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# Radiation Pressure Acceleration in the “Light Sail” regime

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# OUTLOOK

## 1. Light Sail Revisited

- The basic LS concept
- LS improved: Self-Induced Transparency effects
- The “dark mass” puzzle
- Electron and ion dynamics and self-organization

## 2. Radiation Reaction effects

- RR inclusion in PIC via Landau-Lifshitz equation

## THE “LIGHT SAIL” CONCEPT

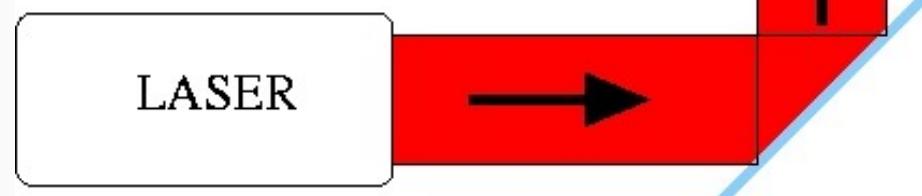
Originally proposed as a way to  
**accelerate a massive mirror**  
by the Radiation Pressure of an Earth-based laser

to  $\alpha$ -Centauri



R.L.Forward, “Roundtrip interstellar travel using  
laser-pushed lightsails”,  
J. Spacecraft and Rockets **21** (1964) 187

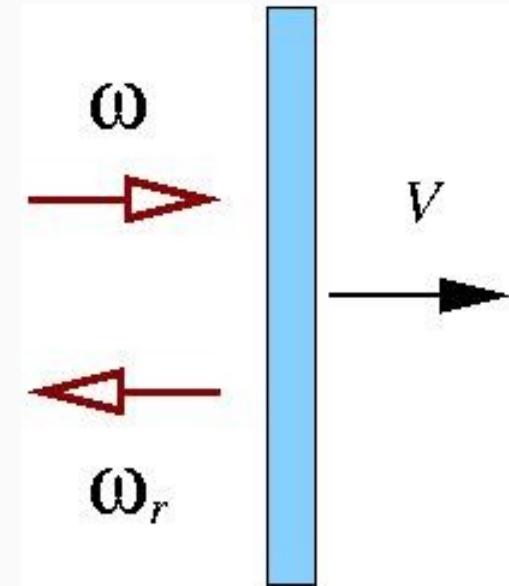
G.Marx, “Interstellar vehicle propelled by terrestrial  
laser beam”, Nature **211** (1966) 22



## ACCELERATING MIRROR MODEL

perfectly reflecting, **rigid mirror** of mass  $M = \rho \ell S$  boosted by a plane light wave

Mirror velocity as a function of the light pulse intensity  $I$  and duration  $\tau$  and of the surface density  $n_e \ell$  of the target:



$$\beta(t) = \frac{(1 + \mathcal{E})^2 - 1}{(1 + \mathcal{E})^2 + 1}, \quad \mathcal{E} = \frac{2F(t)}{\rho \ell c^2} = 2\pi \frac{Z m_e a_0^2 \tau}{A m_p \zeta}$$

$$F(t) = \int_0^t I(t') dt' \propto a_o^2 \tau, \quad \zeta = \pi \frac{n_e \ell}{n_c \lambda}$$

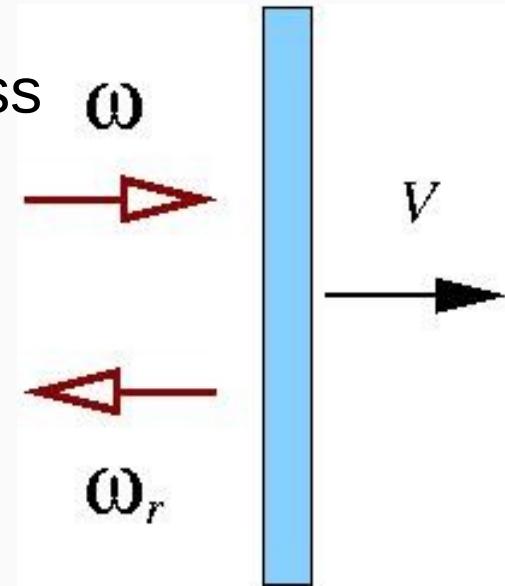
## MECHANICAL EFFICIENCY

The efficiency  $\eta$  of the acceleration process can be obtained by a simple argument of conservation of “number of photons” plus the **Doppler shift** of the reflected light:

$$N = \frac{IS\tau}{\hbar\omega}, \quad \omega_r = \omega \frac{1 - \beta}{1 + \beta}$$

$$\eta = \frac{\mathcal{E}_{\text{abs}}}{\mathcal{E}_{\text{laser}}} = \frac{N\hbar(\omega - \omega_r)}{IS\tau} = \frac{2\beta}{1 + \beta}$$

$$\beta \rightarrow 1 \Rightarrow \eta \rightarrow 1$$



**100% efficiency** in the relativistic limit!

# SCALING TO LASER-SOLID INTERACTIONS

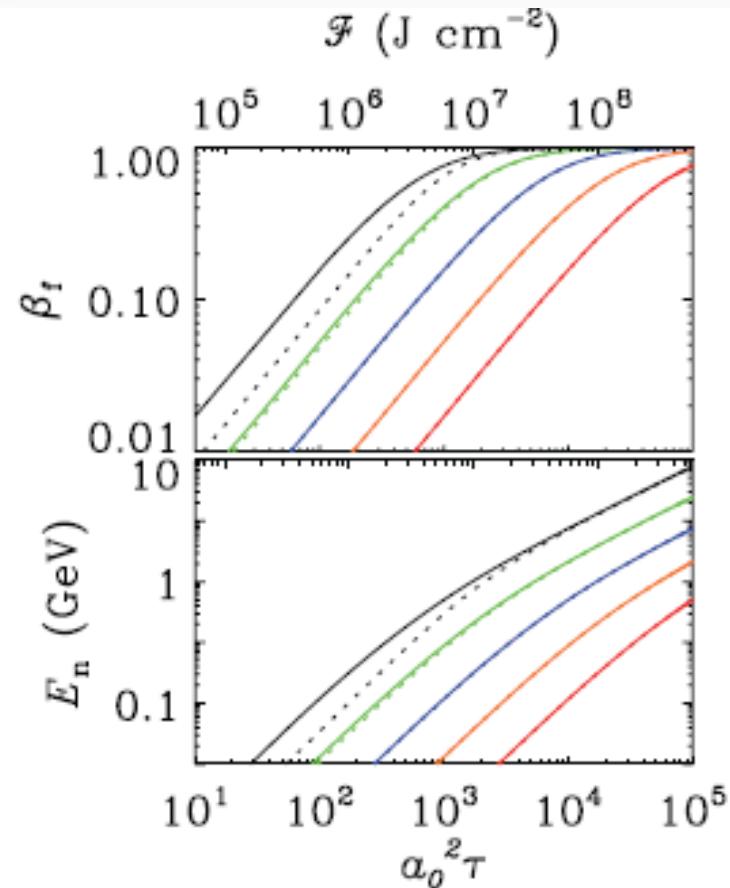
Velocity and energy/nucleon for LS-RPA of a ultrathin solid target

vs. laser pulse fluence  $\mathcal{F}$   
for (dimensionless) surface  
target densities

$$\zeta = 1, \textcolor{green}{3.16}, 10, \textcolor{red}{31.6}, 100$$

Experimental requirements:

- nm foil targets (e.g. DLC)
- ultrahigh contrast (plasma mirrors)
- **circular polarization**



$a_0$  : dimensionless amplitude,

$\tau$  : duration in cycles

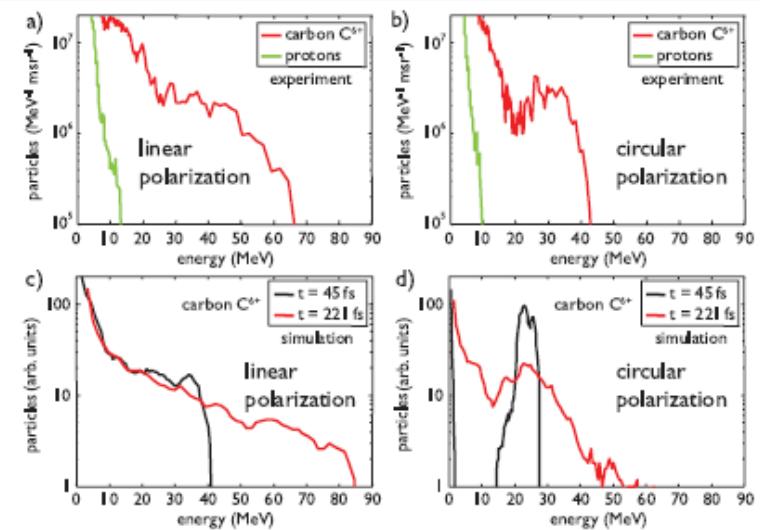
# WHY CIRCULAR POLARIZATION?

Using CP and normal incidence fast electron generation is strongly suppressed, maximizing radiation pressure and preventing foil expansion of the foil target

Early study in “thick” targets:  
 Macchi et al, PRL **94** (2005) 165003

Proposal of CP-RPA of ultrathin foils for efficient and monoenergetic acceleration:  
 Zhang et al, PoP **14** (2007) 073101  
 Robinson et al, NJP **10** (2008) 013201;  
 Klimo et al, PRST-AB **11** (2008) 031301.

First experimental study reported:  
 Henig et al, PRL **103** (2009) 245003



# THEORETICAL INTEREST IN CP-RPA...

Thick (semi-infinite) targets

(“Hole Boring”):

Liseikina & Macchi, APL **94** (2007) 165003;  
Naumova et al, PRL **102** (2009) 025002;  
Schlegel et al, PoP **16** (2009) 083103;  
Robinson et al, PPCF **51** (2009) 024004 & 095006;  
Macchi & Benedetti, NIM A (2010), in press

Ultrathin (sub-wavelength) targets

(“Light sail”):

Yan et al, PRL **100**, (2008) 135003 ;  
Qiao et al, PRL **102** (2009) 145002;  
Tripathi et al, PPCF **51** (2009) 024014;  
Eliasson et al. NJP **11** (2009) 073006;  
Yan et al, PRL **103** (2009) 135001;  
**Macchi et al, PRL **103** (2009) 085003;**  
**Macchi et al, NJP **12** (2010) 045013.**

Variations on the theme

(side effects, structured targets, ...):

Liseikina et al, PPCF **50** (2008) 124033;  
Rykovanov et al., NJP **10**, (2008) 113005;  
Ji et al, PRL **101** (2008) 164802;  
Yin et al, PoP **15** (2008) 093106;  
Holkundkara and Gupta, PoP **15** (2008)  
123104;  
Chen et al, PoP **15** (2008) 113103;  
Zhang et al, PRST-AB **12** (2009) 021301;  
Gonoskov et al, PRL **102** (2009) 145002;  
Chen et al, PRL **103** (2009) 024801  
Grech et al, NJP **11** (2009) 093035

## LS MODEL VS 1D PIC SIMULATIONS - I

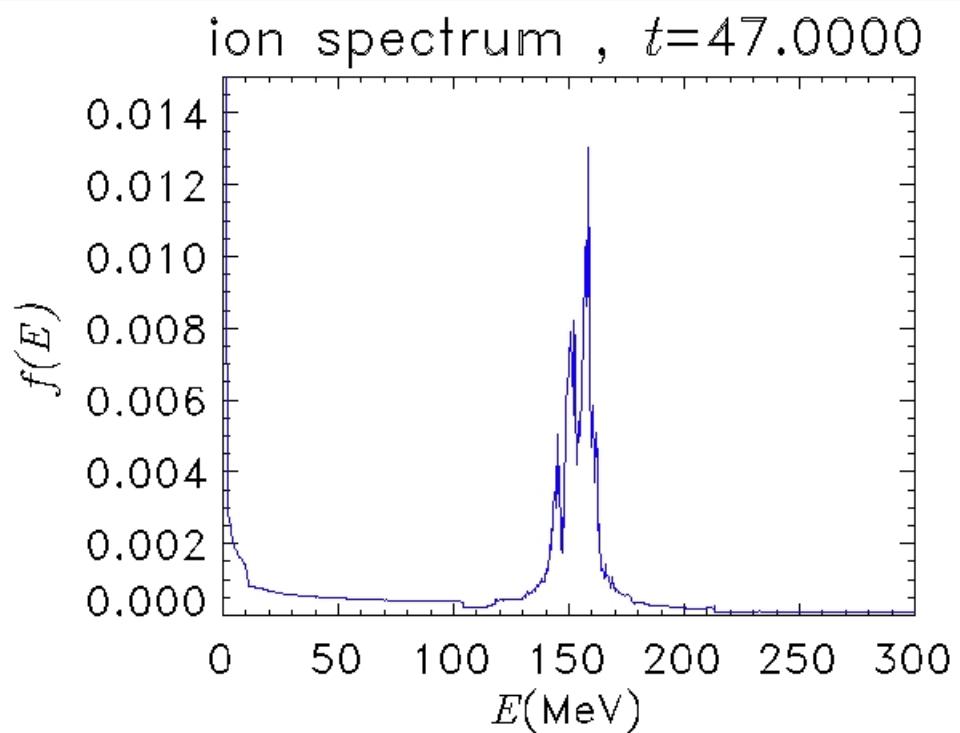
Laser pulse:  $a_0 = 5-50$ ,  $\tau=8$  cycles (“flat-top” envelope)

Thin foil target:  $n_e = 250n_c$ ,  $\ell=0.01-0.1\lambda$  ( $\zeta=7.8-78.5$ )

A narrow spectral peak is observed for  $a_0 < \zeta$ .

The energy of the peak is in good agreement with the LS formula

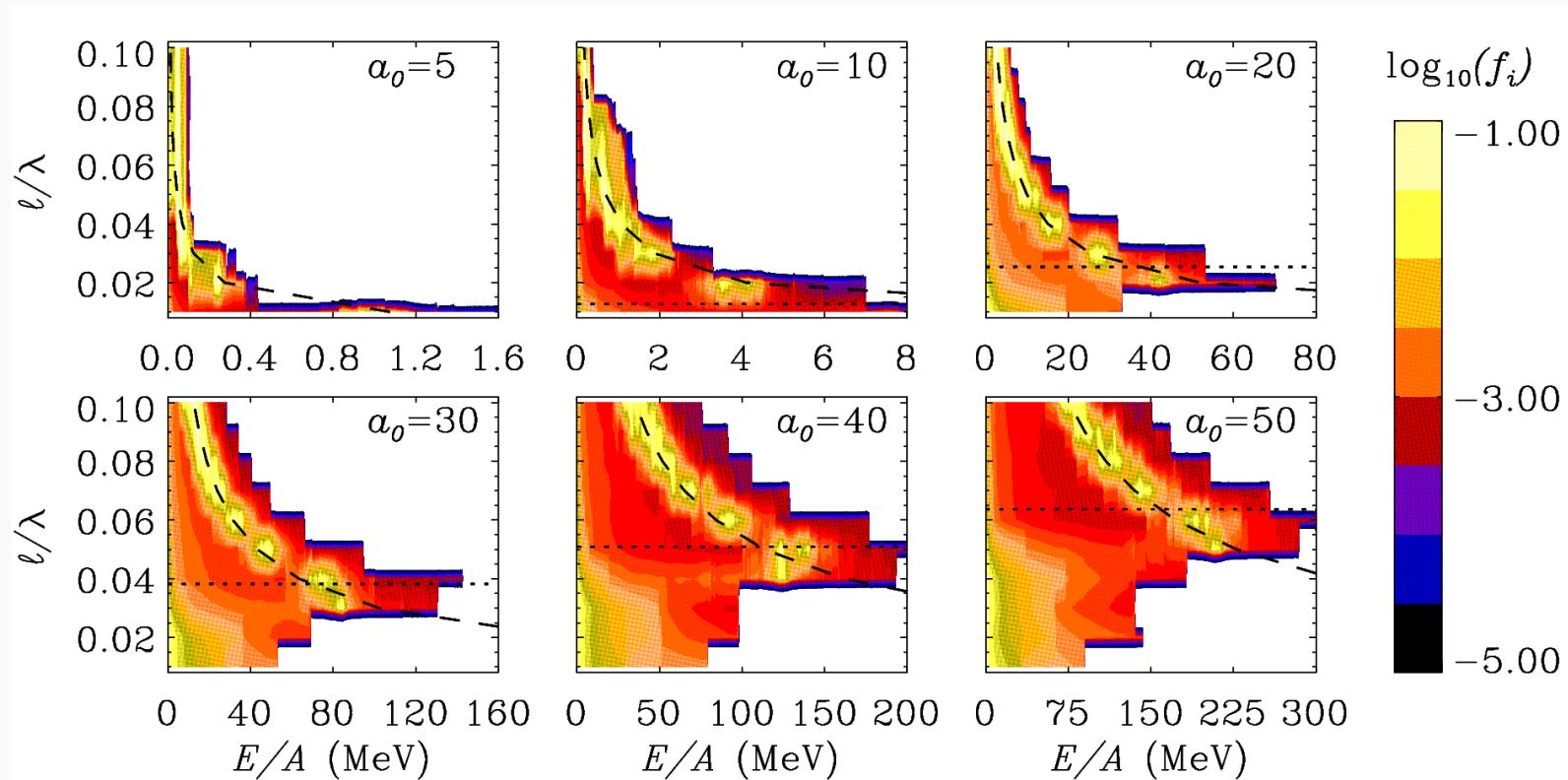
For  $a_0 > \zeta$ , the dynamics is dominated by a Coulomb explosion of the foil



## LS MODEL VS 1D PIC SIMULATIONS - II

Energy spectra vs.  $a_0$  and  $\ell$ :

(Dashed line: LS model prediction, dotted line:  $a_0 = \zeta$ )

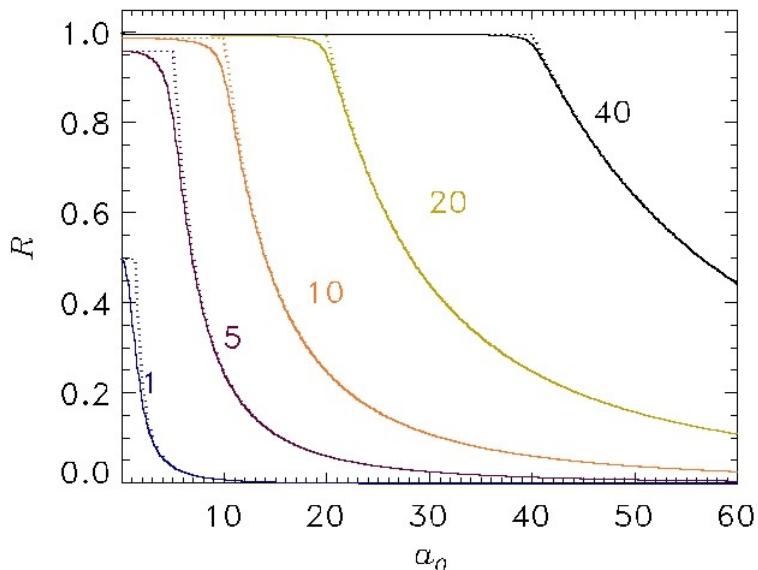


# TRANSPARENCY AND “OPTIMAL” THICKNESS

Ultrathin slab model:  $n_e(x)=n_0 \ell \delta(x)$ , foil thickness  $\ell \ll \lambda$

Total radiation pressure in rest frame  $P_{\text{rad}}=(2I/c)R$

Nonlinear reflectivity  $R=R(\zeta, a_0)$  includes Self-Induced Transparency



$$R \approx \zeta^2 / (\zeta^2 + 1) \quad (a_0 < \zeta)$$

$$R \approx \zeta^2 / a_0^2 \quad (a_0 > \zeta)$$

$P_{\text{rad}}$  does not depend on  $a_0$  for  $a_0 > \zeta$ ! (since  $I \sim a_0^2$ )

The maximum boost of the foil is at  $a_0 \approx \zeta$

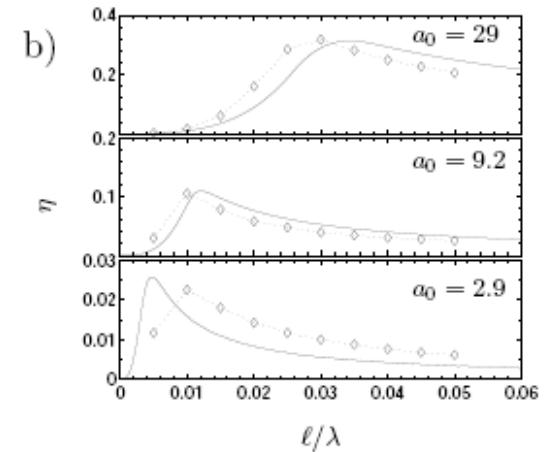
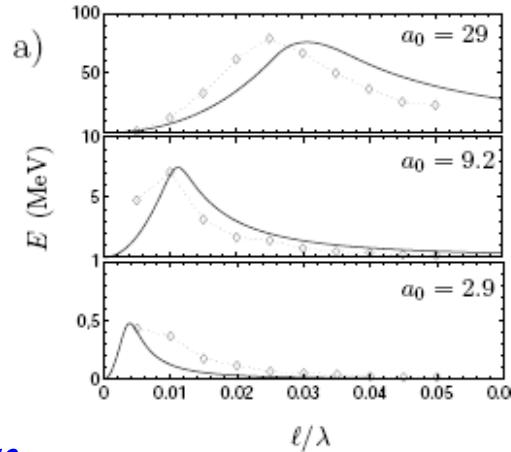
## LS MODEL WITH SIT INCLUDED

Modified foil velocity formula for  $R < 1$ ,  $a_0 < \zeta$

$$\beta(t) = \frac{(1 + \mathcal{E} - \zeta^{-2})^2 + (1 + \mathcal{E} - \zeta^{-2})\sqrt{(1 + \mathcal{E} - \zeta^{-2})^2 + 4\zeta^{-2}} + 2\zeta^{-2} - 2}{(1 + \mathcal{E} - \zeta^{-2})^2 + (1 + \mathcal{E} - \zeta^{-2})\sqrt{(1 + \mathcal{E} - \zeta^{-2})^2 + 4\zeta^{-2}} + 2\zeta^{-2} + 2}$$

$$\mathcal{E} = \frac{2F(t)}{\rho \ell c^2} = 2\pi \frac{Z m_e}{A m_p} \frac{a_0^2 \tau}{\zeta}$$

Ion energy and conversion efficiency vs. intensity and thickness  
(solid: theory, points: PIC sims.)



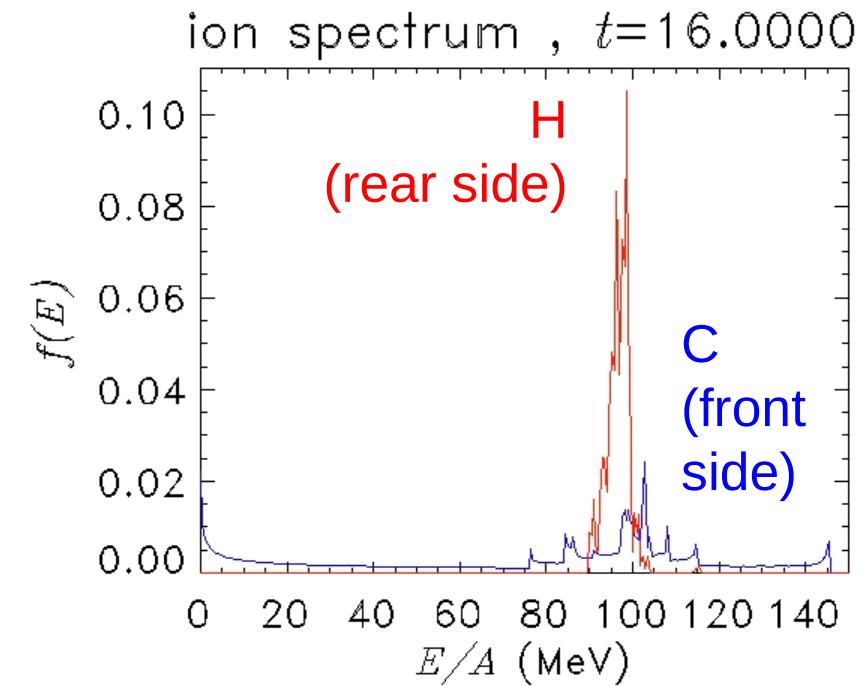
9 cycles pulse,  $n_e = 250 n_c$

## A PUZZLING ISSUE: “DARK” MASS

The RPA peak contains only  
 ~30% of all the ions  
 (and ~64% of their energy)

Only the **rear side** of the foil  
 is accelerated (thus LS RPA  
 may work for double-layer  
 targets!)

→ *Why there is very good  
 Agreement of the energy with  
 the LS formula when inserting  
 there the whole mass of the target  
 (and not ~30% of it)?*



# RADIATION VS ELECTROSTATIC PRESSURE

**Radiation pressure** drives electron depletion and generates back-holding electrostatic pressure

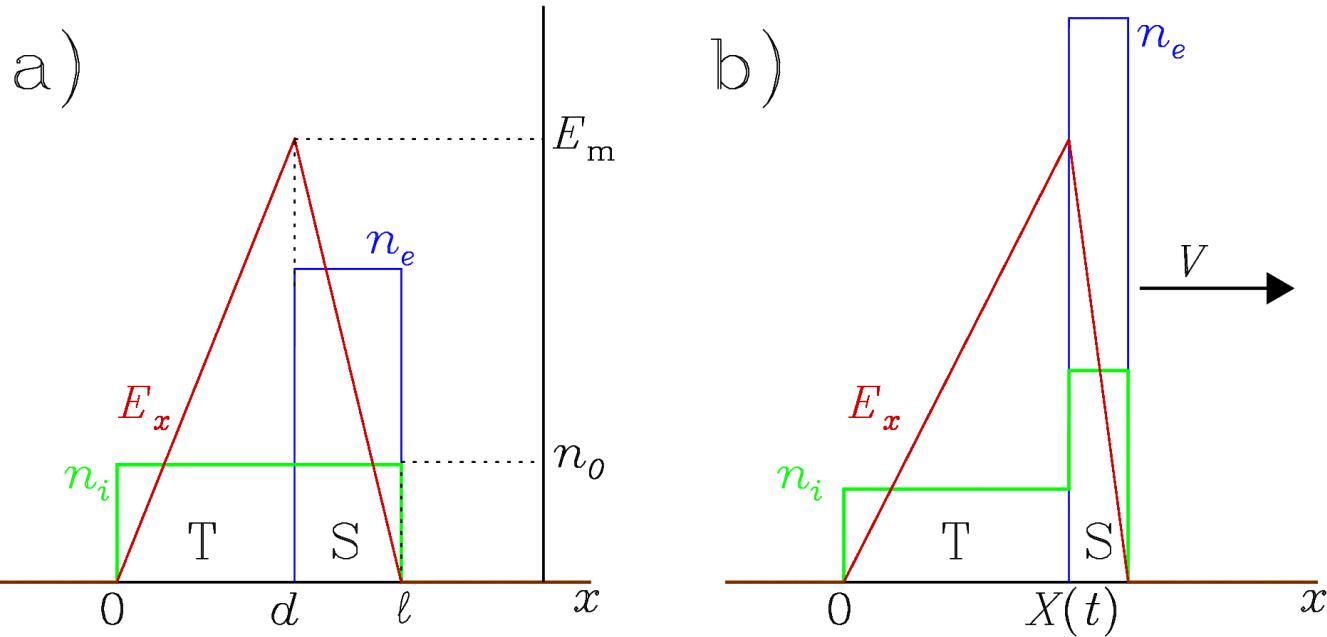
$$P_{\text{rad}} = (2I/c)R \leq P_{\text{es}} = 2\pi(en_0\ell)^2 \quad \text{for } a_0 \leq \zeta$$

If  $a_0 < \zeta$  and  $\zeta \gg 1$ ,  $R \approx 1$  and no electrons are pushed away

For  $a_0 \rightarrow \zeta$  all electrons must pile up near the rear surface in order that  $P_{\text{rad}} \simeq P_{\text{es}}$ .

- the electron pile-up layer is much thinner than the foil
- only a fraction of the foil is accelerated

## TWO ION POPULATIONS: TAIL & SAIL



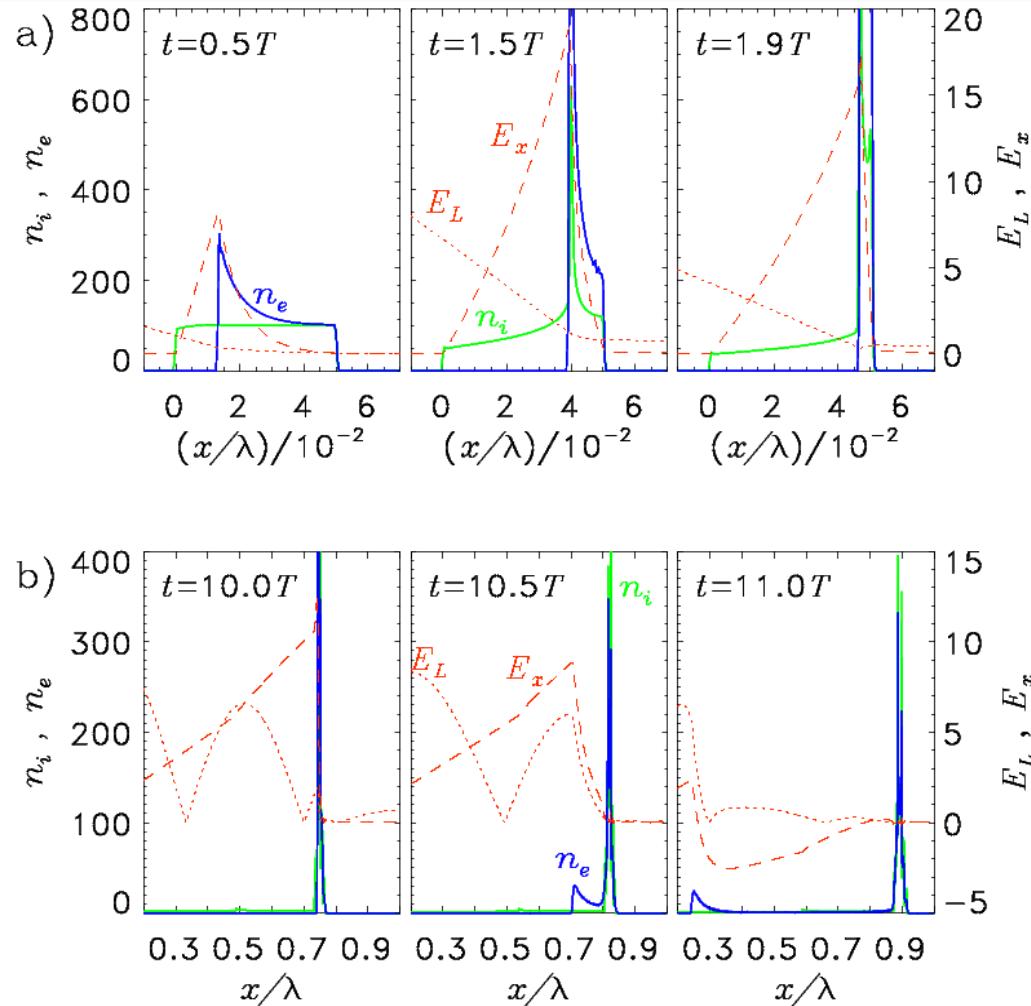
**Sail (S):** ions are bunched accelerated by  $E_x = f_p / e$  and move coherently as a “foil” : **monoenergetic component**

**Tail (T):** ions are accelerated by their own space-charge field and “Coulomb explode”: **broad spectrum component**

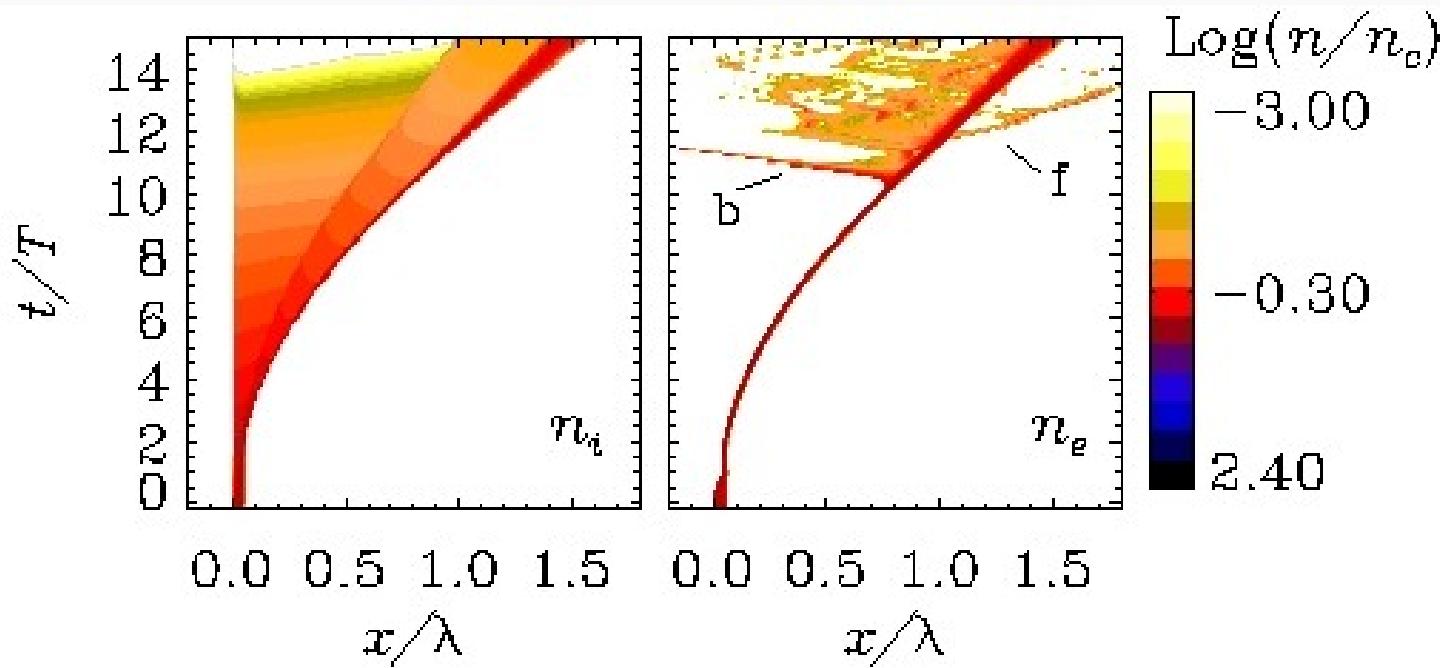
# SAIL CHARGING/DISCHARGING

PIC simulations show ions in the compression layer to form a “Sail” thinner than the original foil and negatively charged (excess of electrons)

The excess electrons “detach” from the Sail near the end of the laser pulse, moving backwards and leaving a neutral plasma bunch



# COLLAPSE OF ELECTRON EQUILIBRIUM



Near the end of the acceleration stage the pressure balance for electrons breaks down: formation of bunches in forward & backward directions and heating

Ion spectrum broadens in the post-acceleration stage

# MISSING INERTIA IS LOWER PRESSURE

Equilibrium of radiation  
and electrostatic  
pressure on *electrons*:

$$P_{\text{rad}} \doteq \int (-e)n_e E_x dx = \int n_e f_p dx$$

Electrostatic pressure  
on *ions*:

$$P_{\text{es}} = \int Z n_i E_x dx < P_{\text{rad}} \quad (Z n_i < n_e)$$

Calculation on equilibrium  
profiles yields:

$$P_{\text{es}} = \frac{M_{\text{Sail}}}{M_{\text{Foil}}} P_{\text{rad}}$$

Equation of motion:

$$P_{\text{es}} = \frac{d}{dt} (M_{\text{Sail}} \mathbf{V}) \iff P_{\text{rad}} = \frac{d}{dt} (M_{\text{Foil}} \mathbf{V})$$

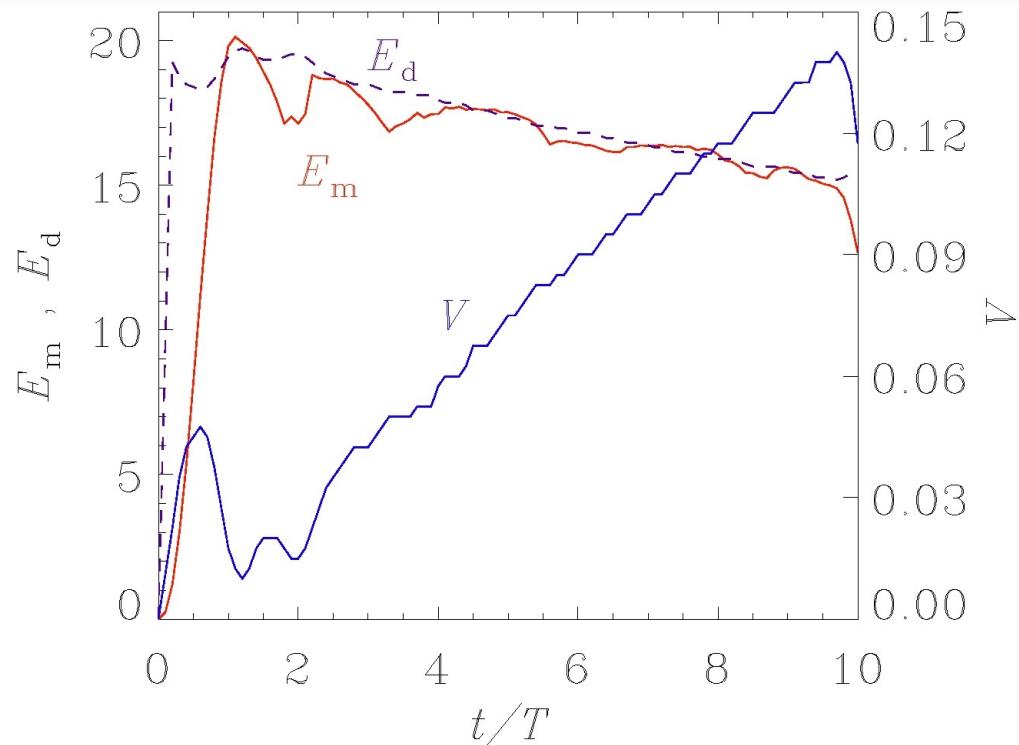
→ The Sail moves as if it had the total mass of the foil

# DYNAMIC PRESSURE BALANCE

$P_{\text{rad}}$  decreases with velocity in the Lab frame  
 $(P_{\text{rad}})^L = (1-\beta)/(1+\beta) P_{\text{rad}}$

To keep pressure equilibrium there is a **mass flow** (ion current) from

$M_{\text{tail}}$  to  $M_{\text{sail}}$



## ENERGY BALANCE

Efficiency depends only on  $\beta$  (the Sail velocity)

BUT the kinetic energy of the Sail is less than the total!

Energy stored in the electrostatic field  $E_x$ :

“Conversion efficiency” into electrostatic energy  $\eta_{es}$ :

For  $a_0 = \zeta$ , the depletion width  $d \approx \ell$  thus  $\eta_{es} \approx 2\beta$ :  
 most of the stored energy is converted into electrostatic energy and eventually goes to Tail ions

$$U_{es} = U_{es}(t) = \int_0^{X(t)} \frac{E_x^2(x, t)}{8\pi} dx$$

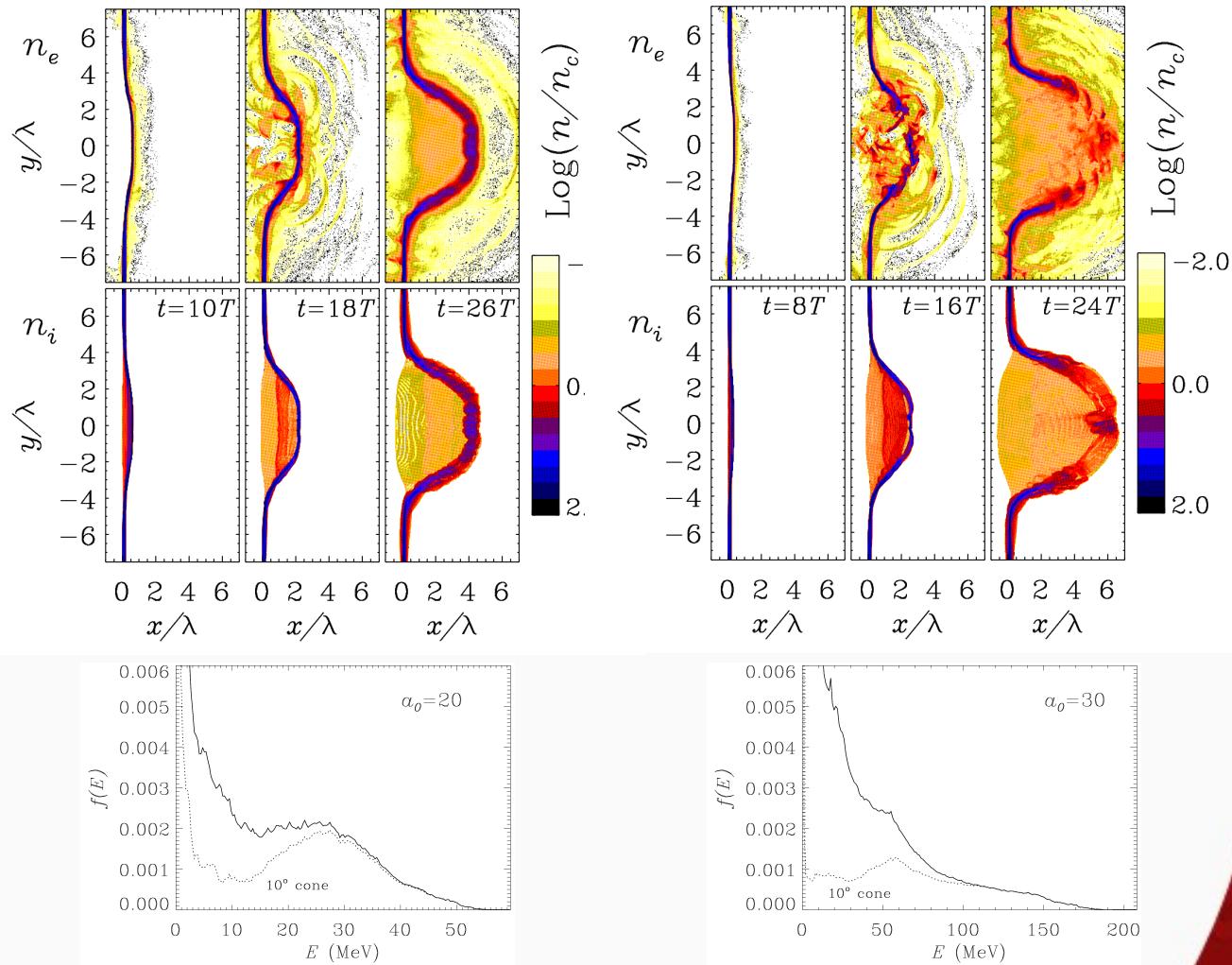
$$\frac{dU_{es}}{dt} = \frac{1}{8\pi} E_x^2[X(t), t] \frac{dX}{dt} = \frac{1}{8\pi} E_0^2 \beta c$$

$$\eta_{es} = \frac{1}{I} \frac{dU_{es}}{dt} = 2\beta \left(\frac{d}{\ell}\right)^2 \left(\frac{\zeta}{a_0}\right)^2$$

# TWO-DIMENSIONAL SIMULATIONS

2D sims for  
 $\zeta=31.4$  and  
 $a_0 = 20$  (left)  
 $a_0 = 30$  (right)

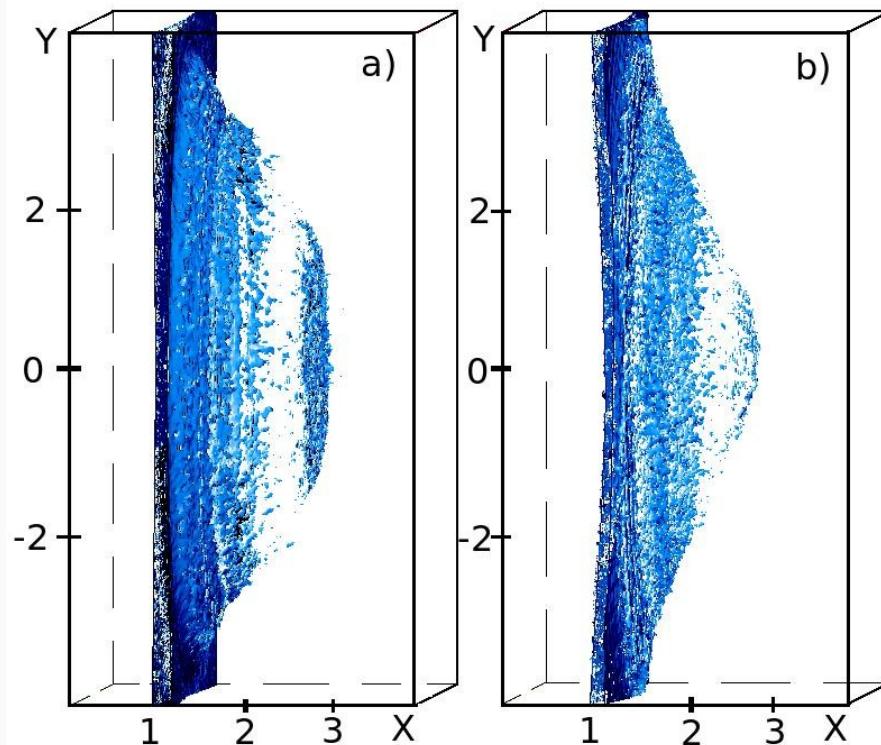
stronger electron heating and lower  
 “penetration” threshold with  
 respect to 1D:  
 ion spectrum is broad



## THREE-DIMENSIONAL SIMULATIONS

3D sims for  $\zeta=15$ ,  
 $a_0=5$ ,  $\tau=18$  cycles

left: Supergaussian  
spot profile  
right: Gaussian



Supergaussian “flat-top” profiles keep a “quasi-1D” geometry and prevent early breakthrough of laser pulse due to lateral expansion

## RADIATION REACTION EFFECTS

Motivation: RR is important for ultra-relativistic particles in EM fields and is thus expected to play a strong role in next generation experiments at ultra-high intensities.

The typical intensity for relevant RR effects is estimated to be  $\sim 10^{23} \text{ W/cm}^2$ . This corresponds, e.g., to the foreseen regime of) RPA dominance (even for Linear Polarization)  
[Esirkepov et al, PRL **92** (2004) 175003]

Two RPA simulation studies (for CP at lower intensity) suggest a “beneficial” effect of “electron cooling” by RR  
[Schlegel et al, PoP **16** (2009) 083103;  
Chen et al, arXiv:0909.5144]

# RADIATION REACTION FORCES

$$mc \frac{du^\mu}{d\tau} = f_L^\mu + f_{RR1}^\mu + f_{RR2}^\mu + f_S^\mu$$

$$f_L^\mu = e F^{\mu\nu} u_\nu$$

$$f_{RR1}^\mu = e \tau_0 (\partial_\alpha F^{\mu\nu} u_\nu u^\alpha)$$

$$f_{RR2}^\mu = \frac{e^2}{mc} \tau_0 \left( F^{\mu\nu} F_{\nu\alpha} u^\alpha + (F^{\nu\beta} u_\beta F_{\nu\alpha} u^\alpha) u^\mu \right)$$

$$f_S^\mu = -\frac{1}{2c} S^{\gamma\delta} \partial^\mu F_{\gamma\delta} + \frac{1}{2c} \left( S^{\gamma\delta} \partial_\alpha F_{\gamma\delta} u^\alpha \right) u^\mu$$

$$u^\alpha = \left( \gamma, \gamma \frac{\mathbf{v}}{c} \right) \quad \partial^\mu \equiv \left( \frac{\partial}{\partial t}, -c \nabla \right)$$

EoM of classical particle  
with spin in EM field

## EoM with Landau-Lifshitz force in non-covariant notation

$$\begin{aligned}
 \frac{d\mathbf{p}}{dt} = & -(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \\
 & - \left( \frac{4}{3}\pi \frac{r_e}{\lambda} \right) \gamma \left[ \left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{E} + \mathbf{v} \times \left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{B} \right] \\
 & + \left( \frac{4}{3}\pi \frac{r_e}{\lambda} \right) \left[ (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \times \mathbf{B} + (\mathbf{v} \cdot \mathbf{E}) \mathbf{E} \right] \\
 & - \left( \frac{4}{3}\pi \frac{r_e}{\lambda} \right) \gamma^2 \left[ (\mathbf{E} + \mathbf{v} \times \mathbf{B})^2 - (\mathbf{v} \cdot \mathbf{E})^2 \right] \mathbf{v}
 \end{aligned}$$

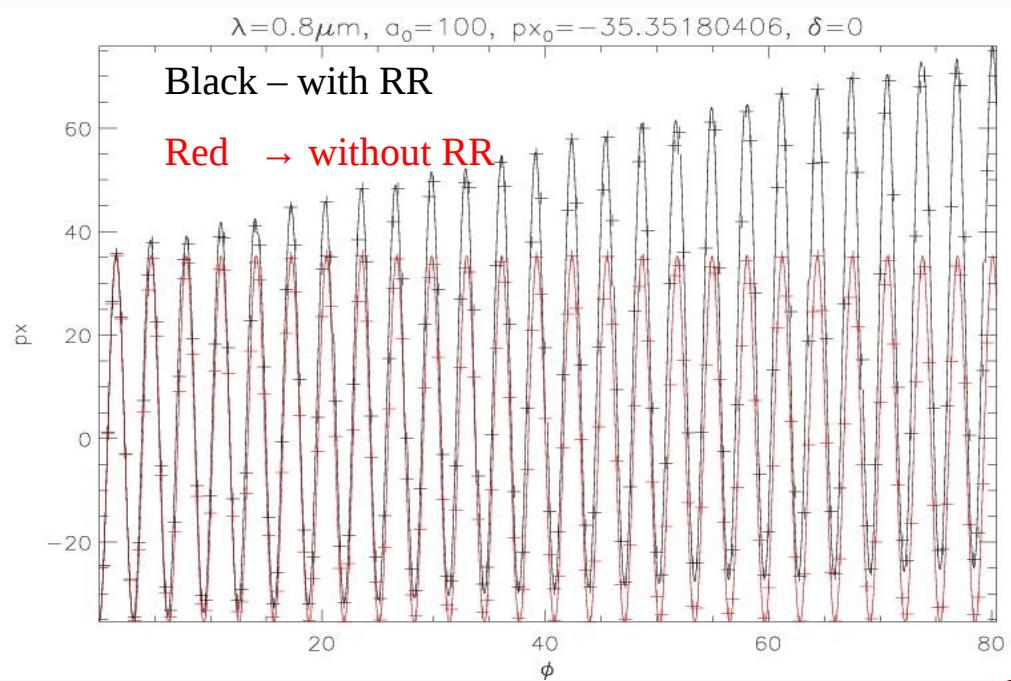
The last “friction” term is the dominant one  
(first two terms are smaller than spin contribution)

# BENCHMARK WITH EXACT SOLUTIONS

Exact solution of the Landau-Lifshitz equation in a plane wave  
 A.Di Piazza, Lett.Math.Phys. **83** (2008) 305

Test of particle pusher algorithm with “reduced” LL included:

- excellent agreement for intensities up to  $10^{24}$  W/cm<sup>2</sup>
- straightforward to include in a “standard” PIC code (based on Boris pusher)
- only ~10% increase in CPU time



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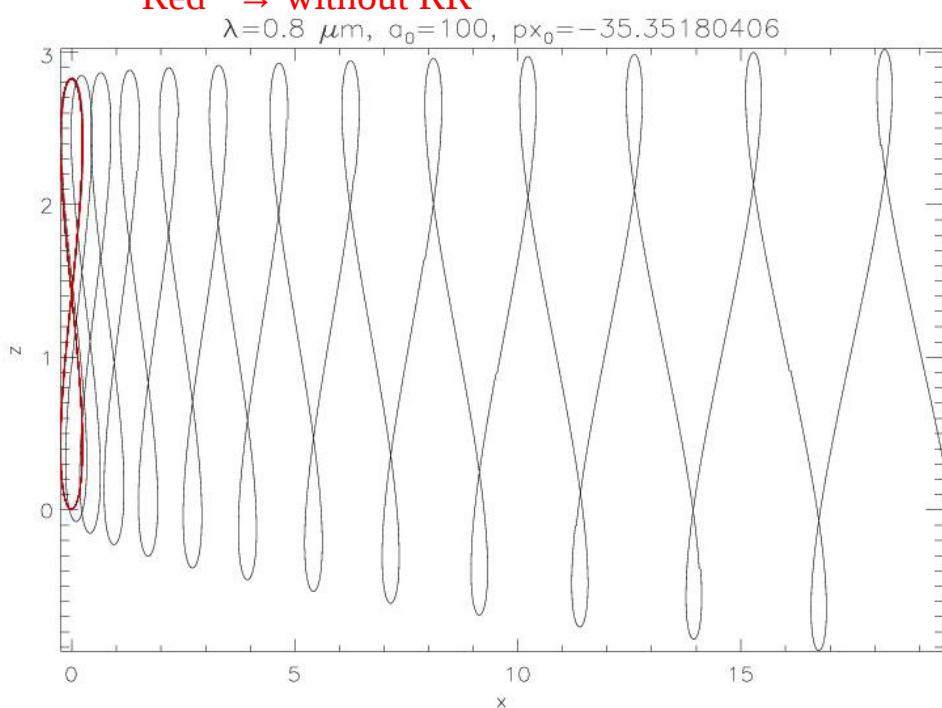
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Black – with RR

Red → without RR

$\lambda=0.8 \mu\text{m}$ ,  $a_0=100$ ,  $px_0=-35.35180406$



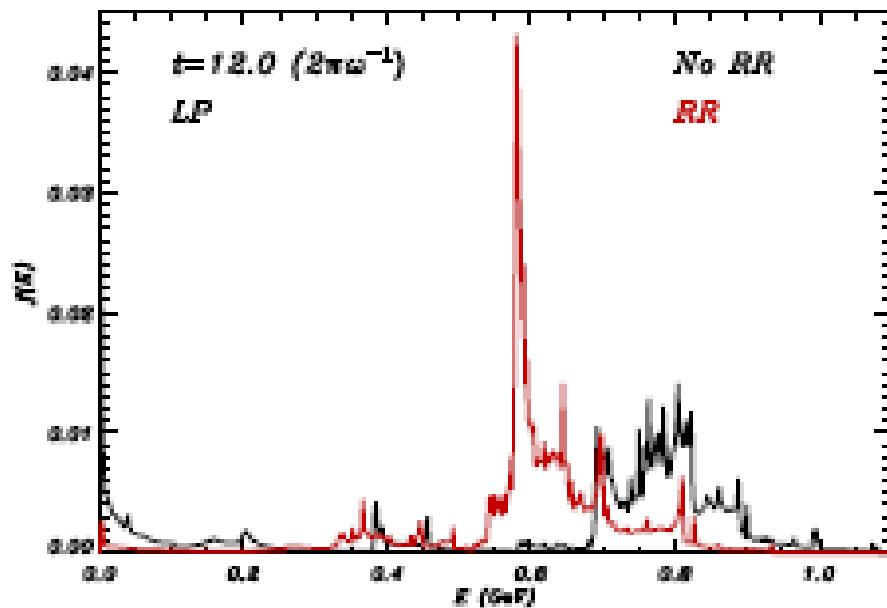
## RR EFFECTS ON ION SPECTRA – I (LP)

RP-dominated regime:  $10^{24} \text{ W/cm}^2$ , 11 cycles pulse

1 um foil,  $100n_c$ , linear polarization

Lower energy,  
narrower spectrum  
with RR included

~20% energy  
“dissipated” by RR



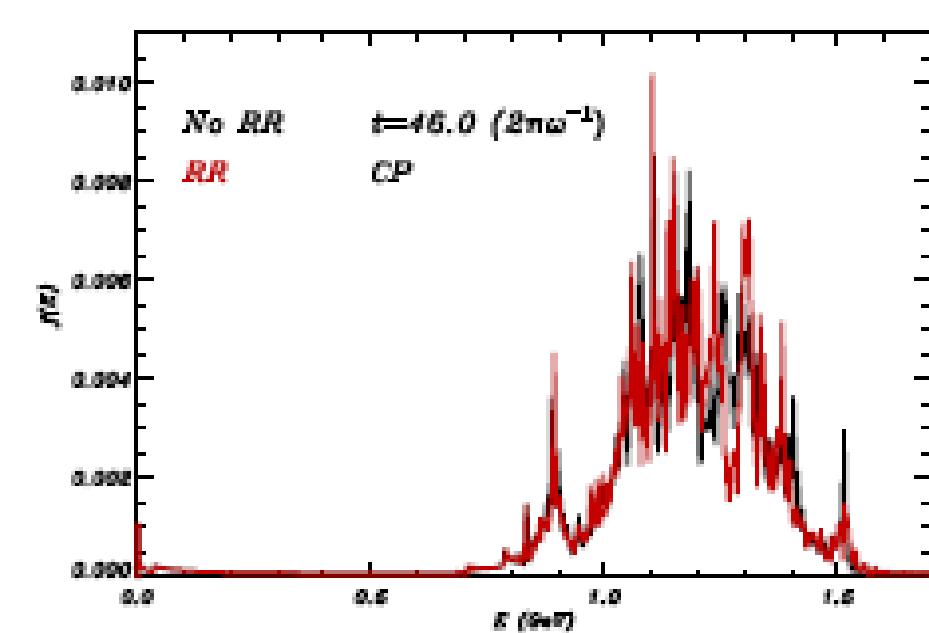
## RR EFFECTS ON ION SPECTRA – II (CP)

RP-dominated regime:  $10^{24} \text{ W/cm}^2$ , 11 cycles pulse

1 um foil,  $100n_c$ , circular polarization

Negligible RR effects  
on ion spectrum!

Higher energy than in  
LP case



## CONCLUSIONS

- The simple Light Sail model correctly predicts the ion energy and conversion efficiency observed in simulations
- Not all the ions are accelerated as a monoenergetic bunch
- Dynamics and self-organization underlying LS has been unfolded
- Keeping a monoenergetic spectrum is difficult
- Radiation Reaction effects at ultrahigh intensities are important only for Linear Polarization

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