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# Modeling Radiation Pressure and Radiation Friction Effects in Superintense Laser-Plasma Interactions

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**Seminar,  
Max Planck Institute for  
Nuclear Physics, Heidelberg  
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**CNR Research Area, Pisa, Italy**



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# OUTLOOK

1. Radiation Pressure Acceleration: concept and some recent results
  - “Light Sail” acceleration revisited
  - LS improved: Self-Induced Transparency effects
  - The “dark mass” puzzle
  - Electron and ion dynamics and self-organization
  
2. Radiation Reaction effects on RPA
  - Motivations
  - RR modeling via Landau-Lifshitz equation
  - Single particle tests and inclusion in PIC codes
  - Effects on RPA: role of laser polarization



# TWO RPA-BASED VISIONS (1996 - 2010)

22

NATURE

JULY 2, 1966 VOL. 211

## INTERSTELLAR VEHICLE PROPELLED BY TERRESTRIAL LASER BEAM

By PROF. G. MARX

Institute of Theoretical Physics, Roland Eötvös University, Budapest

PRL 104, 135003 (2010)

PHYSICAL REVIEW LETTERS

week ending  
2 APRIL 2010

### Unlimited Ion Acceleration by Radiation Pressure

S. V. Bulanov,<sup>1,\*</sup> E. Yu. Echkina,<sup>2</sup> T. Zh. Esirkepov,<sup>1</sup> I. N. Inovenkov,<sup>2</sup> M. Kando,<sup>1</sup> F. Pegoraro,<sup>3</sup> and G. Korn<sup>4</sup>

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(Received 18 November 2009; published 2 April 2010)



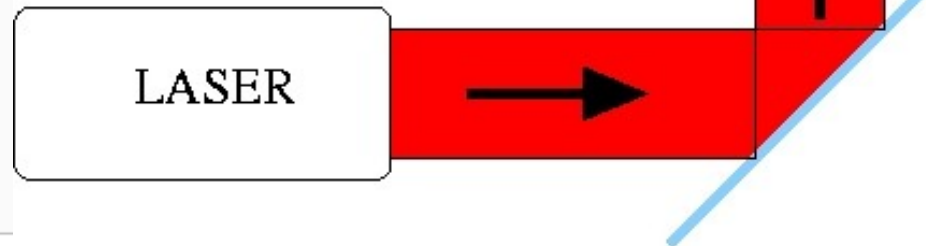
## THE “LIGHT SAIL” CONCEPT

Originally proposed as a way to  
**accelerate a massive mirror**  
by the Radiation Pressure of an Earth-based laser

R.L.Forward, “Roundtrip interstellar travel using  
laser-pushed lightsails”,

J. Spacecraft and Rockets **21** (1964) 187

G.Marx, “Interstellar vehicle propelled by terrestrial  
laser beam”, Nature **211** (1966) 22

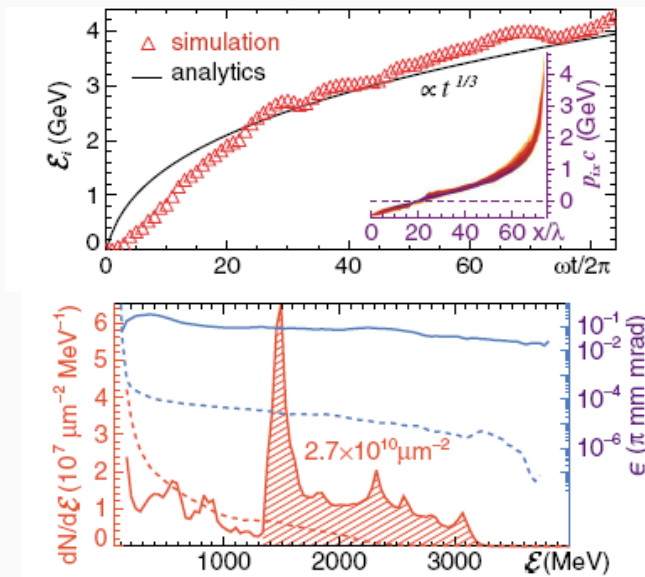
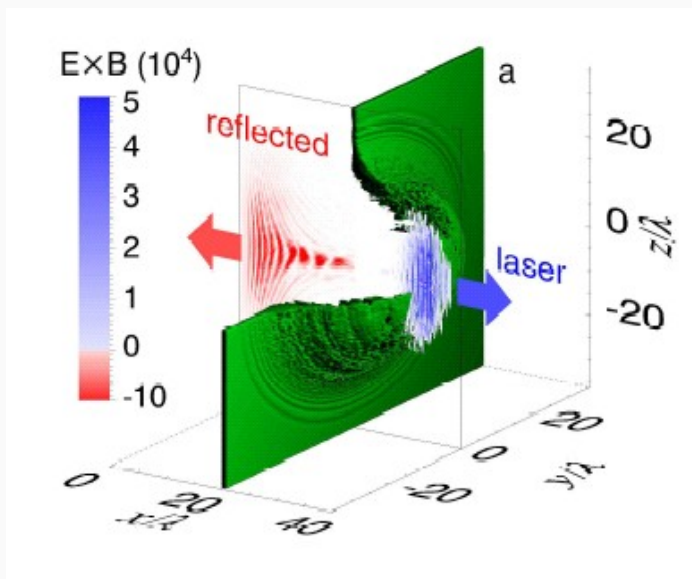


to  $\alpha$ -Centauri





# “LIGHT SAIL” AND LASER ION ACCELERATION



3D simulations suggest “Radiation Pressure Dominance”

in interactions at  $I \geq 10^{23} \text{ W/cm}^2$  with thin plasma foils

Modeling based on simple LS or “accelerating mirror” model

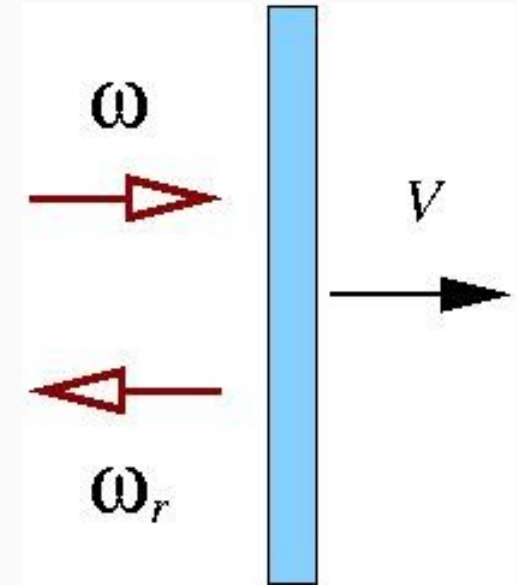
T.Esirkepov, M.Borghesi, S.V.Bulanov, G.Mourou, T.Tajima,  
PRL **92**, 175003 (2004)



## ACCELERATING MIRROR MODEL

perfectly reflecting, **rigid mirror** of mass  $M = \rho \ell S$  boosted by a plane light wave

Mirror velocity as a function of the light pulse intensity  $I$  and duration  $\tau$  and of the surface density  $n_e \ell$  of the target:



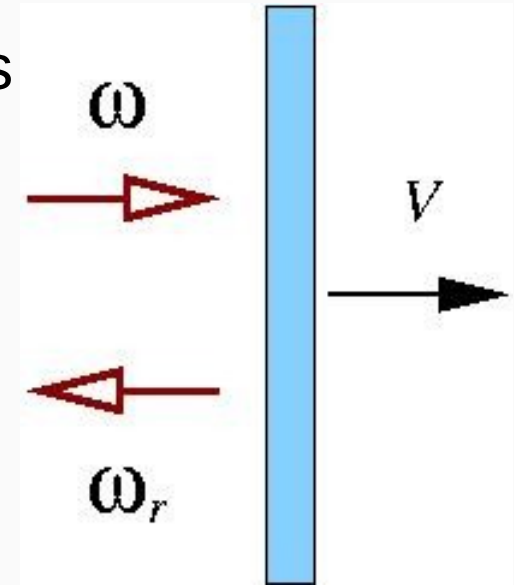
$$\beta(t) = \frac{(1 + \mathcal{E})^2 - 1}{(1 + \mathcal{E})^2 + 1}, \quad \mathcal{E} = \frac{2F(t)}{\rho \ell c^2} = 2\pi \frac{Z m_e a_0^2 \tau}{A m_p \zeta}$$

$$F(t) = \int_0^t I(t') dt' \propto a_0^2 \tau, \quad \zeta = \pi \frac{n_e \ell}{n_c \lambda}$$



## MECHANICAL EFFICIENCY

The efficiency  $\eta$  of the acceleration process can be obtained by a simple argument of conservation of “number of photons” plus the **Doppler shift** of the reflected light:



$$N = \frac{IS\tau}{\hbar\omega}, \quad \omega_r = \omega \frac{1 - \beta}{1 + \beta}$$
$$\eta = \frac{\mathcal{E}_{\text{abs}}}{\mathcal{E}_{\text{laser}}} = \frac{N\hbar(\omega - \omega_r)}{IS\tau} = \frac{2\beta}{1 + \beta}$$
$$\beta \rightarrow 1 \Rightarrow \eta \rightarrow 1$$

**100% efficiency** in the relativistic limit!





# SCALING TO LASER-SOLID INTERACTIONS

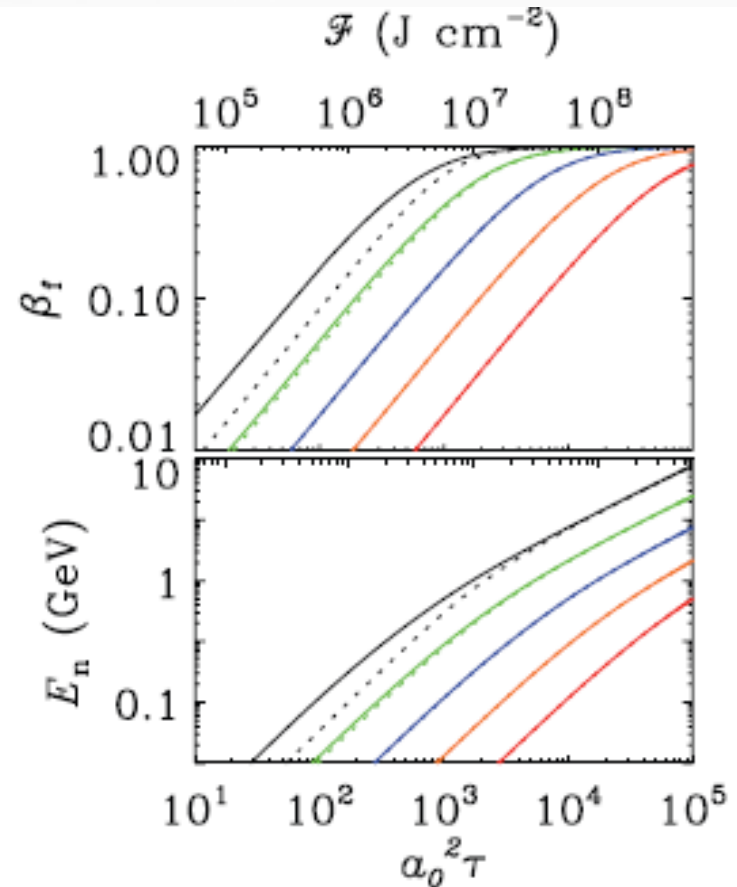
Velocity and energy/nucleon for  
LS-RPA of a ultrathin solid target

vs. laser pulse fluence  $\mathcal{F}$   
for (dimensionless) surface  
target densities

$$\zeta = 1, 3.16, 10, 31.6, 100$$

Experimental requirements:

- nm foil targets (e.g. DLC)
- ultrahigh contrast (plasma mirrors)
- possibly circular polarization?



$a_0$  : dimensionless amplitude,

$\tau$  : duration in cycles



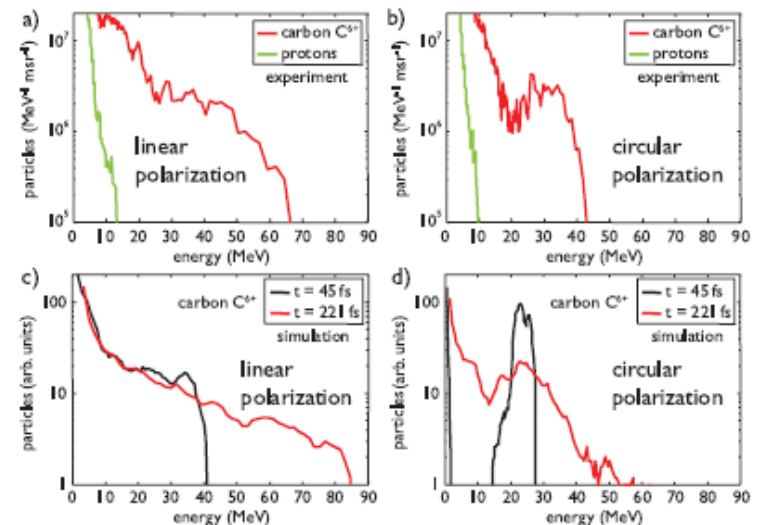
# WHY CIRCULAR POLARIZATION?

Using **CP** and **normal incidence** fast electron generation is strongly **suppressed**, maximizing radiation pressure and preventing foil expansion of the foil target

Early study in “thick” targets:  
[Macchi et al, PRL \*\*94\*\* \(2005\) 165003](#)

Proposal of CP-RPA of **ultrathin foils** for efficient and monoenergetic acceleration:  
[Zhang et al, PoP \*\*14\*\* \(2007\) 073101](#)  
[Robinson et al, NJP \*\*10\*\* \(2008\) 013201](#);  
[Klimo et al, PRST-AB \*\*11\*\* \(2008\) 031301](#).

First **experimental study** reported:  
[Henig et al, PRL \*\*103\*\* \(2009\) 245003](#)





# THEORETICAL INTEREST IN CP-RPA...

## Thick (semi-infinite) targets

("Hole Boring"):

Liseikina & Macchi, APL **94** (2007) 165003;

Naumova et al, PRL **102** (2009) 025002;

Schlegel et al, PoP **16** (2009) 083103;

Robinson et al, PPCF **51** (2009) 024004 & 095006;

Macchi & Benedetti, NIM A **620** (2010) 41

Tikhonchuk et al, Nucl. Fus. **50** (2010) 045003

## Ultrathin (sub-wavelength) targets

("Light sail"):

Yan et al, PRL **100**, (2008) 135003 ;

Qiao et al, PRL **102** (2009) 145002;

Tripathi et al, PPCF **51** (2009) 024014;

Eliasson et al. NJP **11** (2009) 073006;

Yan et al, PRL **103** (2009) 135001;

Macchi et al, PRL **103** (2009) 085003;

Macchi et al, NJP **12** (2010) 045013.

## Variations on the theme

(side effects, multi-species or structured targets, ...):

Liseikina et al, PPCF **50** (2008) 124033;

Rykovanov et al., NJP **10**, (2008) 113005;

Ji et al, PRL **101** (2008) 164802;

Yin et al, PoP **15** (2008) 093106;

Holkundkara & Gupta, PoP **15** (2008) 123104;

Chen et al, PoP **15** (2008) 113103;

Zhang et al, PRST-AB **12** (2009) 021301;

Gonoskov et al, PRL **102** (2009) 145002;

Chen et al, PRL **103** (2009) 024801

Grech et al, NJP **11** (2009) 093035

Yu et al, PRL **105** (2010) 065002

Results presented  
in this talk



## LS MODEL VS 1D PIC SIMULATIONS - I

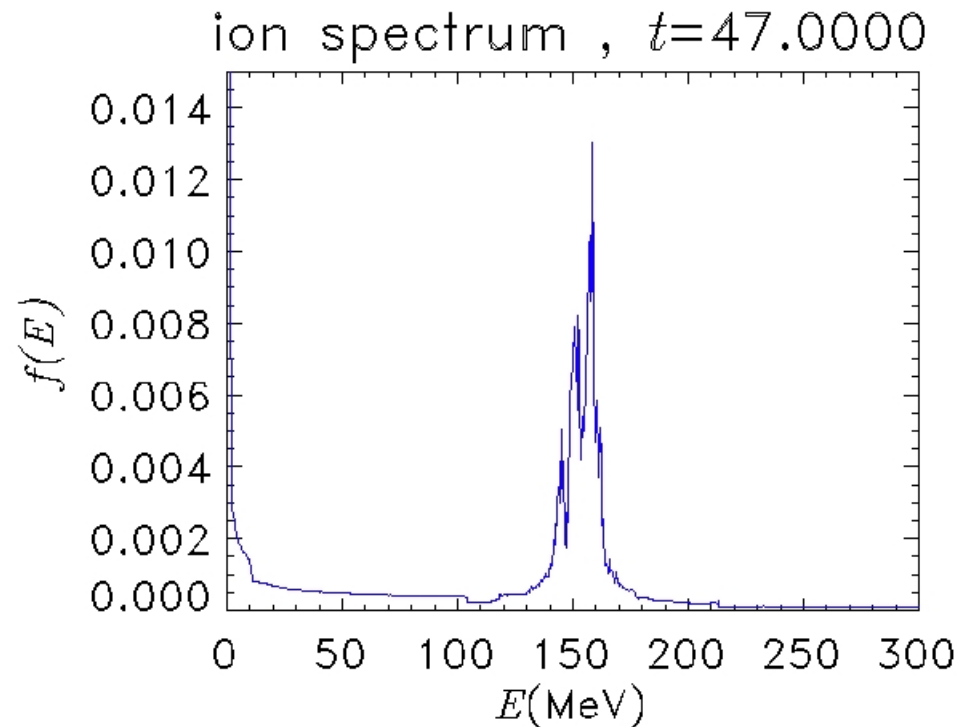
Laser pulse:  $a_0 = \mathbf{5-50}$ ,  $\tau = 8$  cycles (“flat-top” envelope)

Thin foil target:  $n_e = 250n_c$ ,  $\ell = \mathbf{0.01-0.1\lambda}$  ( $\zeta = \mathbf{7.8-78.5}$ )

A narrow spectral peak is observed for  $a_0 < \zeta$ .

The energy of the peak is in **good agreement with the LS formula**

For  $a_0 > \zeta$ , the dynamics is dominated by a **Coulomb explosion** of the foil following a complete **blow-out of electrons**

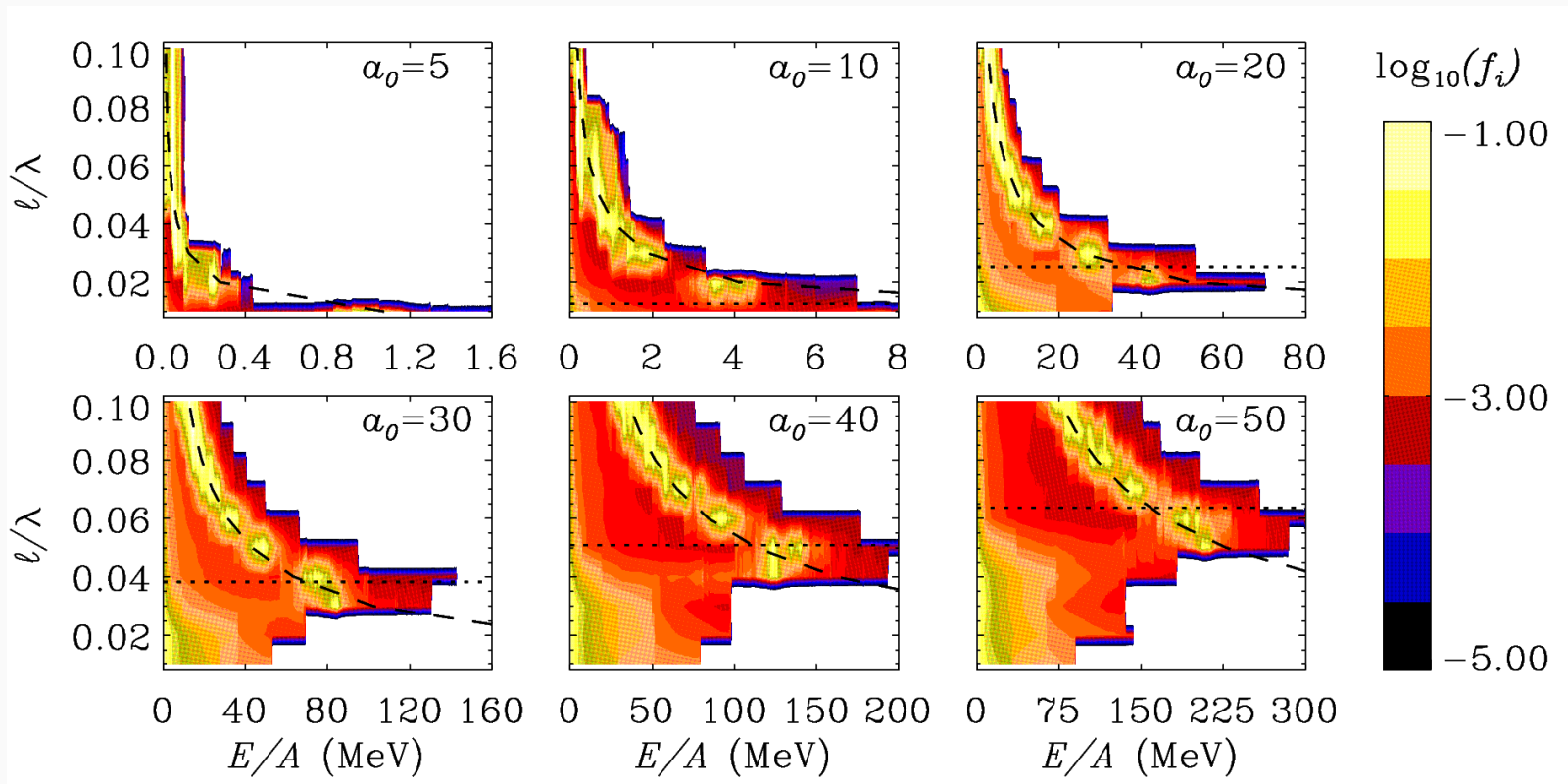




## LS MODEL VS 1D PIC SIMULATIONS - II

Energy spectra vs.  $a_0$  and  $\ell$ :

(Dashed line: LS model prediction, dotted line:  $a_0 = \zeta$ )



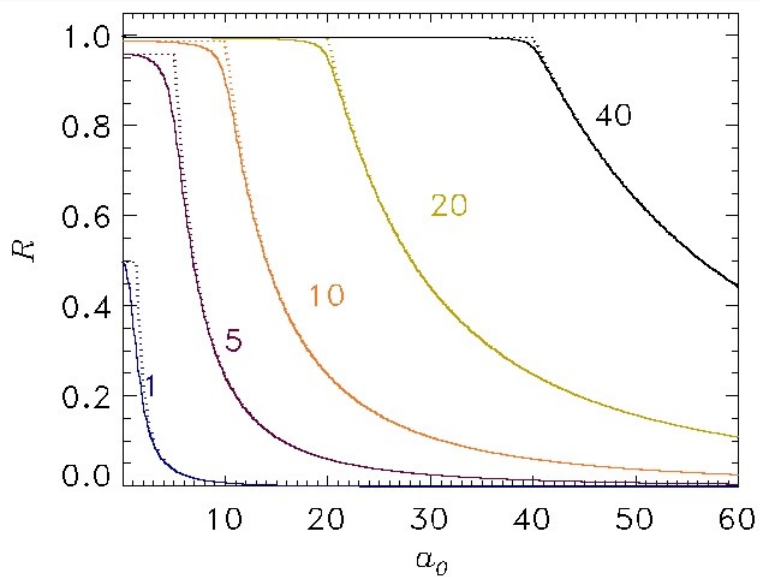


# TRANSPARENCY AND “OPTIMAL” THICKNESS

Ultrathin slab model:  $n_e(x) = n_0 \ell \delta(x)$ , foil thickness  $\ell \ll \lambda$

Total radiation pressure in rest frame  $P_{\text{rad}} = (2I/c)R$

Nonlinear reflectivity  $R = R(\zeta, a_0)$  includes Self-Induced Transparency



$$R \approx \zeta^2 / (\zeta^2 + 1) \quad (a_0 < \zeta)$$

$$R \approx \zeta^2 / a_0^2 \quad (a_0 > \zeta)$$

$P_{\text{rad}}$  does not depend on

$a_0$  for  $a_0 > \zeta$  ! (since  $I \sim a_0^2$ )

The maximum boost of the foil is at  $a_0 \approx \zeta$



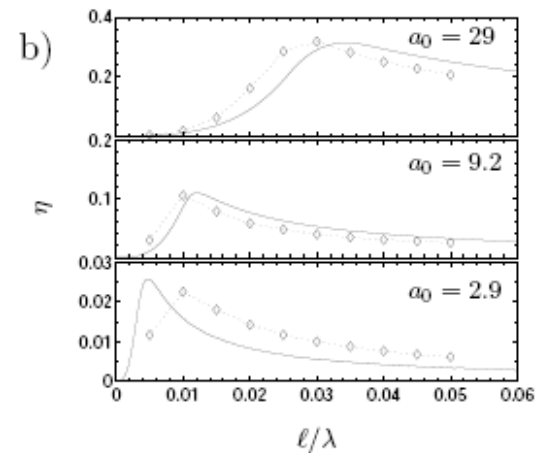
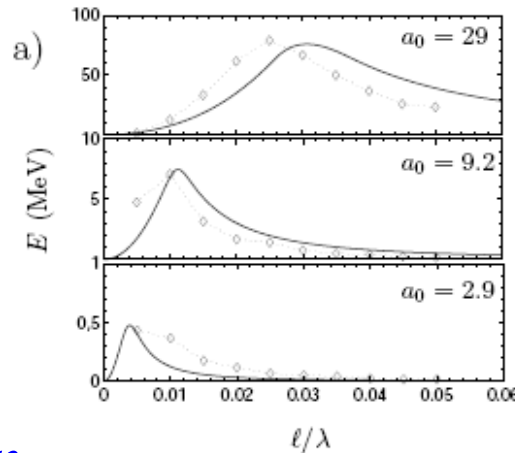
# LS MODEL WITH SIT INCLUDED

Modified foil velocity formula for  $R < 1, a_0 < \zeta$

$$\beta(t) = \frac{(1 + \mathcal{E} - \zeta^{-2})^2 + (1 + \mathcal{E} - \zeta^{-2})\sqrt{(1 + \mathcal{E} - \zeta^{-2})^2 + 4\zeta^{-2}} + 2\zeta^{-2} - 2}{(1 + \mathcal{E} - \zeta^{-2})^2 + (1 + \mathcal{E} - \zeta^{-2})\sqrt{(1 + \mathcal{E} - \zeta^{-2})^2 + 4\zeta^{-2}} + 2\zeta^{-2} + 2}$$

$$\mathcal{E} = \frac{2F(t)}{\rho l c^2} = 2\pi \frac{Z m_e a_0^2 \tau}{A m_p \zeta}$$

Ion energy and conversion efficiency vs. intensity and thickness (solid: theory, points: PIC sims.)



9 cycles pulse,  $n_e = 250 n_c$

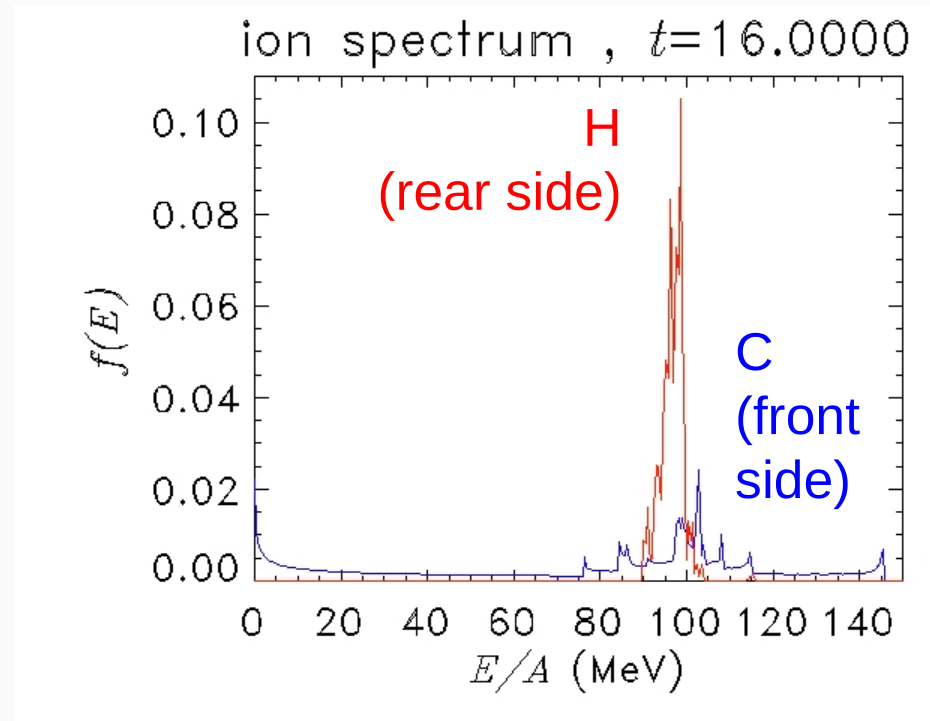


## A PUZZLING ISSUE: “DARK” MASS

The RPA peak contains only  
~30% of all the ions  
(and ~64% of their energy)

Only the **rear side** of the foil  
is accelerated (thus LS RPA  
may work for double-layer  
targets!)

→ *Why there is very good  
Agreement of the energy with  
the LS formula when inserting  
there the whole mass of the target  
(and not ~30% of it)?*







## RADIATION VS ELECTROSTATIC PRESSURE

**Radiation pressure** drives electron depletion and generates back-holding electrostatic pressure

$$P_{\text{rad}} = (2I/c)R \leq P_{\text{es}} = 2\pi(en_0 \ell)^2 \quad \text{for } a_0 \leq \zeta$$

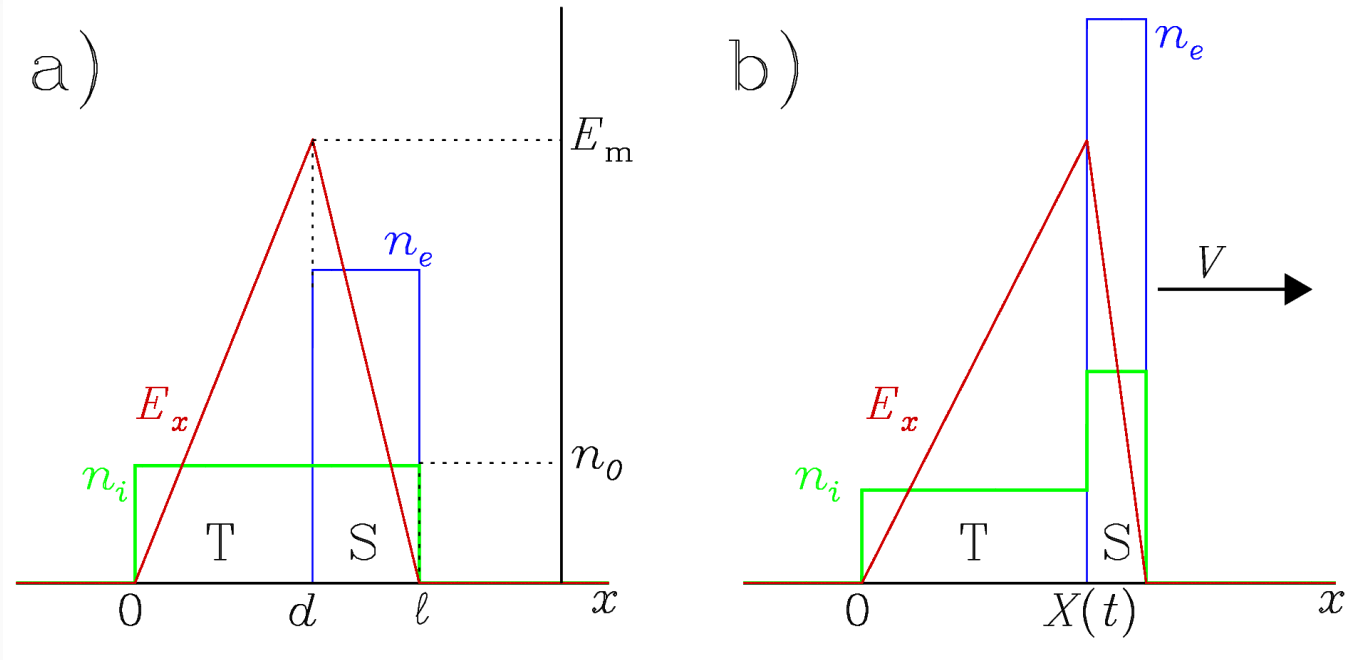
If  $a_0 < \zeta$  and  $\zeta \gg 1$ ,  $R \approx 1$  and **no electrons are pushed away**

For  $a_0 \rightarrow \zeta$  all electrons must pile up near the rear surface in order that  $P_{\text{rad}} \simeq P_{\text{es}}$ .

- the electron pile-up layer is **much thinner than the foil**
- only **a fraction of the foil is accelerated**



## TWO ION POPULATIONS: TAIL & SAIL



**Sail (S):** ions are bunched accelerated by  $E_x = f_p / e$  and move coherently as a “foil” : **monoenergetic component**

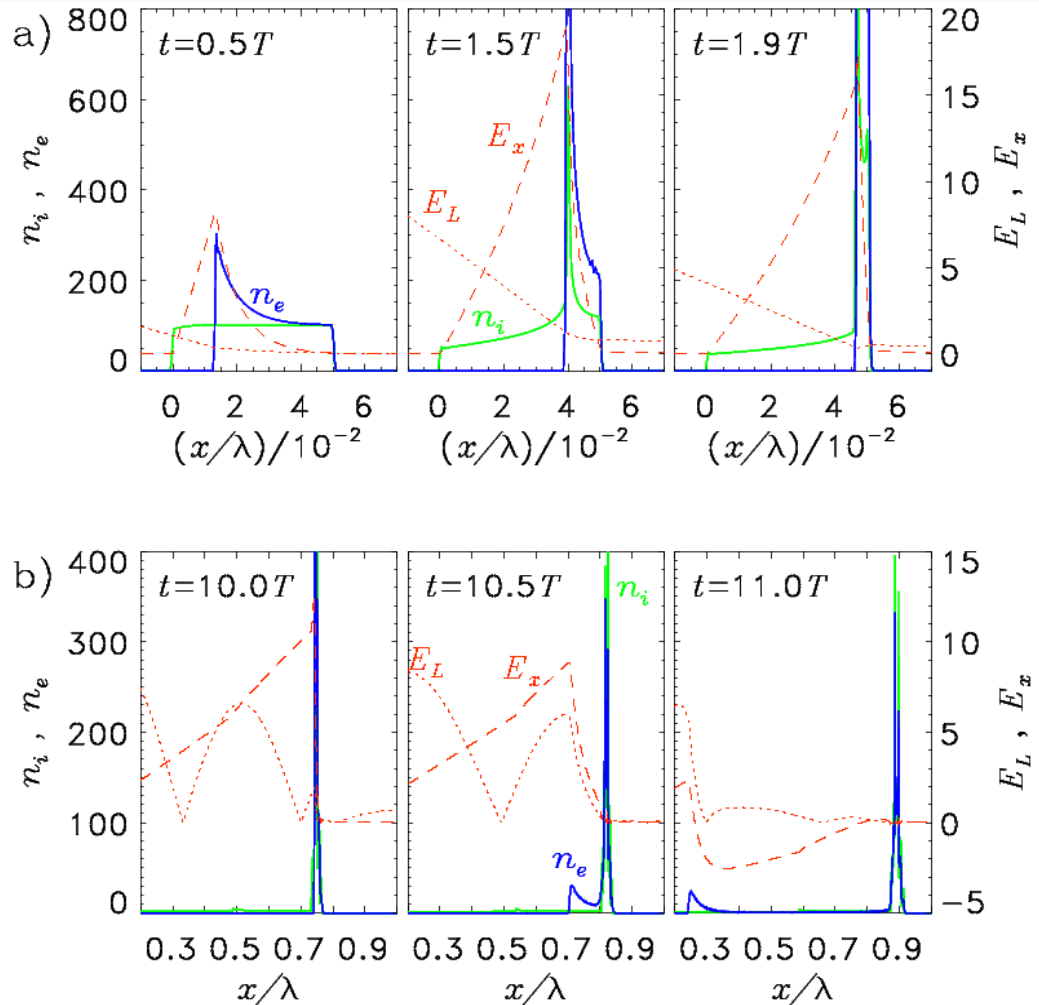
**Tail (T):** ions are accelerated by their own space-charge field and “Coulomb explode”: **broad spectrum component**



## SAIL CHARGING/DISCHARGING

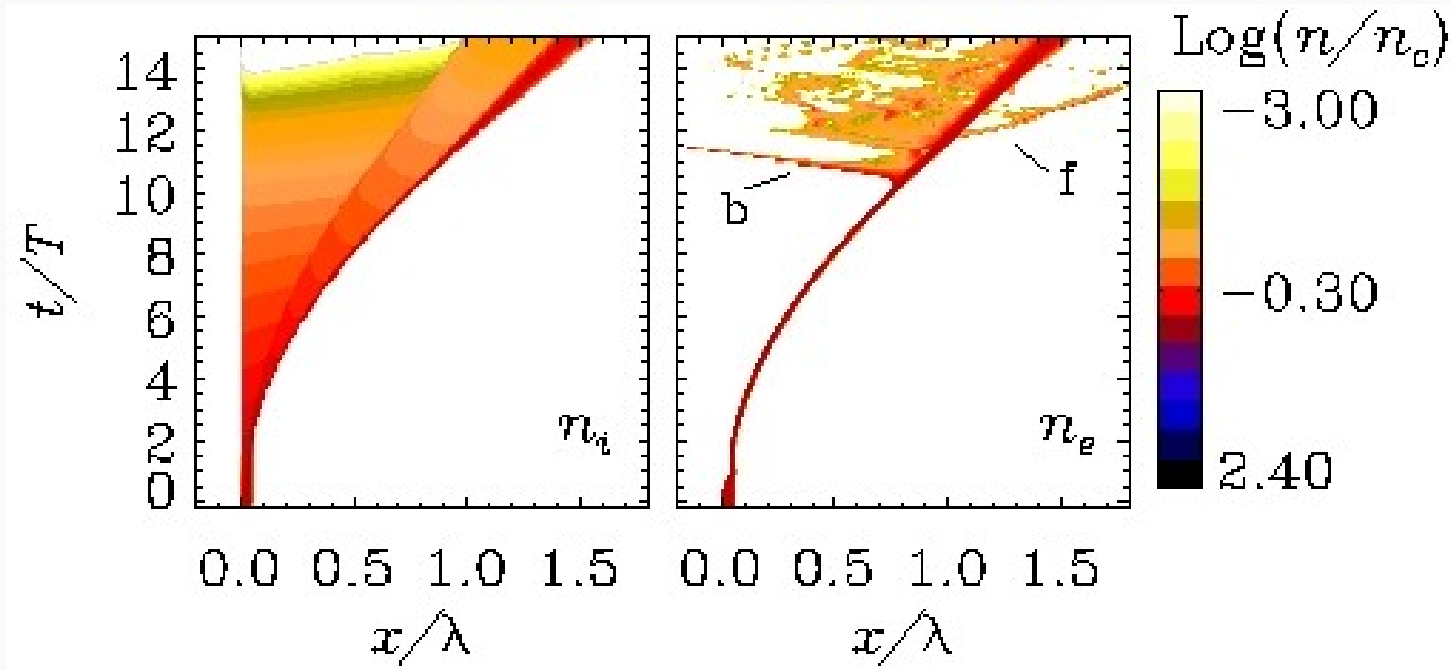
PIC simulations show ions in the compression layer to form a “Sail” thinner than the original foil and negatively charged (excess of electrons)

The excess electrons “detach” from the Sail near the end of the laser pulse, moving backwards and leaving a neutral plasma bunch





# COLLAPSE OF ELECTRON EQUILIBRIUM



Near the end of the acceleration stage the pressure balance for electrons breaks down: formation of bunches in forward & backward directions and heating

Ion spectrum broadens in the post-acceleration stage



## MISSING INERTIA IS LOWER PRESSURE

Equilibrium of radiation  
and electrostatic  
pressure on *electrons*:

$$P_{\text{rad}} \doteq \int (-e)n_e E_x dx = \int n_e f_p dx$$

Electrostatic pressure  
on *ions*:

$$P_{\text{es}} = \int Z n_i E_x dx < P_{\text{rad}} \quad (Z n_i < n_e)$$

Calculation on equilibrium  
profiles yields:

$$P_{\text{es}} = \frac{M_{\text{Sail}}}{M_{\text{Foil}}} P_{\text{rad}}$$

Equation of motion:

$$P_{\text{es}} = \frac{d}{dt} (M_{\text{Sail}} \mathbf{V}) \iff P_{\text{rad}} = \frac{d}{dt} (M_{\text{Foil}} \mathbf{V})$$

→ **The Sail moves as if it had the total mass of the foil**



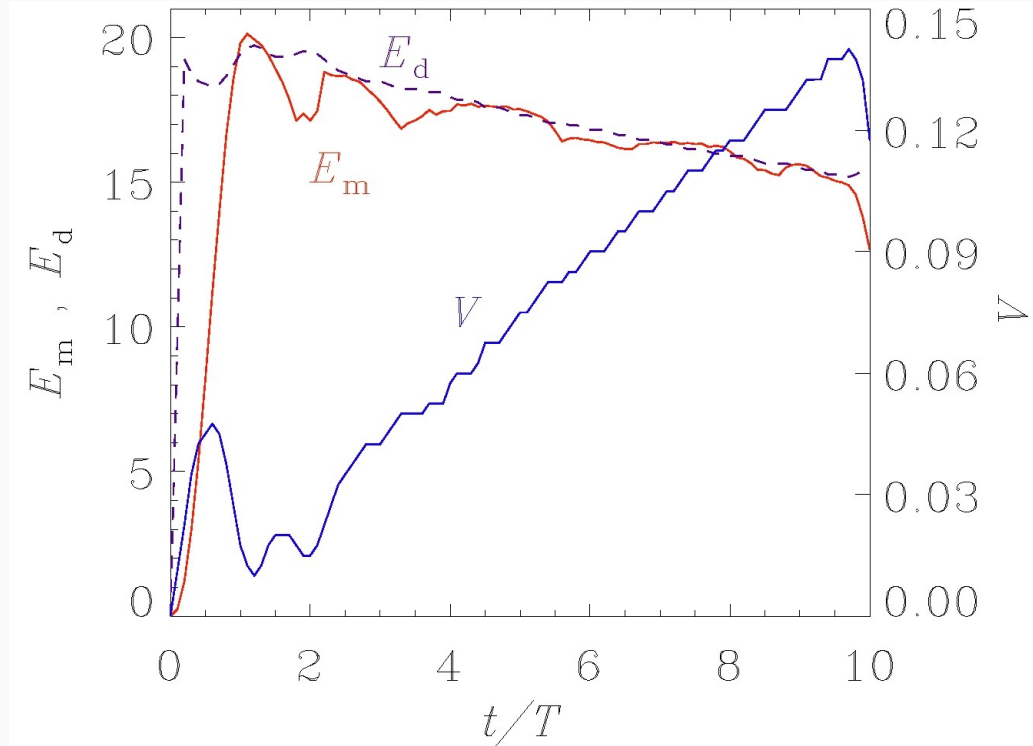
# DYNAMIC PRESSURE BALANCE

$P_{\text{rad}}$  decreases with  
velocity in the Lab frame

$$(P_{\text{rad}})^L = (1-\beta)/(1+\beta) P_{\text{rad}}$$

To keep pressure  
equilibrium there is  
a **mass flow**  
(ion current) from

$$M_{\text{tail}} \text{ to } M_{\text{sail}}$$





## ENERGY BALANCE

Efficiency depends only on  $\beta$  (the Sail velocity)  
BUT the kinetic energy of the Sail is less than the total!

Energy stored in the  
electrostatic field  $E_x$  :

“Conversion efficiency”  
into electrostatic  
energy  $\eta_{es}$  :

$$U_{es} = U_{es}(t) = \int_0^{X(t)} \frac{E_x^2(x, t)}{8\pi} dx$$
$$\frac{dU_{es}}{dt} = \frac{1}{8\pi} E_x^2[X(t), t] \frac{dX}{dt} = \frac{1}{8\pi} E_0^2 \beta c$$
$$\eta_{es} = \frac{1}{I} \frac{dU_{es}}{dt} = 2\beta \left(\frac{d}{\ell}\right)^2 \left(\frac{\zeta}{a_0}\right)^2$$

For  $a_0 = \zeta$ , the depletion width  $d \approx \ell$  thus  $\eta_{es} \approx 2\beta$ :

most of the stored energy is converted into electrostatic energy  
and eventually goes to Tail ions



# TWO-DIMENSIONAL SIMULATIONS

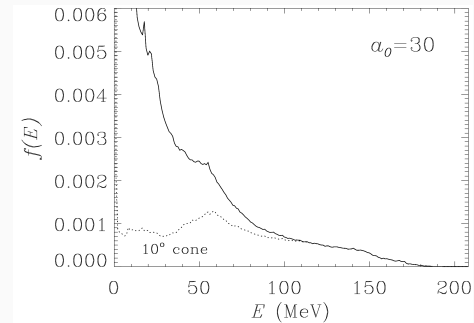
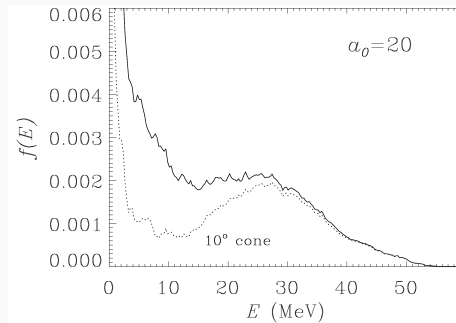
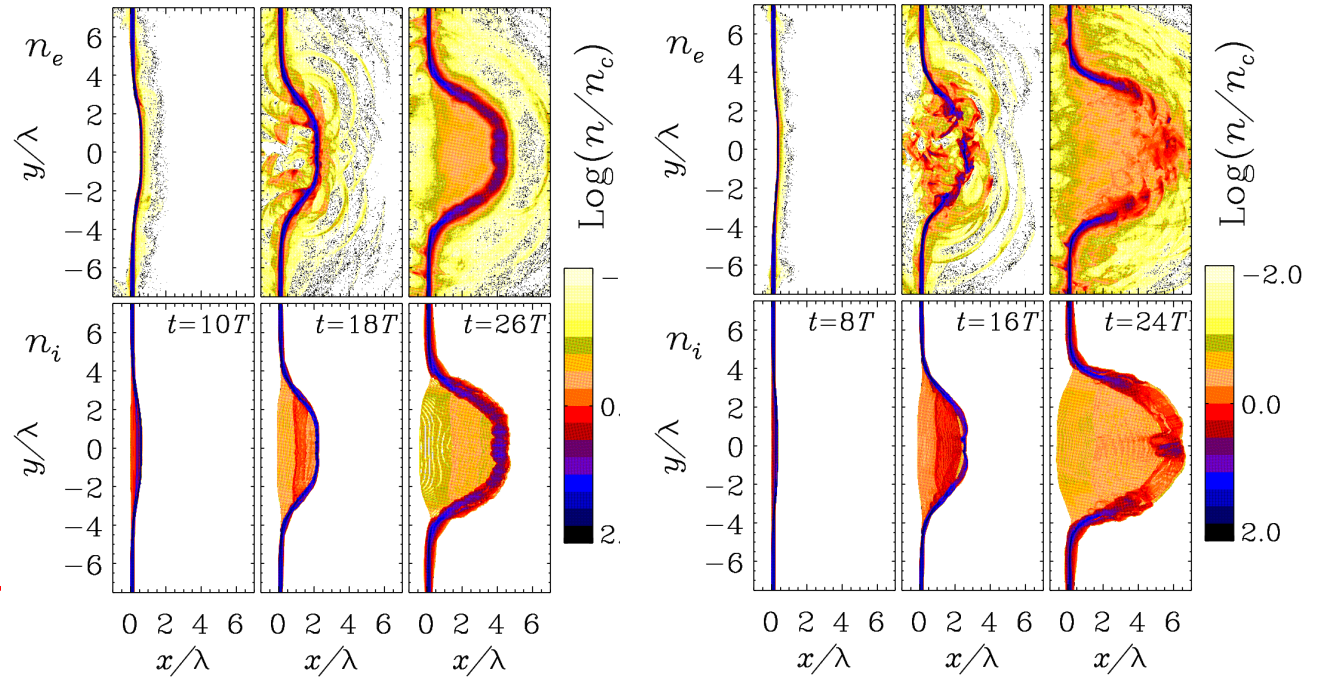
2D sims for  
 $\zeta=31.4$  and

$a_0=20$  (left)

$a_0=30$  (right)

stronger electron  
heating and lower  
“penetration”

threshold with  
respect to 1D:  
ion spectrum is  
broad





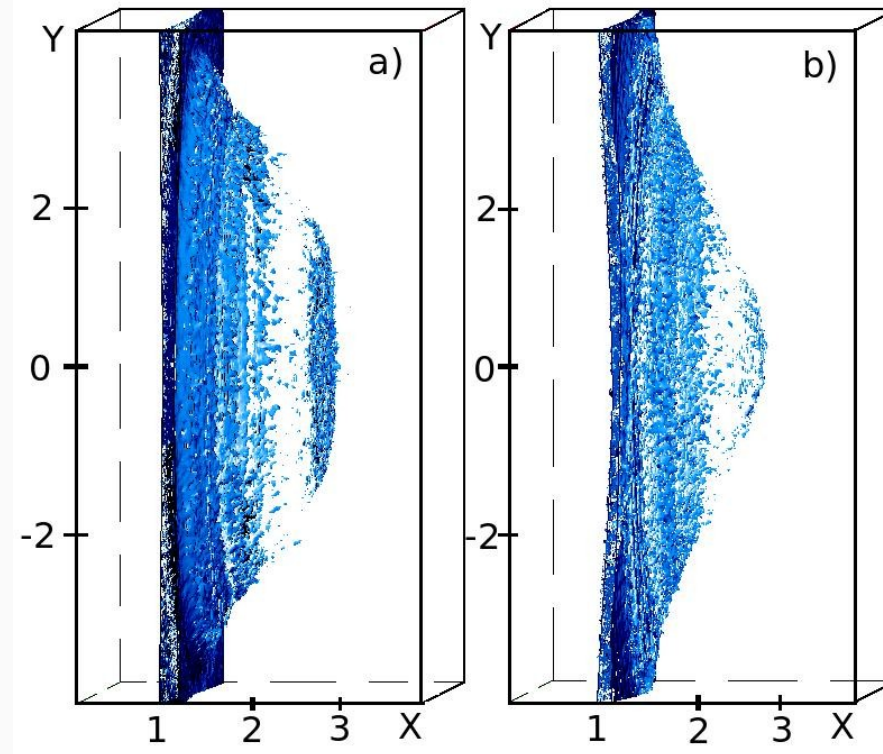


## THREE-DIMENSIONAL SIMULATIONS

3D sims for  $\zeta=15$  ,  
 $a_0=5$  ,  $\tau=18$  cycles

left: Supergaussian  
spot profile  
right: Gaussian

Note that only in 3D  
angular momentum  
conservation is taken  
into account



Supergaussian “flat-top” profiles keep a “quasi-1D” geometry and prevent early breakthrough of laser pulse due to lateral expansion



## RADIATION REACTION EFFECTS

Motivation: **Radiation Reaction** is important for **ultra-relativistic particles** in EM fields and is thus expected to play a strong role in next generation experiments at ultra-high intensities.

The typical intensity for relevant RR effects is estimated to be  $\sim 10^{23} \text{ W/cm}^2$ . This corresponds, to the foreseen regime of RPA dominance (for **Linear Polarization**)  
[Esirkepov et al, PRL **92** (2004) 175003]

Two RPA simulation studies (for **Circular Polarization** at lower intensity) suggest a “beneficial” effect of “electron cooling” by RR  
[Schlegel et al, PoP **16** (2009) 083103;  
Chen et al, arXiv:0909.5144,  
to appear in Plasma Phys. Contr. Fus.]



## RADIATION REACTION FORCES

$$mc \frac{du^\mu}{d\tau} = f_L^\mu + f_{RR1}^\mu + f_{RR2}^\mu + f_S^\mu$$

$$f_L^\mu = eF^{\mu\nu} u_\nu$$

$$f_{RR1}^\mu = e\tau_0 (\partial_\alpha F^{\mu\nu} u_\nu u^\alpha)$$

$$f_{RR2}^\mu = \frac{e^2}{mc} \tau_0 \left( F^{\mu\nu} F_{\nu\alpha} u^\alpha + (F^{\nu\beta} u_\beta F_{\nu\alpha} u^\alpha) u^\mu \right)$$

$$f_S^\mu = -\frac{1}{2c} S^{\gamma\delta} \partial^\mu F_{\gamma\delta} + \frac{1}{2c} \left( S^{\gamma\delta} \partial_\alpha F_{\gamma\delta} u^\alpha \right) u^\mu$$

$$u^\alpha = \left( \gamma, \gamma \frac{\mathbf{v}}{c} \right) \quad \partial^\mu \equiv \left( \frac{\partial}{\partial t}, -c\nabla \right)$$

EoM of classical particle  
with spin in EM field:  
Landau-Lifshitz formula  
for RR term  $f_{RR}^\mu$



EoM with Landau-Lifshitz force in non-covariant notation

$$\begin{aligned} \frac{d\mathbf{p}}{dt} = & -\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \\ & - \left(\frac{4}{3}\pi\frac{r_e}{\lambda}\right) \gamma \left[ \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right) \mathbf{E} + \mathbf{v} \times \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right) \mathbf{B} \right] \\ & + \left(\frac{4}{3}\pi\frac{r_e}{\lambda}\right) \left[ \left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \times \mathbf{B} + \left(\mathbf{v} \cdot \mathbf{E}\right) \mathbf{E} \right] \\ & - \left(\frac{4}{3}\pi\frac{r_e}{\lambda}\right) \gamma^2 \left[ \left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)^2 - \left(\mathbf{v} \cdot \mathbf{E}\right)^2 \right] \mathbf{v} \end{aligned}$$

The last “friction” term is the dominant one  
(the first terms is ordinarily smaller than spin contribution)



## BENCHMARK WITH EXACT SOLUTIONS

Exact solution of the Landau-Lifshitz equation in a plane wave

[A.Di Piazza, Lett.Math.Phys. **83** (2008) 305]

Based on this test case we identify suitable approximations to the electron EoM with RR included:

- the **spin force** is  $\sim 137\gamma$  X the **first LL term** in the RR force

- the **second LL term** is  $\sim a_0 \omega \tau / 137$  X the spin force

→ for intensities  $\gg 10^{22}$  W/cm<sup>2</sup> it is consistent to **neglect both the 1<sup>st</sup> LL term and the spin force**

[M.Tamburini, F.Pegoraro, A. Di Piazza, C.H.Keitel, A.Macchi, preprint [arxiv:1008.1685](https://arxiv.org/abs/1008.1685) ]

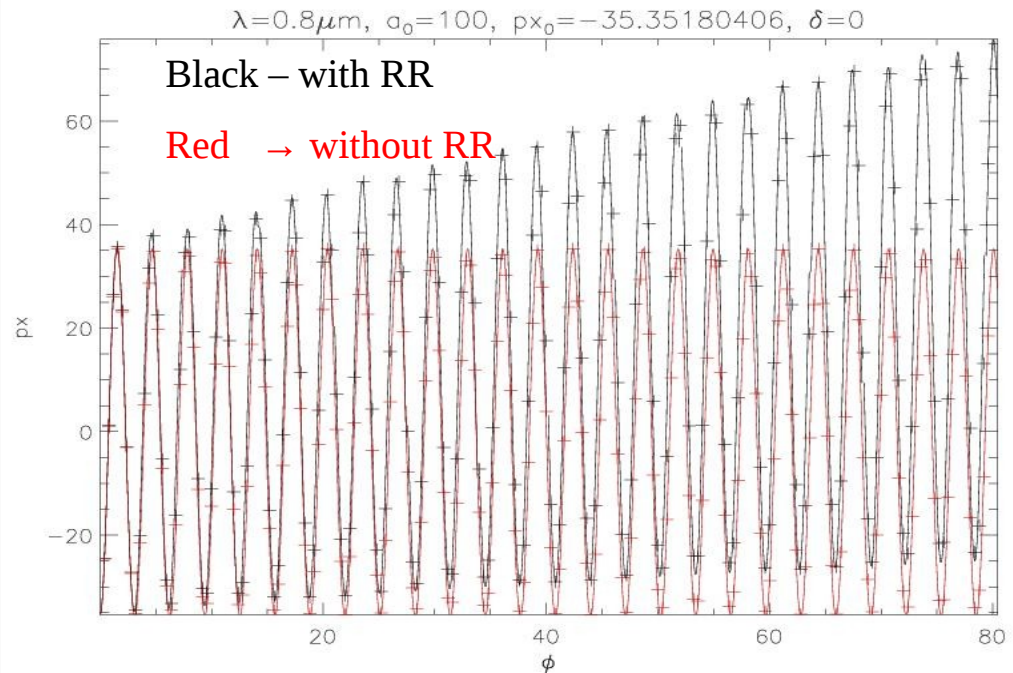


## TEST OF PARTICLE PUSHER - I

A numerical solution of motion in a plane wave based on simple 2<sup>nd</sup> order leap-frog method has been compared with the exact solution and with 4<sup>th</sup> order Runge-Kutta integration

Crosses: analytical  
Line: numerical

- excellent agreement for intensities up to  $10^{24}$  W/cm<sup>2</sup>
- straightforward to include in a “standard” PIC code (based on Boris particle pusher)
- only ~10% increase in CPU time





## TEST OF PARTICLE PUSHER - II

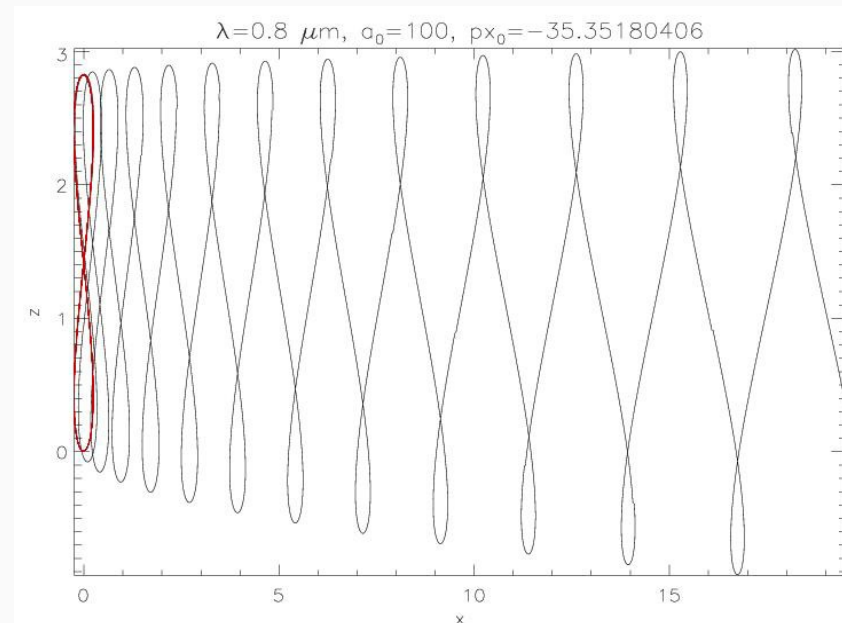
A numerical solution of motion in a plane wave based on simple 2<sup>nd</sup> order leap-frog method has been compared with the exact solution and with 4<sup>th</sup> order Runge-Kutta integration

“Figure of Eight” drifts away when RR is included

- excellent agreement for intensities up to  $10^{24}$  W/cm<sup>2</sup>
- straightforward to include in a “standard” PIC code (based on Boris particle pusher)
- only ~10% increase in CPU time

Black – with RR

Red → without RR





## TEST OF PARTICLE PUSHER - III

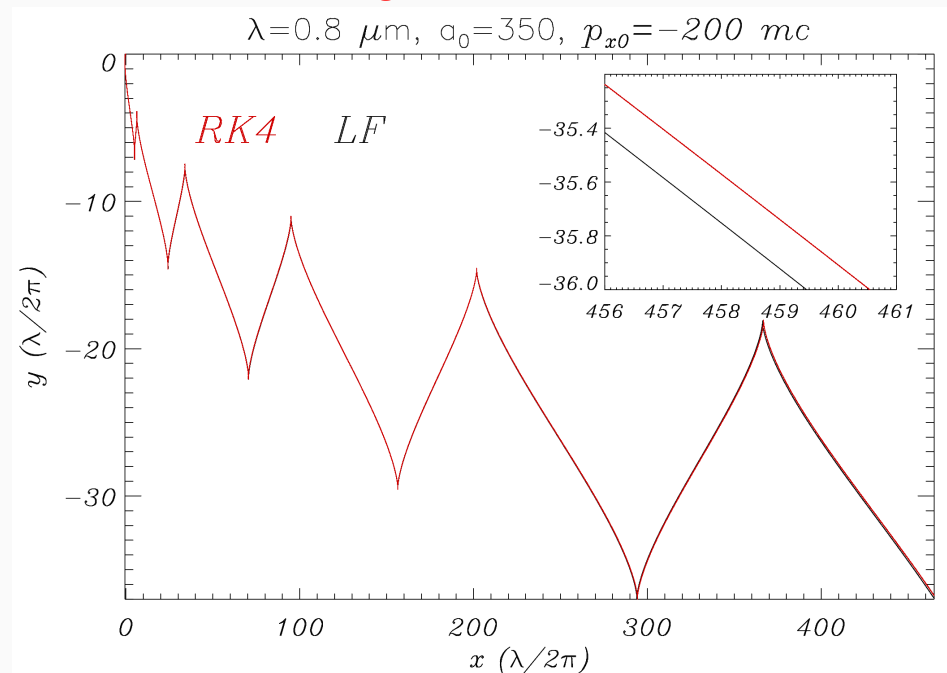
A numerical solution of motion in a plane wave based on simple 2<sup>nd</sup> order leap-frog method has been compared with the exact solution and with 4<sup>th</sup> order Runge-Kutta integration

Crosses: analytical  
Line: numerical

- excellent agreement for intensities up to  $10^{24}$  W/cm<sup>2</sup>
- straightforward to include in a “standard” PIC code (based on Boris particle pusher)
- only ~10% increase in CPU time

Black – Leap Frog

Red → Runge-Kutta





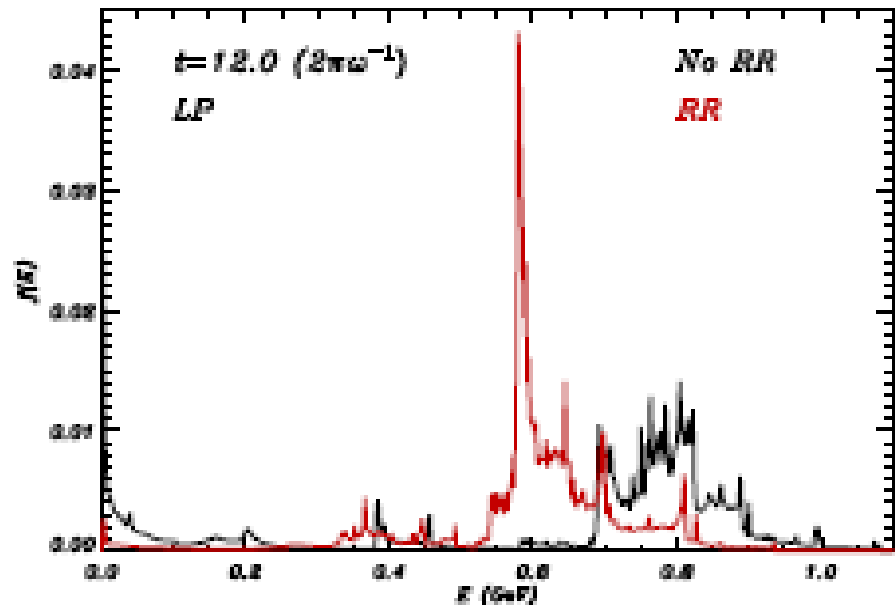


## RR EFFECTS ON ION SPECTRA – I (LP)

RP-dominated regime:  $2.3 \times 10^{23} \text{ W/cm}^2$  , 11 cycles pulse  
1  $\mu\text{m}$  foil,  $100n_c$  , linear polarization

Lower energy,  
narrower spectrum  
with RR included

~25% reduction in  
“peak” ion energy “  
due to RR effects



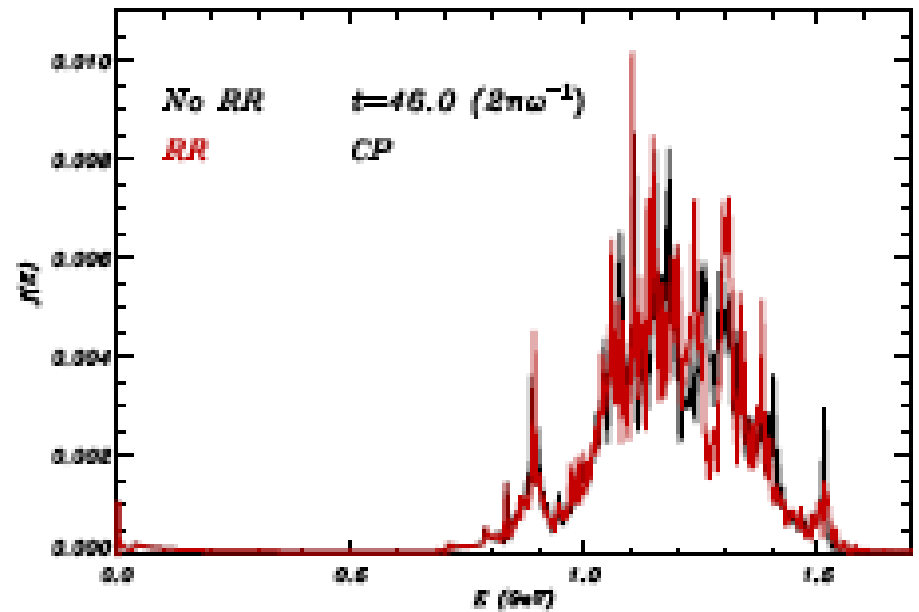


## RR EFFECTS ON ION SPECTRA – II (CP)

RP-dominated regime:  $2.3 \times 10^{23} \text{ W/cm}^2$  , 11 cycles pulse  
1  $\mu\text{m}$  foil,  $100n_c$  , circular polarization

Negligible RR effects  
on ion spectrum!

Higher energy than in  
LP case



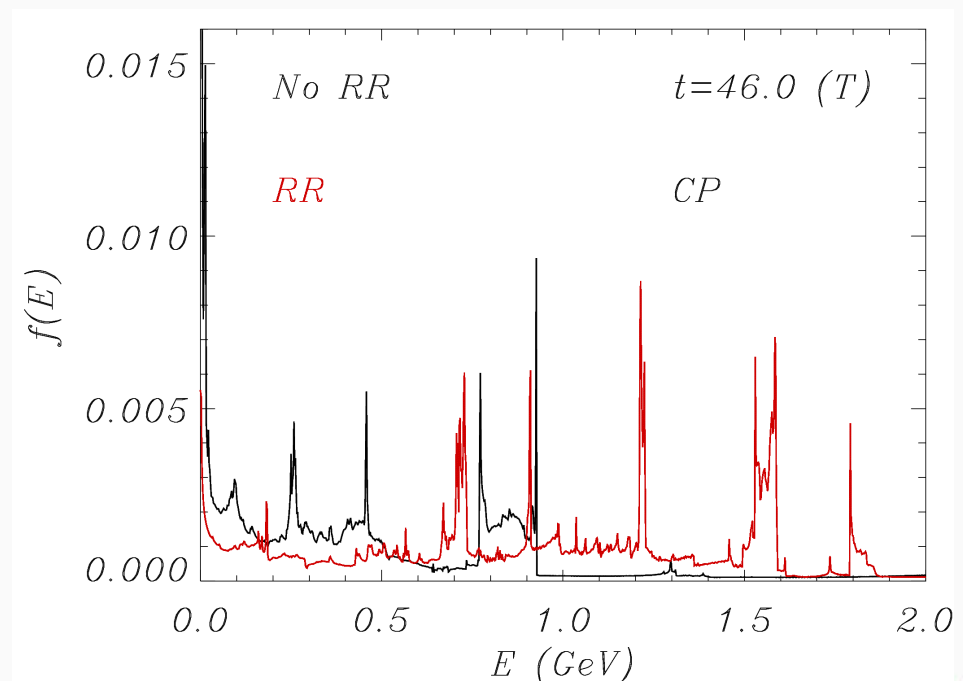


## RR EFFECTS ON ION SPECTRA – III (CP)

RP-dominated regime:  $2.3 \times 10^{23} \text{ W/cm}^2$  , 11 cycles pulse  
0.3  $\mu\text{m}$  foil,  $100n_c$  , circular polarization

The pulse penetrates through the foil due to “relativistic” Self-Induced Transparency

RR effects are now important for CP and *increase* the ion energy, but the regime is *not* optimal for ion acceleration

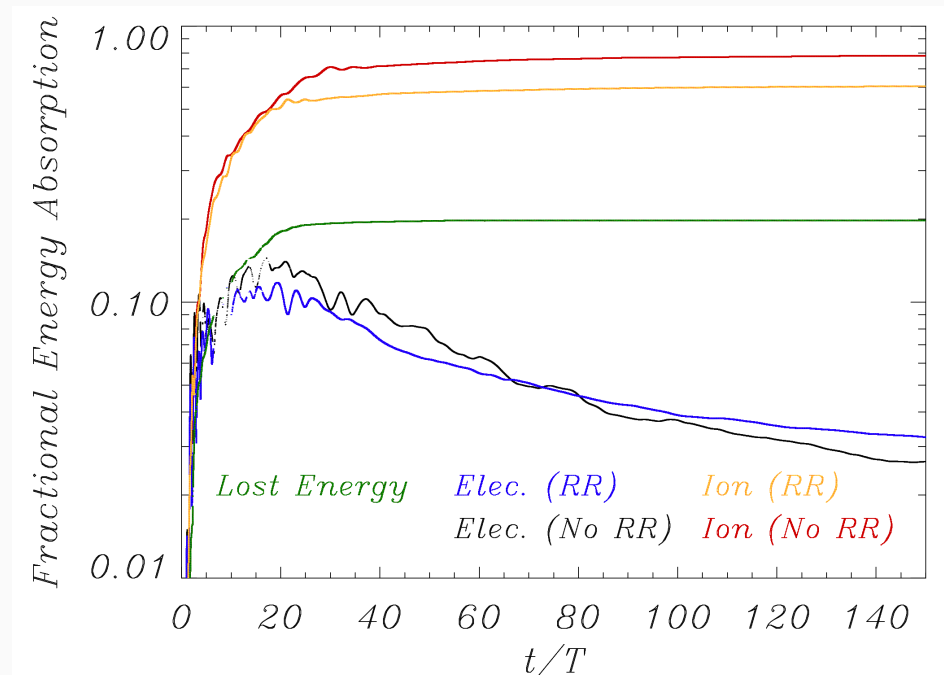




## ENERGY BALANCE (LP)

RP-dominated regime:  $2.3 \times 10^{23} \text{ W/cm}^2$  , 11 cycles pulse  
1  $\mu\text{m}$  foil,  $100n_c$  , linear polarization

~20% energy  
“dissipated” by RR  
as incoherent, high  
frequency radiation  
escaping from the  
plasma





## CONCLUSIONS

“Light Sail” regime of Radiation Pressure Acceleration:

- Underlying dynamics and self-organization have been studied in detail
- Formation of two ion populations and “puzzling” issues in pressure and energy balance have been unfolded

Radiation Reaction (or Friction) effects:

- Development and test of a simple model for inclusion in PIC codes via Landau-Lifshitz equation
- RR effects on RPA at ultrahigh intensities are important only for Linear Polarization or in the Self-Induced Transparency regime



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