

Radiation Friction Effects on Ion Acceleration and Magnetic Field Generation in Superintense Laser-Plasma Interactions

Andrea Macchi

National Institute of Optics, National Research Council (CNR/INO), Adriano
Gozzini research unit, Pisa, Italy

Enrico Fermi Department of Physics, University of Pisa, Italy



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Coworkers

Matteo Tamburini^{1,2,3}, Francesco Pegoraro^{1,2},
Antonino Di Piazza³, Christoph H. Keitel³,
Tatyana V. Liseykina⁴, Sergey V. Propruzhenko⁵

¹Enrico Fermi Department of Physics, University of Pisa, Pisa, Italy

²CNR/INO, Pisa, Italy

³Max Planck Institute for Nuclear Physics, Heidelberg, Germany

⁴Institute for Physics, University of Rostock, Germany

⁵National Research Nuclear University, Moscow Engineering Physics
Institute, Russia

Outline

- ▶ Radiation friction (RF) basics
- ▶ RF modeling in laser-plasma interactions
- ▶ RF effect on radiation-pressure dominated ion acceleration
 - thin target (Light Sail regime)
 - thick target (Hole Boring regime)
- ▶ Magnetic field generation induced by RF losses

Introducing Radiation Friction

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Pedagogical example:
electron in a magnetic field \mathbf{B}_0

$$\mathbf{f}_L = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}/c) \text{ Lorentz force}$$

$$m_e \frac{d\mathbf{v}}{dt} = \mathbf{f}_L = -\frac{e}{c} \mathbf{v} \times \mathbf{B}_0$$

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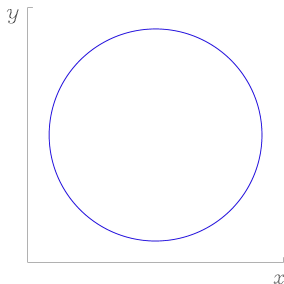
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Solution: uniform circular motion

$$|\mathbf{v}| = v = \text{const.}$$

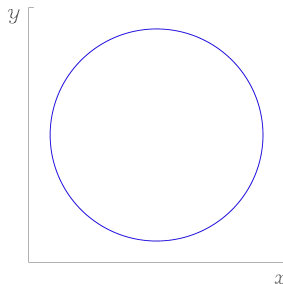
$$K = \frac{1}{2} m_e v^2 = \text{constant} \quad \omega_c = \frac{eB_0}{m_e c} \quad r = \frac{v}{\omega_c}$$



Introducing Radiation Friction

BUT the electron radiates:

$$P_{\text{rad}} = \frac{2e^2}{3c^3} \left| \frac{d\mathbf{v}}{dt} \right|^2 = \frac{2e^2}{3c^3} \omega_c^2 v^2$$



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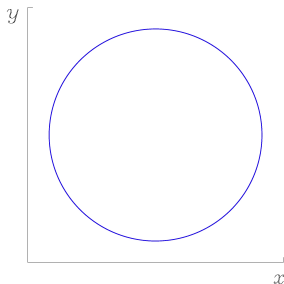
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Energy loss due to radiation:

$$\frac{dK}{dt} = -P_{\text{rad}} \longrightarrow v(t) = v(0)e^{-t/\tau}$$

$$\tau = \frac{3m_e c^3}{2e^2 \omega_c^2} = \frac{3c}{2r_c \omega_c^2}$$



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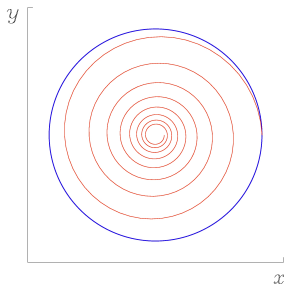
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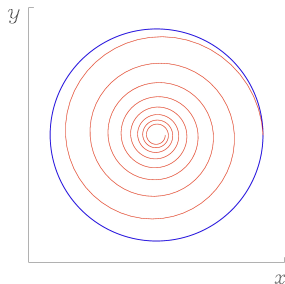
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If $r(t) \simeq v(t)/\omega_c$, electron “falls” along a spiral



Introducing Radiation Friction

The Lorentz force does not describe the electron motion consistently:
need to include an extra force



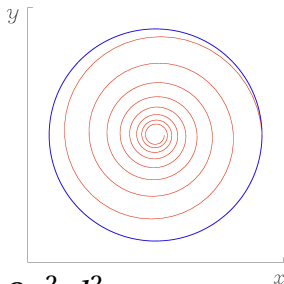
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$$m_e \frac{d\mathbf{v}}{dt} = \mathbf{f}_L + \mathbf{f}_{\text{rad}}$$

Work done by extra force = energy loss

$$\int_0^t \mathbf{f}_{\text{rad}} \cdot \mathbf{v} dt = - \int_0^t P_{\text{rad}} dt \longrightarrow \mathbf{f}_{\text{rad}} = - \frac{2e^2}{3c^3} \frac{d^2\mathbf{v}}{dt^2}$$



Introducing Radiation Friction

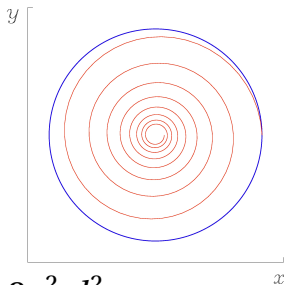
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Physical interpretation: the electron is affected by the self-generated radiation field (radiation *reaction* or *self-force*)



Landau-Lifshitz approach

$$\mathbf{f}_{\text{rad}} = -\frac{2e^2}{3c^3} \frac{d^2 \mathbf{v}}{dt^2} \text{ is unsatisfying:}$$

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LL iterative approach brings $\mathbf{f}_{\text{rad}} = \mathbf{f}_{\text{rad}}(\mathbf{E}, \mathbf{B})$:

$$\mathbf{f}_{\text{rad}} \simeq -\frac{2e^2}{3c^3} \left(-\frac{e}{m_e} \frac{d}{dt} \mathbf{f}_L \right) = \frac{2e^3}{3m_e c^3} \left(\dot{\mathbf{E}} - \frac{e}{m_e c} \mathbf{E} \times \mathbf{B} \right)$$

in the “instantaneous” frame where $\mathbf{v} = 0$

L.L.Landau, E.M.Lifshitz, *The Classical Theory of Fields*
(Elsevier, 1975), 2nd Ed., par.76

Relativistic Landau-Lifshitz RF force

$$f_{\text{rad}}^{\mu} = -\frac{2r_c^2}{3} \left[F^{\mu\nu} F_{\alpha\nu} u^{\alpha} - F^{\alpha\nu} u_{\nu} F_{\alpha\beta} u^{\beta} u^{\mu} + \frac{m_e c}{e} \partial_{\alpha} F^{\mu\nu} u^{\alpha} u_{\nu} \right]$$

Spatial component in the laboratory frame:

$$\begin{aligned} \mathbf{f}_{\text{rad}} = & -\frac{2r_c^2}{3} \left\{ \gamma^2 \left[\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right)^2 - \left(\frac{\mathbf{v}}{c} \cdot \mathbf{E} \right)^2 \right] \frac{\mathbf{v}}{c} \right. \\ & \left. - \left[\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right) \times \mathbf{B} + \left(\frac{\mathbf{v}}{c} \cdot \mathbf{E} \right) \mathbf{E} \right] + \gamma \frac{m_e c}{e} \left(\dot{\mathbf{E}} + \frac{\mathbf{v}}{c} \times \dot{\mathbf{B}} \right) \right\} \end{aligned}$$

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Quantum ElectroDynamics limits are more stringent:

$$E < E_s = \frac{m_e c^2}{\lambda_c} = \frac{m_e^2 c^3}{e\hbar} < \frac{m_e c^2}{r_c} \quad \frac{c}{\omega} > \lambda_c = \frac{\hbar}{m_e c}$$

$$E_s = 1.3 \times 10^{16} \text{ V cm}^{-1} \text{ (Schwinger field)}$$

$$\lambda_c = 4 \times 10^{-11} \text{ cm (Compton wavelength)}$$

Why worry about Radiation Friction?

The relevant fields seem out of reach, but . . .

- ▶ Depending on the interaction geometry the field amplitudes and frequencies are much higher in the rest frame of the electron

Example: collision of an electron with $\gamma \gg 1$ and a plane wave

$$F = \frac{2}{3} \left(\frac{e^2}{m_e c^2} \right) |\mathbf{E} \times \mathbf{B}| = \frac{8\pi}{3} r_c^2 I \longrightarrow F' = \frac{8\pi}{3} r_c^2 (4\gamma^2 I) \gg F$$

- ▶ From another point of view: so far RF effects has *never* been characterized in experiments despite >100 years of theoretical work!

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[J.Koga, T.Esirkepov, S.V. Bulanov, PoP **12**, 093106 (2005)]

RDR in a laser field: (radiation loss) \simeq (initial energy)

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Thomson scattering geometry “enhances” RF effects

$$P_{\text{rad}} \frac{2\pi}{\omega} \simeq \mathcal{E}_{\text{osc}} = m_e c^2 \left[\left(1 + \frac{\mathbf{p}^2}{m_e c^2} \right)^{1/2} - 1 \right]$$

$$R \equiv \frac{2r_c \omega}{3c} \gamma_0 (1 + \beta_0) a^2 \simeq 1 \quad a = \frac{eE}{m_e \omega c}$$

($\beta_0 = v_0/c$, $\gamma_0 = (1 - \beta_0^2)^{-1/2}$ initial β - and γ -factor)

$R = 1$ for $\gamma_0 = 300$ (150 MeV) and $a = 336$ ($2.4 \times 10^{23} \text{ W cm}^{-2}$)

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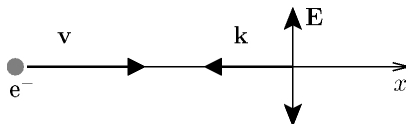
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Thomson Scattering with RF included

TS of a 35 fs pulse by 150 MeV electrons

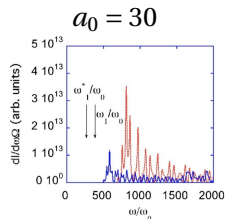


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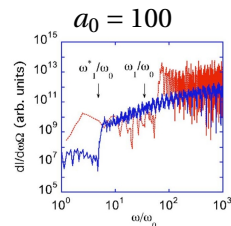
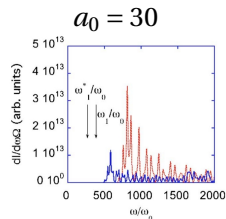
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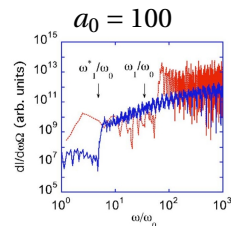
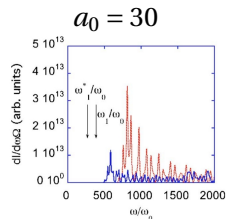
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Total power and low-frequency cut-off
are strongly affected by RF
already when $R \ll 1$

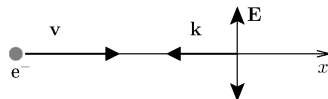


[J.Koga, T.Esirkepov, S.V. Bulanov, PoP **12**, 093106 (2005)]

Angular RF signatures in Thomson Scattering

Key point: the RF decreases v_x
(velocity component anti-parallel to \mathbf{k})

Change in v_x affects
the angular distribution
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“Backward” TS observed with RF

$I = 5 \times 10^{22} \text{ W cm}^{-2}$, 27 fs pulse

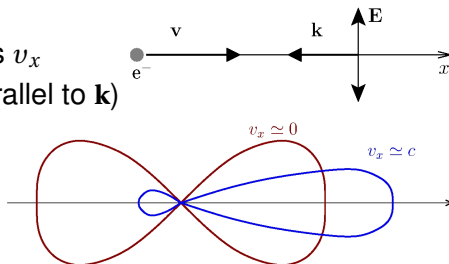
40 MeV electrons, $R \simeq 0.05$

[A.Di Piazza, K.Z.Hatsagortsyan, C.H.Keitel, PRL **102**, 254802 (2009)]

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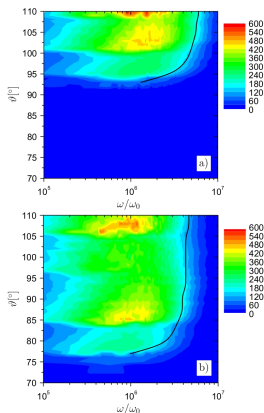
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Radiation Reaction modeling - I

“Reduced” Landau-Lifshitz equation for electrons

[M. Tamburini, F. Pegoraro, A. Di Piazza, C. H. Keitel, A. Macchi,
New J. Phys. **10**, 123005 (2010)]

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Radiation Reaction modeling - II

Kinetic equation for electrons and some properties

$$\partial_t f + \nabla_{\mathbf{r}} \cdot (\mathbf{v} f) + \nabla_{\mathbf{p}} \cdot (\mathbf{F} f) = 0, \quad f = f(\mathbf{r}, \mathbf{p}, t)$$

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$$\frac{d}{dt} \int f \ln f d^3 \mathbf{p} d^3 \mathbf{q} = \int f d^3 \mathbf{p} d^3 \mathbf{q} \nabla_{\mathbf{p}} \cdot (\mathbf{F}_{\text{rad}}) < 0$$

- Entropy decrease and phase space contraction because of RF cooling effect

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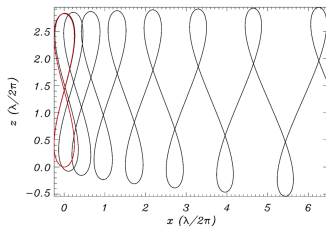
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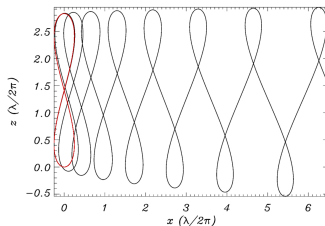
Radiation Reaction modeling - III

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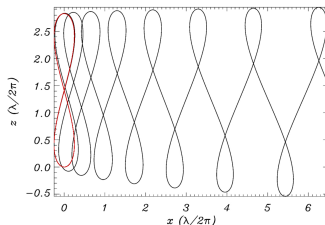
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Radiation Reaction modeling - IV

Assume main contribution to RF losses is at wavelengths
 $\lambda_{\text{RF}} \ll (3/4\pi n_e)^{1/3}$

→ radiation is incoherent and escapes from the plasma; it appears as energy dissipation

In simulations:

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Radiation energetics with RF included

1D PIC simulations of
laser-plasma interaction
with radiation emission
calculated as a diagnostic

[Capdessus et al.,
PRE **86**, 036401 (2012)]

$$n_e = 10n_c, d = 100\lambda, \tau_L = 16T$$
$$1e21 - 1e22 - 8e22 - 3e23 \text{ W cm}^{-2}$$

With RF (left): energy balance is consistent

Without RF (right): radiative loss \simeq input laser energy
for the highest intensity!

\Rightarrow RF inclusion is necessary at intensities $> 10^{22} \text{ W cm}^{-2}$

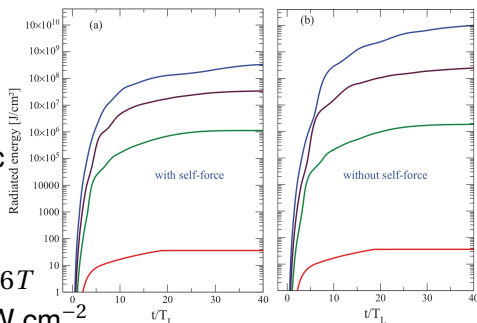
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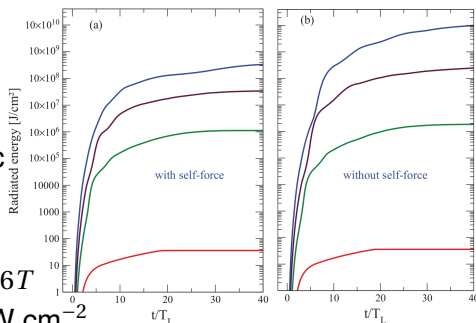
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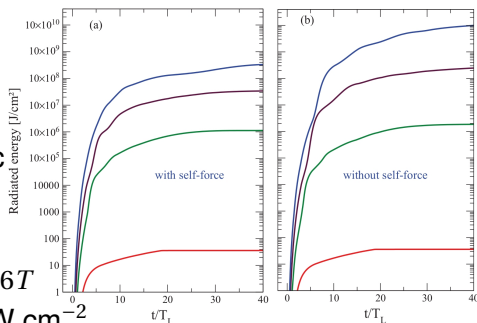
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RF effects on Light Pressure Acceleration

- Motivations:

Radiation Pressure Dominant Acceleration (RPDA) of thin solid foils

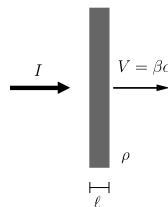
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($I\lambda^2 > 10^{23} \text{ W cm}^{-2} \mu\text{m}^2$) is a possible route to “unlimited” acceleration towards the GeV/nucleon limit

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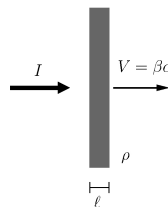
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- ▶ Laser pulse: $(9T) \times (10\lambda)^2$ (FWHM) [$T = \lambda/c$]
Circular (CP) or Linear (LP) polarization
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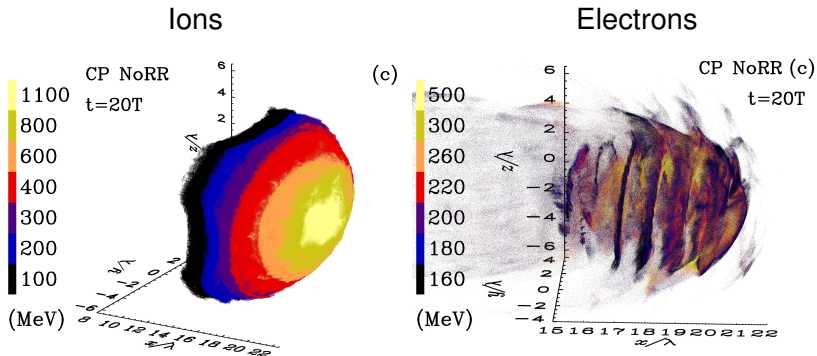
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Space-energy distribution: CP, no RF

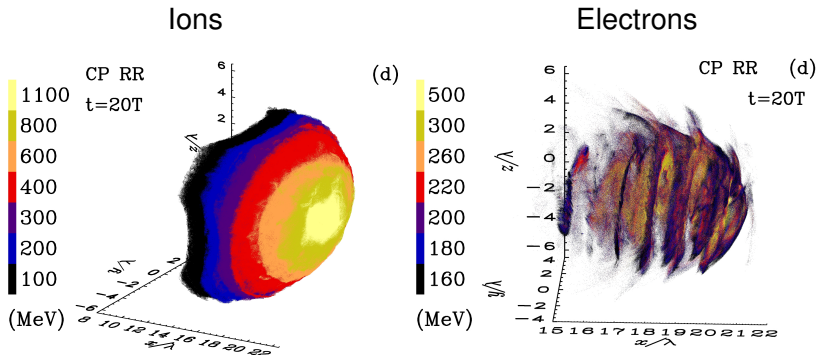


Symmetric, collimated distribution of ions

Cut-off energy of ~ 1.6 GeV at $t = 20T$

[Tamburini et al, PRE **85** (2012) 016407]

Space-energy distribution: CP, with RF

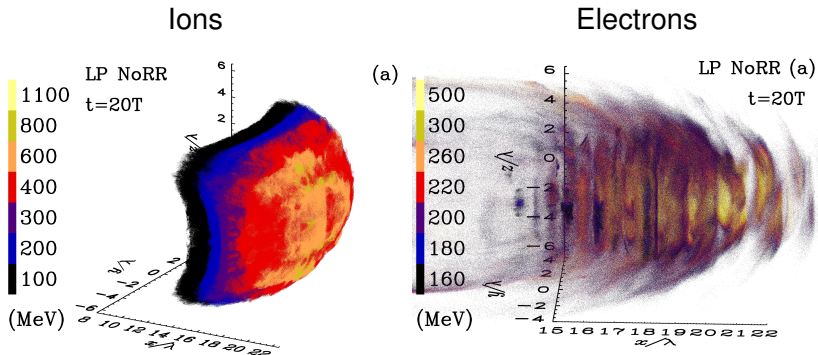


Ion distribution unchanged by RF

Cooling of electrons in pulse tail due to radiative losses

[Tamburini et al, PRE **85** (2012) 016407]

Space-energy distribution: LP, no RF

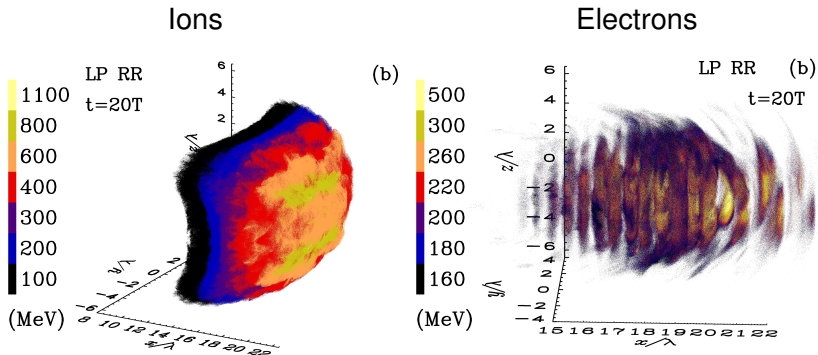


Asymmetric distribution with highest energy ions off-axis

Cut-off energy of ~ 0.9 GeV much lower than for CP

[Tamburini et al, PRE **85** (2012) 016407]

Space-energy distribution: LP, with RF



Strong cooling of electrons by radiative losses

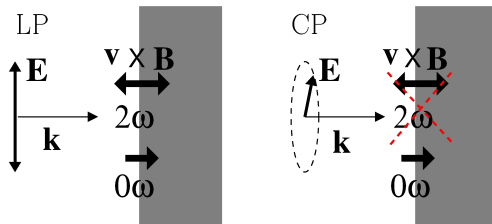
Cut-off energy is *increased* up to ~ 1.1 GeV by RF

[Tamburini et al, PRE **85** (2012) 016407]

Polarization effect on RF losses

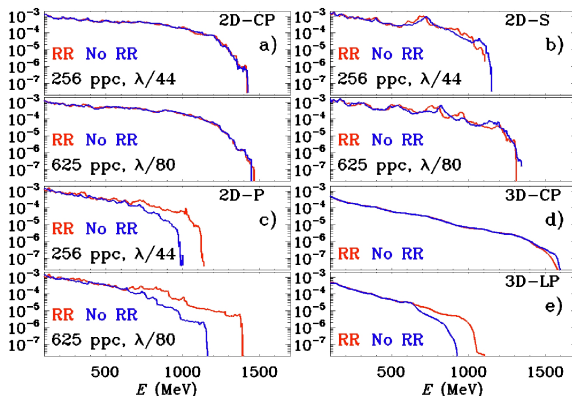
For CP, electrons move coherently with the foil at $v_x \sim c$ and reflection is small ($R \sim (1 - v_x/c)/(1 + v_x/c)$); they almost co-propagate with the laser pulse $\Rightarrow \mathbf{f}_{\text{rad}} = 0$

For LP, the $\mathbf{J} \times \mathbf{B}$ *oscillating* term causes $v_x \sim -c$ periodically \Rightarrow “colliding” geometry maximizes \mathbf{f}_{rad}



Effects of reduced dimensionality and resolution

Comparison of 3D ion spectra with 2D results (both S and P for LP) for both the same and higher resolution

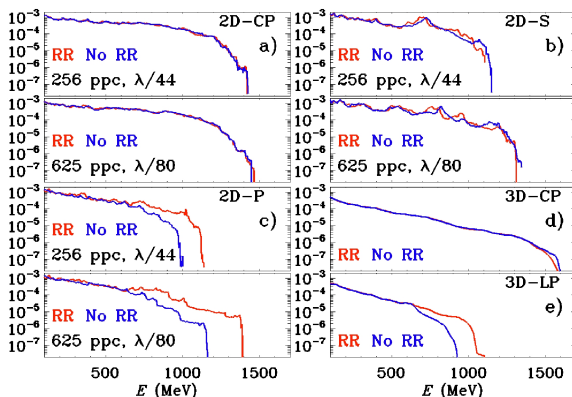


Effects of 2D vs 3D and of limited resolution are evident, but kept below physical effects

The “optimal” CP case is the most robust (but energy is *lower* in 2D vs 3D !)

Effects of reduced dimensionality and resolution

Comparison of 3D ion spectra with 2D results (both S and P for LP) for both the same and higher resolution

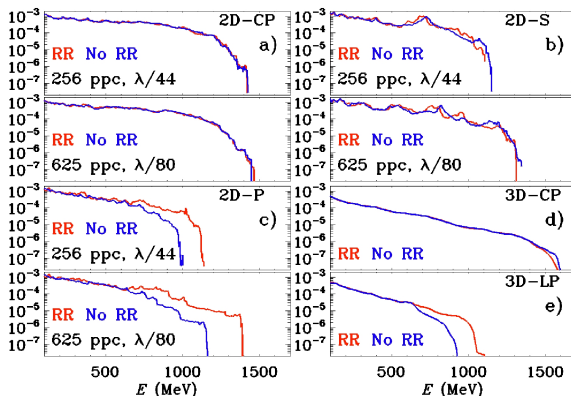


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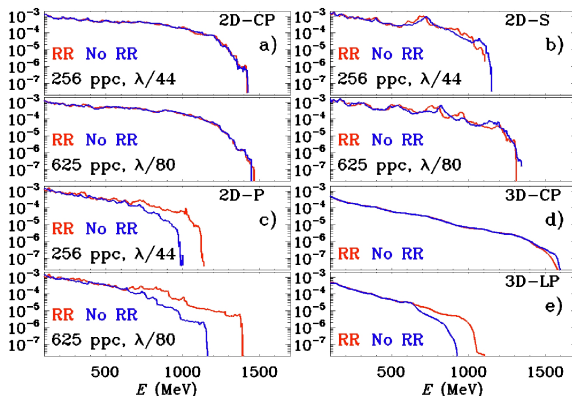


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RF losses in thick targets

- ▶ RF losses are small for **thin** targets pushed by CP pulses
- ▶ But **thick** targets show major RF losses also for CP!
[Naumova et al, PRL **102** (2009) 25002; Schlegel et al, PoP **16** (2009) 83103; Nerush & Kostyukov, PPCF **57** (2015) 35007]
- ▶ “piston oscillations” produce bunches of returning electrons
→ collective “collisions” with the laser pulse → high RF losses

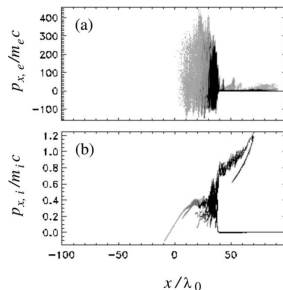
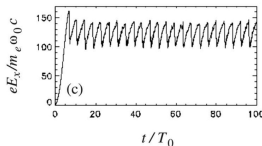
Figures from Schlegel et al.

$$I = 4 \times 10^{22} \text{ W cm}^{-2}$$

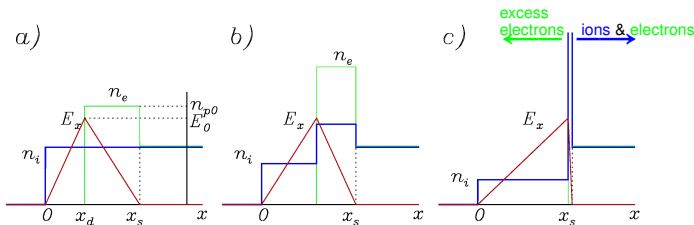
$$a_0 = 100$$

$$n_e = (10 - 20) n_c$$

up to $\sim 40\%$ energy
dissipated by RF

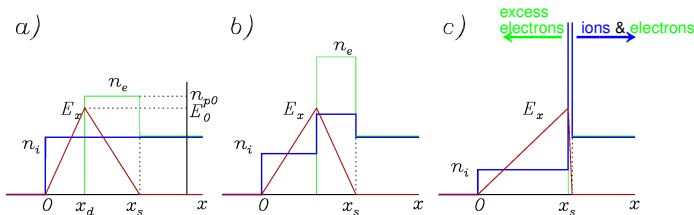


Model for returning electrons in thick targets



- ▶ Light pressure generates an excess of electrons in the skin layer and of ions in the depletion layer
 - ▶ At the end of the acceleration stage, equilibrium between electrostatic tension and ponderomotive force (i.e. local light pressure) is lost
- the excess electrons return quickly towards the laser

Estimating the number of returning electrons



- ▶ $eE_0 n_{p0} \ell_s / 2 \simeq 2I / c$ (pressure balance) ($\ell_s = x_s - x_d$)
 - ▶ $E_0 = 4\pi e n_0 d$ (Poisson-Gauss equation)
 - ▶ $n_{p0} \ell_s = n_0 (d + \ell_s)$ (charge conservation)
- $N_x = (n_{p0} - n_0) \ell_s \simeq n_{p0} \ell_s \simeq a_0 / r_c \lambda$
 returning electrons per unit surface ($r_c = e^2 / m_e c^2$)

Estimating radiation from returning electrons

- ▶ Radiation power for an electron in the laser field

$$P_r = \frac{2e^4 E_L^2}{3m_e^2 c^3} \gamma^2 \left(1 - \frac{v_x}{c}\right)^2 \simeq \frac{8e^4 E_L^2 \gamma^2}{3m_e^2 c^3} = \frac{8e^2 \omega^2 a_0^2}{3c}$$

- ▶ Fraction of laser energy re-emitted as radiation from N_x electrons with $v_x \simeq -1$

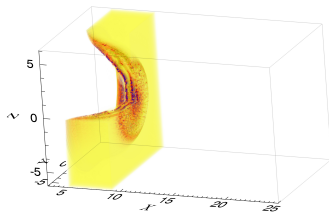
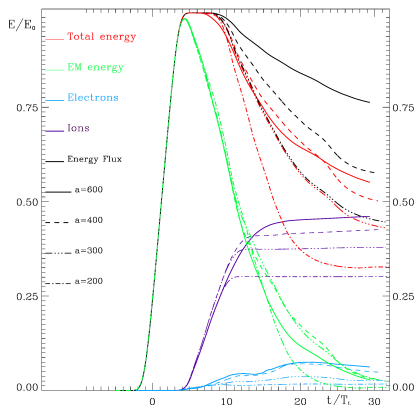
$$\eta \equiv \frac{U_r}{U_L} \simeq \frac{P_r(\tau_L/2)N_x}{E_L^2 c \tau_L/4} \simeq \frac{16}{3} \frac{r_c}{\lambda} \gamma^2 a_0$$

(assuming radiation is emitted for a $\simeq \tau_L/2$ time)

- ▶ If $\gamma = (1 + a_0^2/2)^{1/2} \simeq a_0/\sqrt{2}$ ($|p_x| \ll |p_\perp|$)

$$\eta \simeq \frac{8r_c}{3c\tau_L} a_0^3$$

Radiation losses in 3D thick target simulations



$$n_e = 90n_c = 1.6 \times 10^{23} \text{ cm}^{-3}$$

$$a_0 = (200 - 600)$$

$$I = (0.9 - 7.8) \times 10^{23} \text{ W cm}^{-2}$$

$$\eta \approx 22\% \text{ for } a_0 = 600$$

Scaling $\eta \propto a_0^3$ fairly consistent with the model

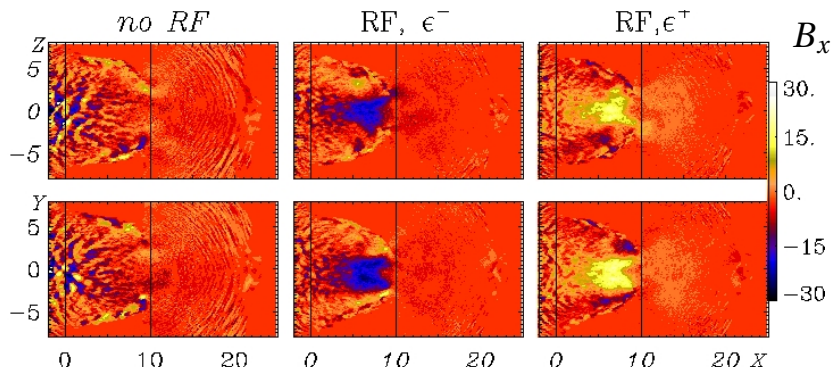
Angular momentum absorption

- ▶ A CP laser pulse carries “spin” angular momentum (AM) that may be absorbed by the target

$$L_z = \int_0^\infty \ell_z(r) 2\pi r dr = - \int_0^\infty \frac{r}{2c\omega} \partial_r I(r) 2\pi r dr$$

- ▶ Reflection from a perfect mirror conserves number of photons and does *not* change sign of “spin”
- No AM absorption if “dissipation” is absent ($A = 0$)
- ▶ First 3D simulations of RPA showed very small AM absorption (AMA)
[T. V. Liseykina et al, PPCF **50** (2008) 124033]
- Can RF provide efficient dissipation for AMA?
- Does RF-induced AMA leads to generation of an axial magnetic field (“Inverse Faraday Effect”)?

Axial magnetic fields induced by RF



Sign of the B_x field depends on laser pulse helicity

Field amplitude $B_{\max} \simeq 4 \times 10^9$ G for $a_0 = 600$

(units: $B_0 = 2\pi m_e c / (e\lambda) = 1.34 \times 10^8$ G)

B-field saturation and AM transfer to ions

Induced electric field $E_\phi = -(r/2c)\partial_t B_x$ leads to saturation of B_x growth and exerts a torque on ions

$$\frac{dL_e}{dt} = M_{\text{abs}} - M_E, \quad \frac{dL_i}{dt} = +M_E$$

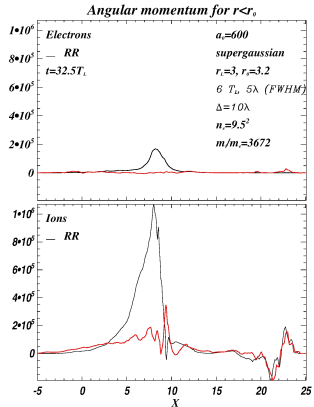
For rapid saturation $M_{\text{abs}} = P_{\text{abs}}/\omega \simeq M_E$
and $L_i \simeq U_{\text{abs}}/\omega \gg L_e$

Magnetic field at saturation

(ℓ : thickness, r_L, r_0 : pulse length & radius)

$$B_{xs} \simeq 4\eta \frac{r_L}{\ell} \left(\frac{c}{r_0\omega} \right)^2 \frac{n_c}{n_e} a_0^2 B_0$$

consistent with $\eta \simeq 0.2$ and $n_e \simeq 0.1 n_0$
(due to axial density depletion)



Conclusions

- ▶ The Landau-Lifshitz “reduced” model is a “good compromise” which allows 3D Particle-In-Cell simulations with Radiation Friction included
 - ▶ Effects on Light Pressure acceleration strongly depend on laser pulse polarization and target thickness
 - ▶ Dissipation due to radiation friction leads to absorption of angular momentum from a circularly polarized pulse and generation of multi-Gigagauss axial magnetic fields
- [T. V. Liseykina, S. V. Propruzhenko, A. Macchi, “*Inverse Faraday Effect driven by Radiation Friction*”, in preparation]