Plasma Waves in a Different Frame: a Tutorial for Plasma-based Electron Accelerators

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Nice example of acceleration by a strong wave

From: T.Katsouleas, Nature **444** (2006) 688

Outline

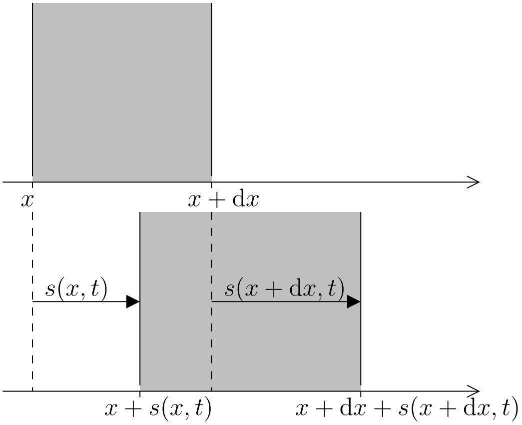
- Motivation: nonlinear, relativistic longitudinal waves are the basis of plasma-based electron accelerators
- Purpose: show to newcomers of the field an “easy route” (with respect to existing literature) to basic properties of nonlinear plasma waves and estimates of the acceleration potential
- Level: absolute beginners

Starting point: cold plasma oscillations

One-dimensional displacement of electrons, described by $s(x,t)$.

(Lagrangian coordinates)

- Ions are immobile
- Assume no overturn between electrons



Starting point: cold plasma oscillations

$$n_0 \mathrm{d}x = n_e(x,t)\left[x + \mathrm{d}x + s(x+\mathrm{d}x,t) - x - s(x,t)\right]$$

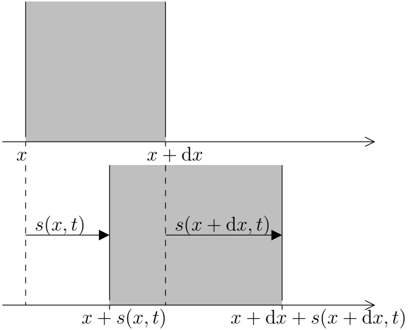
$$n_e(x,t) = \frac{n_0}{1+\partial_x s(x,t)} .$$

assuming small displacements $|\partial_x s(x,t)| \ll 1$

$$n_e(x,t) \simeq n_0 \left[1 - \partial_x s(x,t)\right] .$$

Electric field (from Gauss's law)

$$E_x = E_x(x,t) = 4\pi n_0 e s(x,t) ,$$



Starting point: cold plasma oscillations

Equation of motion and its general solution

$$m_e \partial_t^2 s(x,t) = -eE_x(x,t) = -4\pi n_0 e^2 s(x,t) \, ,$$

$$s(x,t) = \mathrm{Re}[\tilde{s}(x)\mathrm{e}^{-i\omega_p t}] = \frac{1}{2}\left[\tilde{s}(x)\mathrm{e}^{-i\omega_p t} + \tilde{s}^*(x)\mathrm{e}^{+i\omega_p t}\right] \, ,$$

with the plasma frequency $\omega_p = \left(\dfrac{4\pi e^2 n_0}{m_e}\right)^{1/2}$

Oscillations are localized and do *not* propagate

Plasma oscillations from the wave equation

General wave equation for $\mathbf{E}$ from Maxwell's equations[1]

$$\left(\nabla^2 - \frac{1}{c^2}\partial_t^2\right)\mathbf{E} - \nabla(\nabla\cdot\mathbf{E}) = \frac{4\pi}{c^2}\partial_t\mathbf{J}$$

Assume monochromatic fields e.g. $\mathbf{E}(\mathbf{r},t) = \tilde{\mathbf{E}}(\mathbf{r})\mathrm{e}^{-i\omega t}$ and use linearized, non-relativistic equations ($|\mathbf{u}_e| \ll c$)

$$\partial_t\mathbf{u}_e = -\frac{e}{m_e}\mathbf{E}\,, \qquad \mathbf{J} = -en_e\mathbf{u}_e\,, \qquad \tilde{\mathbf{J}} = -\frac{i}{4\pi}\frac{\omega_p^2}{\omega}\tilde{\mathbf{E}}\,,$$

$$\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\tilde{\mathbf{E}} - \nabla(\nabla\cdot\tilde{\mathbf{E}}) = \frac{\omega_p^2}{c^2}\tilde{\mathbf{E}} \qquad \text{(Helmoltz equation)}$$

[1] Eulerian coordinates are used here

Transverse electromagnetic waves

Taking $\tilde{\mathbf{E}}(\mathbf{r}) = E_0 \boldsymbol{\epsilon} \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}}$, $\quad \nabla \cdot \mathbf{E} = 0$, $\quad \mathbf{k} \perp \boldsymbol{\epsilon}$, $\quad \mathbf{B} = \mathbf{k} \times \mathbf{E}/k$

$$\left(\nabla^2 + \varepsilon(\omega)\frac{\omega^2}{c^2}\right)\tilde{\mathbf{E}} = \left(-k^2 + \mathrm{n}^2(\omega)\frac{\omega^2}{c^2}\right)\tilde{\mathbf{E}} = 0$$

$$\varepsilon(\omega) = \mathrm{n}^2(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

$\varepsilon(\omega)$ dielectric function, $\mathrm{n}(\omega)$ refractive index

dispersion relation $\quad k^2 c^2 = \varepsilon(\omega)\omega^2 = \omega^2 - \omega_p^2$

Phase and group velocities (assuming $\omega > \omega_p$ i.e. k real)

$$v_p = \frac{\omega}{k} = c\left(1 - \frac{\omega_p^2}{\omega^2}\right)^{-1/2} > c \qquad v_g = \frac{\partial\omega}{\partial k} = c\left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2} < c$$

Longitudinal electrostatic waves

Taking $\tilde{\mathbf{E}}(\mathbf{r}) = E_0 \boldsymbol{\epsilon} \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}}$, $\mathbf{k} \parallel \boldsymbol{\epsilon}$, $\nabla \cdot \mathbf{E} = \mathbf{k} \cdot \mathbf{E}$, $\mathbf{B} = 0$

$$\nabla(\nabla \cdot \tilde{\mathbf{E}}) = \nabla^2 \tilde{\mathbf{E}} \quad \Rightarrow \quad (\omega^2 - \omega_p^2)\tilde{\mathbf{E}} = 0 \quad \Rightarrow \quad \omega = \omega_p$$

Phase velocity is arbitrary, group velocity is zero

$$v_p = \frac{\omega}{k} = \frac{\omega_p}{k}, \qquad v_g = \frac{\partial \omega}{\partial k} = 0 .$$

Since these ES solutions do neither propagate nor transport energy they may be called “fake” waves ...

From fake to wake

Idea: let plasma oscillations be excited by a moving perturbation

Bodensee at Bad Schachen, Lindau, Germany.
Photo by Daderot, Wikipedia, public domain.

Wake plasma wave

Assume a delta-kick force pulse $f(x,t) = m_e u_0 \delta(t - x/V)$ to travel with velocity V through the plasma

Electrons located at x are displaced (by $s_0 = u_0/\omega_p$) at the overtaking time $t = x/V$:

$$s(x,t) = \begin{cases} 0 & (t < x/V)\,, \\ s_0 \cos\left(\omega_p(t - x/V)\right) & (t > x/V)\,. \end{cases}$$

The phase velocity of the wake wave $v_p = V$ "by construction" and this determines $k = \omega_p/V$.

Wake plasma wave

Example: a charged particle or bunch penetrating a plasma loses its energy to the wake (basic mechanism of collective stopping)

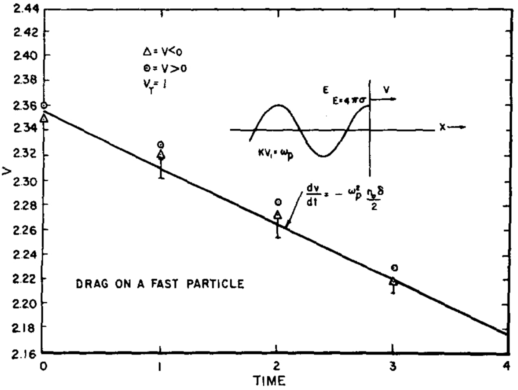


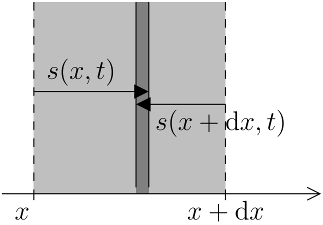
FIG. 6. The drag on a fast sheet.

J. Dawson, Phys. Fluids **5** (1962) 445

Plasma wave breaking

The electron density becomes singular when $\partial_x s(x,t) = -1$, i.e. the trajectories of electrons starting at x overlap with those starting at $x + \mathrm{d}x$

$$n_e(x,t) = \frac{n_0}{1 + \partial_x s(x,t)} \longrightarrow \infty .$$



The regular "hydrodynamic" structure is lost: the wave *breaks*

Note that $\partial_x s(x,t) = -1$ violates the assumption of small amplitude oscillations: a nonlinear analysis is required

A (not-so-correct) wavebreaking criterion

The electron density must obviously remain positive:

$$n_e = n_0 + \delta n_e > 0 \quad \Leftrightarrow \quad |\delta n_e| < n_0$$

For an harmonic wave

$$s(x,t) = s_0 \cos(kx - \omega_p t) = s_0 \cos\big(k(x - v_p t)\big)$$

the $|\delta n_e| \le n_0$ condition is equivalent to

$$ks_0 \le 1\,, \qquad u_0 \le v_p\,, \qquad E_0 \le m_e \omega_p^2 v_p / e$$

(u_0, E_0: amplitudes of electron velocity and electric field)

Objection: when the wave becomes nonlinear, it cannot be considered as harmonic!

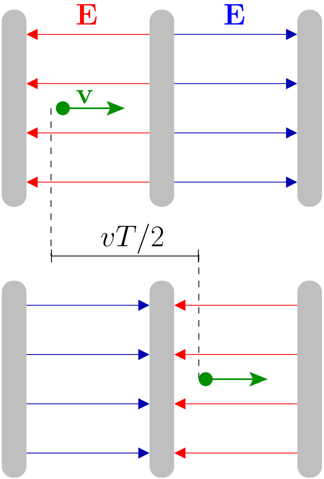
Principle of electron acceleration

LINAC principle: an electron of velocity v crosses a cavity of length L within half the period of oscillation of the electric field T so to "see" the electric field in a constant direction
$\longrightarrow L \simeq vT/2$

For strongly relativistic electrons

$$L \simeq cT/2 = \pi c/\omega$$

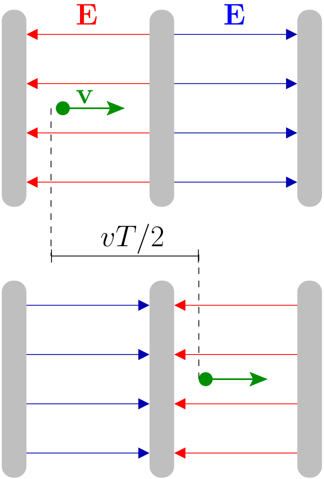
($\approx$ independent of electron energy)



Plasma wake as the perfect wave?

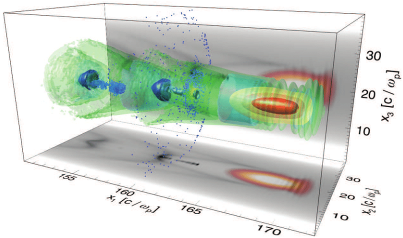
A plasma wave with phase velocity v_p can be seen as a sequence of "cavities" with $T = \pi/\omega_p$ and $L = v_p T_p$ which may accelerate electrons injected with velocity close to v_p.

For a plasma wake v_p is set by the driver and $\mathbf{E}$ is only limited by wavebreaking (not by electrical breakdown of cavity components).

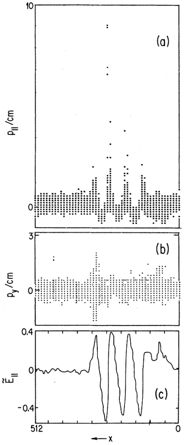


Laser(-Plasma) wakefield accelerator

Tajima & Dawson's famous proposal:
[Phys. Rev. Lett. **43**, 267 (1979)]
use short laser pulses as a driver for wake plasma waves
(as an alternative: use a relativistic particle bunch)

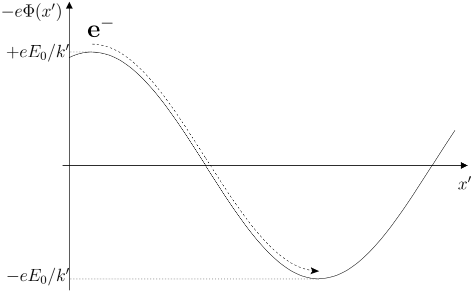


A 3D simulation Fonseca et al, Plasma Phys. Control. Fusion **50**, 124034 (2008)



Estimate the energy gain

In a reference frame S' moving with the phase velocity v_p with respect to the laboratory S the wave field is time-independent and can be derived by a static potential.



A test electron moving from the top to the bottom of the potential hill with initial velocity $v'_{x0} = 0$ (hence $v_{x0} = v_p$ in the lab frame) will get the maximum energy gain possible.

The wave frame

(we use the complex notation for brevity)

- E-field in S: $E_x = E_0 \mathrm{e}^{-i\omega_p(t-x/v_p)} = E_0 \mathrm{e}^{ikx-i\omega_p t}$

  Lorentz boost to S' with $\boldsymbol{\beta} = \beta_p \hat{\mathbf{x}} \quad , \quad \beta_p = v_p/c$:
- frequency & wavevector in S':
  $$\begin{cases} \omega' & = & \gamma_p(\omega_p - k v_p) = 0 \\ k' & = & \gamma_p\left(k - \omega_p v_p/c^2\right) = k/\gamma_p \end{cases}$$
  where $\gamma_p = (1-\beta_p^2)^{-1/2}$
- E-field in S': $E'_x = E_0 \mathrm{e}^{ik'x'} \quad , E'_0 = E_0$

  ($E_\parallel$ is unchanged by Lorentz transformations)
- electric potential in S': $\Phi'(x') = \dfrac{i}{k'} E_0 \mathrm{e}^{ik'x'}$

  ($E'_x = -\partial'_x \Phi(x')$)

Maximum energy gain in the wave frame

Potential energy

$U' = U'(x') = -e\text{Re}(\Phi') = \dfrac{eE_0}{k'}\sin(k'x')$

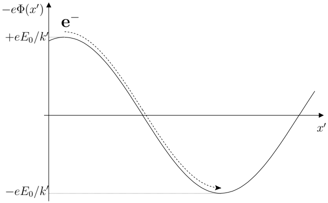
Maximum energy gain in S'

$W' = \max(U') - \min(U') = \dfrac{2eE_0}{k'}$

Energy-momentum in S'

$p'_\mu = (p'_0, p'_x) = \left(W'/c, (W'^2 - m_e^2c^4)^{1/2}/c\right)$

(being $p_\mu p^\mu = p_0^2 - p_x^2 = m_e^2c^2$)



The maximum energy $W_{\max}$ in the lab frame S can be obtained by back-Lorentz transformation of p'_μ .

Tajima & Dawson's maximum energy estimate

- Assume $v \simeq c$ i.e. $\beta_p \simeq 1$
- Assume $E_0 = \frac{m_e \omega_p^2 v_p}{e}$ (wavebreaking limit)
  then $p'_x \gg m_e c \quad , \quad p'_0 \simeq p'_x c \quad ,$
- $p_0 = \gamma_p (p'_0 + \beta_p p'_x) \simeq 2\gamma_p p'_0 = 4 m_e c \gamma_p^2$
  The maximum foreseeable energy for the "luckiest" electron (optimal initial onditions) is

$$W_{\max} = p_0 c = 4 m_e c^2 \gamma_p^2$$

- Objection! The incorrect wavebreaking limit was used ...

Nonlinear relativistic approach

Since $v_p \simeq c$ we expect the electron dynamics near the wavebreaking threshold to be strongly relativistic.
Start from the 1D relativistic equation of motion for the fluid electron momentum $p_{ex} = p_{ex}(x,t)$ in an ES field $E_x = E_x(x,t)$:

$$\frac{\mathrm{d}}{\mathrm{d}t} p_{ex} = (\partial_t + u_{ex}\partial_x) p_{ex} = -eE_x \, ,$$

$$p_{ex} = m_e \gamma_e u_{ex} \, , \quad \gamma_e = (1 - u_{ex}^2/c^2)^{-1/2} = (1 + p_{ex}^2/m_e^2 c^2)^{-1/2} \, .$$

In the wave frame $\partial'_t = 0$ and $u'_{ex} = u'_{ex}(x')$ etc ...

$$u'_{ex}\partial'_x p'_{ex} = -eE'_x = e\partial'_x \Phi'$$

This can be integrated to give

$$m_e c^2 \gamma'_e - e\Phi' = \text{cost.} \, . \tag{1}$$

Zero current condition

In a purely electrostatic ($\mathbf{B} = 0$), time-independent configuration the current density vanishes:

$$\frac{4\pi}{c}\mathbf{J} = \nabla \times \mathbf{B} - \frac{1}{c}\partial_t \mathbf{E} = 0 - 0 = 0\,.$$

We impose $J'_x = J'_{ex} + J'_{ix} = 0$ in the wave frame

$$\begin{aligned} J'_{ex} &= -en'_e u'_{ex} \\ J'_{ix} &= +en'_i u'_{ix} = e(\gamma_p n_0)(-v_p) \end{aligned}$$

(the ion fluid in S' has a density γ_p times the value in S due to Lorentz contraction and a steady velocity $-v_p$). Thus

$$n'_e u'_{ex} + \gamma_p n_0 v_p = 0\,. \tag{2}$$

Complete equations and normalizations

To close the system Poisson's eq. (3) is added to (1) and (2):

$$\partial_x'^2 \Phi' = 4\pi e(n'_e - n'_0) = 4\pi e(n'_e - n_0\gamma_p)$$

It is now convenient to put the system in dimensionless form

$$\tau \equiv \frac{\omega_p x'}{c}\,, \quad N_e \equiv \frac{n'_e}{n_0}\,, \quad p \equiv \frac{p'_x}{m_e c}\,, \quad \phi \equiv \frac{e\Phi'}{m_e c^2}\,.$$

and write simply γ for γ_e:

$$\gamma \equiv \gamma_e = (p^2 + 1)^{1/2}\,.$$

(primes are dropped for brevity).

Pseudopotential equation

Three equations for ϕ, p and γ

all functions of $\tau = \frac{\omega_p x'}{c}$:

$$\begin{cases} \partial_\tau^2 \phi &= N_e - \gamma_p \,, \\ \gamma - \phi &= \text{cost.}\,, \\ N_e \frac{p}{\gamma} &= -\gamma_p \beta_p \,, \end{cases}$$

Eliminate ϕ and p as functions of γ using $p = -(\gamma^2 - 1)^{1/2}$
($p < 0$ because electrons flow in the direction opposite to the boost)
to obtain an equation for $\gamma = \gamma(\tau)$:

$$\partial_\tau^2 \gamma = \gamma_p \beta_p \frac{\gamma}{(\gamma^2 - 1)^{1/2}} - \gamma_p \equiv -\frac{\partial}{\partial \gamma} U(\gamma) \,,$$

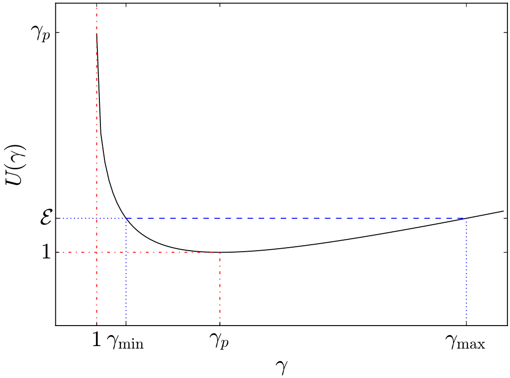
$$U(\gamma) = \gamma_p \left(\gamma - \beta_p (\gamma^2 - 1)^{1/2} \right) .$$

[A. C. L. Chian, Plasma Phys. **21**, 509 (1979)]

Pseudopotential analysis

$$\partial_\tau^2 \gamma = -\frac{\partial}{\partial \gamma} U(\gamma) \qquad (3)$$

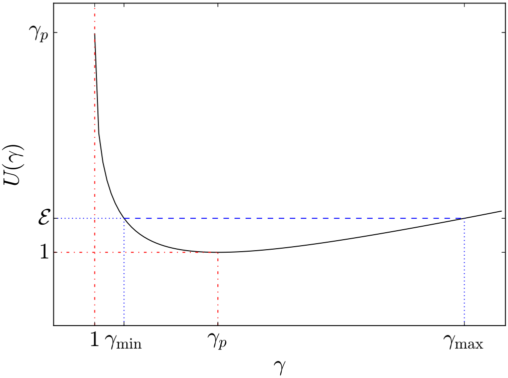
$\equiv$ Newton's equation of motion for a "pseudoparticle" of coordinate $\gamma = \gamma(\tau)$ in the potential $U = U(\gamma)$.



"Pseudo-energy" conservation: $\frac{1}{2}(\partial_\tau \gamma)^2 + U(\gamma) = \mathscr{E}$

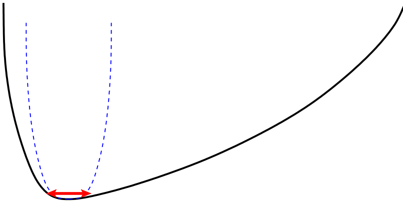
Pseudopotential analysis - II

The value of "pseudoenergy" $\mathcal{E}$ determines the type of trajectory $\gamma(\tau)$ which corresponds to the waveform $\gamma\left(\frac{\omega_p}{c}x'\right)$.



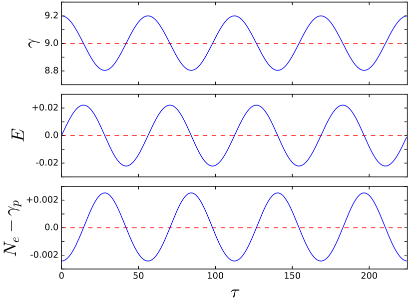
If $\mathcal{E} < \max(U) = \gamma_p$ the pseudoparticle bounces back and forth between $\gamma_{\rm min}$ and $\gamma_{\rm max}$: the waveform is periodic

Linear harmonic wave



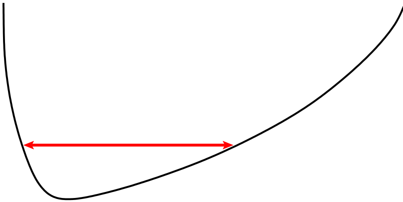
If $\mathscr{E} \gtrsim \min(U) = 1$ the pseudoparticle performs small amplitude oscillations around $\gamma = \gamma_p$ (bottom of potential well):
harmonic waveform

Note that the electric field $E = -\partial_\tau \phi = -\partial_\tau \gamma$ is (minus the) pseudovelocity $\partial_\tau \gamma$



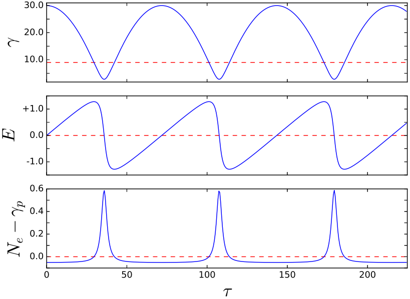
Numerical solution of EoM (3)
for $\gamma_p = 9$ and $\mathscr{E} = 1.0 + 2.445 \times 10^{-4}$

Nonlinear anharmonic wave - I



Increasing $\mathscr{E}$ the pseudoparticle performs anharmonic, asymmetric oscillations in the well with strong acceleration on one side and weak on the other:

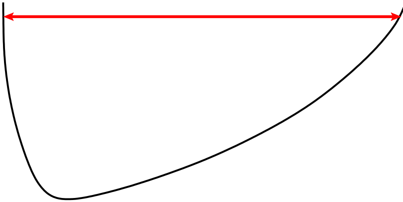
"sawtooth" profile for pseudovelocity (E) and spiky profile for $N_e = \gamma_p - \partial_\tau E$



Numerical solution of EoM (3)

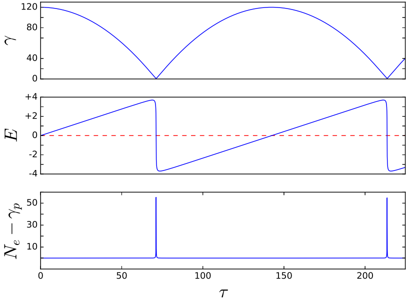
for $\gamma_p = 9$ and $\mathscr{E} = 1.8$

Nonlinear anharmonic wave - II



When $\mathscr{E}$ approaches $\max(U) = \gamma_p$ the pseudoparticle has extremely different accelerations and half-oscillation times on either side of the well:

the pseudovelocity (E) becomes discontinous (rebound on the steep potential hill) and N_e becomes singular



Numerical solution of EoM (3)

for $\gamma_p = 9$ and $\mathscr{E} = 7.8$

Relativistic wavebreaking threshold

For $\mathcal{E} > \max(U) = \gamma_p$ no regular solutions are found.
At threshold ($\mathcal{E} = \gamma_p$) the pseudoparticle "falls" from $\max(U) = U(1) = \gamma_p$ down to $\min(U) = U(\gamma_p) = 1$ acquiring a maximum pseudovelocity $-E_{\max}$:

$$E_{\max} = \sqrt{2}(U(1) - U(\gamma_p))^{1/2} = \sqrt{2}(\gamma_p - 1)^{1/2} .$$

In standard dimensional units the maximum possible field is

$$E_{x,\max} = \frac{m_e c\omega_p}{e}\sqrt{2}(\gamma_p - 1)^{1/2} \equiv E_{\mathrm{WB}} \quad \left(> \frac{m_e v_p \omega_p}{e}\right) .$$

[Akhiezer & Polovin, Sov. Phys. JETP **3** (1956) 696]

Improved estimate of energy gain - I

Since $m_e c^2\gamma'_e - e\Phi' = $ cost., the maximum energy gain for a test electron is obtained by the extrema in the γ-oscillation $(\gamma_{\rm min}\,, \gamma_{\rm max})$

$$W' = e\left[\max(\Phi') - \min(\Phi')\right] \quad = \quad m_e c^2(\gamma_{\rm max} - \gamma_{\rm min})\,,$$

At the wavebreaking threshold $U(\gamma) = \gamma_p \ (\gg 1)$ which yields two solutions

$$U(\gamma) \doteq \gamma_p \quad \longrightarrow \quad \gamma = \begin{cases} 2\gamma_p^2 & \equiv & \gamma_{\rm max} \\ 1 & \equiv & \gamma_{\rm min} \end{cases}$$

$$W'_{\rm max} \equiv \max(W') = m_e c^2(2\gamma_p^2 - 1) \simeq 2m_e c^2\gamma_p^2$$

Improved estimate of energy gain - II

Evaluate $W_{\max}$ in the laboratory S:

$$p'_0 = W'_{\max}/c \simeq 2m_e c\gamma_p^2 \,, \qquad p'_x \simeq p'_0 \,,$$

$$\begin{aligned} W_{\max} &= p_0 c = \gamma_p(p'_0 c + v_p p'_x) \simeq m_e c^2 \gamma_p (2\gamma_p^2 + 2\gamma_p^2) \\ &= 4m_e c^2 \gamma_p^3 \end{aligned}$$

Tajima & Dawson's estimate $W_{\max}^{(\mathrm{TD})} = p_0 c = 4m_e c^2 \gamma_p^2$ is increased by a substantial factor γ_p, in agreement with detailed (more complex) calculations

[E. Esarey and M. Pilloff, Phys. Plasmas **2**, 1432 (1995)]

Acceleration length

L_{acc} = how long must my plasma wave be to allow the maximum energy gain = (max energy)/(max force)

$$L_{\mathrm{acc}} = \frac{W_{\mathrm{max}}}{eE_{\mathrm{WB}}} \simeq 2\sqrt{2}\frac{c}{\omega_p}\gamma_p^{5/2} .$$

For $\gamma_p = 100$ and 10^{17} cm^{-3} (SLAC experiments)

$$W_{\mathrm{max}} = 2 \text{ TeV} \qquad \text{for} \qquad L_{\mathrm{acc}} \simeq 90 \text{ m}$$

Note that in S' the acceleration length is half the plasma wavelength

$$L'_{\mathrm{acc}} = \frac{1}{2}\lambda'_p , \qquad L_{\mathrm{acc}} = \gamma_p L'_{\mathrm{acc}}$$

the last relation comes from Lorentz contraction

Other plasma wave features

From the pseudopotential analysis it is easy to estimate other quantities at the WB threshold, e.g.

$$\lambda'_p \simeq 4\sqrt{2}\gamma_p^{3/2}\frac{c}{\omega_p}$$

or the minimum electron density (which does not vanish)

$$\min(n_e) \simeq \frac{1}{2}n_0$$

[see Macchi, Am. J. Phys. **88**, 723 (2020) for details]

All these results are known from previous publications but the present approach is apparently easier on the mathematical side

“Dawson’s sheet” model - I

First plasma simulation model ever published!

J. Dawson, Phys. Fluids **5** (1962) 445

Figure from Birdsall & Langdon, *Plasma Physics via Computer Simulation* (Taylor & Francis, 1975/2004)

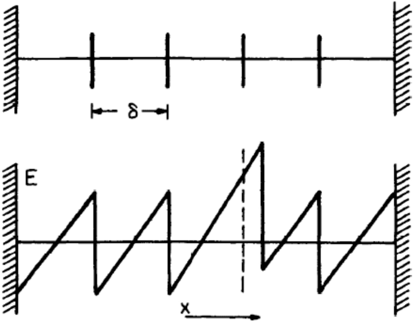
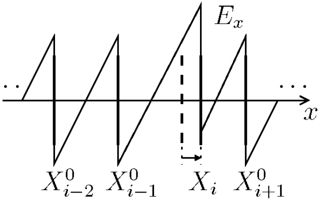


Figure 13-2a Original *Dawson* (1962) model, with thin electron sheets spaced $\delta = 1/n$ apart (in equilibrium) in a uniform positive ion background. The lower part shows $E(x)$ with one sheet displaced.

"Dawson's sheet" model - II

Each charged sheet (of position $X_i(t)$, $i = 1, \ldots, N$) is a "macro-electron" (over a neutralizing background of density n_0)



E-field on each sheet (Gauss):

$$E_i = E_x[x = X_i(t)] = 4\pi e n_0 \left[X_i(t) - X_i^{\text{eq}}\right]$$

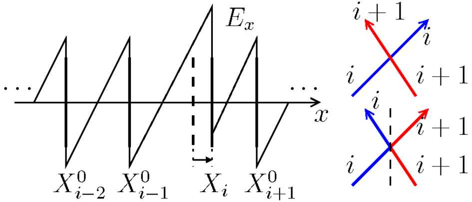
if $X_i < X_{i+1}\ \forall i$

Equation of motion (with external force f_{ext})

$$\frac{\mathrm{d}^2 X_i}{\mathrm{d}t^2} = -\frac{e}{m_e} E_x(X_i) + \frac{f_{\text{ext}}}{m_e} = -\omega_p^2 \left(X_i - X_i^{\text{eq}}\right) + \frac{f_{\text{ext}}}{m_e}$$

“Dawson’s sheet” model - III

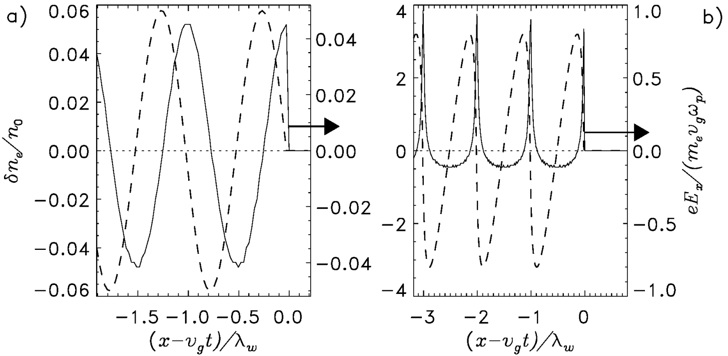
Sheet crossing is equivalent to a “reindexing” of sheets
$\longrightarrow$ swapping of indices at each timestep keeps the ordering



All nonlinear effects are in the swap!

Simulating wakes with Dawson's sheet model

Simulation with impulsive force $f_{\text{ext}} = m_e u_0 \delta(t - x/v_g)$

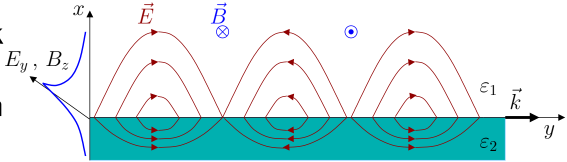


weak forcing

strong forcing

Surface Plasmon (aka Surface Plasma Wave)

SP: a building block of plasmonics (mostly studied in the *linear* regime)



SP excitation $\longrightarrow$ EM field confinement and enhancement

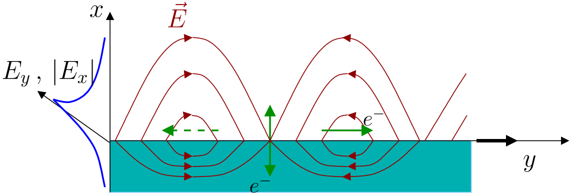
Interface between vacuum and "simple metal" (cold plasma):

$$\varepsilon_1 = 1 \qquad \varepsilon_2 = 1 - \frac{\omega_p^2}{\omega^2} = 1 - \frac{n_e}{n_c(\omega)} < -1$$

$$k = \frac{\omega}{c}\left(\frac{\omega_p^2 - \omega^2}{\omega_p^2 - 2\omega^2}\right)^{1/2} \qquad \omega < \frac{\omega_p}{\sqrt{2}} \qquad v_p = \frac{\omega}{k} < c$$

Surfin' the Surface Wave

Can a SP accelerate electrons like a "bulk" plasma wave?



- longitudinal E-component (E_y)
- sub-luminal phase velocity $v_\mathrm{p} < c$

  (with $v_\mathrm{p} \to c$ when $\omega_p \gg \omega$)

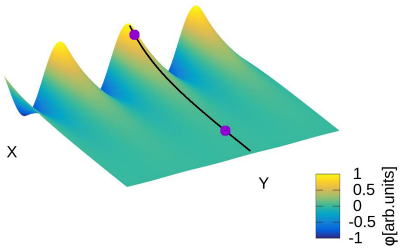
→ electrons may "surf" the SP

Simple model of SP acceleration - I

SP field on the vacuum side is electrostatic in the wave frame S' moving with phase velocity $\beta_\mathrm{p} = v_\mathrm{p}/c$ with respect to S (lab)
Electrostatic potential in S':

$$\Phi' = -\left(\frac{\gamma_\mathrm{p} E_\mathrm{SP}}{k}\right) \mathrm{e}^{k' x} \sin k' y' \qquad k' = \frac{k}{\gamma_\mathrm{p}} \qquad \gamma_\mathrm{p} = (1-\beta_\mathrm{p}^2)^{-1/2}$$

The motion is 2D: the energy gain depends on the “kick angle” from the top of the potential hill



Simple model of SP acceleration - II

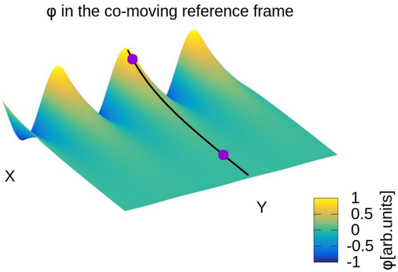
Assume as the most likely case an electron going downhill along the x-direction and acquiring an energy $W' = eE_{\mathrm{SP}}/k'$

$$W \simeq \gamma_{\mathrm{p}} W' \simeq m_e c^2 a_{\mathrm{SP}} \frac{\omega_p^2}{\omega^2} \qquad (a_{\mathrm{SP}} = eE_{\mathrm{SP}}/m_e \omega c)$$

with ejection angle in L (for $W' \gg m_e c^2$)

$$\tan\phi_e = \frac{p_x}{p_y} \simeq \frac{1}{\gamma_{\mathrm{p}}}$$

→ high energy electrons are beamed near the surface ($\tan\phi_e \ll 1$)



Observation of “surfing” acceleration on a SP

PRL **116,** 015001 (2016) PHYSICAL REVIEW LETTERS week ending 8 JANUARY 2016

Electron Acceleration by Relativistic Surface Plasmons in Laser-Grating Interaction

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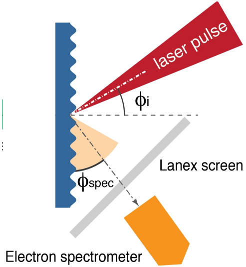
(Received 30 June 2015; published 7 January 2016)

LaserLAB experiment at SLIC, CEA Saclay

UHI laser: 25 fs pulse, 5×10^{19} Wcm^{-2}, $a_0 = 4.8$

contrast $\gtrsim 10^{12}$ at 5 ps

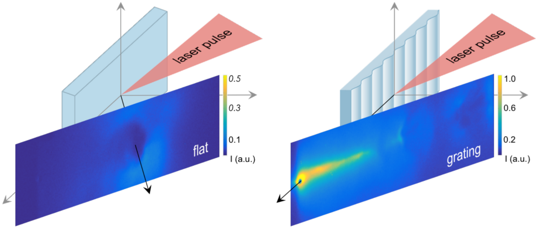
Experimental results

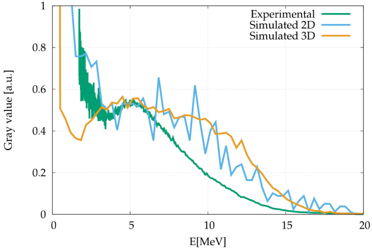


collimated ($\simeq 20^\circ$ cone) electron emission near the surface tangent ($\phi \simeq 2^\circ$)

multi-MeV energy, total charge $\simeq 100$ pC

Excellent agreement with 3D simulations





References

- A. Macchi, *Plasma waves in a different frame*, Am. J. Phys. **88** (2020) 723
- A. Macchi, *A Superintense Laser-Plasma Interaction Theory Primer* (Springer, 2013), sec. 2.3, 3.6 & 4.1
- A. Macchi, G. Cantono, L. Fedeli, F. Pisani, T. Ceccotti, *Extreme high field plasmonics: electron acceleration and XUV harmonic generation from ultrashort surface plasmons*, Phys. Plasmas **26** (2019) 042114