Radiation Pressure and Radiation Reaction Effects in Laser-Solid Interactions

INO-CNR
Istituto Nazionale di Ottica

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OUTLOOK

1. Radiation Pressure Acceleration:
   concept and overview of some recent results

2. Radiation Reaction effects on RPA
   - Motivations
   - RR modeling via Landau-Lifshitz equation
   - Single particle tests and inclusion in PIC codes
   - Effects on RPA: role of laser polarization

TWO RPA-BASED VISIONS (1996 - 2010)

22 NATURE JULY 2, 1966 VOL. 211

INTERSTELLAR VEHICLE PROPELLED BY TERRESTRIAL LASER BEAM

By PROF. G. MARX

Institute of Theoretical Physics, Roland Eötvös University, Budapest

PRL 104, 135003 (2010) PHYSICAL REVIEW LETTERS week ending 2 APRIL 2010

Unlimited Ion Acceleration by Radiation Pressure

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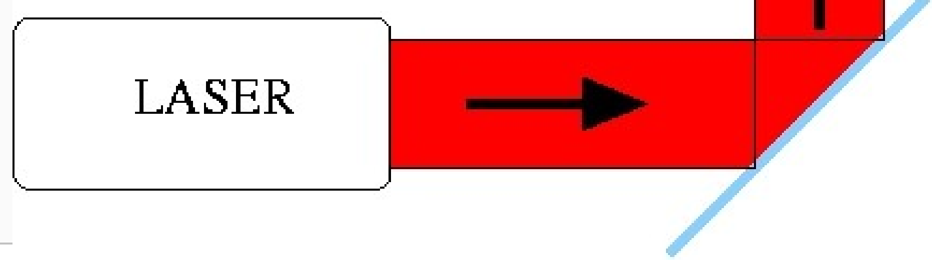
THE "LIGHT SAIL" CONCEPT

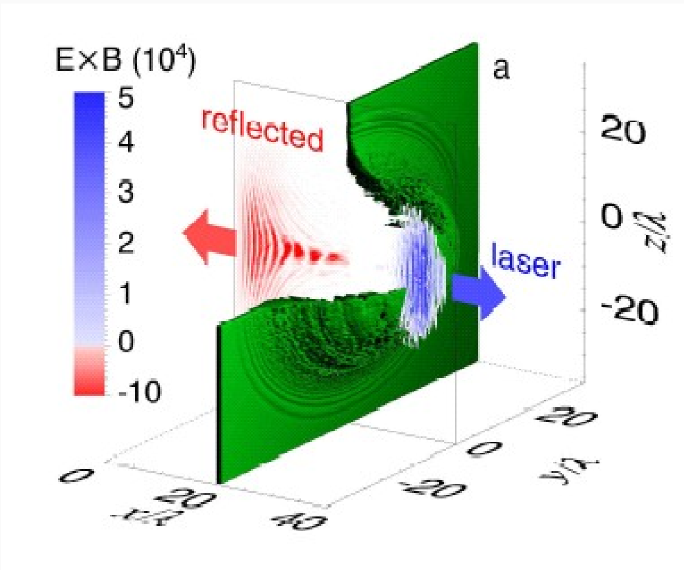
Originally proposed as a way to
accelerate a massive mirror
by the Radiation Pressure of an Earth-based laser

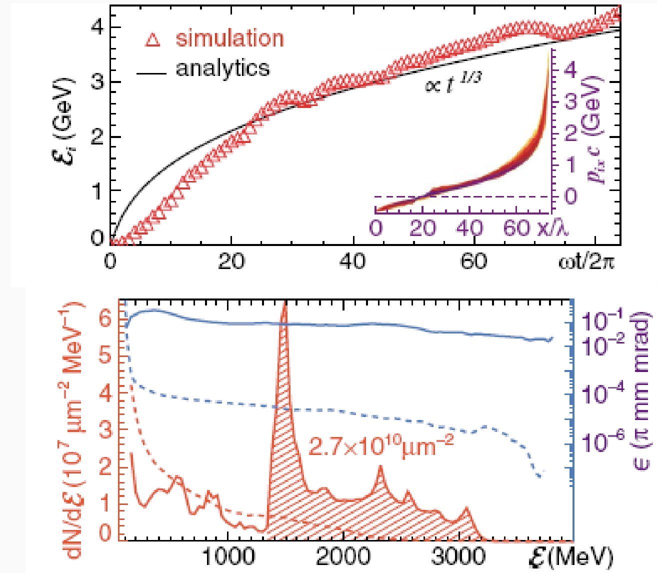
R.L.Forward, "Roundtrip interstellar travel using laser-pushed lightsails",
J. Spacecraft and Rockets **21** (1964) 187

G.Marx, "Interstellar vehicle propelled by terrestrial laser beam", Nature **211** (1966) 22



“LIGHT SAIL” AND LASER ION ACCELERATION





3D simulations suggest “Radiation Pressure Dominance”

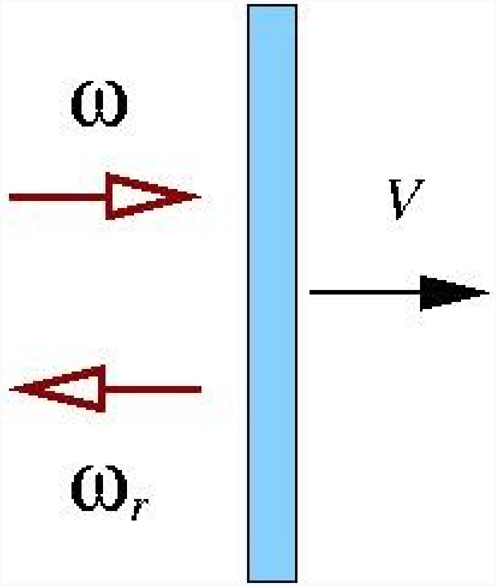
in interactions at $I \geq 10^{23}\ \mathrm{W/cm^2}$ with thin plasma foils

Modeling based on simple LS or “accelerating mirror” model

T.Esirkepov, M.Borghesi, S.V.Bulanov,G.Mourou, T.Tajima,
PRL **92**, 175003 (2004)

ACCELERATING MIRROR MODEL

perfectly reflecting, rigid mirror of
mass $M=\rho \ell S$ boosted by a plane light wave



Mirror velocity as a function of the light pulse intensity I and duration τ and of the surface density $n_e \ell$ of of the target:

$$\beta(t) = \frac{(1+\mathcal{E})^2 - 1}{(1+\mathcal{E})^2 + 1}, \qquad \mathcal{E} = \frac{2F(t)}{\rho \ell c^2} = 2\pi \frac{Z}{A}\frac{m_e}{m_p}\frac{a_0^2 \tau}{\zeta}$$

$$F(t) = \int_0^t I(t')dt' \propto a_o^2 \tau, \qquad \zeta = \pi \frac{n_e}{n_c}\frac{\ell}{\lambda}$$

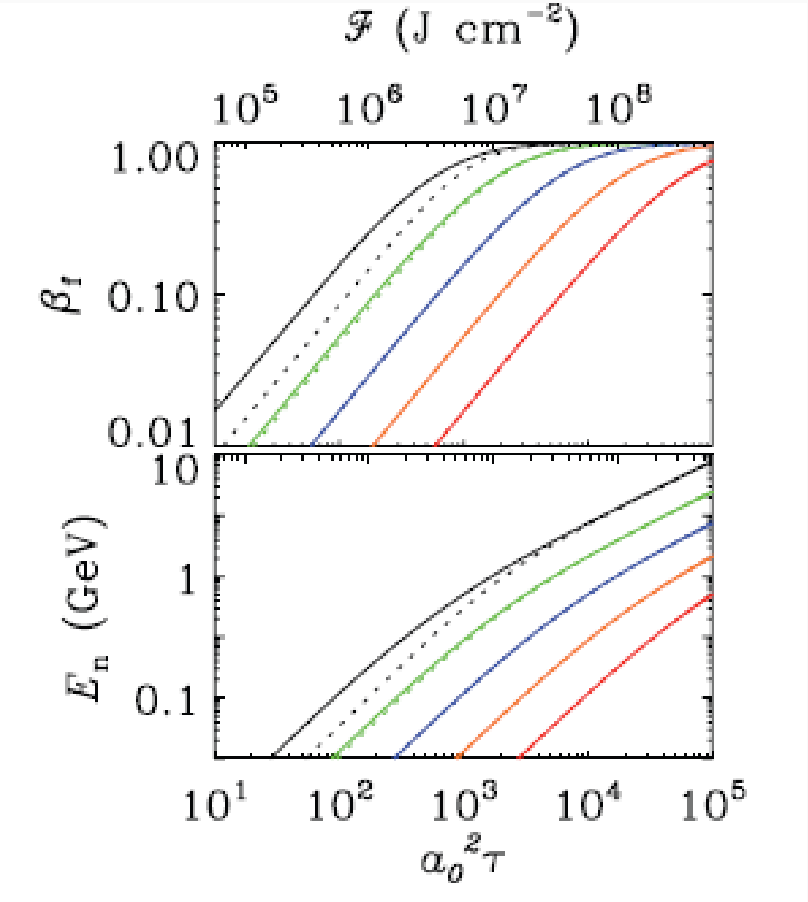
SCALING TO LASER-SOLID INTERACTIONS

Velocity and energy/nucleon for LS-RPA of a ultrathin solid target

vs. laser pulse fluence $\mathcal{F}$
for (dimensionless) surface target densities

$\zeta = 1, 3.16, 10, 31.6, 100$

Experimental requirements:
- nm foil targets (e.g. DLC)
- ultrahigh contrast (plasma mirrors)
- possibly circular polarization?



a_0 :dimensionless amplitude,

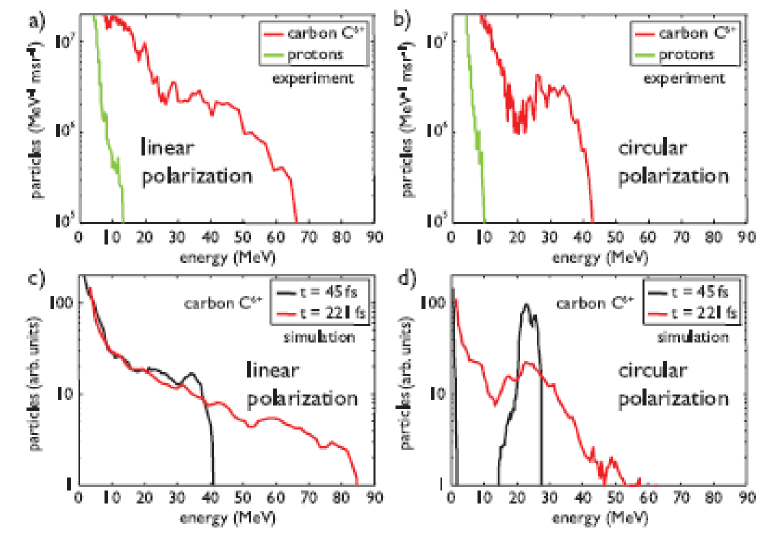
τ : duration in cycles

WHY CIRCULAR POLARIZATION?

Using CP and normal incidence fast electron generation is strongly suppressed, maximizing radiation pressure and preventing foil expansion of the foil target

Early study in "thick" targets:
Macchi et al, PRL **94** (2005) 165003

Proposal of CP-RPA of ultrathin foils for efficient and monoenergetic acceleration:
Zhang et al, PoP **14** (2007) 073101
Robinson et al, NJP **10** (2008) 013201;
Klimo et al, PRST-AB **11** (2008) 031301.



First experimental study reported:
Henig et al, PRL **103** (2009) 245003

"LIGHT SAIL" REVISITED

Recent Results on thin foil RPA with CP pulses:

- Improved formula accounting for relativistic Self-Induced Transparency effects
- Determination of "optimal" thickness
- Dynamics and self-organization underlying the LS picture
- Solution of somewhat puzzling issues concerning energy and pressure balance

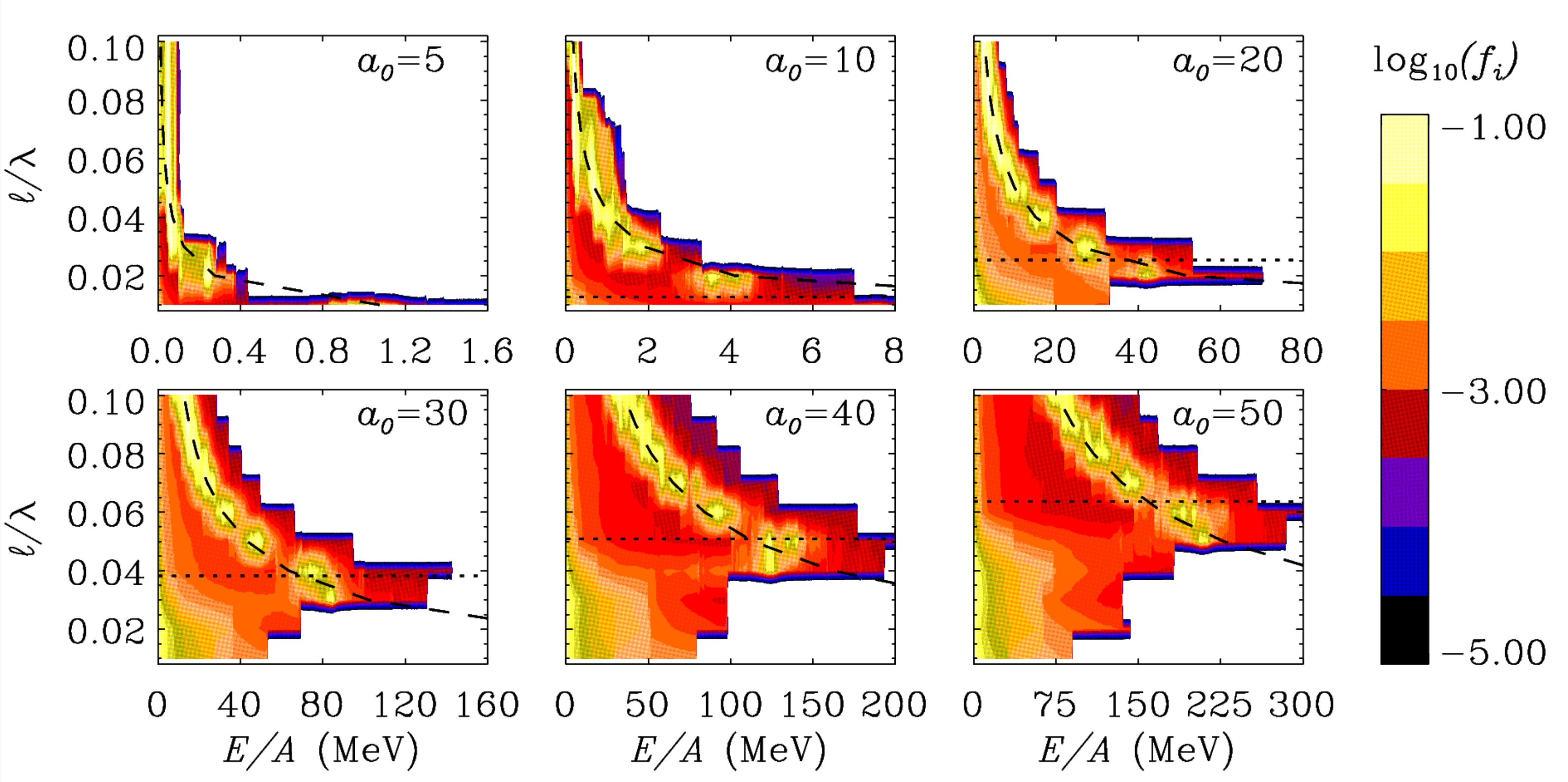
See:

Macchi et al, PRL **103** (2009) 085003;
Macchi et al, NJP **12** (2010) 045013.

LS MODEL VS 1D PIC SIMULATIONS - II

Energy spectra vs. a_0 and ℓ:

(Dashed line: LS model prediction, dotted line: $a_0=\zeta$)

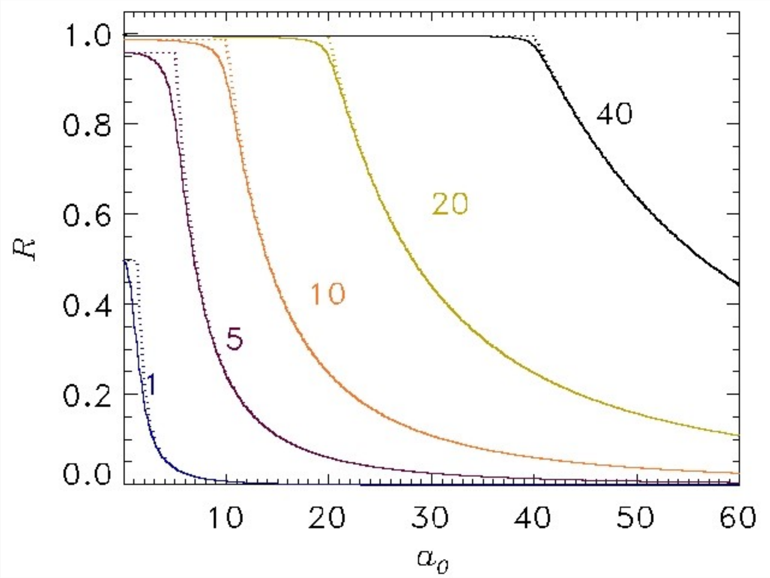


TRANSPARENCY AND "OPTIMAL" THICKNESS

Ultrathin slab model: $n_e(x)=n_0\ell\delta(x)$, foil thickness $\ell<<\lambda$

Total radiation pressure in rest frame $P_{rad}=(2I/c)R$

Nonlinear reflectivity $R=R(\zeta,a_0)$ includes Self-Induced Transparency



$R\approx\zeta^2/(\zeta^2+1)$ $(a_0<\zeta)$

$R\approx\zeta^2/a_0^2$ $(a_0>\zeta)$

P_{rad} does not depend on a_0 for $a_0>\zeta$! (since $I\sim a_0^2$)

The maximum boost of the foil is at $a_0\approx\zeta$

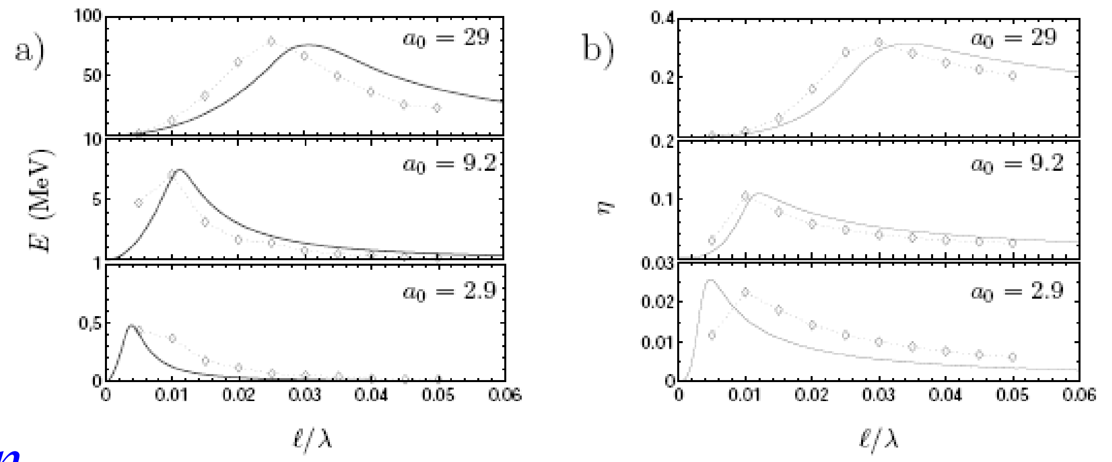
LS MODEL WITH SIT INCLUDED

Modified foil velocity formula for $R<1$, $a_0<\zeta$

$$\beta(t) = \frac{(1+\mathcal{E}-\zeta^{-2})^2 + (1+\mathcal{E}-\zeta^{-2})\sqrt{(1+\mathcal{E}-\zeta^{-2})^2+4\zeta^{-2}} + 2\zeta^{-2} - 2}{(1+\mathcal{E}-\zeta^{-2})^2 + (1+\mathcal{E}-\zeta^{-2})\sqrt{(1+\mathcal{E}-\zeta^{-2})^2+4\zeta^{-2}} + 2\zeta^{-2} + 2}$$

$$\mathcal{E} = \frac{2F(t)}{\rho \ell c^2} = 2\pi \frac{Z}{A}\frac{m_e}{m_p}\frac{a_0^2 \tau}{\zeta}$$

Ion energy and conversion efficiency vs. intensity and thickness (solid: theory, points: PIC sims.)



9 cycles pulse, $n_e = 250 n_c$

TWO-DIMENSIONAL SIMULATIONS

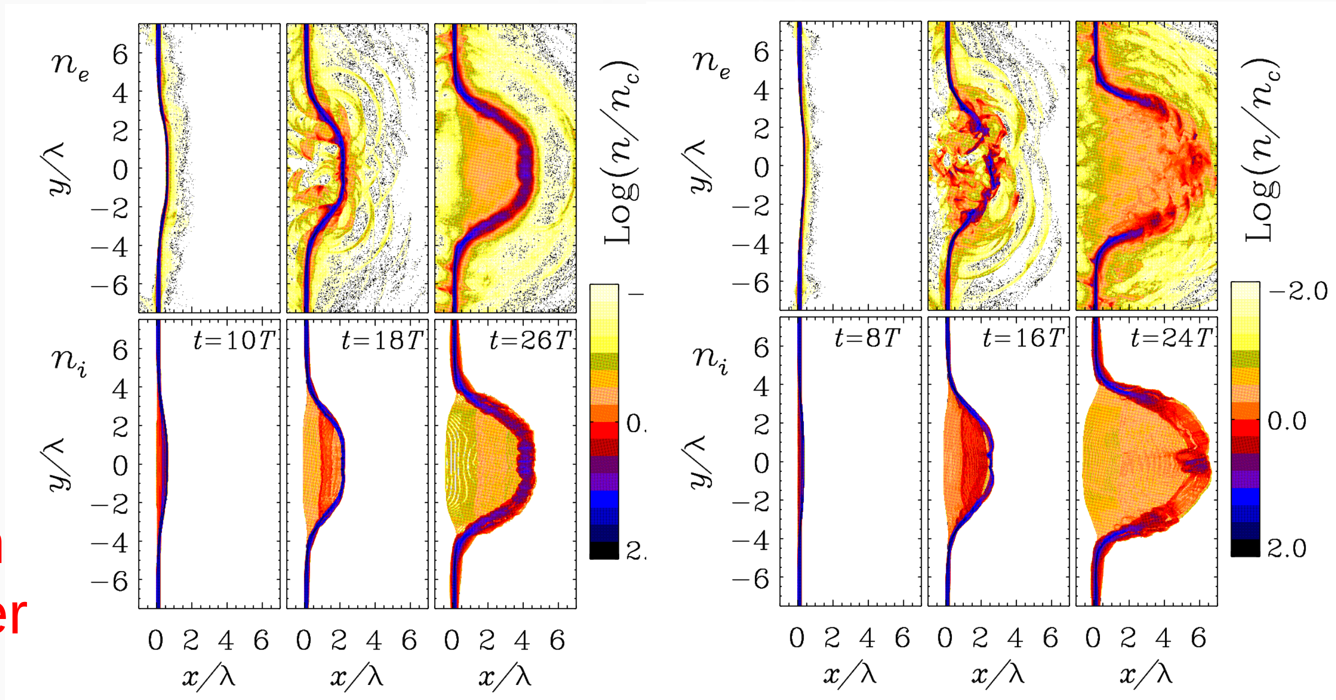
2D sims for

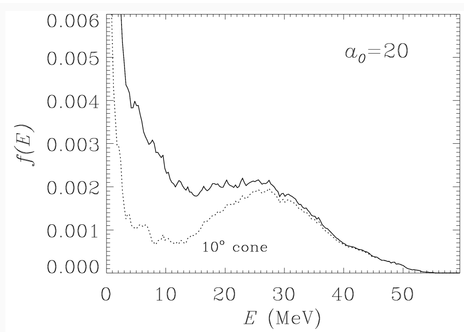
ζ=31.4 and

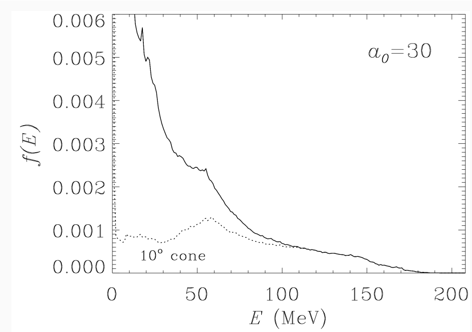
a_0 =20 (left)

a_0 =30 (right)

stronger electron heating and lower "penetration" threshold with respect to 1D: ion spectrum is broad



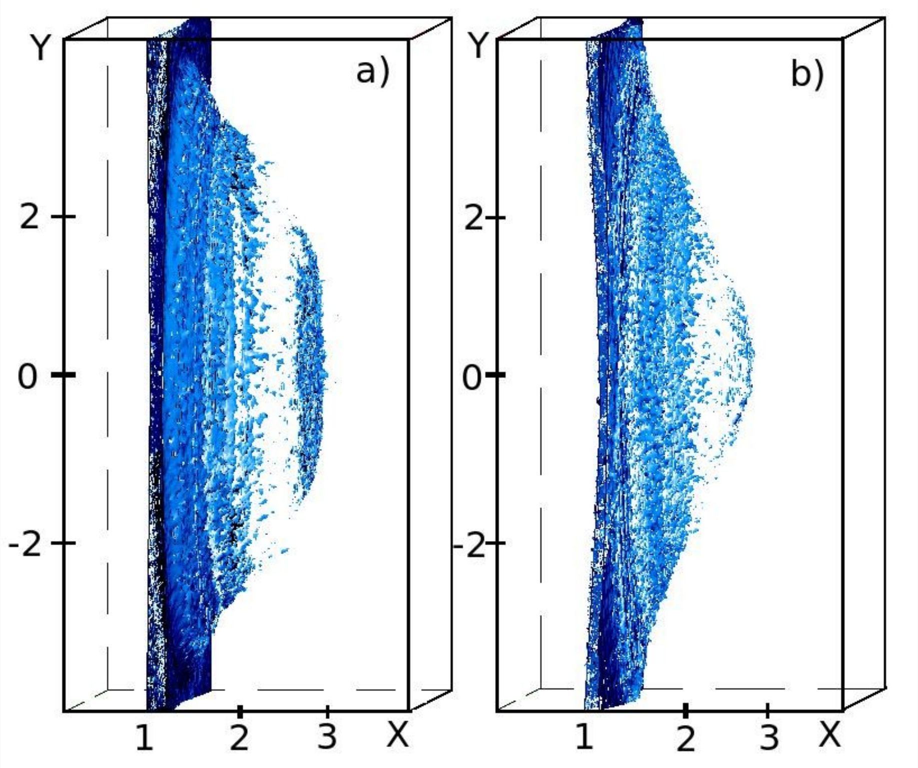




THREE-DIMENSIONAL SIMULATIONS

3D sims for $\zeta=15$,
$a_0=5$, $\tau=18$ cycles
left: Supergaussian spot profile
right: Gaussian

Note that only in 3D angular momentum conservation is taken into account



Supergaussian “flat-top” profiles keep a “quasi-1D” geometry and prevent early breakthrough of laser pulse due to lateral expansion

RADIATION REACTION EFFECTS

Motivation: Radiation Reaction is important for ultra-relativistic particles in EM fields and is thus expected to play a strong role in next generation experiments at ultra-high intensities.

The typical intensity for relevant RR effects is estimated to be $\sim 10^{23}$ W/cm^2 . This corresponds, to the foreseen regime of RPA dominance (for Linear Polarization)
[Esirkepov et al, PRL **92** (2004) 175003]

Two RPA simulation studies (for Circular Polarization at lower intensity) suggest a “beneficial” effect of “electron cooling” by RR
[Schlegel et al, PoP **16** (2009) 083103;
Chen et al, arXiv:0909.5144,
to appear in Plasma Phys. Contr. Fus.]

RADIATION REACTION FORCES

$$mc\frac{du^\mu}{d\tau} = f_L^\mu + f_{RR\,1}^\mu + f_{RR\,2}^\mu + f_S^\mu$$

$$f_L^\mu = eF^{\mu\nu}u_\nu$$

$$f_{RR\,1}^\mu = e\tau_0\left(\partial_\alpha F^{\mu\nu}u_\nu u^\alpha\right)$$

$$f_{RR\,2}^\mu = \frac{e^2}{mc}\tau_0\left(F^{\mu\nu}F_{\nu\alpha}u^\alpha + (F^{\nu\beta}u_\beta F_{\nu\alpha}u^\alpha)u^\mu\right)$$

$$f_S^\mu = -\frac{1}{2c}S^{\gamma\delta}\partial^\mu F_{\gamma\delta} + \frac{1}{2c}\left(S^{\gamma\delta}\partial_\alpha F_{\gamma\delta}u^\alpha\right)u^\mu$$

$$u^\alpha = \left(\gamma, \gamma\frac{\mathbf{v}}{c}\right) \qquad \partial^\mu \equiv \left(\frac{\partial}{\partial t}, -c\nabla\right)$$

EoM of classical particle with spin in EM field: Landau-Lifshitz formula for RR term $f^\mu{}_{RR}$

RADIATION REACTION MODELING

EoM with Landau-Lifshitz force in non-covariant notation

$$\frac{d\mathbf{p}}{dt} = -\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) - \left(\frac{4}{3}\pi\frac{r_e}{\lambda}\right)\gamma\left[\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\mathbf{E} + \mathbf{v} \times \left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\mathbf{B}\right] + \left(\frac{4}{3}\pi\frac{r_e}{\lambda}\right)\left[\left(\mathbf{E} + \mathbf{v}\times\mathbf{B}\right)\times\mathbf{B} + \left(\mathbf{v}\cdot\mathbf{E}\right)\mathbf{E}\right] - \left(\frac{4}{3}\pi\frac{r_e}{\lambda}\right)\gamma^2\left[\left(\mathbf{E} + \mathbf{v}\times\mathbf{B}\right)^2 - \left(\mathbf{v}\cdot\mathbf{E}\right)^2\right]\mathbf{v}$$

The last “friction” term is the dominant one
(the first terms is ordinarily smaller than spin contribution)

BENCHMARK WITH EXACT SOLUTIONS

Exact solution of the Landau-Lifshitz equation in a plane wave
[A.Di Piazza, Lett.Math.Phys. **83** (2008) 305]

Based on this test case we identify suitable approximations to the electron EoM with RR included:

- the spin force is $\sim 137\gamma$ X the first LL term in the RR force
- the second LL term is $\sim a_0 \omega\tau / 137$ X the spin force

→ for intensities >> 10^{22} W/cm^2 it is consistent to neglect both the 1st LL term and the spin force
[M.Tamburini, F.Pegoraro, A. Di Piazza, C.H.Keitel, A.Macchi, preprint arxiv:1008.1685]

TEST OF PARTICLE PUSHER

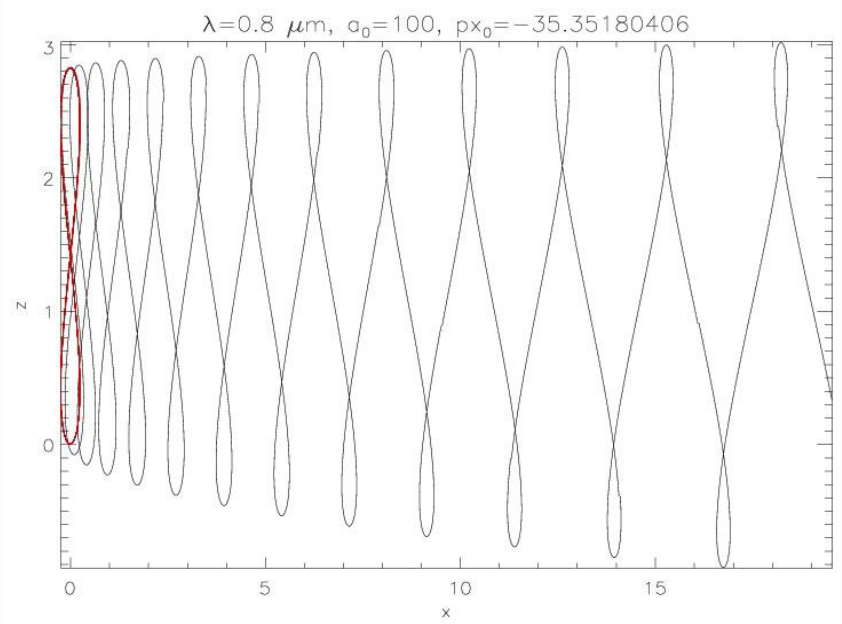
A numerical solution of motion in a plane wave based on simple 2nd order leap-frog method has been compared with the exact solution and with 4th order Runge-Kutta integration

"Figure of Eight" drifts away when RR is included

Black – with RR

Red → without RR

- excellent agreement for intensities up to 10^{24} W/cm^2
- straightforward to include in a "standard" PIC code (based on Boris particle pusher)
- only ~10% increase in CPU time



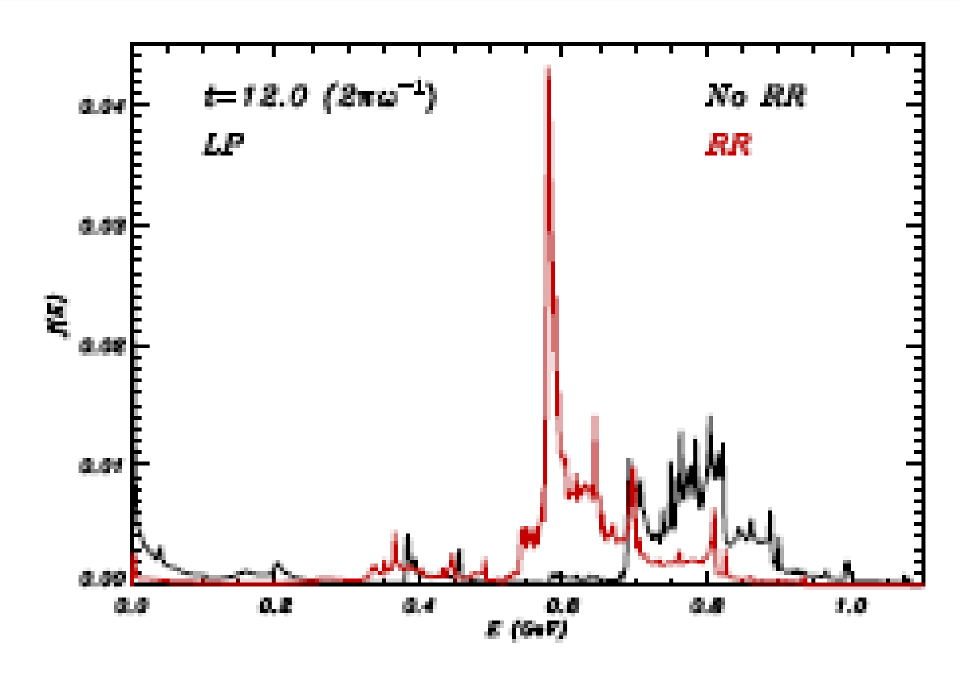
RR EFFECTS ON ION SPECTRA – I (LP)

RP-dominated regime: 2.3 x 10^{23} W/cm^2 , 11 cycles pulse

1 um foil, 100n_c , linear polarization

Lower energy, narrower spectrum with RR included

~25% reduction in "peak" ion energy " due to RR effects



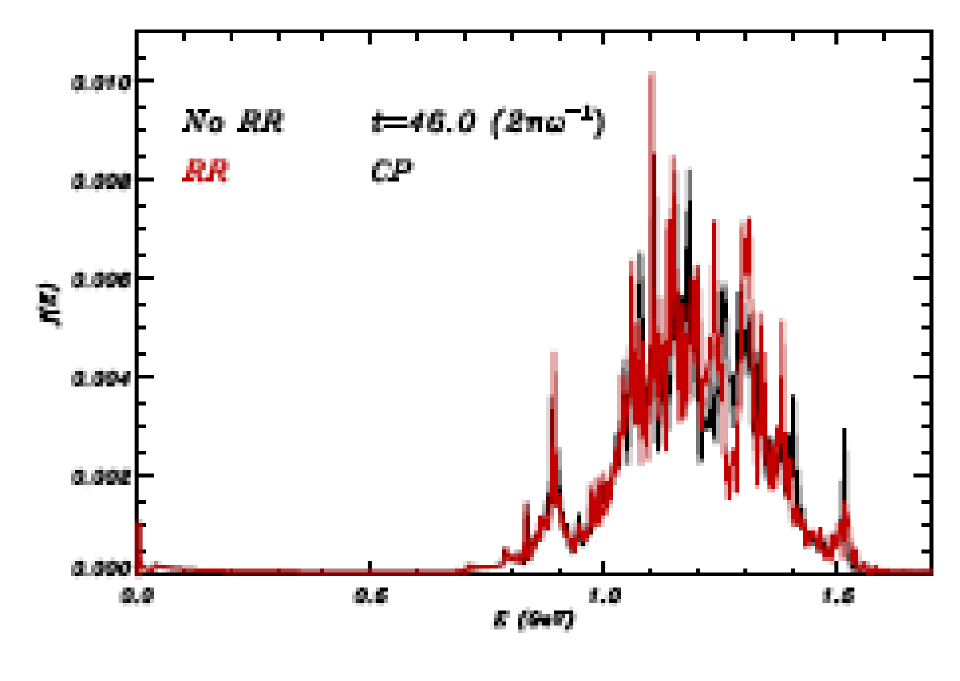
RR EFFECTS ON ION SPECTRA – II (CP)

RP-dominated regime: 2.3×10^{23} W/cm^2 , 11 cycles pulse

1 um foil, $100n_c$, circular polarization

Negligible RR effects on ion spectrum!

Higher energy than in LP case



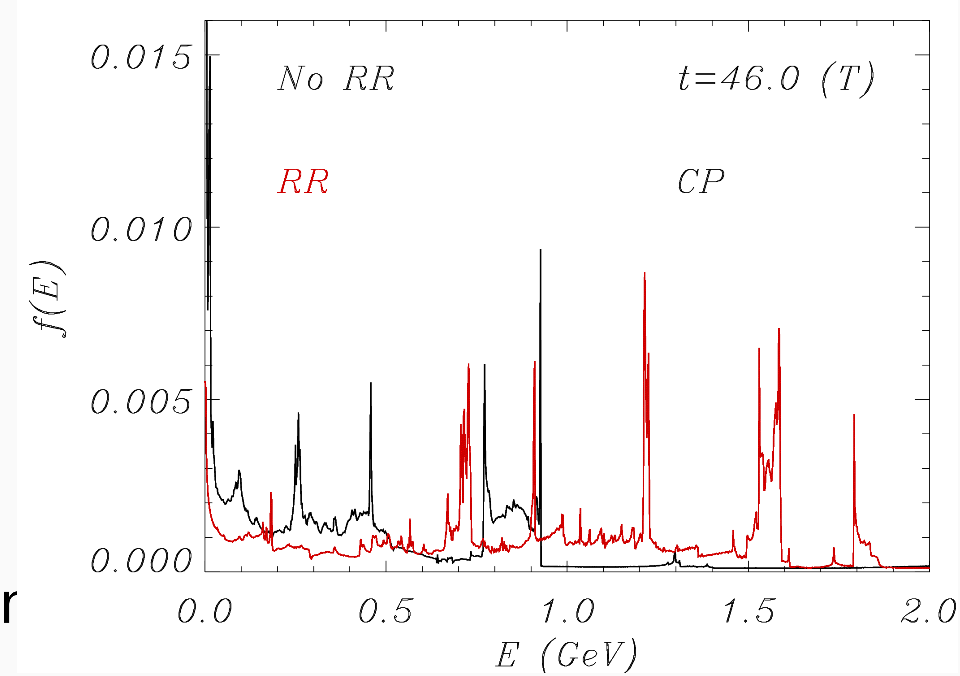
RR EFFECTS ON ION SPECTRA – III (CP)

RP-dominated regime: 2.3×10^{23} W/cm^2 , 11 cycles pulse

0.3 um foil, $100n_c$, circular polarization

The pulse penetrates through the foil due to "relativistic" Self-Induced Transparency

RR effects are now important for CP and *increase* the ion energy, but the regime is *not* optimal for ion acceleration

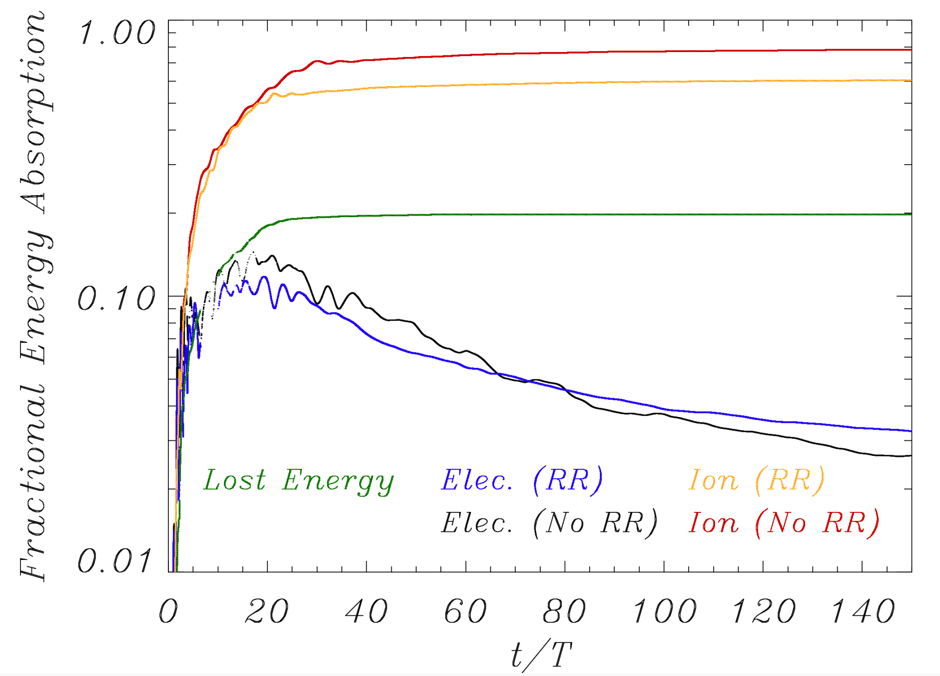


ENERGY BALANCE (LP)

RP-dominated regime: 2.3 x 10^{23} W/cm^2 , 11 cycles pulse

1 um foil, 100n_c , linear polarization

~20% energy "dissipated" by RR as incoherent, high frequency radiation escaping from the plasma



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SUMMARY AND CONCLUSIONS

- Superintense Radiation Pressure Acceleration is appealing
- The simple “Light Sail” model provides promising scalings and is in agreement with 1D simulation predictions (although the dynamics is much more complex than suggested by the model)
- Circular Polarization affects RPA even at extreme intensities (Radiation Pressure dominance)
- Radiation Reaction (or Friction) effects have been included in a PIC code via the Landau-Lifshitz equation
- RR effects on RPA at ultrahigh intensities are important only for Linear Polarization or in the Self-Induced Transparency regime

ACKNOWLEDGMENTS

Thanks to Sergey Propuzhenko for intriguing discussions on RR modeling, angular momentum absorption, and so on

Use of Supercomputing facilities at CINECA (Bologna, Italy) is greatly acknowledged

WANT TO SEE MORE?

EXTRA SLIDES

THEORETICAL INTEREST IN CP-RPA...

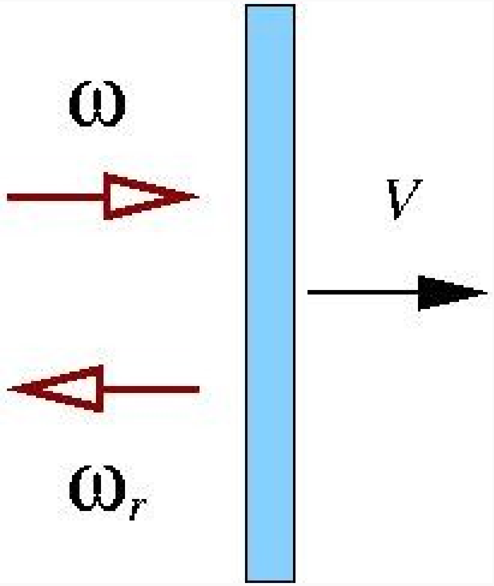
Thick (semi-infinite) targets
("Hole Boring"):
Liseikina & Macchi, APL **94** (2007) 165003;
Naumova et al, PRL **102** (2009) 025002;
Schlegel et al, PoP **16** (2009) 083103;
Robinson et al, PPCF **51** (2009) 024004 & 095006;
Macchi & Benedetti, NIM A **620** (2010) 41
Tikhonchuk et al, Nucl. Fus. **50** (2010) 045003

Ultrathin (sub-wavelength) targets
("Light sail"):
Yan et al, PRL **100**, (2008) 135003 ;
Qiao et al, PRL **102** (2009) 145002;
Tripathi et al, PPCF **51** (2009) 024014;
Eliasson et al. NJP **11** (2009) 073006;
Yan et al, PRL **103** (2009) 135001;
Macchi et al, PRL **103** (2009) 085003;
Macchi et al, NJP **12** (2010) 045013.

Variations on the theme
(side effects,multi-species or
structured targets, ...):
Liseikina et al, PPCF **50** (2008) 124033;
Rykovanov et al., NJP **10**, (2008) 113005;
Ji et al, PRL **101** (2008) 164802;
Yin et al, PoP **15** (2008) 093106;
Holkundkara & Gupta, PoP **15** (2008) 123104;
Chen et al, PoP **15** (2008) 113103;
Zhang et al, PRST-AB **12** (2009) 021301;
Gonoskov et al, PRL **102** (2009) 145002;
Chen et al, PRL **103** (2009) 024801
Grech et al, NJP **11** (2009) 093035
Yu et al, PRL **105** (2010) 065002

Results presented in this talk

MECHANICAL EFFICIENCY



The efficiency η of the acceleration process can be obtained by a simple argument of conservation of "number of photons" plus the Doppler shift of the reflected light:

$$N = \frac{IS\tau}{\hbar\omega}, \qquad \omega_r = \omega\frac{1-\beta}{1+\beta}$$

$$\eta = \frac{\mathcal{E}_{\rm abs}}{\mathcal{E}_{\rm laser}} = \frac{N\hbar(\omega-\omega_r)}{IS\tau} = \frac{2\beta}{1+\beta}$$

$$\beta \to 1 \Rightarrow \eta \to 1$$

100% efficiency in the relativistic limit!

LS MODEL VS 1D PIC SIMULATIONS - I

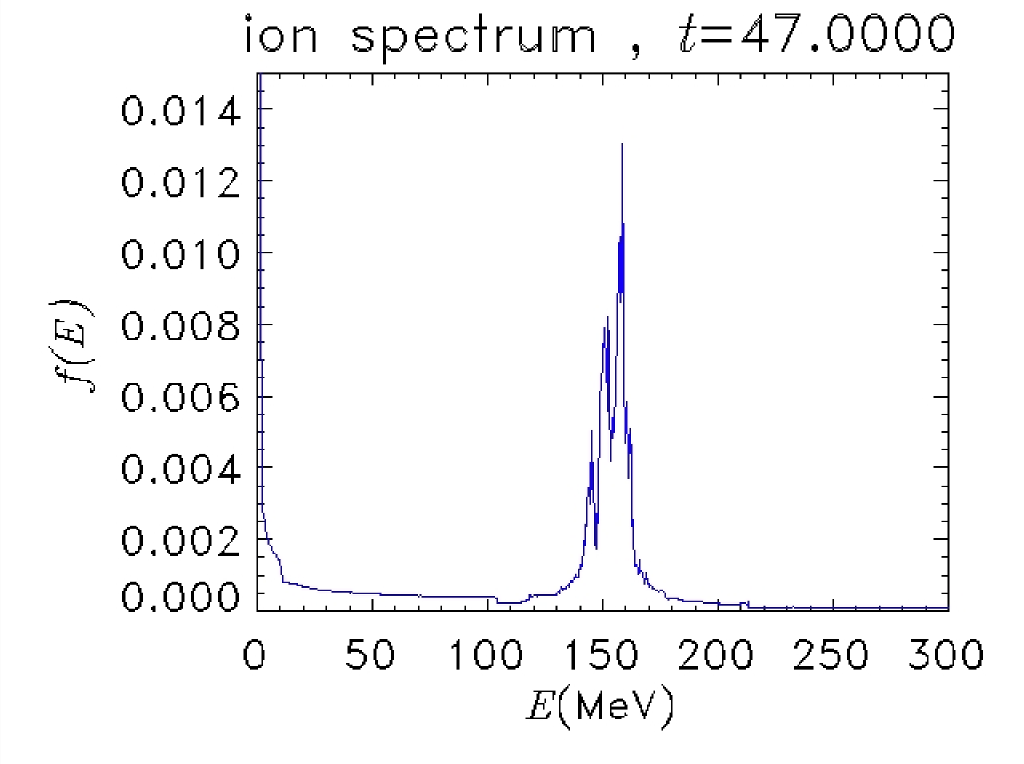
Laser pulse: a_0=**5-50**, τ=8 cycles ("flat-top" envelope)

Thin foil target: $n_e=250n_c$, ℓ=**0.01-0.1**λ (ζ=**7.8-78.5**)

A narrow spectral peak is observed for $a_0<\zeta$.

The energy of the peak is in good agreement with the LS formula

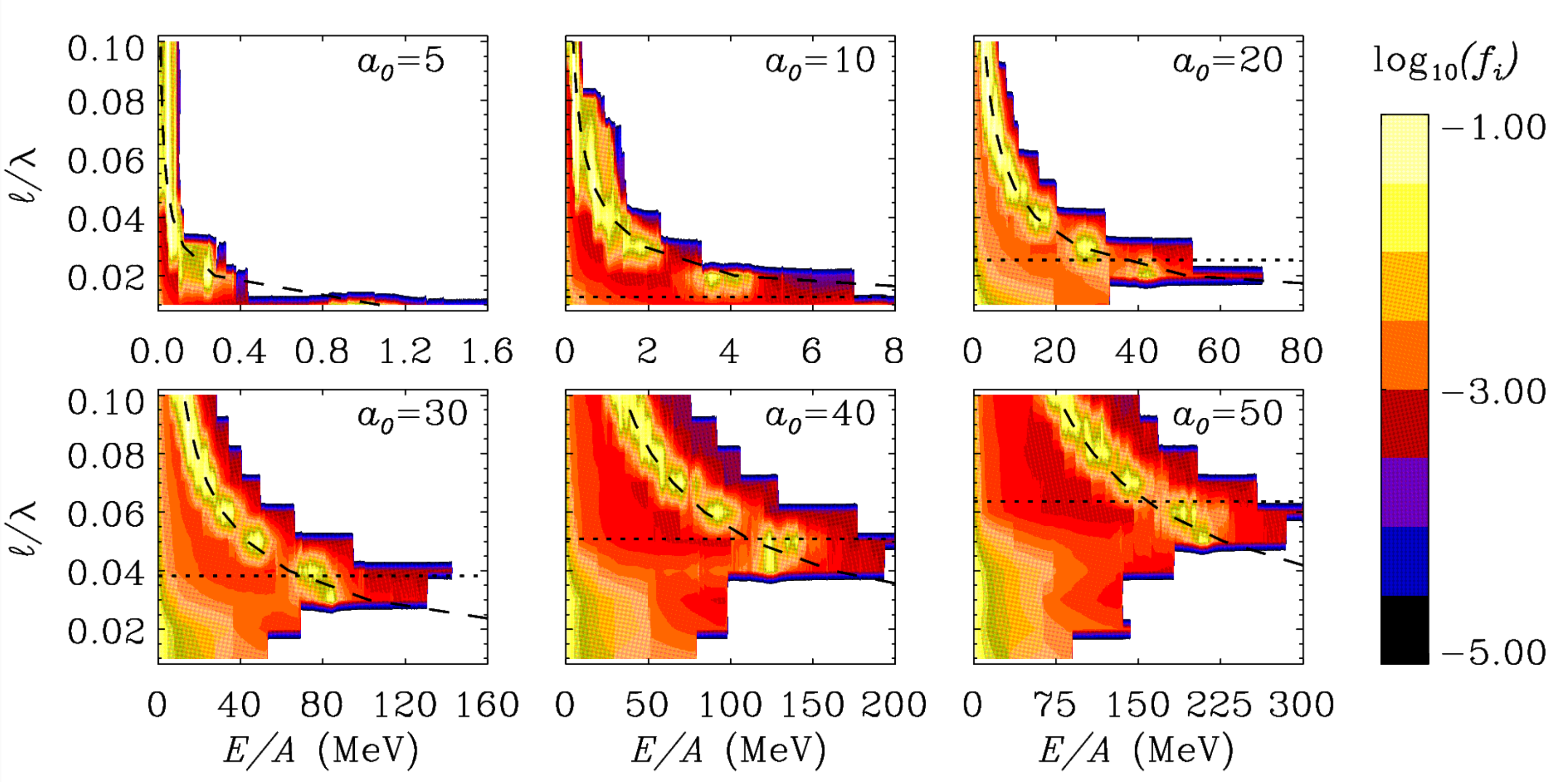
For $a_0>\zeta$, the dynamics is dominated by a Coulomb explosion of the foil following a complete blow-out of electrons



LS MODEL VS 1D PIC SIMULATIONS - II

Energy spectra vs. a_0 and ℓ:

(Dashed line: LS model prediction, dotted line: $a_0=\zeta$)

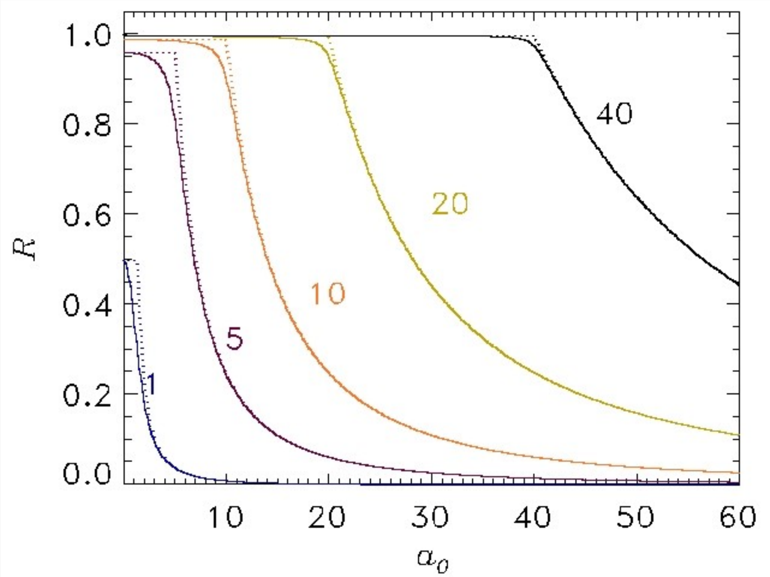


TRANSPARENCY AND “OPTIMAL” THICKNESS

Ultrathin slab model: $n_e(x)=n_0\ell\delta(x)$, foil thickness $\ell<<\lambda$

Total radiation pressure in rest frame $P_{rad}=(2I/c)R$

Nonlinear reflectivity $R=R(\zeta,a_0)$ includes Self-Induced Transparency



$R\approx\zeta^2/(\zeta^2+1)$ $(a_0<\zeta)$

$R\approx\zeta^2/a_0^2$ $(a_0>\zeta)$

P_{rad} does not depend on a_0 for $a_0>\zeta$! (since $I\sim a_0^2$)

The maximum boost of the foil is at $a_0\approx\zeta$

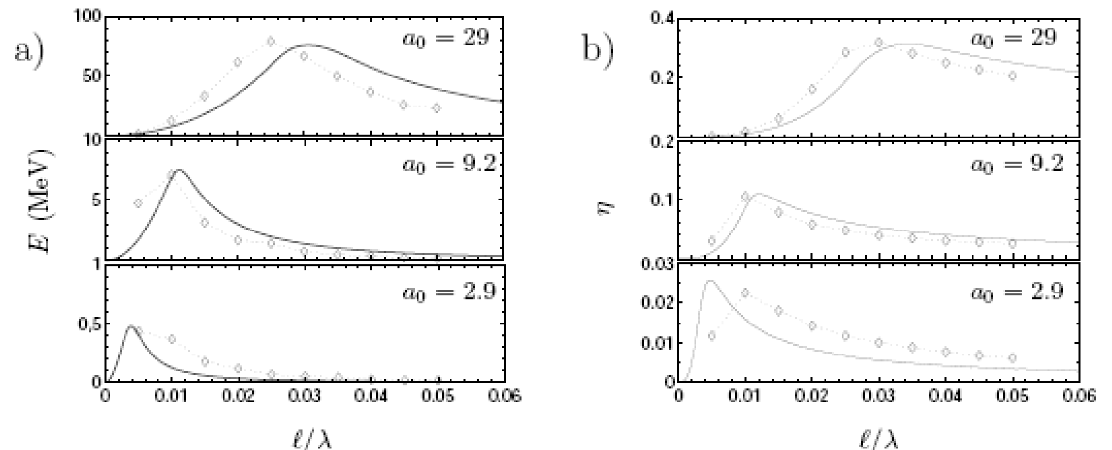
LS MODEL WITH SIT INCLUDED

Modified foil velocity formula for $R<1,\ a_0<\zeta$

$$\beta(t) = \frac{(1+\mathcal{E}-\zeta^{-2})^2 + (1+\mathcal{E}-\zeta^{-2})\sqrt{(1+\mathcal{E}-\zeta^{-2})^2+4\zeta^{-2}} + 2\zeta^{-2} - 2}{(1+\mathcal{E}-\zeta^{-2})^2 + (1+\mathcal{E}-\zeta^{-2})\sqrt{(1+\mathcal{E}-\zeta^{-2})^2+4\zeta^{-2}} + 2\zeta^{-2} + 2}$$

$$\mathcal{E} = \frac{2F(t)}{\rho\ell c^2} = 2\pi\frac{Z}{A}\frac{m_e}{m_p}\frac{a_0^2\tau}{\zeta}$$

Ion energy and conversion efficiency vs. intensity and thickness (solid: theory, points: PIC sims.)



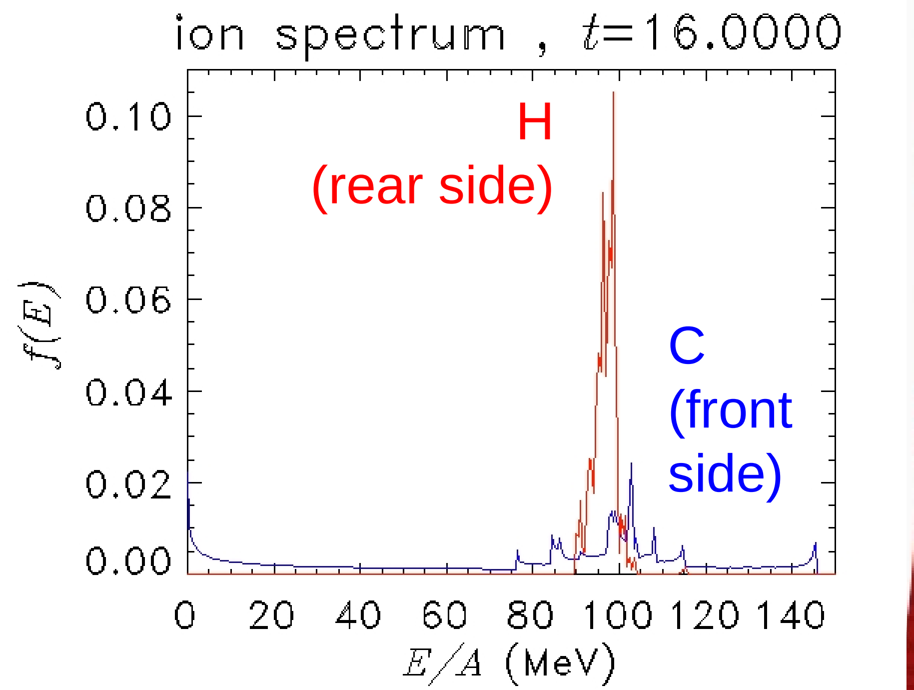
9 cycles pulse, $n_e=250n_c$

A PUZZLING ISSUE: "DARK" MASS

The RPA peak contains only ~30% of all the ions (and ~64% of their energy)

Only the rear side of the foil is accelerated (thus LS RPA may work for double-layer targets!)

→ *Why there is very good Agreement of the energy with the LS formula when inserting there the whole mass of the target (and not ~30% of it)?*



RADIATION VS ELECTROSTATIC PRESSURE

Radiation pressure drives electron depletion and generates back-holding electrostatic pressure

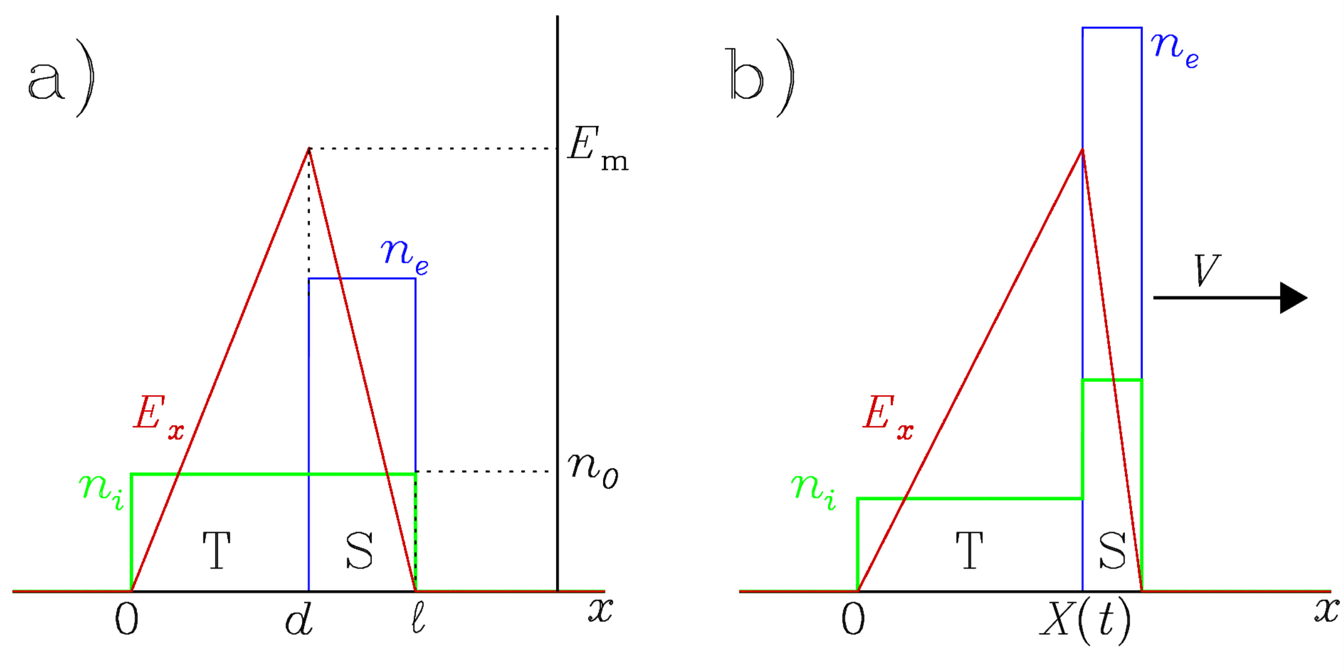
$$P_{\mathrm{rad}} = (2I/c)R \leq P_{\mathrm{es}} = 2\pi(en_0\ell)^2 \quad \text{for } a_0 \leq \zeta$$

If $a_0 < \zeta$ and $\zeta >> 1$, $R \approx 1$ and no electrons are pushed away

For $a_0 \rightarrow \zeta$ all electrons must pile up near the rear surface in order that $P_{\mathrm{rad}} \simeq P_{\mathrm{es}}$.

→ the electron pile-up layer is much thinner than the foil
→ only a fraction of the foil is accelerated

TWO ION POPULATIONS: TAIL & SAIL



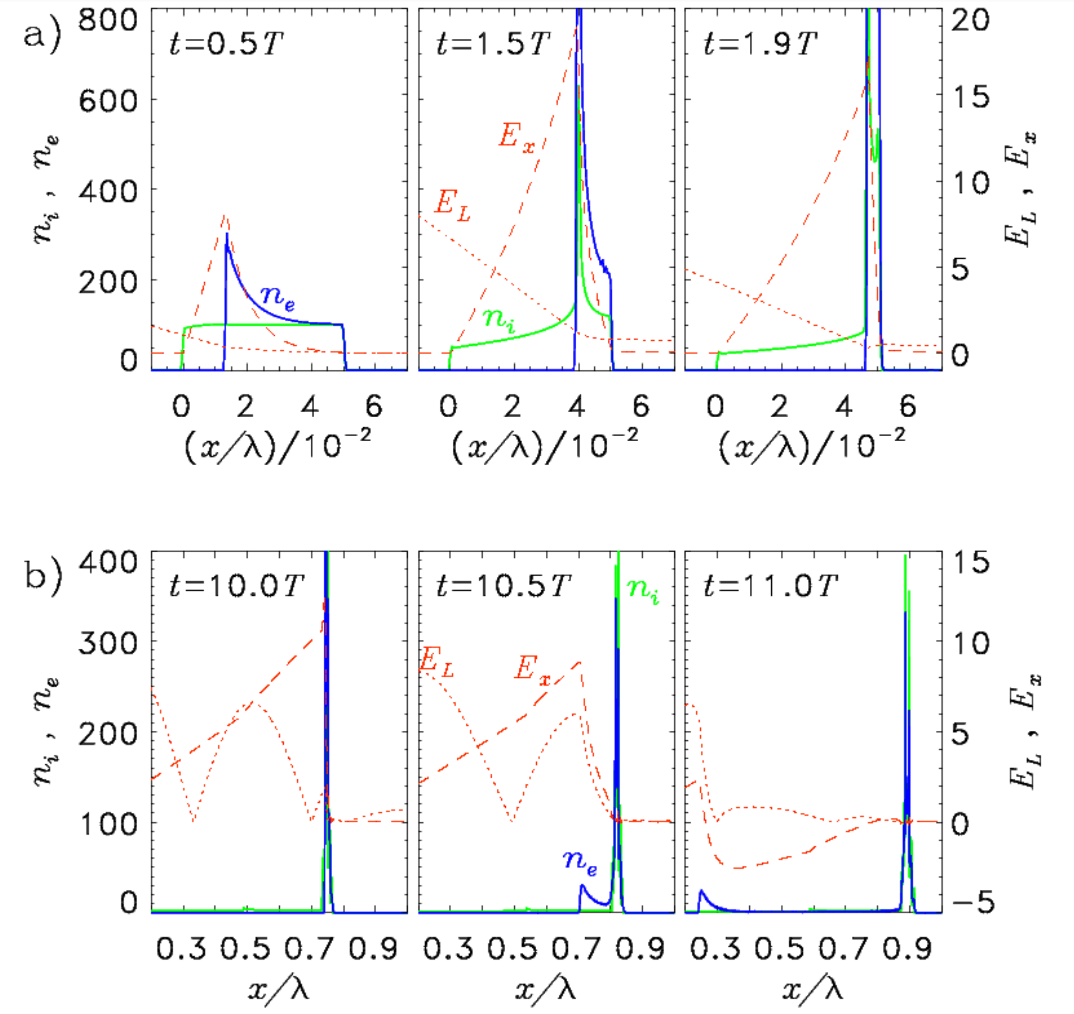
Sail (S): ions are bunched accelerated by $E_x = f_p / e$ and move coherently as a "foil" : monoenergetic component

Tail (T): ions are accelerated by their own space-charge field and "Coulomb explode": broad spectrum component

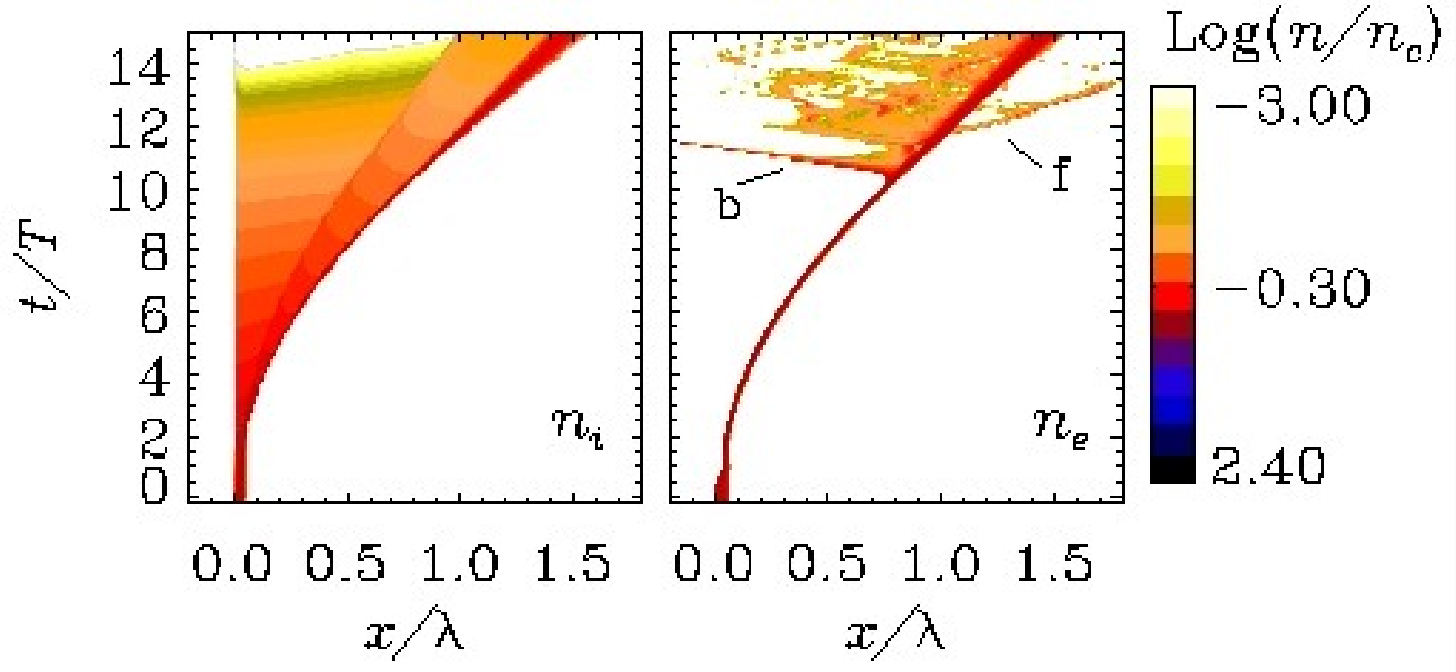
SAIL CHARGING/DISCHARGING

PIC simulations show ions in the compression layer to form a “Sail” thinner than the original foil and negatively charged (excess of electrons)

The excess electrons “detach” from the Sail near the end of the laser pulse, moving backwards and leaving a neutral plasma bunch



COLLAPSE OF ELECTRON EQUILIBRIUM



Near the end of the acceleration stage the pressure balance for electrons breaks down: formation of bunches in forward & backward directions and heating

Ion spectrum broadens in the post-acceleration stage

MISSING INERTIA IS LOWER PRESSURE

Equilibrium of radiation and electrostatic pressure on *electrons*:

$$P_{\rm rad} \doteq \int (-e) n_e E_x dx = \int n_e f_p dx$$

Electrostatic pressure on *ions*:

$$P_{\rm es} = \int Z e n_i E_x dx < P_{\rm rad} \qquad (Z n_i < n_e)$$

Calculation on equilibrium profiles yields:

$$P_{\rm es} = \frac{M_{\rm Sail}}{M_{\rm Foil}} P_{\rm rad}$$

Equation of motion:

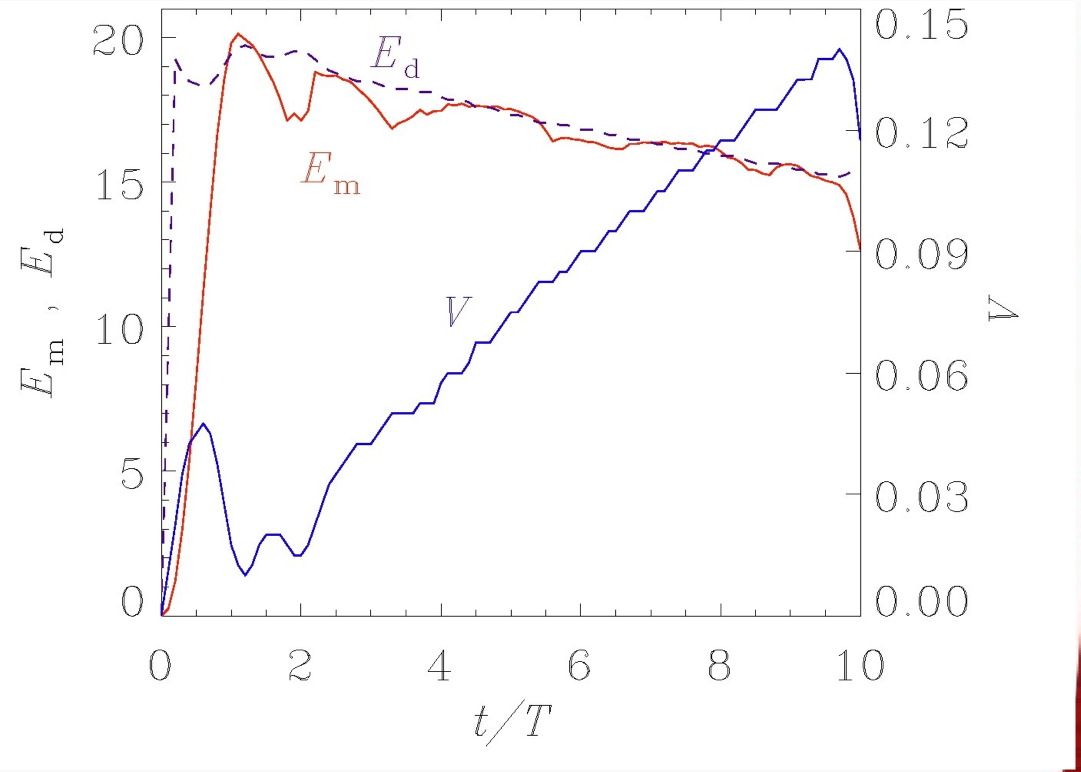
$$P_{\rm es} = \frac{d}{dt}(M_{\rm Sail}\mathbf{V}) \Longleftrightarrow P_{\rm rad} = \frac{d}{dt}(M_{\rm Foil}\mathbf{V})$$

$\rightarrow$The Sail moves as if it had the total mass of the foil

DYNAMIC PRESSURE BALANCE

P_{rad} decreases with velocity in the Lab frame
$(P_{rad})^L=(1-\beta)/(1+\beta)P_{rad}$

To keep pressure equilibrium there is a mass flow (ion current) from M_{tail} to M_{sail}



ENERGY BALANCE

Efficiency depends only on β (the Sail velocity)
BUT the kinetic energy of the Sail is less than the total!

Energy stored in the electrostatic field E_x :

$$U_{es} = U_{es}(t) = \int_0^{X(t)} \frac{E_x^2(x,t)}{8\pi} dx$$

$$\frac{dU_{es}}{dt} = \frac{1}{8\pi} E_x^2[X(t),t] \frac{dX}{dt} = \frac{1}{8\pi} E_0^2 \beta c$$

"Conversion efficiency" into electrostatic energy η_{es} :

$$\eta_{es} = \frac{1}{I}\frac{dU_{es}}{dt} = 2\beta \left(\frac{d}{\ell}\right)^2 \left(\frac{\zeta}{a_0}\right)^2$$

For $a_0 = \zeta$, the depletion width $d \approx \ell$ thus $\eta_{es} \approx 2\beta$:

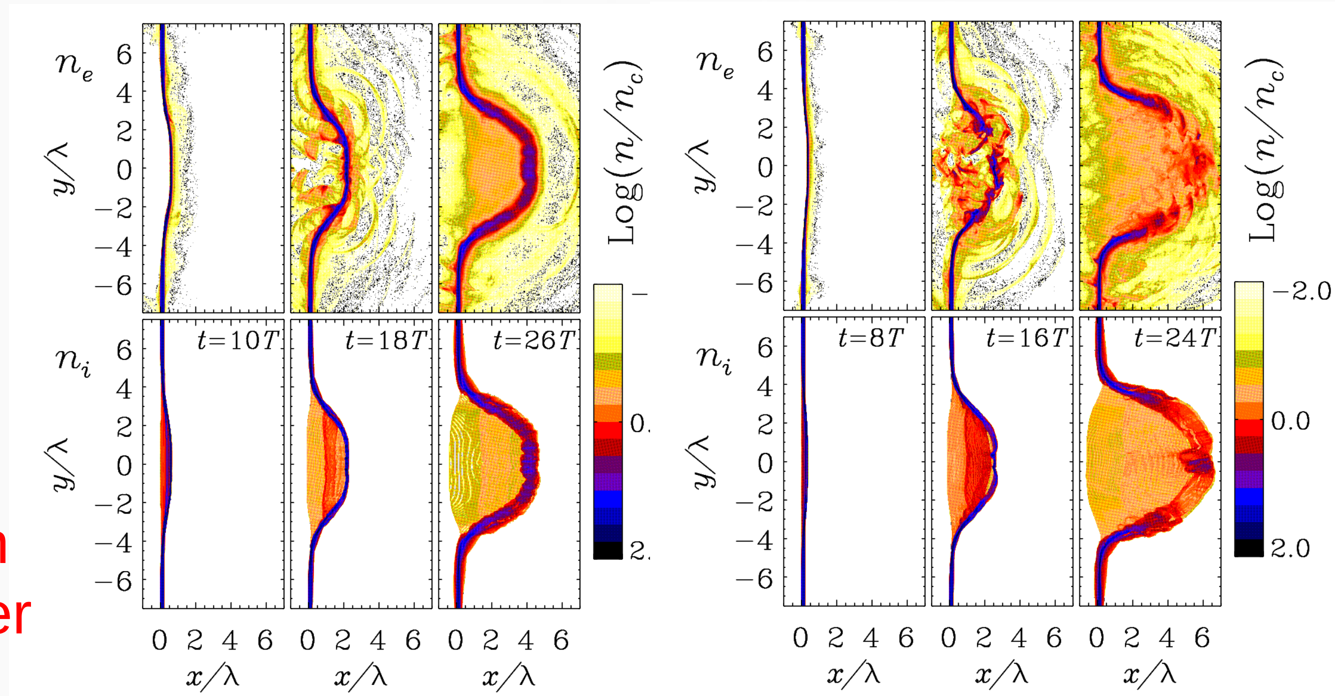
most of the stored energy is converted into electrostatic energy and eventually goes to Tail ions

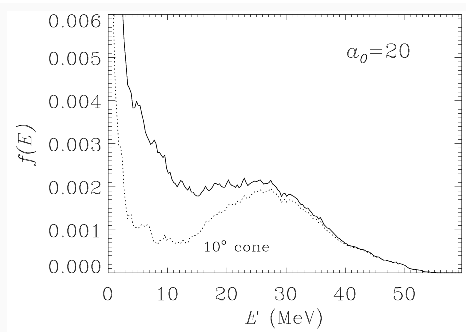
TWO-DIMENSIONAL SIMULATIONS

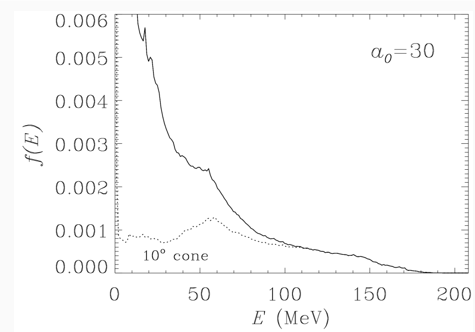
INO-CNR
ISTITUTO NAZIONALE DI OTTICA

2D sims for
ζ=31.4 and
a_0 =20 (left)
a_0 =30 (right)

stronger electron heating and lower "penetration" threshold with respect to 1D: ion spectrum is broad







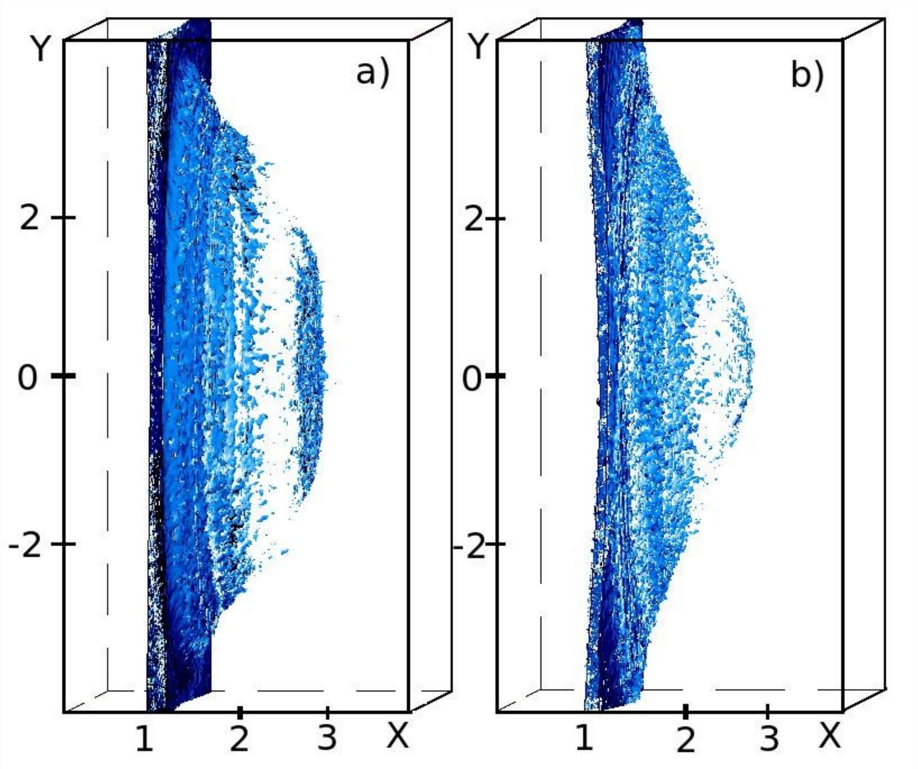
THREE-DIMENSIONAL SIMULATIONS

3D sims for $\zeta=15$, $a_0=5$, $\tau=18$ cycles

left: Supergaussian spot profile

right: Gaussian

Note that only in 3D angular momentum conservation is taken into account



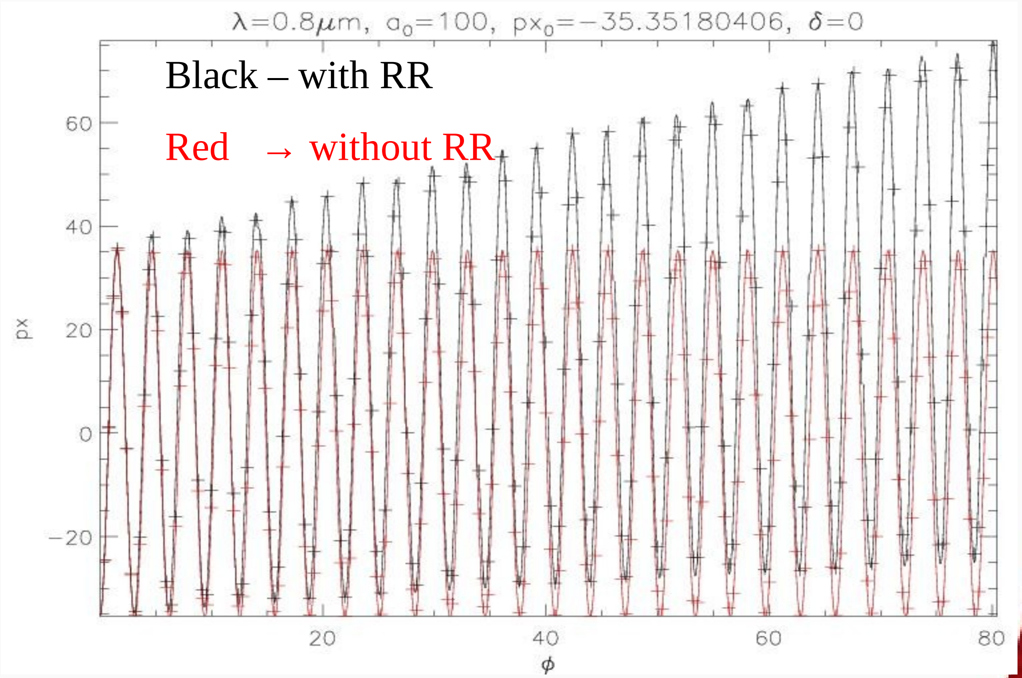
Supergaussian “flat-top” profiles keep a “quasi-1D” geometry and prevent early breakthrough of laser pulse due to lateral expansion

TEST OF PARTICLE PUSHER - II

A numerical solution of motion in a plane wave based on simple 2nd order leap-frog method has been compared with the exact solution and with 4th order Runge-Kutta integration

Crosses: analytical
Line: numerical

- excellent agreement for intensities up to 10^{24} W/cm^2
- straightforward to include in a "standard" PIC code (based on Boris particle pusher)
- only ~10% increase in CPU time



TEST OF PARTICLE PUSHER - III

A numerical solution of motion in a plane wave based on simple 2nd order leap-frog method has been compared with the exact solution and with 4th order Runge-Kutta integration

Crosses: analytical
Line: numerical

- excellent agreement for intensities up to 10^{24} W/cm^2
- straightforward to include in a “standard” PIC code (based on Boris particle pusher)
- only ~10% increase in CPU time

Black – Leap Frog

Red → Runge-Kutta