Radiative losses and inverse Faraday effect in ultraintense laser-plasma interaction

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1st Chalmers Workshop on Laser Plasma Theory (LPT)
Gothenburg, 5-7 October 2016

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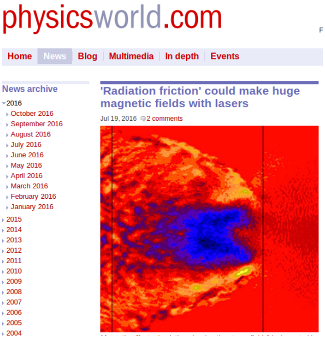
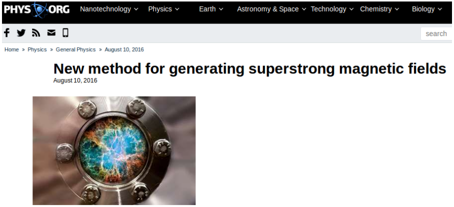
TVL, SVP, AM, "Inverse Faraday effect driven by radiation friction", New Journal of Physics **18** (2016) 072001

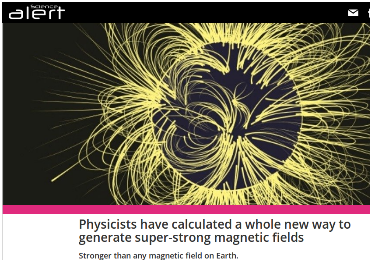
Outline

- Radiation friction (RF) in laser-plasma interaction simulations: the Landau-Lifshitz (LL) “reduced” model
- RF effects in radiation pressure acceleration (RPA) regimes:
  - thin target (Light Sail): polarization dependence
  - thick target (Hole Boring): model for high losses
- Inverse Faraday effect (IFE) and multi-Gigagauss magnetic fields induced by RF
  - observation in 3D simulations
  - analytical model

Unexpected (and embarassing) hype . . .







Relativistic Landau-Lifshitz RF force

Aim: making electron dynamics consistent with radiation emission (i.e. self-generated fields)

$$
\begin{aligned}
\frac{dp^\mu}{d\tau} &= -eF^{\mu\nu}u_\nu + f^\mu_{\mathrm{rad}} \\
f^\mu_{\mathrm{rad}} &= -\frac{2r_c^2}{3}\left[F^{\mu\nu}F_{\alpha\nu}u^\alpha - F^{\alpha\nu}u_\nu F_{\alpha\beta}u^\beta u^\mu + \frac{m_e c}{e}\partial_\alpha F^{\mu\nu}u^\alpha u_\nu\right]
\end{aligned}
$$

Spatial component in the laboratory frame:

$$
\begin{aligned}
\mathbf{f}_{\mathrm{rad}} = & -\frac{2r_c^2}{3}\left\{\gamma^2\left[\left(\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B}\right)^2 - \left(\frac{\mathbf{v}}{c}\cdot\mathbf{E}\right)^2\right]\frac{\mathbf{v}}{c}\right. \\
& \left. -\left[\left(\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B}\right)\times\mathbf{B} + \left(\frac{\mathbf{v}}{c}\cdot\mathbf{E}\right)\mathbf{E}\right] + \gamma\frac{m_e c}{e}\left(\frac{\mathrm{d}\mathbf{E}}{\mathrm{d}t} + \frac{\mathbf{v}}{c}\times\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\right)\right\}
\end{aligned}
$$

“Reduced” LL model

[M. Tamburini, F. Pegoraro, A. Di Piazza, C. H. Keitel, A. Macchi, New J. Phys. **10**, 123005 (2010)]

$$\mathbf{f}_{\text{rad}} = -\left(\frac{2r_c^2}{3}\right)\times\left\{\gamma^2\left[\left(\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B}\right)^2-\left(\frac{\mathbf{v}}{c}\cdot\mathbf{E}\right)^2\right]\frac{\mathbf{v}}{c} - \left[\left(\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B}\right)\times\mathbf{B}+\left(\frac{\mathbf{v}}{c}\cdot\mathbf{E}\right)\mathbf{E}\right]\right\}$$

- Spin force and smaller term containing $\partial_t\mathbf{E}$, $\partial_t\mathbf{B}$ are neglected in $\mathbf{f}_{\text{rad}}$
- Chosen by other groups, e.g. OSIRIS team after extensive comparison [Vranic et al, Comp. Phys. **204** (2016) 141]

Kinetic equation with RF included

Equation for distribution function $f_e = f_e(\mathbf{r}, \mathbf{p}, t)$

$$\partial_t f_e + \nabla_{\mathbf{r}} \cdot (\mathbf{v} f_e) + \nabla_{\mathbf{p}} \cdot (\mathbf{f} f_e) = 0 \qquad (1)$$

$$\mathbf{f} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}/c) + \mathbf{f}_{\mathrm{rad}}, \qquad \nabla_{\mathbf{p}} \cdot (\mathbf{f}_{\mathrm{rad}} f_e) = 0 \neq \mathbf{f}_{\mathrm{rad}} \cdot \nabla_{\mathbf{p}} f_e$$

$\longleftarrow$ the PIC approach provides a solution of (1)

Entropy decrease and phase space contraction because of RF cooling effect

$$\frac{\mathrm{d}}{\mathrm{d}t} \int f_e \ln f_e \mathrm{d}^3\mathbf{p}\mathrm{d}^3\mathbf{q} = \int f_e \nabla_{\mathbf{p}} \cdot (\mathbf{f}_{\mathrm{rad}}) \mathrm{d}^3\mathbf{p}\mathrm{d}^3\mathbf{q} < 0$$

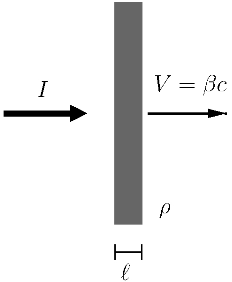
[M. Tamburini et al, NIMA **653** (2011) 181]

RF effects on Radiation Pressure Acceleration

- Motivations: RPA of thin solid foils in the ultra-relativistic regime ($I\lambda^2 > 10^{23}$ W cm$^{-2}\mu$m^2) may allow "unlimited" acceleration towards the GeV/nucleon limit

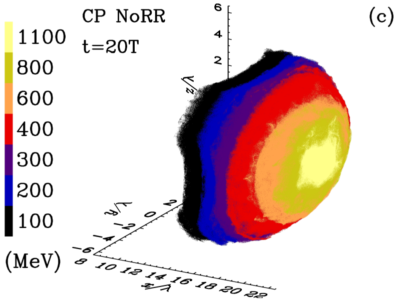
  [T. Esirkepov et al, PRL **92** (2004) 175003
  S. V. Bulanov et al, PRL **104** (2010) 135003]

→ at such extreme intensities
RF effects should be considered

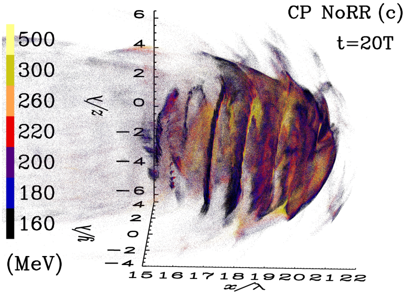


Space-energy distribution: CP, no RF

Ions



Electrons



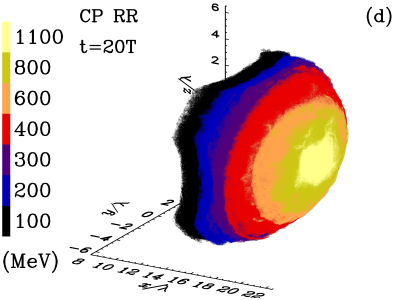
Symmetric, collimated distribution of ions

Cut-off energy of ~ 1.6 GeV at $t = 20T$

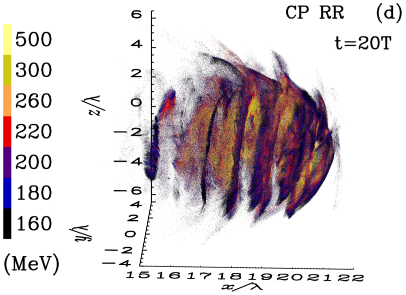
[Tamburini et al, PRE **85** (2012) 016407]

Space-energy distribution: CP, with RF

Ions



Electrons



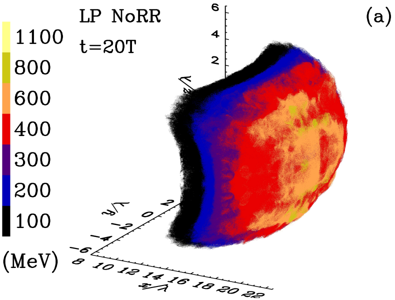
Ion distribution unchanged by RF

Cooling of electrons in pulse tail due to radiative losses

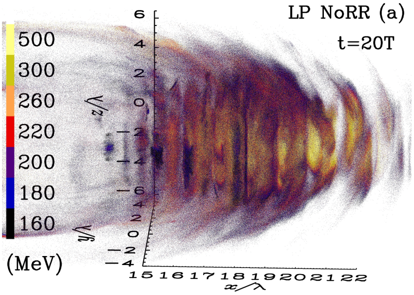
[Tamburini et al, PRE **85** (2012) 016407]

Space-energy distribution: LP, no RF

Ions



Electrons



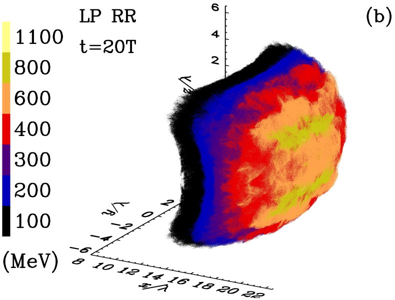
Asymmetric distribution with highest energy ions off-axis

Cut-off energy of ~ 0.9 GeV much lower than for CP

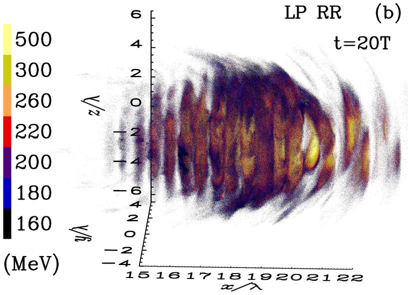
[Tamburini et al, PRE **85** (2012) 016407]

Space-energy distribution: LP, with RF

Ions



Electrons



Strong cooling of electrons by radiative losses

Cut-off energy is *increased* up to ~ 1.1 GeV by RF

[Tamburini et al, PRE **85** (2012) 016407]

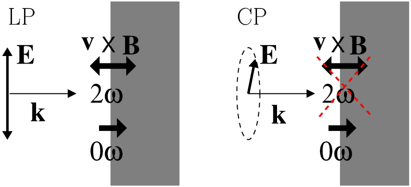
Polarization effect on RF losses

For CP, electrons move coherently with the foil at $v_x \sim c$ and reflection is small

$$R \sim \frac{1 - v_x/c}{1 + v_x/c} \simeq 0$$

$\longrightarrow$ almost copropagation with the laser pulse $\Rightarrow \mathbf{f}_{\text{rad}} = 0$

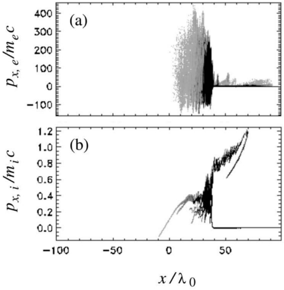
For LP, the *oscillating* term in the $\mathbf{v} \times \mathbf{B}$ force causes $v_x \sim -c$ periodically $\Rightarrow$ "colliding" geometry maximizes $\mathbf{f}_{\text{rad}}$



RF losses in thick targets

- RF losses are small for thin targets pushed by CP pulses
- But thick targets show major RF losses also for CP!

  [Naumova et al, PRL **102** (2009) 25002; Schlegel et al, PoP **16** (2009) 83103; Nerush & Kostyukov, PPCF **57** (2015) 35007]
- “piston oscillations” produce bunches of returning electrons → collective “collisions” with the laser pulse → high RF losses



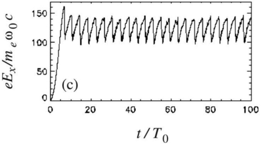
Figures from Schlegel et al.

$I = 4\times10^{22}$ W cm^{-2}

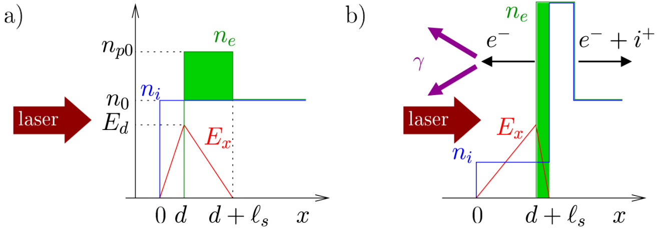
$a_0 = 100$

$n_e = (10-20)\,n_c$

up to $\sim 40\%$ energy dissipated by RF



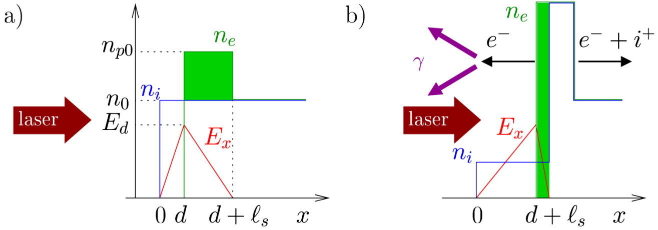
Model for returning electrons in thick targets



- Light pressure generates an excess of electrons in the skin layer ($d < x < d + \ell_s$) and of ions in the depletion layer ($0 < x < d$)
- At the end of the acceleration stage ($t = \tau_i$), equilibrium between electrostatic tension and ponderomotive force (i.e. local light pressure) is lost

$\rightarrow$ the excess electrons return towards the laser in a time τ_e

Estimating the number of returning electrons



- $eE_0 n_{p0}\ell_s/2 \simeq 2I/c$ (pressure balance)
- $E_0 = 4\pi e n_0 d$ (Poisson-Gauss equation)
- $n_{p0}\ell_s = n_0(d+\ell_s)$ (charge conservation)

$\rightarrow$ $N_x = (n_{p0} - n_0)\ell_s \simeq n_{p0}\ell_s \simeq a_0/r_c\lambda$
returning electrons per unit surface ($r_c = e^2/m_e c^2$)

Estimating radiation from returning electrons

- Radiation power for an electron in the laser field (=work done per unit time by $\mathbf{f}_{\text{rad}}$)

$$P_{\text{rad}} = \frac{2e^2\omega^2\gamma^2 a_0^2}{3c}\left(1-\frac{v_x}{c}\right)^2$$

- Fraction of laser energy re-emitted as radiation from N_x electrons with $(1-v_x/c) \gtrsim 1$

$$\eta_{\text{rad}} \equiv \frac{P_{\text{rad}} N_x}{I_L}\frac{\tau_e}{\tau_e+\tau_i} \approx \frac{4\pi}{3}\frac{r_c}{\lambda}\gamma^2 a_0 \qquad (\text{if } \tau_e \approx \tau_i)$$

- If $\gamma = (1+a_0^2)^{1/2} \simeq a_0$ ($a_0 \gg 1$, $|p_x| \ll |p_\perp|$)

$$\eta_{\text{rad}} \simeq \frac{4\pi}{3}\frac{r_c}{\lambda}a_0^3 \sim 1 \text{ for } a_0 = 400 \ (\lambda = 0.8\mu\text{m})$$

Angular momentum absorption

- A CP laser pulse carries "spin" angular momentum (AM) that may be absorbed by the target

$$L_z = \int_0^\infty \ell_z(r) 2\pi r \mathrm{d}r = -\int_0^\infty \frac{r}{2c\omega} \partial_r I(r) 2\pi r \mathrm{d}r$$

- Reflection from a perfect mirror conserves number of photons and does *not* change sign of "spin"

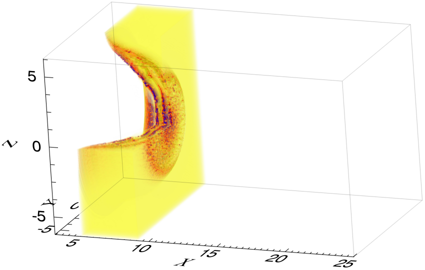
→ No AM absorption if "dissipation" is absent

- First 3D simulations of RPA showed very small AM absorption (AMA)
  [T. V. Liseykina et al, PPCF **50** (2008) 124033]

• Can RF provide efficient dissipation for AMA?

• Does RF-induced AMA leads to generation of an axial magnetic field ("Inverse Faraday Effect")?

3D thick target simulations



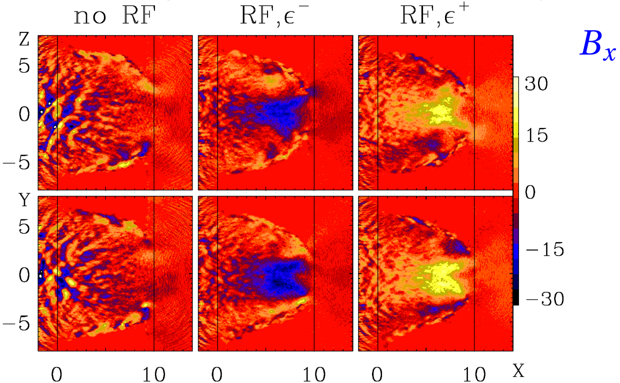
$\lambda = 0.8\ \mu$m
$n_e = 90 n_c = 1.6 \times 10^{23}$ cm^{-3}
$a_0 = (200 - 600)$
$I = (0.9 - 7.8) \times 10^{23}$ W cm^{-2}

Laser pulse at waist (target boundary)

$$\mathbf{a}(x=0,r,t) = a_0 \left(\hat{\mathbf{y}}\cos(\omega t) \pm \hat{\mathbf{z}}\sin(\omega t)\right) \mathrm{e}^{-(r/r_0)^n - (ct/r_l)^4}$$

$n = 2$ (Gaussian profile) or $n = 4$ (super-Gaussian)
$r_l = 3\lambda$, $r_0 = 3.8\lambda$

Gigagauss axial magnetic fields induced by RF -I

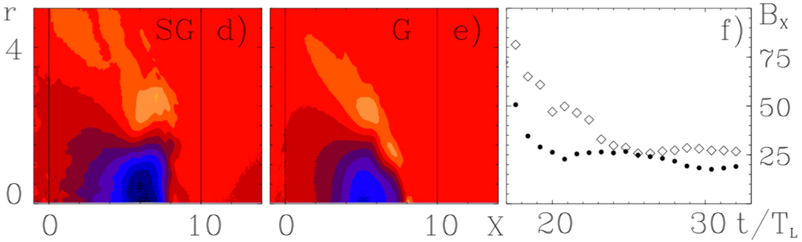


No axial field without RF

Sign of B_x changes with laser pulse helicity

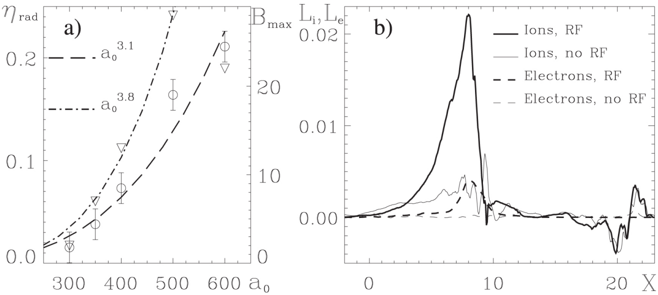
(units: $B_0 = m_e c\omega/e = 1.34 \times 10^8$ G)

Gigagauss axial magnetic fields induced by RF -II



Similar results for Gaussian and Super-Gaussian profiles
The axial $\mathbf{B}$-field is quasi-steady after the laser pulse
Peak amplitude $B_{\text{max}} \simeq 28B_0 \simeq 5 \times 10^9$ G for $a_0 = 500$

Scaling with a_0 and AM absorption



Scaling with a_0:
Radiative loss $\eta_{\text{max}} \sim a_0^3$
(max value $\sim 25\%$)
Peak field $B_{\text{max}} \sim a_0^4$

Mechanical AM
$L_{\text{e/i}}(x) = \int n_{e/i}(\mathbf{r} \times \mathbf{p})\text{d}y\text{d}z$
$L_i \gg L_e$: most of AM eventually goes to ions

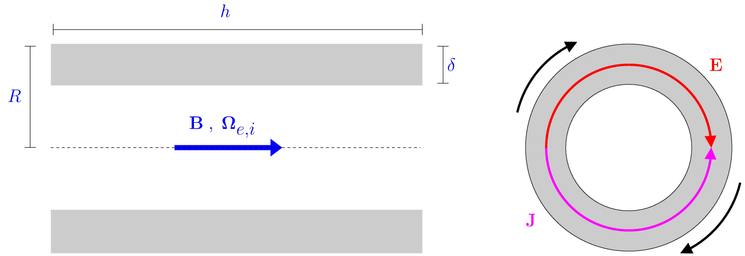
Analytical theory of IFE - 1

Angular momentum density peaks at the beam edge ($r = R \simeq r_0$)

$$\mathscr{L}_x = (\mathbf{r} \times (\mathbf{E} \times \mathbf{B}))_x / 4\pi c = -r\partial_r I_L(r)/(2c\omega)$$

$\longrightarrow$ ion and electron fluids modeled as thin rigid cylinders with momenta of inertia $\mathscr{I}_e = 2\pi R^3 \delta h m_e n_e$, $\mathscr{I}_i = (Am_p/Zm_e)\mathscr{I}_e$

Torque by AMA on electrons $M_{\rm abs} \simeq P_{\rm abs}/\omega \simeq P_{\rm rad}/\omega$



Analytical theory of IFE - 2

Induced $\mathbf{E}$-field exerts opposite torques on ions and electrons

$$\frac{dL_e}{dt} = \mathcal{I}_e \frac{d\Omega_e}{dt} = M_{\text{abs}} - M_E \qquad \frac{dL_i}{dt} = \mathcal{I}_i \frac{d\Omega_i}{dt} = M_E$$

$$E_\phi = -(r/2c)\partial_t B_x \qquad M_E = \int n_e e E_\phi r \mathrm{d}^3 r \simeq \frac{eE_\phi(R)}{m_e R}\mathcal{I}_e$$

Electron current density $j_{e\phi} \simeq -en_e\Omega_e R$

“Solenoid” approximation $B_x \simeq 4\pi j_{e\phi}\delta/c$

$$\longrightarrow \; M_E \simeq \frac{\omega_p^2 R\delta}{2c^2}\mathcal{I}_e \frac{d\Omega_e}{dt} \equiv \mathcal{I}'_e \frac{d\Omega_e}{dt}$$

$\longrightarrow$ Induction effects are equivalent to additional inertia

Analytical theory of IFE - 3

$$\Omega_e(t) = \frac{1}{\mathcal{I}_e + \mathcal{I}'_e} \int_0^t M_{\text{abs}}(t')dt'$$

"Induction inertia" dominates for electrons

$$\mathcal{I}'_e \sim (\omega_p^2/\omega^2)\mathcal{I}_e = (n_e/n_c)\mathcal{I}_e \gg \mathcal{I}_e$$

$$\longrightarrow M_{\text{abs}} - M_E = \mathcal{I}_e \frac{d\Omega_e}{dt} \simeq 0$$

$$\longrightarrow L_i \simeq \int_0^t M_{\text{abs}}(t')dt' \simeq \frac{\mathcal{I}'_e}{\mathcal{I}_e} L_e \gg L_e$$

in agreement with simulations •

Analytical theory of IFE - 4

Posing $M_{\text{abs}} \simeq M_E$ allows to obtain E_ϕ and $\partial_t B_x$ from M_{abs}
Peak magnetic field $B_{xm} = B_x(r=0, t=\infty)$

$$\frac{\pi e}{c} n_e h R^3 \delta B_{xm} \simeq \int_0^\infty M_{\text{abs}}(t)\,dt \simeq \frac{U_{\text{abs}}}{\omega} = \eta_{\text{rad}} \frac{U_{\text{L}}}{\omega}$$

$n_e h \simeq 2 n_c a_0 c/\omega$ from HB model

$$\frac{B_{xm}}{B_0} \simeq \frac{0.2}{\pi} \eta_{\text{rad}} \frac{r_l \lambda}{R\delta} a_0 \quad \propto a_0^4$$

scaling in agreement with simulations

Analytical theory of IFE - 5

Estimate of B_{xm} from empirical value $\eta_{\text{rad}} \simeq 0.24$ and using "vacuum" laser parameters $R \simeq r_0 = 3.8\lambda$ and $\delta \simeq \lambda$

$$B_{xm} \simeq 4.8B_0$$

Considering dynamical evolution of the pulse (self-channeling and focusing, radial profile steepening) yielding $R \simeq 2\lambda$, $\delta \simeq 0.5\lambda$ and $a(r=0) \simeq 1.2a_0$

$$B_{xm} \simeq 23B_0$$

fair agreement with numerical observation

Conclusions

- A simple model accounts for high radiation friction (RF) losses in thick targets and provides $\sim a_0^3$ scaling
- 3D simulations give evidence of multi-Gigagauss axial magnetic field generation by the "inverse Faraday" effect (IFE)
- Another simple model for IFE fairly agrees with simulation results

Possible developments:

- improved modeling
- effect of the $\mathbf{B}$-field on the dynamics
- observable to test RF models?