac{I}{c}$

EoM for a plane mirror of finite mass

$$\frac{\mathrm{d}(\gamma\beta)}{\mathrm{d}t} = \frac{2}{\rho\ell c^2} I\left(t - \frac{X}{c}\right)\frac{1-\beta}{1+\beta} \qquad \frac{\mathrm{d}X}{\mathrm{d}t} = \beta c$$



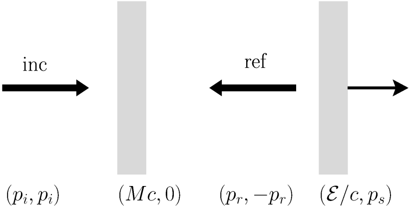
Full analytical solution exists for constant I

Final velocity can be calculated for arbitrary pulse $I(t)$

Note the dependence on “retarded” time $t - X(t)/c$ in the EoM

Light Sail energy from conservation laws

Conservation of 4-momenta in "collision" between laser pulse and moving mirror (mass $M = \rho\ell$)



$$\begin{aligned} p_i + mc &= p_r + \mathcal{E}/c \\ p_i &= -p_r + p_s \end{aligned}$$

Using $\mathcal{E}^2 = M^2c^2 + p_s^2$ and $p_i = \int_0^\infty \frac{I(t')}{c}\mathrm{d}t' \equiv \frac{Mc}{2}\mathcal{F}$

$$\frac{\mathcal{E}}{Mc^2} = \frac{\mathcal{F}^2}{2(\mathcal{F}+1)} \qquad \left(\simeq \frac{\mathcal{F}^2}{2} \text{ for } \beta = \frac{p_s c}{\mathcal{E}} \ll 1\right)$$

efficiency $\eta = \mathcal{E}/p_i c = 2\beta/(1+\beta)$