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Radiation Friction Modeling and Effects in Laser-Plasma Interaction at Extreme Intensities

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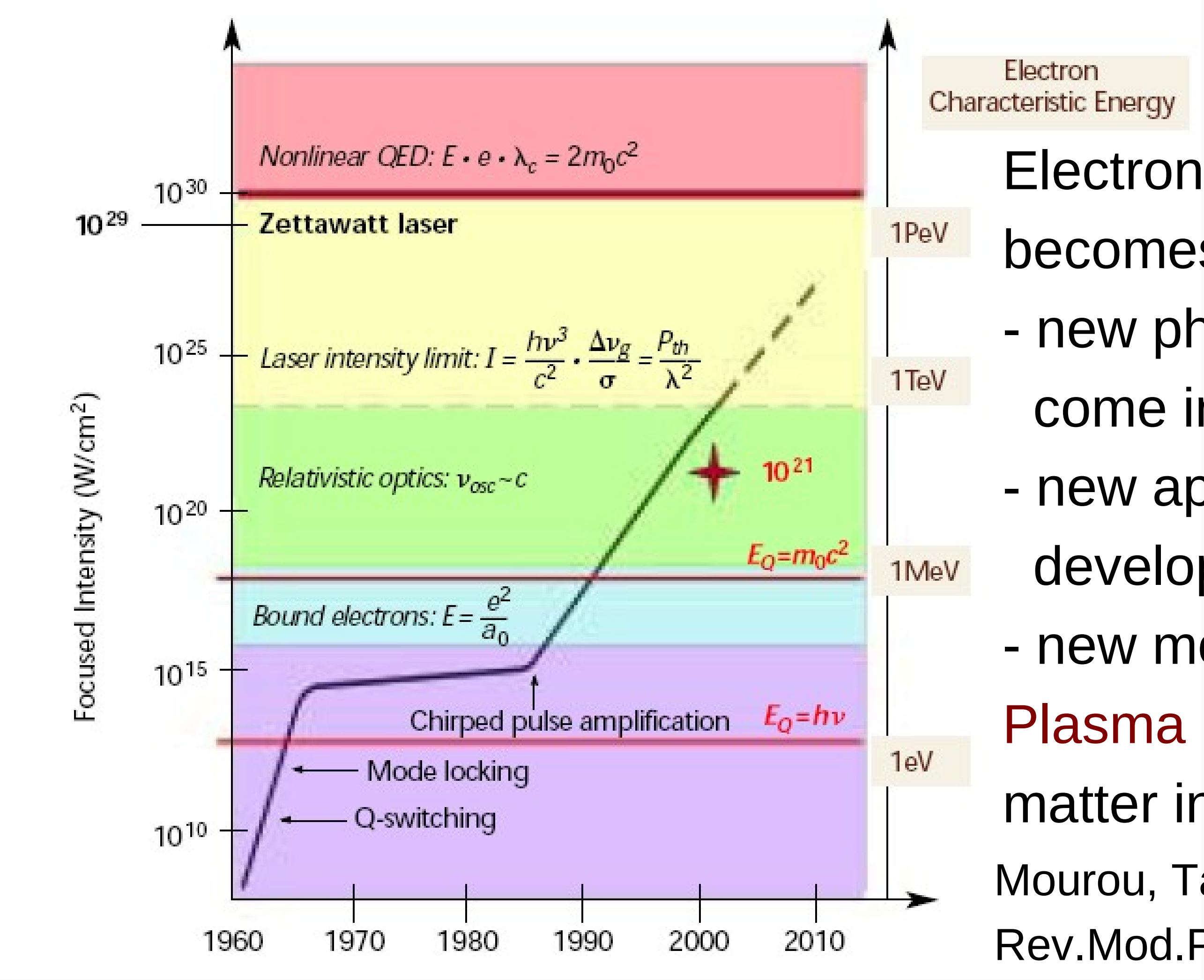
OUTLOOK

1. Motivations
- Ultra-relativistic laser-plasma interactions
- Radiation Pressure Acceleration: a route to GeV ions?
- Radiation Friction (or Reaction)

2. Modeling
- Landau-Lifshitz equation
- Single particle tests and inclusion in PIC codes

3. Simulations
- Importance of Radiation Friction for Radiation Pressure Acceleration
- Polarization, anisotropy, and numerical effects

ULTRARELATIVISTIC LASER PLASMAS



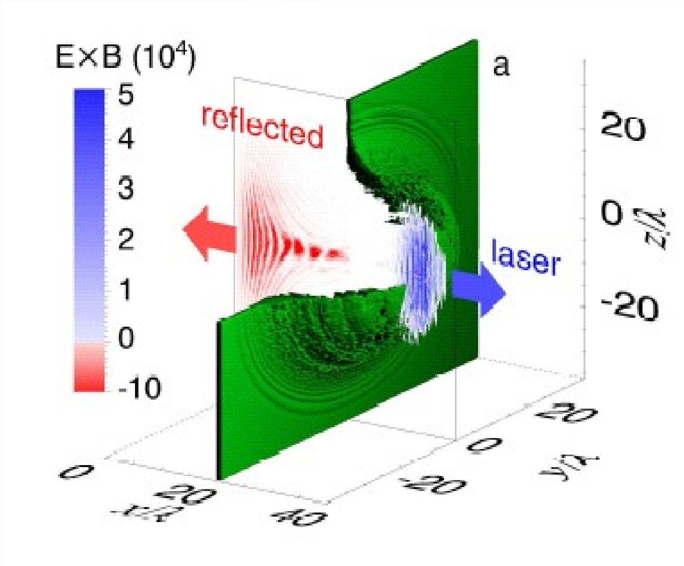
Electron quiver motion becomes ultrarelativistic:

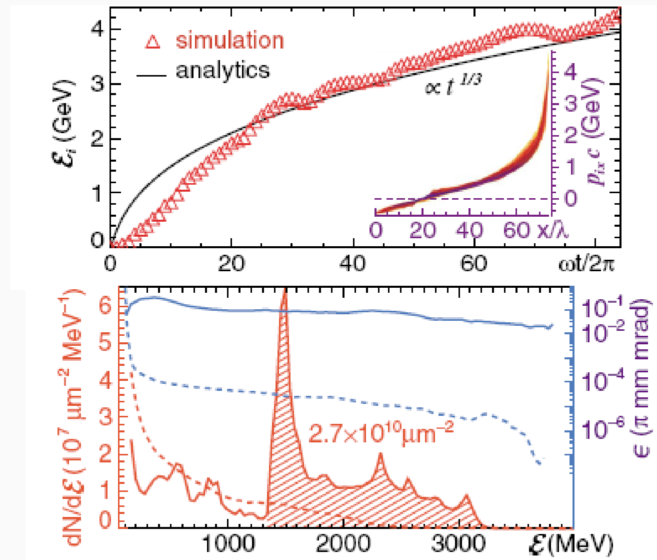
- new physical phenomena come into play
- new applications can be developed
- new models are needed

Plasma is the “only” state of matter in such fields!

Mourou, Tajima, Bulanov, Rev.Mod.Phys. **78** (2006) 309

RELATIVISTIC ION ACCELERATION





Simulations show dominant Radiation Pressure Acceleration (RPA) in interactions at $I > 10^{23}\ \mathrm{W/cm^2}$ with thin plasma foils leading to efficient acceleration of relativistic (GeV) ions

T.Esirkepov et al, *PRL* **92** (2004) 175003

(It may be possible at lower intensities with circularly polarized pulses: see e.g. Macchi et al, *New J. Phys*. **12** (2010) 045013 and references therein)

TWO RPA-BASED VISIONS (1996 - 2010)

22 NATURE JULY 2, 1966 VOL. 211

INTERSTELLAR VEHICLE PROPELLED BY TERRESTRIAL LASER BEAM

By PROF. G. MARX
Institute of Theoretical Physics, Roland Eötvös University, Budapest

PRL 104, 135003 (2010) PHYSICAL REVIEW LETTERS week ending 2 APRIL 2010

Unlimited Ion Acceleration by Radiation Pressure

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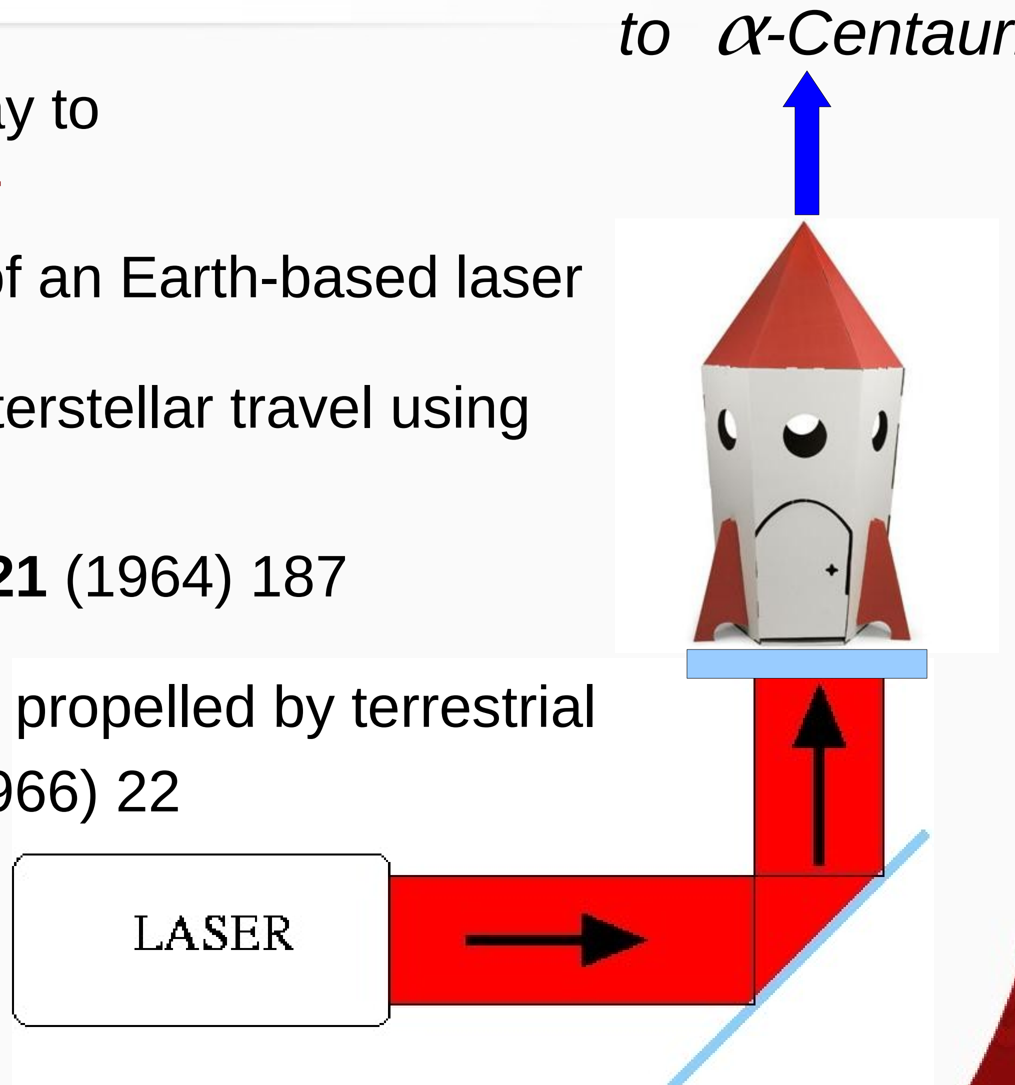
(Received 18 November 2009; published 2 April 2010)

THE “LIGHT SAIL” CONCEPT

Originally proposed as a way to
accelerate a massive mirror
by the Radiation Pressure of an Earth-based laser

R.L.Forward, “Roundtrip interstellar travel using laser-pushed lightsails”,
J. Spacecraft and Rockets **21** (1964) 187

G.Marx, “Interstellar vehicle propelled by terrestrial laser beam”, Nature **211** (1966) 22



RADIATION FRICTION (REACTION)

Radiation Reaction (RR): back-action on a point-like charge (electron) of the EM fields generated by the charge itself (important for extreme accelerations and ultrarelativistic particles)

[Jackson, *Classical Electrodynamics*, 3rd Ed., Chap.16;*
Landau & Lifshitz, *The Classical Theory of Fields,* Chap.9, Sec.75-76]

A "RR" term must be added to the Lorentz force to make the motion consistent with the Larmor formula for the radiated power

RR is a long-lasting and controversial problem which has attracted new interest recently:

See e.g. F. Rohrlich, Phys. Lett. A **303** (2002) 307 ;
Gralla et al, *PRD* **80** (2009) 24031;
Sokolov et al, *Phys.Plasmas* **16** (2009) 93115

*"*a completely satisfactory classical treatment does not exist ... the fundamental problem is not solved*" (Jackson)

RADIATION REACTION EFFECTS

RR is important for ultra-relativistic particles in EM fields:
in next generation experiments at ultra-high intensities RR can both play an important role and be diagnosed for the first time
[see e.g. Di Piazza et al, *PRL* **102** (2009) 254802;
Hadad et al, *PRD* **82** (2010) 096012]

The typical intensity for relevant RR effects is estimated to be $\sim 10^{23}$ W/cm^2 . This corresponds, to the foreseen regime of RPA dominance (for Linear Polarization)
[Esirkepov et al, PRL **92** (2004) 175003]

Two RPA simulation studies (for Circular Polarization at lower intensity) suggested a “beneficial” RR effect of “electron cooling”
[Schlegel et al, PoP **16** (2009) 083103;
Chen et al, Plasma Phys. Contr. Fus. **53** (2011) 014004]

RADIATION REACTION MODELING

Search for an approach suitable for inclusion in laser-plasma Particle-In-Cell (PIC) simulations:

- Consistent with a classical, mean field approach (i.e. Vlasov-Maxwell system)
- Reasonable compromise between simplicity and accuracy
- Possible benchmark with some exact solutions
- Inclusion in PIC in a modular way – no strong modification of the code needed
- Computationally cheap (not to let CPU time explode ...)

Our “textbook” choice: Landau-Lifshitz approach

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THE LANDAU-LIFSHITZ FORCE -I

“Naivest” approach to RR leads to the Lorentz-Abraham-Dirac equation with pathological solutions

LL approach: approximate (iterative) procedure to write the RR force as a function of the EM fields only

$$f^{\mu}_{\mathrm{RR}} = \frac{2e^3}{3mc^2}\left(\partial_\alpha F^{\mu\nu} u_\nu u^\alpha\right) + \frac{2e^4}{3m^2c^4}\left(F^{\mu\nu}F_{\nu\alpha}u^\alpha + (F^{\nu\beta}u_\beta F_{\nu\alpha}u^\alpha)u^\mu\right)$$

Notice: in relativistic regimes in principle one has also to take the coupling with the electron *spin* into account

$$f^{\mu}_{S} = -\frac{1}{2}Q^{\gamma\delta}\partial^\mu F_{\gamma\delta} + \frac{1}{2}\left(Q^{\gamma\delta}\partial_\lambda F_{\gamma\delta}u^\lambda\right)u^\mu$$

$$Q^{\gamma\delta} = \varepsilon^{\gamma\delta\alpha\beta}u_\alpha m_\beta\,, \qquad m_\beta \text{ magnetic moment}$$

THE LANDAU-LIFSHITZ FORCE - II

Equation of Motion with LL force in non-covariant notation (and dimensionless units)

$$\frac{d\mathbf{p}}{dt} = -(\mathbf{E}+\mathbf{v}\times\mathbf{B}) + \mathbf{F}_{\mathrm{RR}}$$

$$\begin{aligned}\mathbf{F}_{\mathrm{RR}} = & -\left(\frac{4\pi}{3}\frac{r_e}{\lambda}\right)\times \\ & \left\{\gamma^2\left[(\mathbf{E}+\mathbf{v}\times\mathbf{B})^2-(\mathbf{v}\cdot\mathbf{E})^2\right]\mathbf{v}\right. \\ & -\left[(\mathbf{E}+\mathbf{v}\times\mathbf{B})\times\mathbf{B}+(\mathbf{v}\cdot\mathbf{E})\mathbf{E}\right] \\ & \left.+\gamma\left[(\partial_t+\mathbf{v}\cdot\nabla)\mathbf{E}+\mathbf{v}\times(\partial_t+\mathbf{v}\cdot\nabla)\mathbf{B}\right]\right\}\end{aligned}$$

The first “friction” term ($\sim -\gamma^2\mathbf{v}$) is usually the dominant one

KINETIC EQUATION

$$d_t f_e = \partial_t f_e + \boldsymbol{\nabla}_{\mathbf{x}} \cdot (\dot{\mathbf{x}} f_e) + \boldsymbol{\nabla}_{\mathbf{p}} \cdot (\dot{\mathbf{p}} f_e) = 0$$

$$\dot{\mathbf{p}}_e = -e(\mathbf{E}+\mathbf{v}\times\mathbf{B})+\mathbf{F}_{\mathrm{RR}}, \qquad \dot{\mathbf{x}}_e = \frac{\mathbf{p}_e}{m_e\gamma_e} = \frac{\mathbf{p}_e}{\sqrt{\mathbf{p}_e^2 + m_e^2}},$$

$$\boldsymbol{\nabla}_{\mathbf{p}} \cdot (\mathbf{F}_{\mathrm{RR}} f_e) \neq \mathbf{F}_{\mathrm{RR}} \cdot \boldsymbol{\nabla}_{\mathbf{p}} f_e$$

This is *not* the usual Vlasov (collisionless) equation, but the PIC method provides a solution anyway

It can be shown that $\boldsymbol{\nabla}_{\mathbf{p}} \cdot \mathbf{F}_{\mathrm{RR}} < 0$
and leads to phase space volume “contraction”, i.e. “cooling”

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OTHER APPROACHES - I

Sokolov et al, *Phys.Plasmas* **16** (2009) 93115

$$d_t\mathbf{p} = \mathbf{F}_L - \delta\mathbf{v}\times\mathbf{B} - \gamma^2(\delta\mathbf{v}\cdot\mathbf{F}_L)\mathbf{v}, \qquad d_t\mathbf{x} = \mathbf{v} + \delta\mathbf{v}$$

$$\mathbf{F}_L = -(\mathbf{E}+\mathbf{v}\times\mathbf{B}), \qquad \delta\mathbf{v} = \tau_0\frac{\mathbf{F}_L - \mathbf{v}(\delta\mathbf{v}\cdot\mathbf{F}_L)}{1-\tau_0(\delta\mathbf{v}\cdot\mathbf{F}_L)}$$

Equations of Motion for “ ‘particles’ consisting of a large number of electrons, radiating independently and incoherently“

notice: $d_t\mathbf{x} \neq \mathbf{v}$

- The EoMs are solved for the *computational* particles in the PIC approach

OTHER APPROACHES - II

Chen et al, *Plasma Phys. Contr. Fus.* **53** (2011) 014004

$$\mathbf{F}_{\mathrm{RR}} = d_t \mathbf{p}_{\mathrm{RR}} = -\frac{2r_c}{3\lambda}\gamma^2 \left(d_t \mathbf{p}\right)^2 \frac{\mathbf{v}}{v}, \qquad d_t \mathbf{p} = -(\mathbf{E}+\mathbf{v}\times\mathbf{B})$$

“Synchrotron” approximation for the emitted radiation, equivalent to the main “force” term in Sokolov et al.’s

$$d_t \mathbf{p} \simeq -\gamma^2 (\delta \mathbf{v} \cdot \mathbf{F}_L)\mathbf{v}$$

BENCHMARK WITH ANALYTICAL SOLUTION

Exact solution of the Landau-Lifshitz equation in a plane wave
[A.Di Piazza, Lett.Math.Phys. **83** (2008) 305]

Based on this test case we identify suitable approximations to the electron EoM with RR included:

- the spin force is $\sim 137\gamma$ X the last LL term in the RR force
- the first term is $\sim a_0 \omega\tau / 137$ X the spin force

→ for intensities >> 10^{22} W/cm^2 it is consistent to neglect both the last term and the spin force ; we keep only two terms (which include Chen et al's term and conserve the *mass-shell* condition)

Warning: at ~10^{24} W/cm^2 quantum effects begin to play a role

NUMERICAL IMPLEMENTATION

The advancement of electron momentum (“particle pushing”) is split into two “leap-frog” steps : one for the Lorentz force and one for the RR

$$\frac{\mathbf{p}^{(n+1/2)} - \mathbf{p}^{(n-1/2)}}{\Delta t} = \mathbf{f}^{(n)} = \mathbf{f}_L^{(n)} + \mathbf{f}_{\mathrm{RR}}^{(n)}$$

$$\mathbf{p}_L^{(n+1/2)} \equiv \mathbf{p}^{(n-1/2)} + \mathbf{f}_L^{(n)}\Delta t\,, \quad \mathbf{p}_{\mathrm{RR}}^{(n+1/2)} \equiv \mathbf{p}^{(n-1/2)} + \mathbf{f}_{\mathrm{RR}}^{(n)}\Delta t$$

$$\mathbf{p}^{(n+1/2)} = \mathbf{p}_L^{(n+1/2)} + \mathbf{p}_{\mathrm{RR}}^{(n+1/2)} - \mathbf{p}^{(n-1/2)}$$

The Lorentz step is based on the standard PIC integrator (“Boris pusher”)

In the RR step, we use the momentum updated by the Lorentz contribution to estimate the velocity at intermediate step

The approach is simple, modular, and suitable for all spatial dimensions (the momentum space is three-dimensional in all PIC codes)

TEST OF PARTICLE PUSHER - I

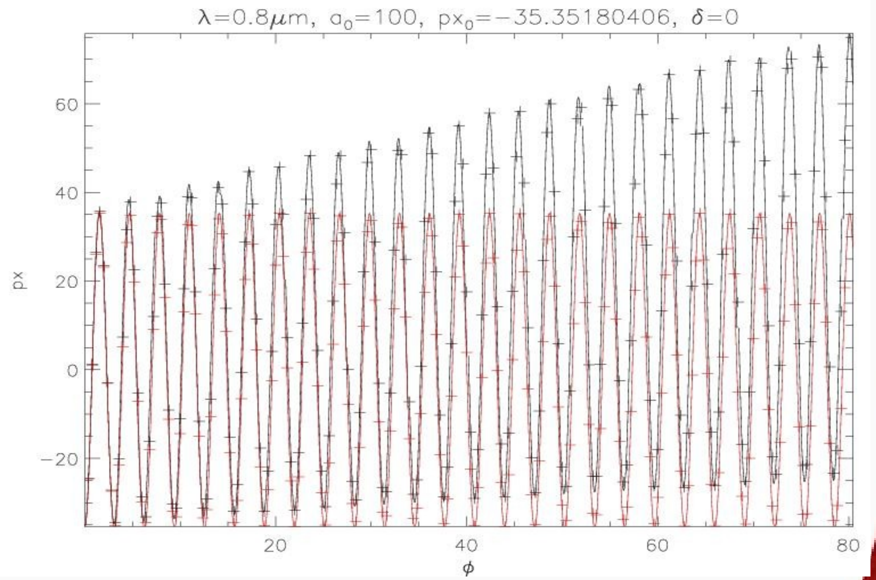
A numerical solution of motion in a plane wave based on simple 2nd order leap-frog method has been compared with the exact solution and with 4th order Runge-Kutta integration

- excellent agreement for intensities up to 10^{24} W/cm^2
- straightforward to include in a "standard" PIC code (based on Boris particle pusher)
- only ~10% increase in CPU time

[M.Tamburini, F.Pegoraro, A. Di Piazza, C.H.Keitel, A.Macchi,
New J. Phys. **10** (2010) 123005]

Black – with RR   Line: analytical

Red → without RR   Crosses: numerical



TEST OF PARTICLE PUSHER - II

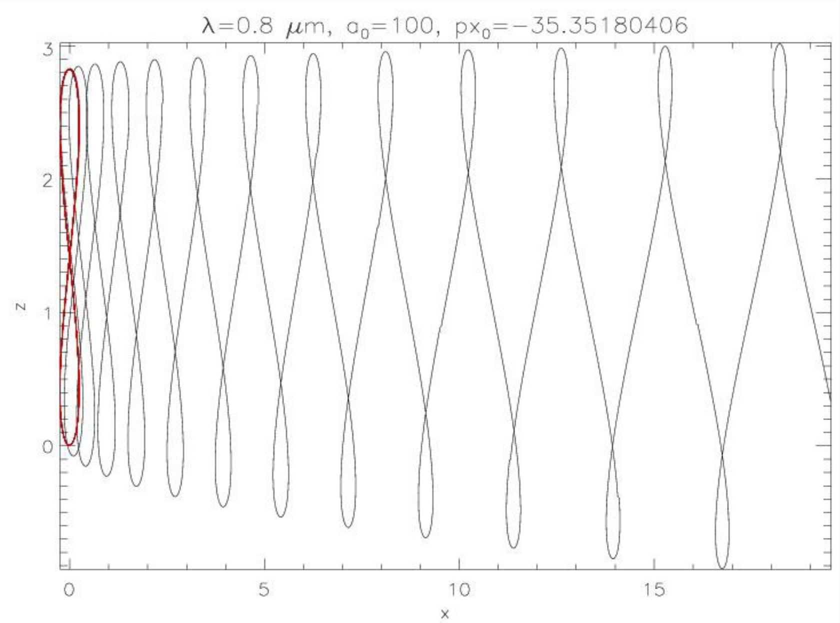
A numerical solution of motion in a plane wave based on simple 2nd order leap-frog method has been compared with the exact solution and with 4th order Runge-Kutta integration

- excellent agreement for intensities up to 10^{24} W/cm^2
- straightforward to include in a “standard” PIC code
(based on Boris particle pusher)
- only ~10% increase in CPU time

[M.Tamburini, F.Pegoraro, A. Di Piazza, C.H.Keitel, A.Macchi,
New J. Phys. **10** (2010) 123005]

Black – with RR

Red → without RR



TEST OF PARTICLE PUSHER - III

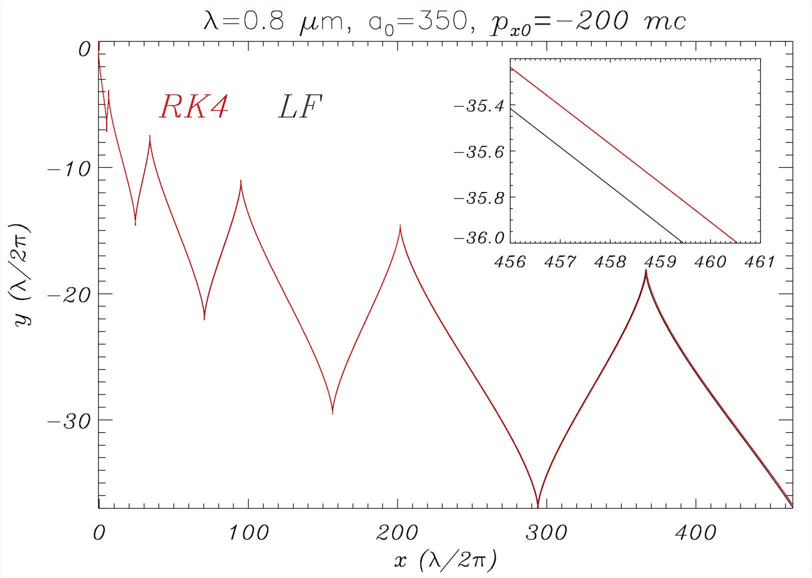
A numerical solution of motion in a plane wave based on simple 2nd order leap-frog method has been compared with the exact solution and with 4th order Runge-Kutta integration

- excellent agreement for intensities up to 10^{24} W/cm^2
- straightforward to include in a "standard" PIC code (based on Boris particle pusher)
- only ~10% increase in CPU time

[M.Tamburini, F.Pegoraro, A. Di Piazza, C.H.Keitel, A.Macchi,
New J. Phys. **10** (2010) 123005]

Black – Leap Frog

Red → Runge-Kutta



THE ENERGY BALANCE

The RR force acts a a friction: energy is ***not*** conserved in the system.

The friction effect physically corresponds to the emission of high-frequency radiation by ultra-relativistic electrons.

The high-frequency radiation is consistently assume to be incoherent and escape from the plasma

The “energy loss by radiative effects” is evaluated by comparing the energy balance in simulations with and without RR (energy is typically conserved by ~1% in the 1D PIC code without RR)

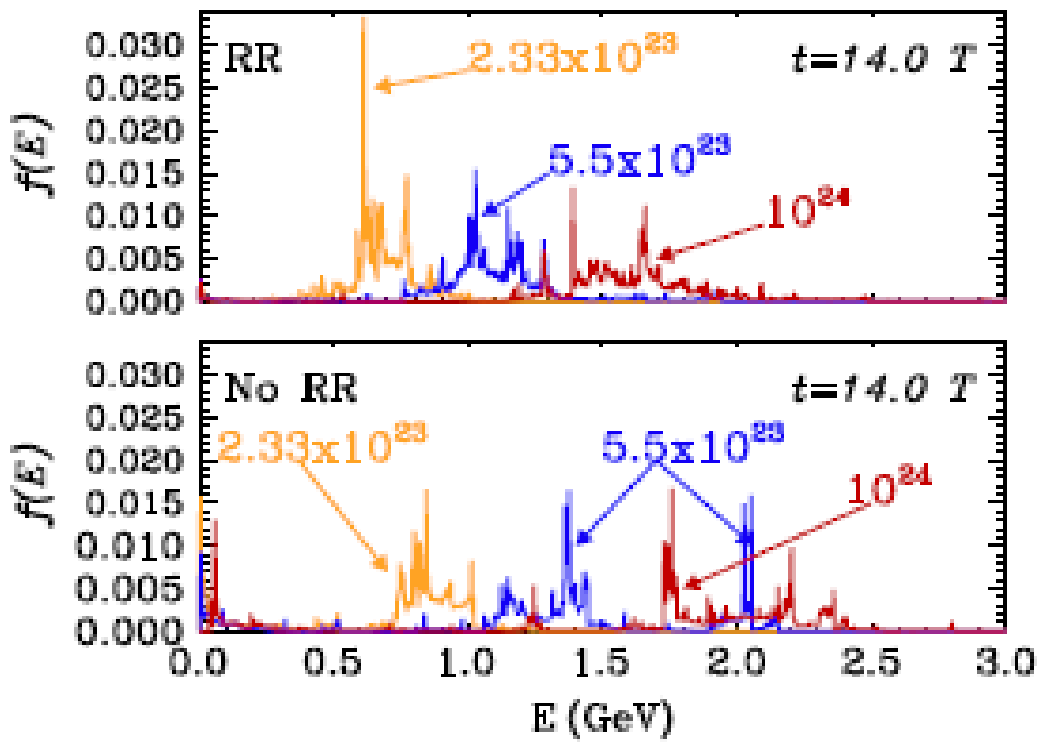
Note: energy and momentum are ***not*** conserved exactly in the LL EoM for the *single* electron, but the neglected terms are smaller than quantum contributions

1D PIC SIMS: LINEAR POLARIZATION

Radiation Pressure Acceleration of a thin foil

(0.2-1.0)x 10^{24} W/cm^2 , 11 cycles pulse, 1μm foil, 100n_c ,

linear polarization (LP)



Lower energy, narrower spectrum when RR included

Warning:
highest intensity case (10^{24} W/cm^2) possibly beyond classical limit

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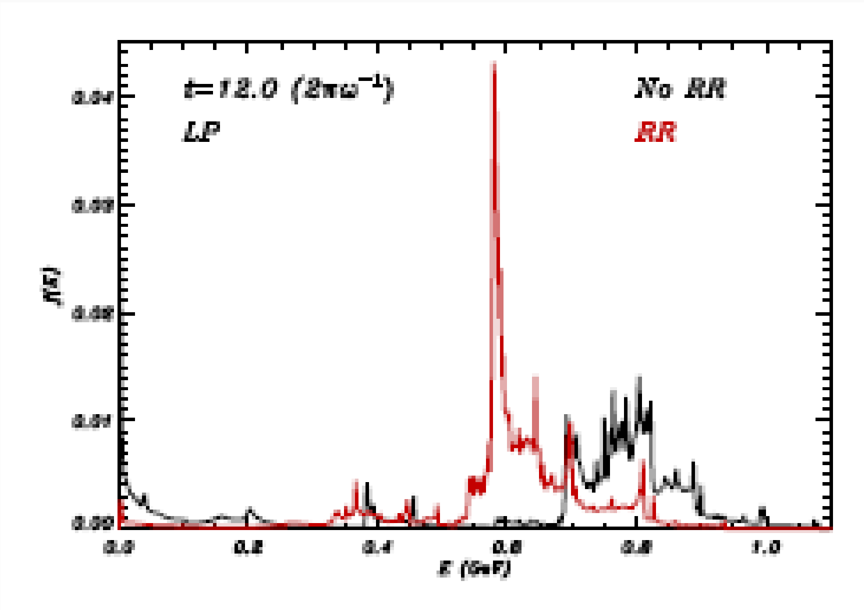
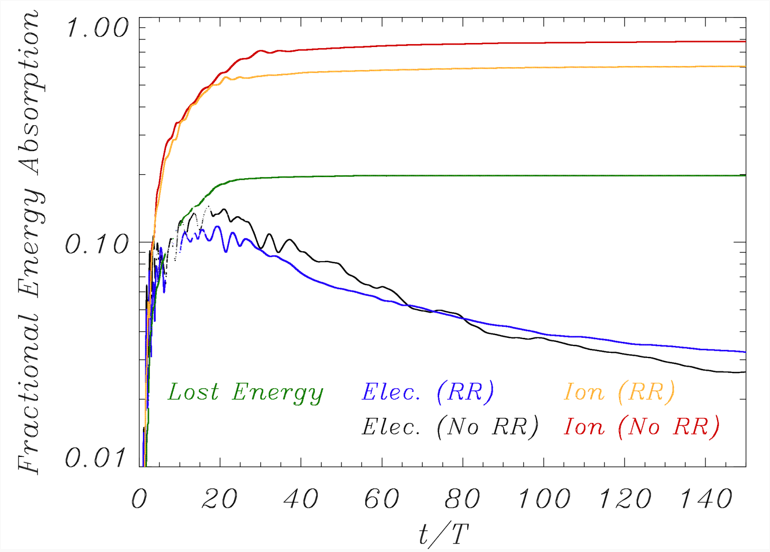
ENERGY LOSS FOR LINEAR POLARIZATION

2.3x 10^{23} W/cm^2 , 11 cycles pulse, 1μm foil, 100n_c , LP





~25% reduction in “peak” ion energy “ due to RR effects

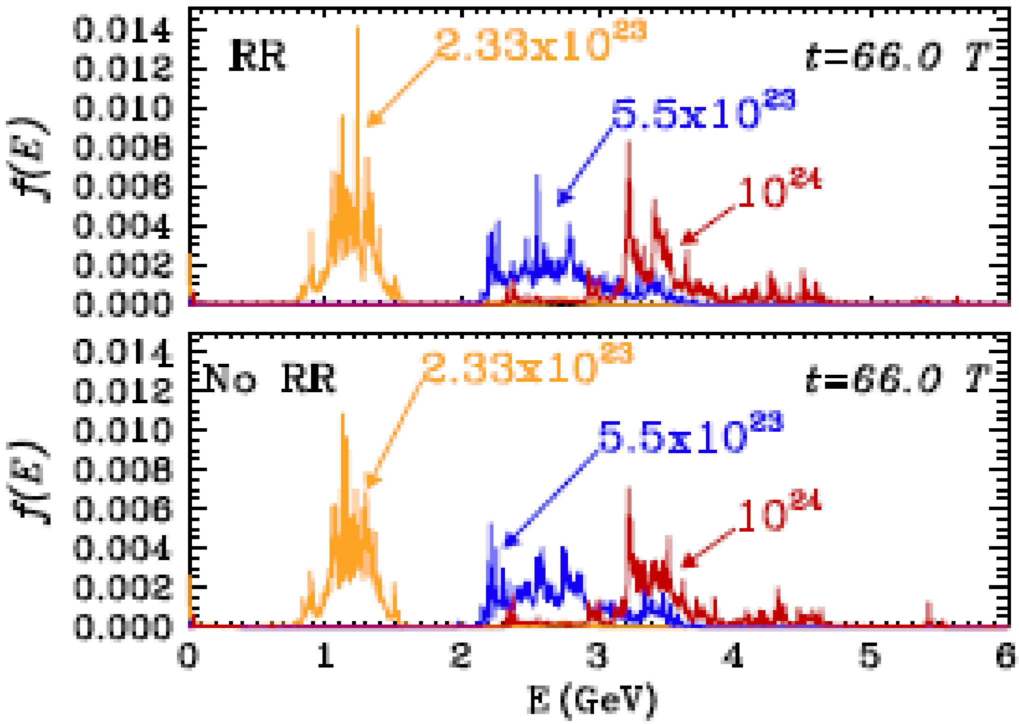
~20% pulse energy “radiated away” when RR is included

CIRCULAR POLARIZATION

Same parameters as Linear Polarization

(0.2-1.0)x 10^{24} W/cm^2 , 11 cycles pulse, 1µm foil, 100n_c ,

- circular polarization
- *below* threshold for pulse breakthrough

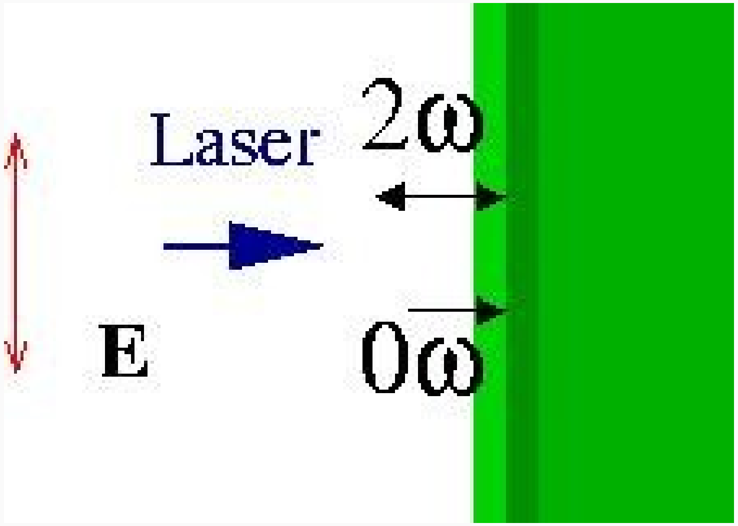


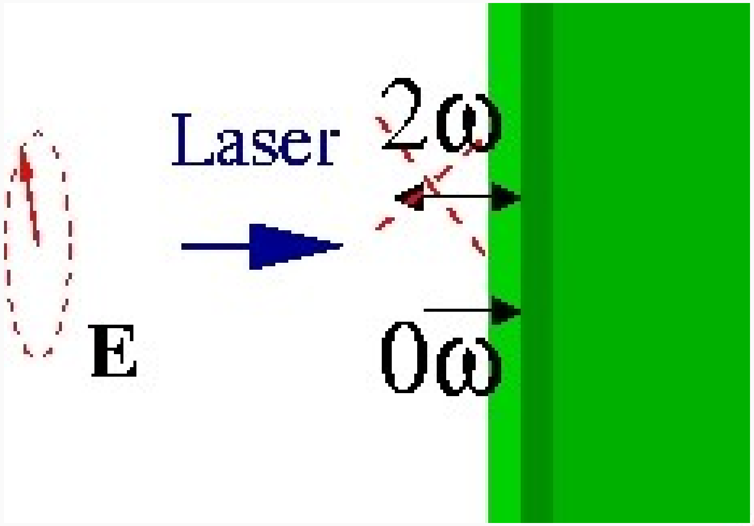
- Negligible RR effects on spectrum
- Higher ion energy than in LP case

DIFFERENCES BETWEEN LP AND CP

At normal incidence, for Linear Polarization the **v**×**B** force has a 2ω longitudinal component driving a push-and-pull motion of electrons across the surface, with strong heating

For Circular Polarization, the 2ω force is absent: steady push of the target





DISCUSSION: CP VS LP

Conjecture by Esirkepov et al: RR effects are not important in relativistic RPA because the RR strength parameter $\sim r_e/\lambda$ decreases in the frame of the foil target

For CP the foil moves coherently at $v_x \sim c$

The EM field is similar to a copropagating plane wave (reflection is small) and the LL force vanishes when $v_x = c$

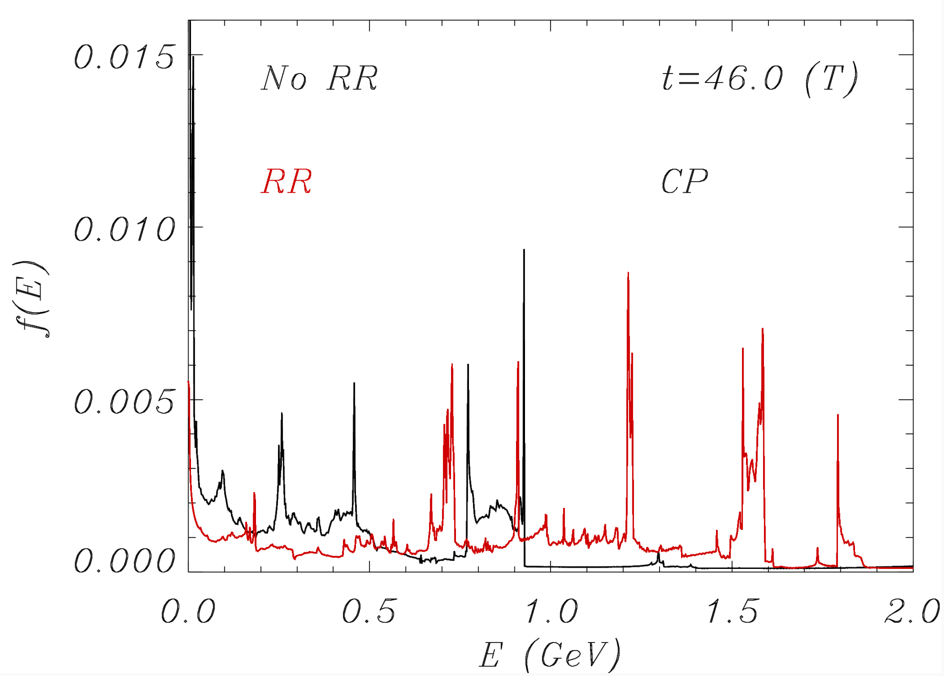
For LP the **v**X**B** force at 2ω drives strong longitudinal oscillations of electrons increasing the RR effect (the LL force in a plane wave is maximum for counterpropagating electrons: $v_x = -c$)

CASE OF PULSE BREAKTHROUGH

2.3x 10^{23} W/cm^2 , 11 cycles pulse, 0.3µm foil, 100n_c , CP

The pulse now penetrates through the thinner foil due to "relativistic" Self-Induced Transparency, driving strong longitudinal motion also for CP

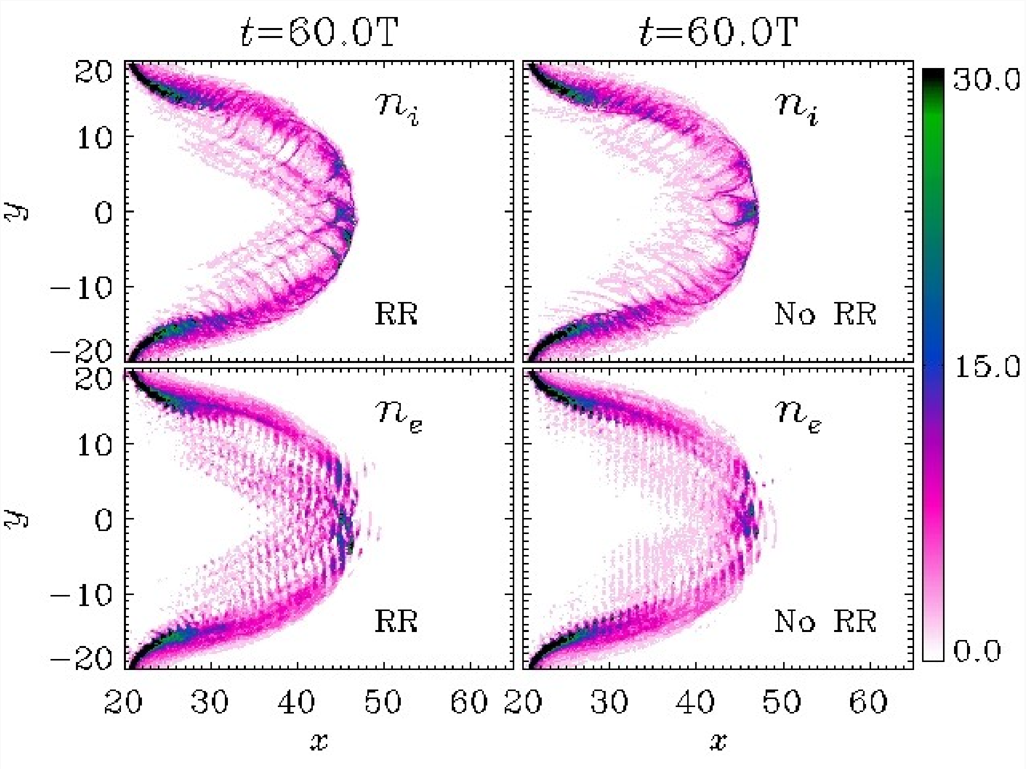
RR effects are now important for CP and *increase* the ion energy, but the parameters are *not* optimal (thicker foil has higher energy and narrower spectrum



TWO-DIMENSIONAL SIMULATIONS - CP

10^{23} W/cm^2 , 11 cycles pulse, 0.5μm foil, 169n_c ,

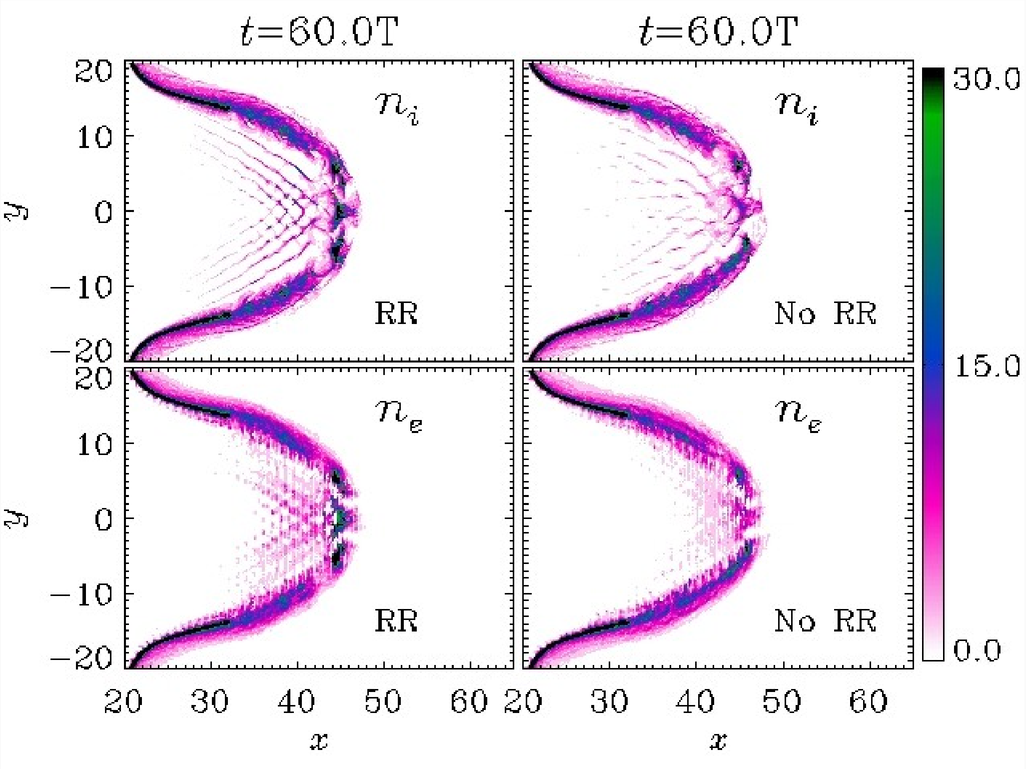
Circular Polarization



TWO-DIMENSIONAL SIMULATIONS – LP “S”

10^{23} W/cm^2 , 11 cycles pulse, 0.5μm foil, 169n_c ,

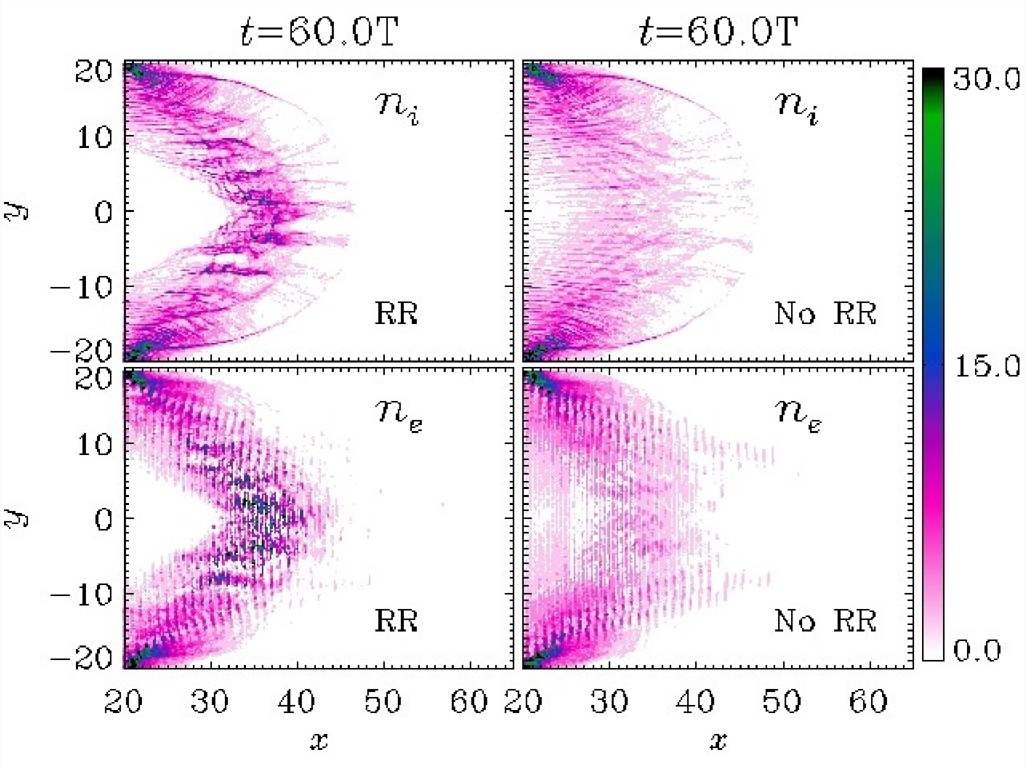
Linear “*S*” Polarization



TWO-DIMENSIONAL SIMULATIONS – LP "*P*"

10^{23} W/cm^2 , 11 cycles pulse, 0.5µm foil, 169n_c ,

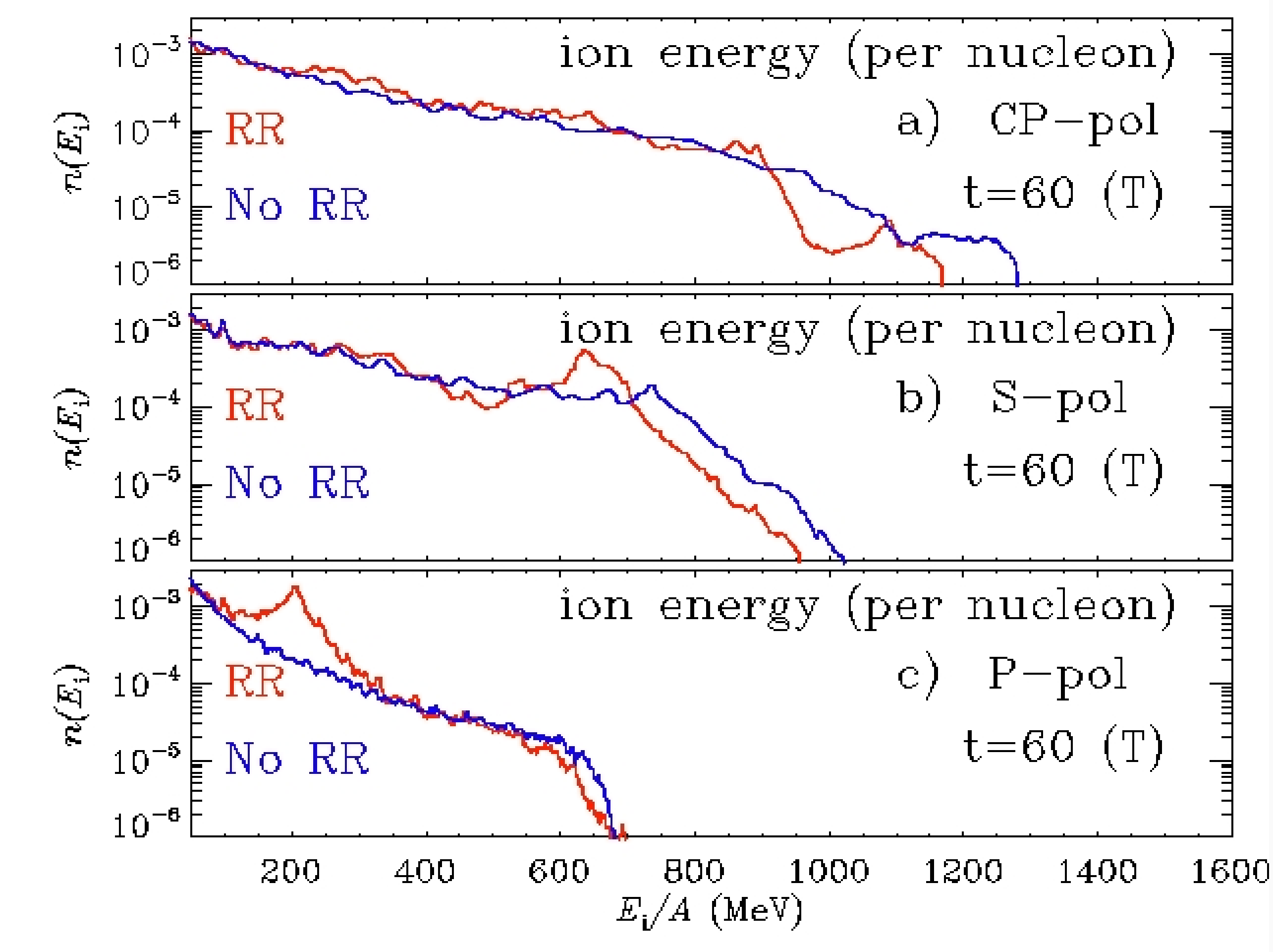
Linear *"P"* Polarization



ENERGY SPECTRA VS POLARIZATIONS

"Best" case for CP (higher cut-off energy)

For LP differences between *S* and *P* suggest strong anisotropy in the "real" 3D case



NUMERICAL EFFECTS

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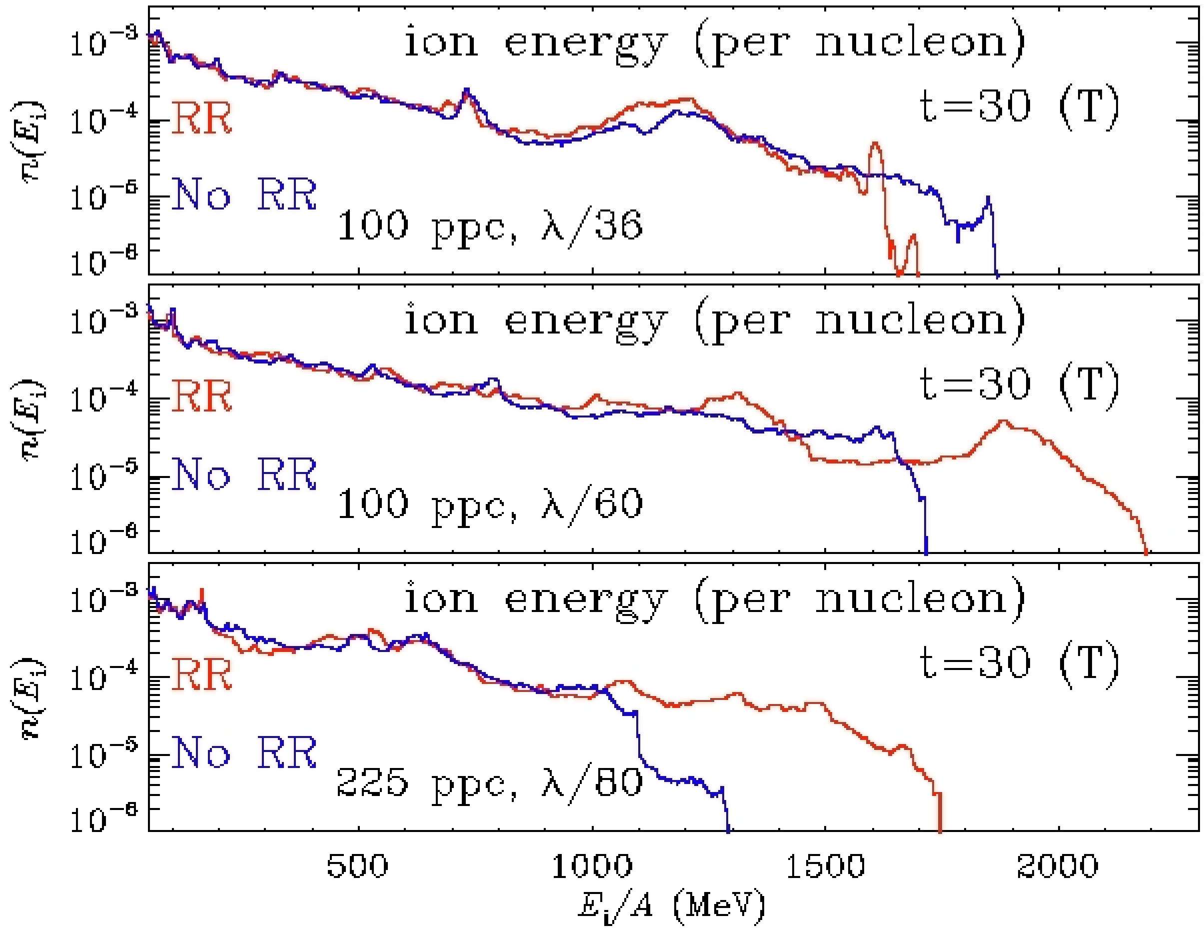
Some "convergence tests" preliminary to 3D simulations

ppc= particles per cell

$\lambda/\Delta x$=(35,60,80)

Results are sensitive to the total number of particles!!!



(RR effects *emphasize* the differences)

CONCLUSIONS

- A relatively “simple and cheap” model for inclusion of Radiation Reaction in PIC codes via the Landau-Lifshitz equation has been developed and tested
- in 1D, RR effects on Radiation Pressure Acceleration at ultrahigh intensities are important for Linear Polarization or in the Self-Induced Transparency regime; RR is negligible for Circular Polarization and “optimal” target thickness
- 2D simulations suggest strong anisotropy and polarization effects
- on the route to 3D simulations, numerical effects for too low number of particles must be considered with care

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(Italian SuperComputing Resource Allocation /
TOwards FUll-Scale Simulations of laser-plasma Experiments)

www.df.unipi.it/~macchi/TALKS/