

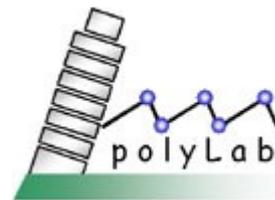
# *Laser-Plasma Physics Probed by Protons*

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CLPU, Universidad de Salamanca, January 13, 2010

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# Outline

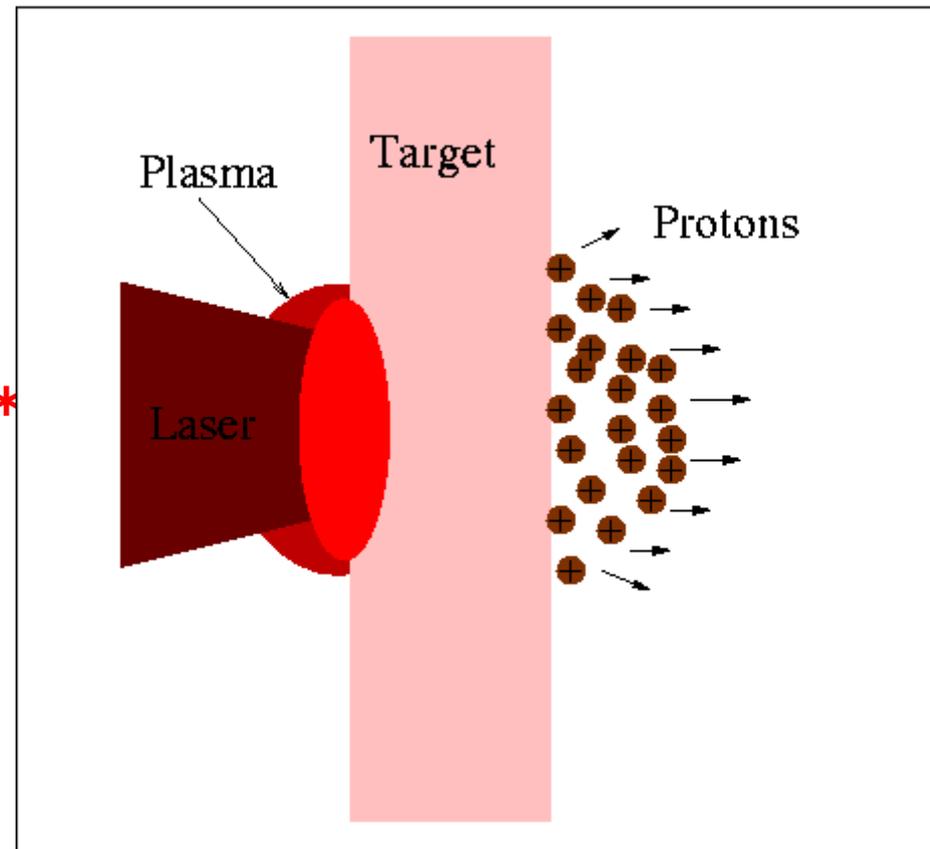
- Laser-Accelerated Protons as a probe of Laser-Plasma interactions
- Proton probing-based investigations
  - Plasma expansion and sheath acceleration
  - Pulse self-channeling in the charge-displacement regime
  - Ultrafast charging dynamics

# The discovery of MeV proton emission in superintense interaction with *metallic* targets

Reported in 2000  
by three experimental groups

[Clark et al, PRL **84** (2000) 670;  
Maksimchuk et al, *ibid.*, 4108;  
Snively et al, PRL 85 (2000) 2945\*

**\* -  $2 \times 10^{13}$  protons with  
energy  $> 10$  MeV,  
58 MeV cut-off,  
good collimation ( $20^\circ$ )  
conversion efficiency  
12% of laser energy  
@  $I=3 \times 10^{20}$  W/cm<sup>2</sup>**

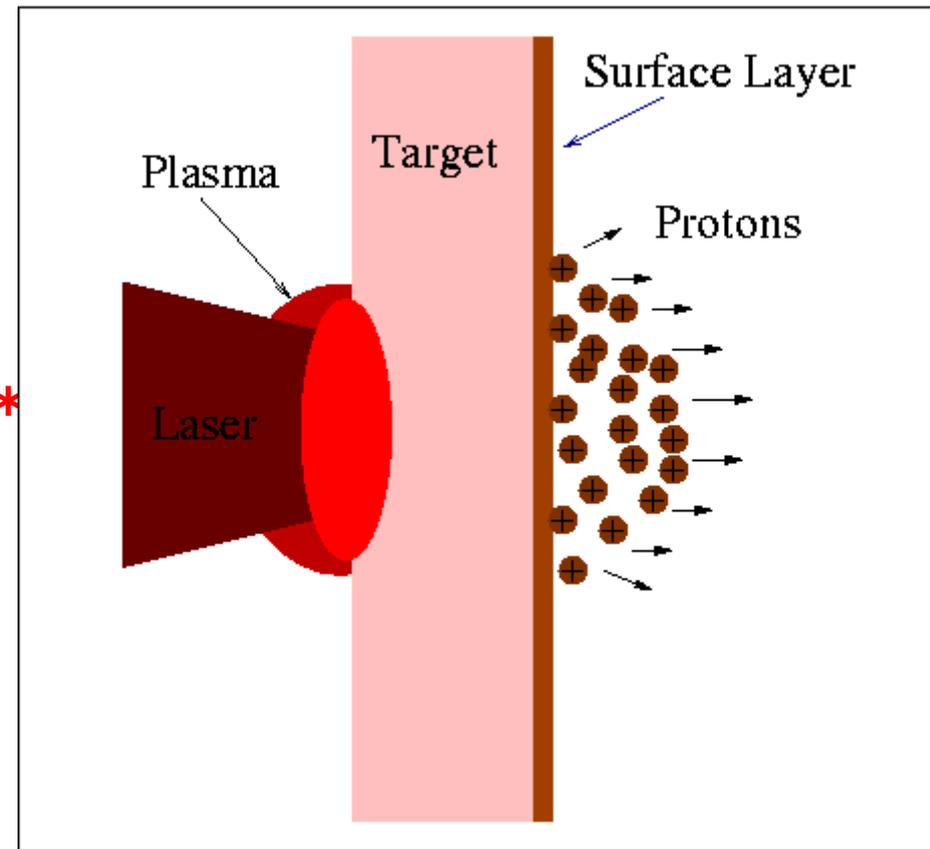


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*Question:* origin of protons from metal?

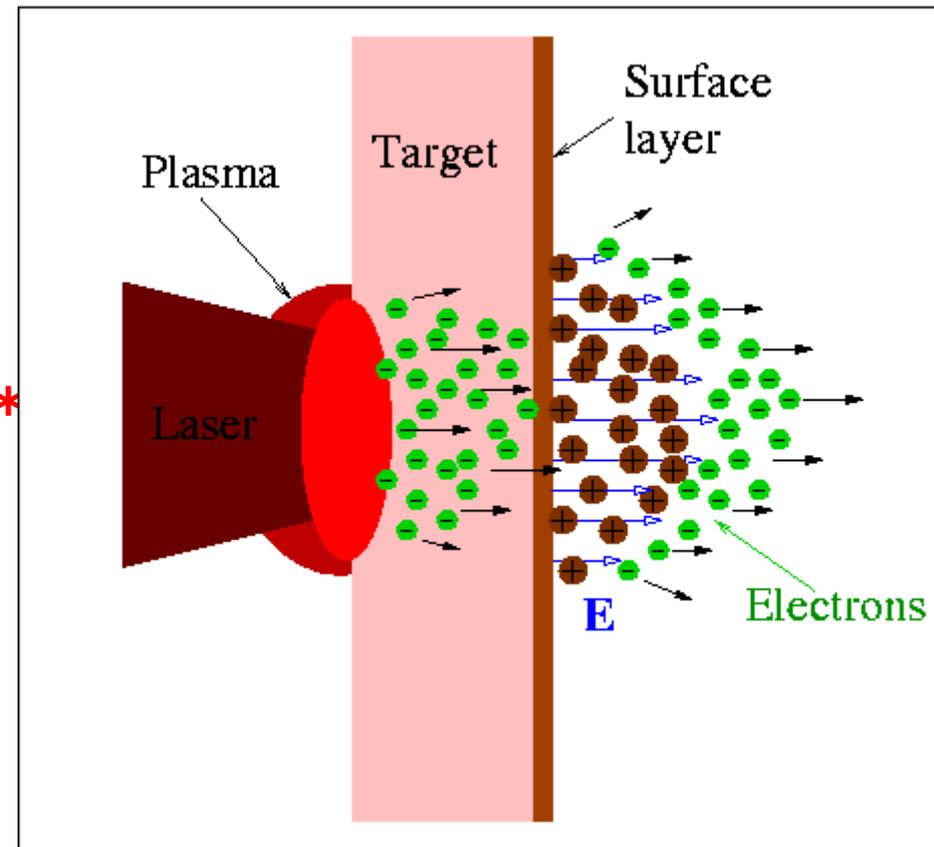
*Answer:* hydrocarbon or water impurities  
on the target surface  
(e.g. from vacuum pump ...)

# The discovery of MeV proton emission in superintense interaction with *metallic* targets

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*Physical mechanism:*  
acceleration in the space-charge  
electric field generated by  
“fast” electrons escaping from the target  
(TNSA, Target Normal Sheath Acceleration)

# TNSA (Target Normal Sheath Acceleration) model

After an intense debate on the Origin of Protons  
(*more in the following...*)

TNSA has been accepted as the basic mechanism leading to proton acceleration in **most of the reported experiments** and provided a framework for **source development and optimization**

Schworer et al, Nature **439** (2006) 445

Hegelich et al, Nature **439** (2006) 441

Toncian et al, Science **312** (2006) 410

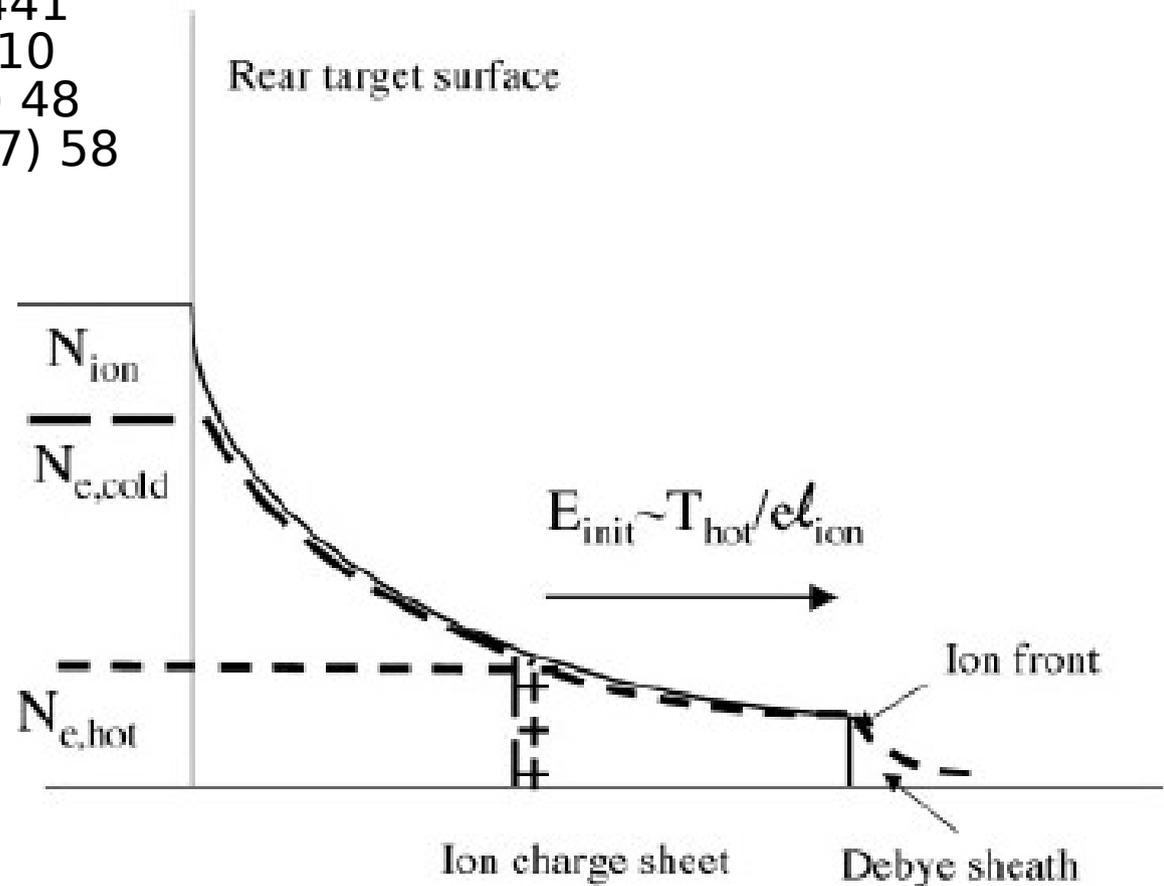
Fuchs et al, Nature Physics **2** (2005) 48

Robson et al, Nature Physics **3** (2007) 58

Kar et al, PRL **100** (2008) 105004

The TNSA picture accounts for the observed properties of the proton source

[picture from S. Wilks et al, Phys. Plasmas **8** (2001) 542]



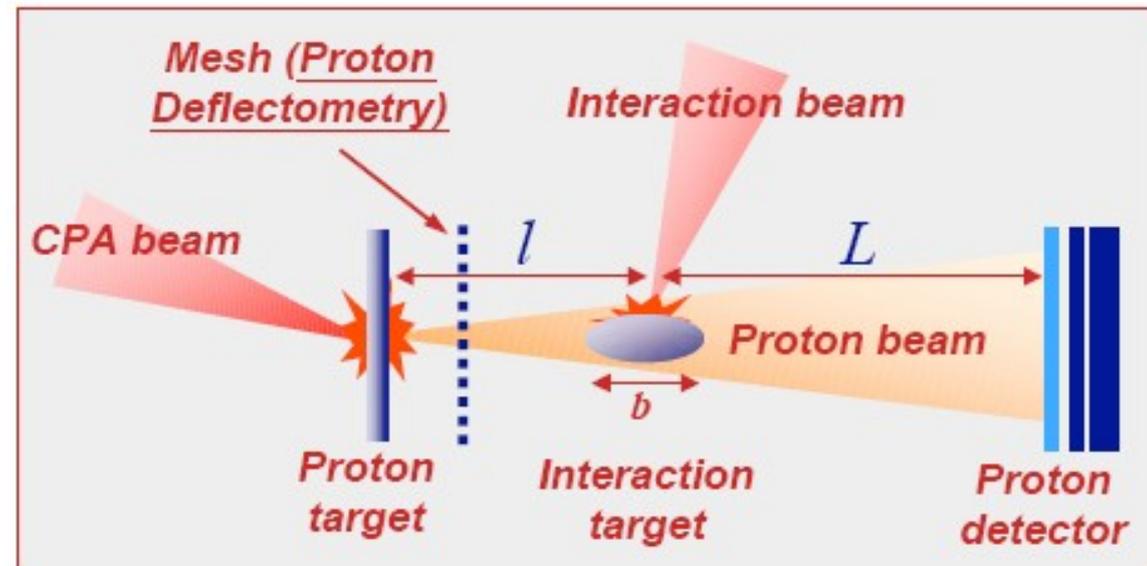
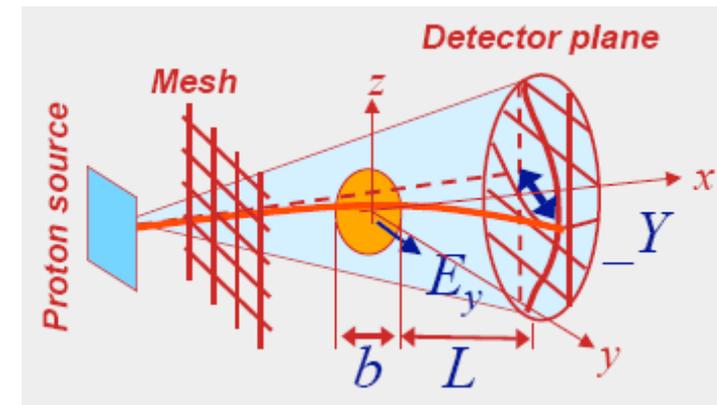
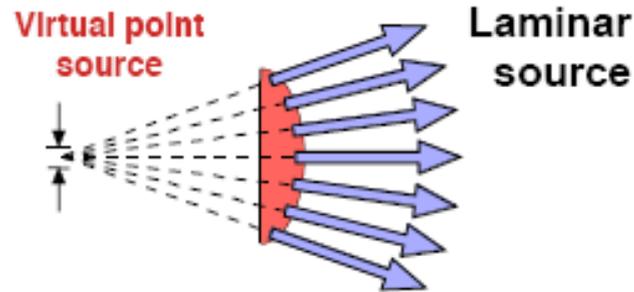
# Use of laser-accelerated protons as a probe of EM fields in laser-plasma interactions

Due to high laminarity the proton beam has **imaging properties**

The short duration of the proton burst allows **picosecond temporal resolution**

Protons of a given energy will cross the probed object at a particular time. An energy-resolving detector (e.g. Radiochromic Film) thus provides **multiframe capability**

In a laser-plasma experiment the proton probe is easily **synchronized with the interaction**

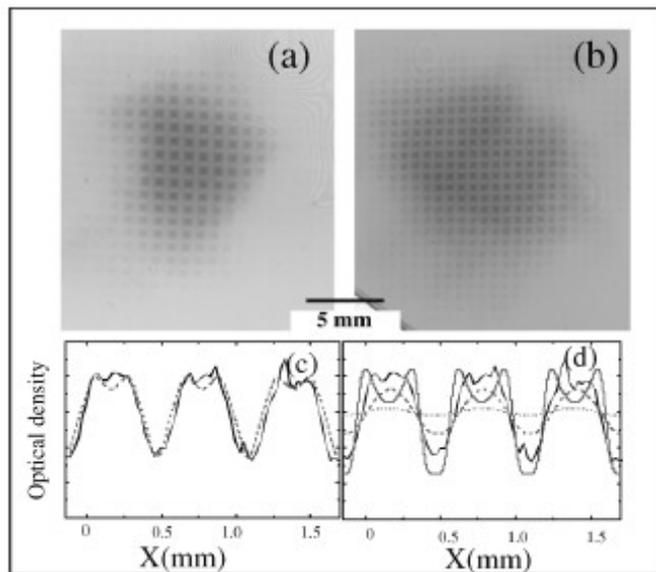
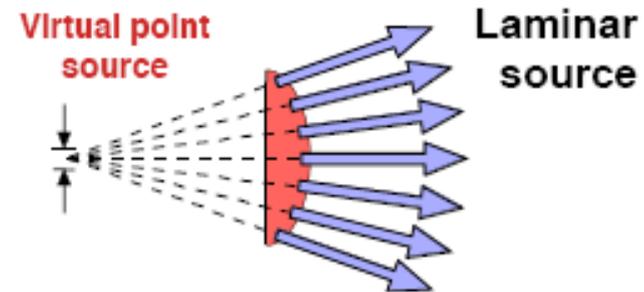


Borghesi et al, Phys.Plasmas **9** (2002) 2214  
Borghesi et al, Phys.Rev.Lett. **92** (2004) 055003  
Cowan et al, Phys.Rev.Lett. **92** (2004) 204851

# Properties of the proton source

Imaging properties of the emitted protons are those of a **point-like virtual source**

Proton beam is **quasi-laminar** (**ultra-low emittance**)



$\mu$  m grid mesh image

Shadow of **micrometric grid mesh** with **15 MeV** protons provides magnification test and estimate of source dimensions  $\alpha < 10 \mu\text{m}$  [Borghesi et al, PRL **92** (2004) 055003]

$\alpha = 1.4 \mu\text{m}$  reported at lower energy [Cobble et al, J.Appl.Phys. **92** (2002) 1775]

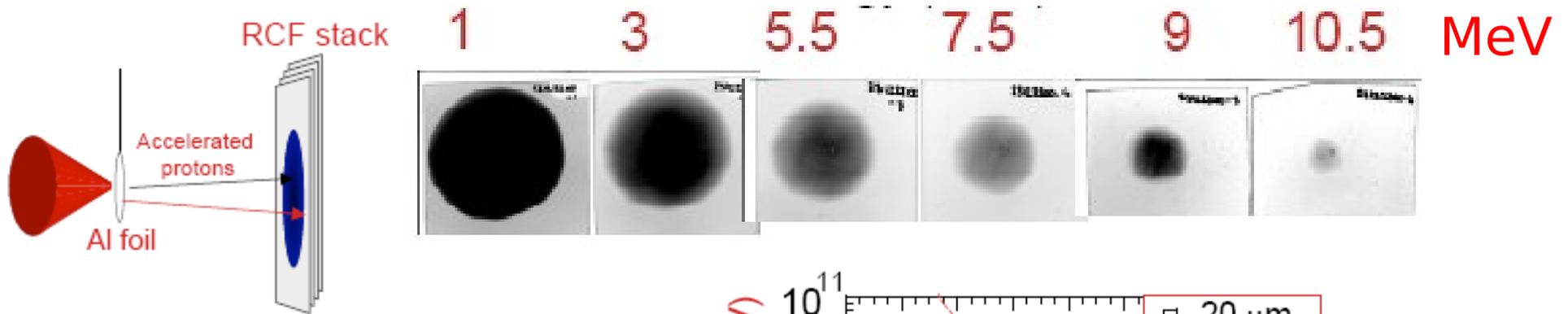
Imaging of thin objects possible due to multiple small-angle scattering of protons [West & Sherwood, Nature **239** (1972) 157]

Transverse emittance measurement for **>10 MeV** protons [Cowan et al, PRL **92** (2004) 204801]

$$\epsilon < 4 \times 10^{-3} \text{ mm mrad}$$

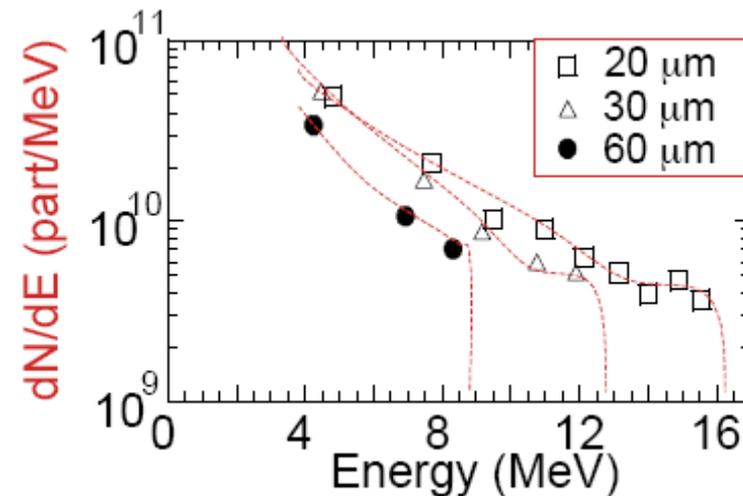
# Energy resolving allows temporal resolution

Use of RadioChromic Film (RCF) stack as a detector allows **single-shot**, **spatial** and **energy** resolution of the proton beam



Typical data example

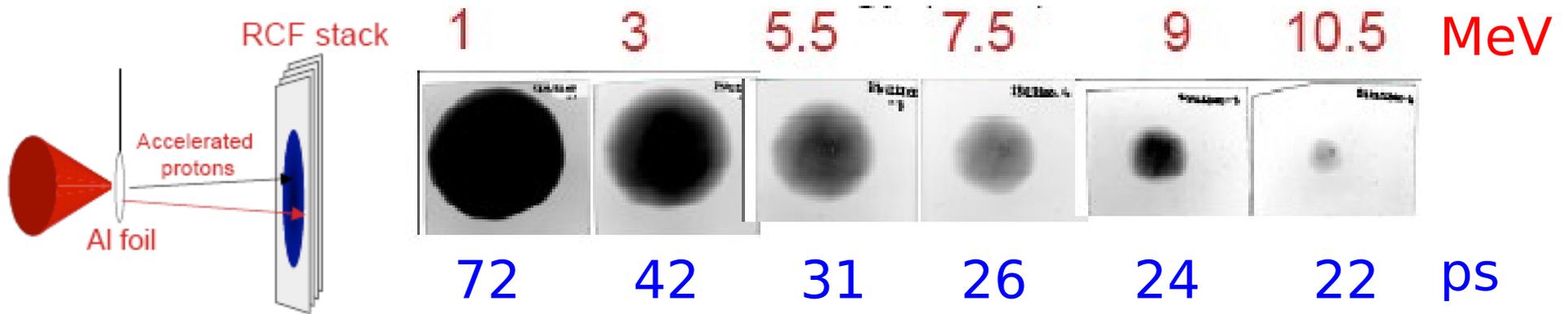
(taken at the LULI 100 TW facility,  
École Polytechnique,  
Palaiseau, France)



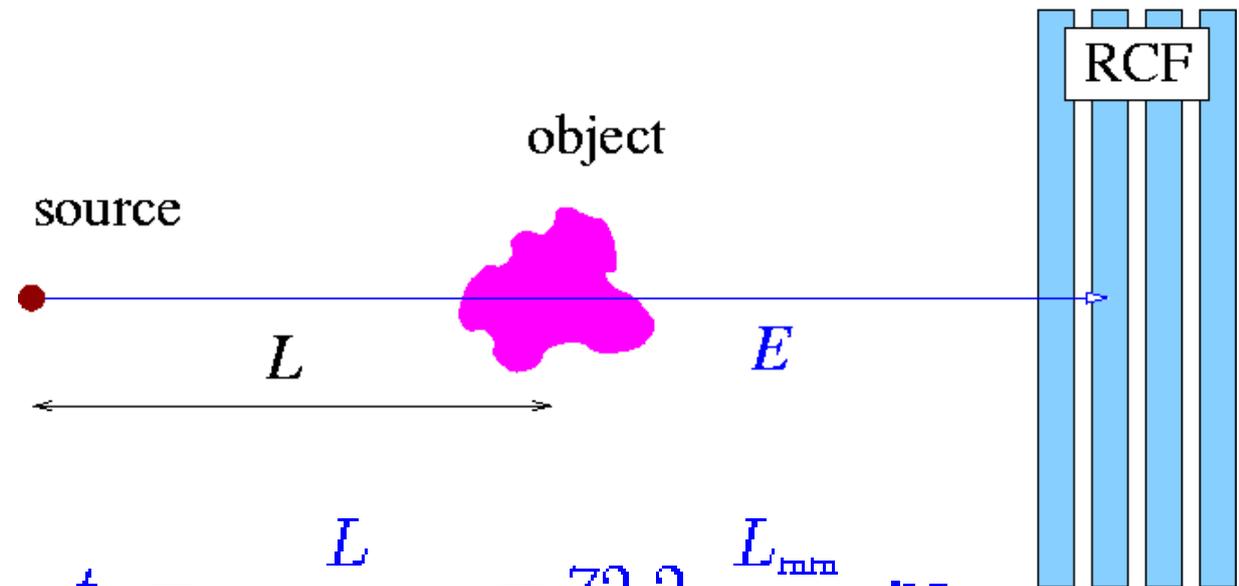
RCF energy selection capability is a consequence of Bragg peak deposition

# Energy resolving allows temporal resolution

Use of RadioChromic Film (RCF) stack as a detector allows **single-shot, spatial** and **energy** resolution of the proton beam



In a **time-of-flight** arrangement protons of different energies cross the object at different times: **imaging of transient objects** possible

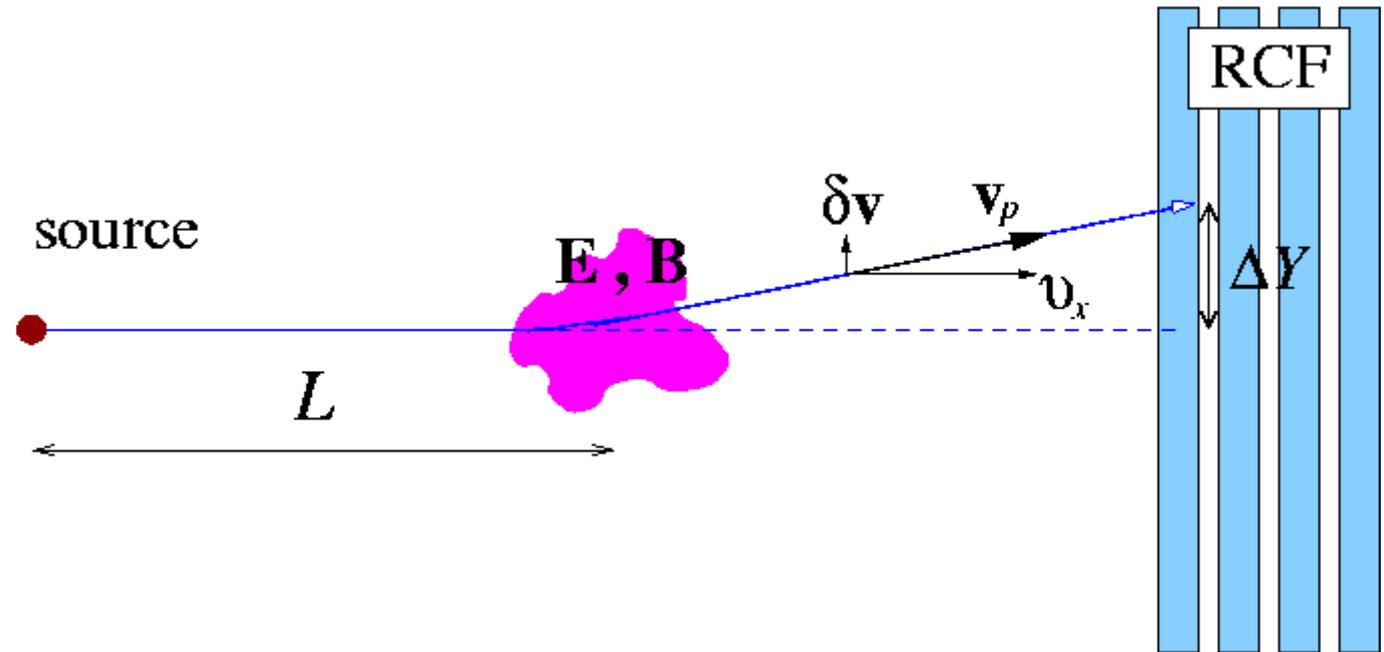


RCF energy bandwidth allows **picosecond resolution**

$$t_p = \frac{L}{\sqrt{2E/m_p}} = 72.2 \frac{L_{\text{mm}}}{\sqrt{E_{\text{MeV}}}} \text{ ps}$$

# Using protons to detect EM fields

A proton crossing an EM field distribution acquires a transverse velocity  $\delta \mathbf{v}$



$$\delta \mathbf{v} = \frac{e}{m_p} \int (\mathbf{E} + \mathbf{v}_p \times \mathbf{B}) dt$$

For *weak* deflections (1<sup>st</sup> order Born approximation)

$$v_x = \frac{dx}{dt} \simeq v_p \quad \rightarrow \quad \delta Y = |\delta \mathbf{v}_\perp| \Delta t \simeq \frac{eL}{2\varepsilon_p} \int (\mathbf{E} + \mathbf{v}_p \times \mathbf{B})_\perp dx$$

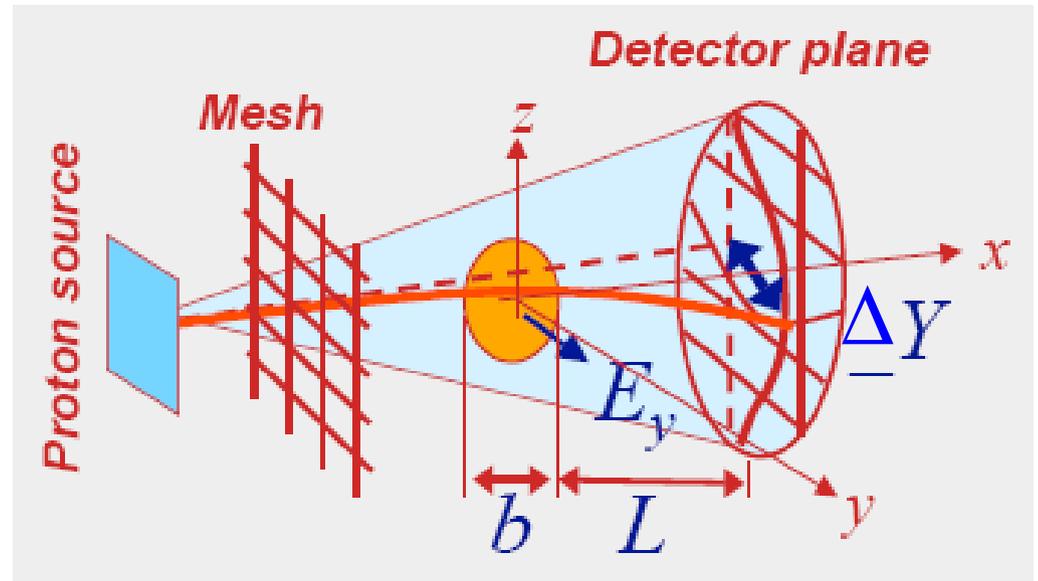
Concept: estimate  $\mathbf{E}$  (and/or  $\mathbf{B}$ ) from the measurement of  $\Delta Y$

# Proton "Deflectometry"

The proton deflection  $\Delta Y$  can be measured directly by the deformation of the imprint of a **stopping mesh**

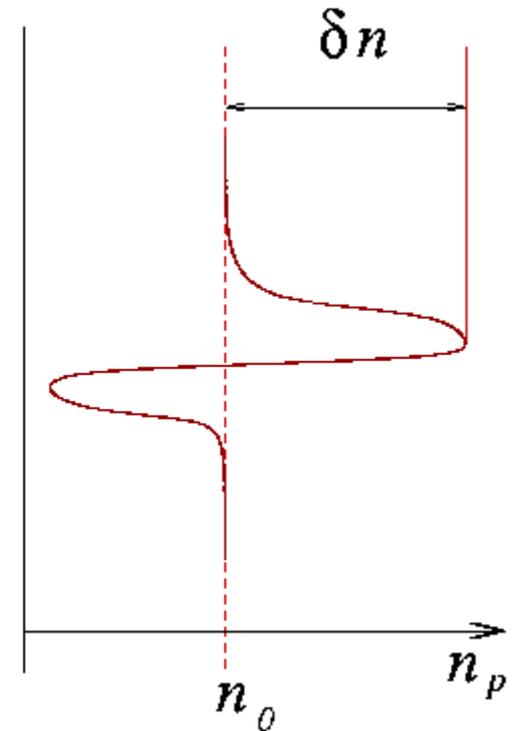
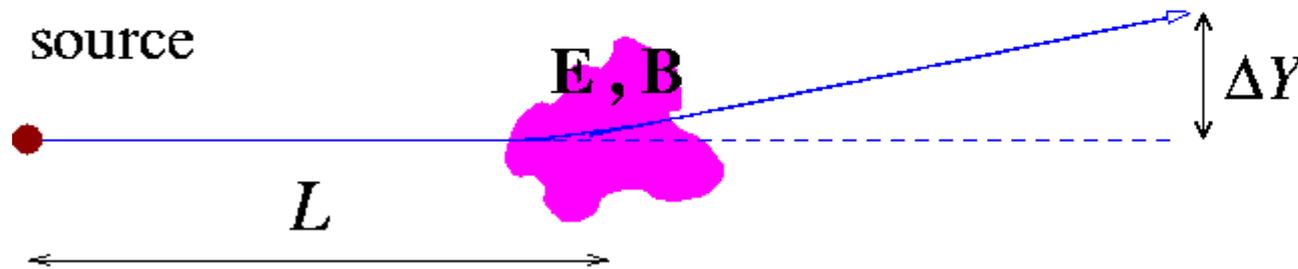
Assuming that only  $\mathbf{E}$  contributes we estimate the average field as

$$\langle \mathbf{E} \rangle \simeq \frac{1}{b} \int_{-b/2}^{+b/2} \mathbf{E}_{\perp} dx \simeq \frac{2\epsilon_p}{eLb} \Delta Y$$



# Proton "Imaging"

When **fields gradients** are present deflections produce a modulation  $\delta n$  in the **proton density**  $n_p$  on the detector plane:



The observed modulation  $\delta n$  is proportional to the **line integral** of "source" terms

( $M$  : geometrical magnification)

$$\frac{\delta n}{n_0} \simeq -\frac{1}{M} \nabla_{\perp} \cdot \Delta Y \simeq -\frac{2\pi e L b}{\epsilon_p M} \int_{-b/2}^{+b/2} \left( \rho - \frac{1}{c^2} \mathbf{v}_p \cdot \mathbf{J} \right) dx$$

If electric fields are dominant  $\delta n$  is proportional to the **line integral of the charge density**  $\rho$  along the proton path.

# Breakthroughs and limitations (experimental)

Proton Probing gives an **unprecedented possibility** to detect electric and magnetic fields with high spatial and temporal resolution in low- and high-density plasmas

However...

- an accurate “**tomographic**” **3D reconstruction** of field patterns is **not possible** (at least with a single proton beam)
- using a single beam the observed effects **cannot be attributed** with absolute unambiguity **to electric or to magnetic fields alone**
- evaluating the **spatial and temporal resolution** of the technique is **not straightforward**; several effects need to be considered
- > The experimental data must be interpreted
  - 1) within a theoretical framework (starting assumptions)
  - 2) with the help of numerical modeling of proton probing: **particle tracing** simulations with  **$\mathbf{E}(x,t)$**  and/or  **$\mathbf{B}(x,t)$**  as input

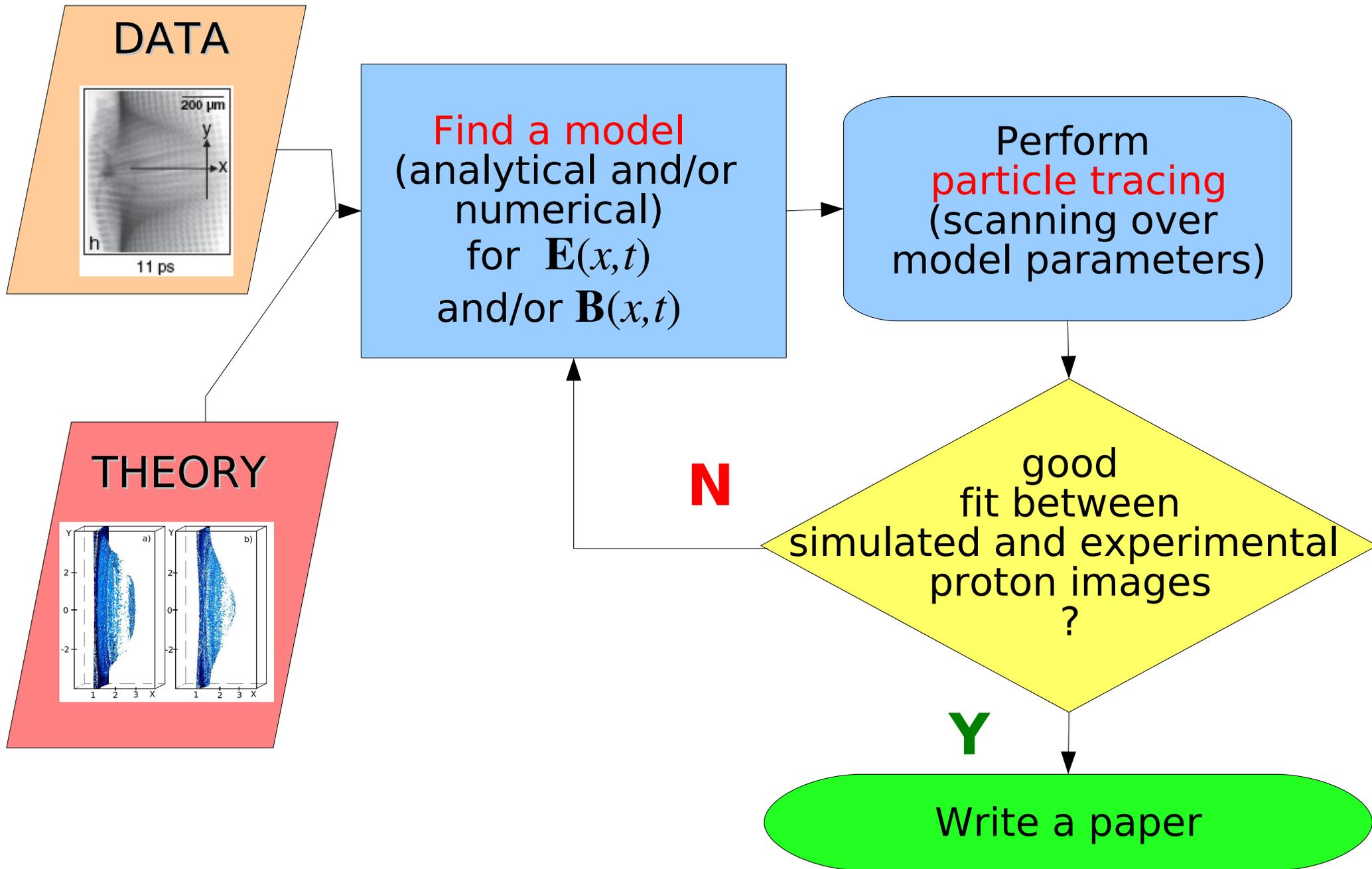
# Breakthroughs and limitations (theoretical)

Proton Probing allows to observe “**plasma physics in action**” via self-generated electric and magnetic fields, allowing for the first time to **observe a large set of phenomena** and **validate related theories**

However...

- the **temporal scale accessible to proton probing** (ps) is often **much larger than the “fast” scale** of the investigated phenomena, whose observation is thus based on “**remnants**” of the fields
- fully self-consistent “**ab initio**” **simulations** (e.g. with PIC codes) **in 3D** and for spatial and temporal scales corresponding to the experiment are often **beyond available (super)computing power**
- > **Models** are needed to bridge the gap within “feasible” (e.g. short-scale, 2D, symmetry assumptions ...) simulations and the experimentally relevant regimes

# Working on “proton probing experiments”



# Several basic phenomena observed by proton probing

Bubble-like structures interpreted as remnants of **relativistic solitons** (“post-solitons”)

[Borghesi et al., Phys. Rev. Lett. **88** (2002) 135002]

Ion modulations resulting from onset and evolution of **Buneman instability** in the late evolution of a plasma wake

[Borghesi et al., Phys. Rev. Lett. **94** (2005) 195003]

**Collisionless shock waves** in the plasma blow-off

[Romagnani et al., Phys. Rev. Lett. **101** (2008) 025004]

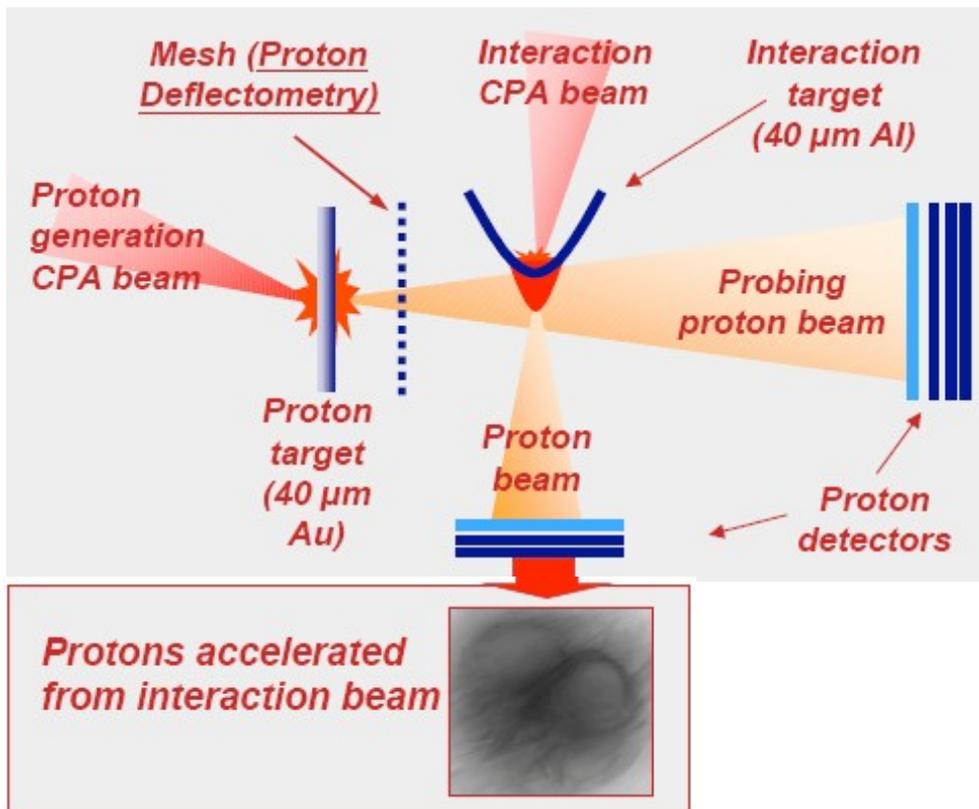
Observation of **phase space holes**

[Sarri et al., [arXiv:0908.4527](https://arxiv.org/abs/0908.4527)]

... and other including the ones discussed in the following  
(**plasma sheath expansion, relativistic self-channeling, “Hybrid” magnetic structures, ultrafast charging ...**)

Additional applications in experiments of relevance for **Inertial Confinement Fusion** (radiography of implosions and shocks, detection of self-generated fields) have been demonstrated by several groups (QUB, Rutherford, LULI, Livermore, Rochester, ...)

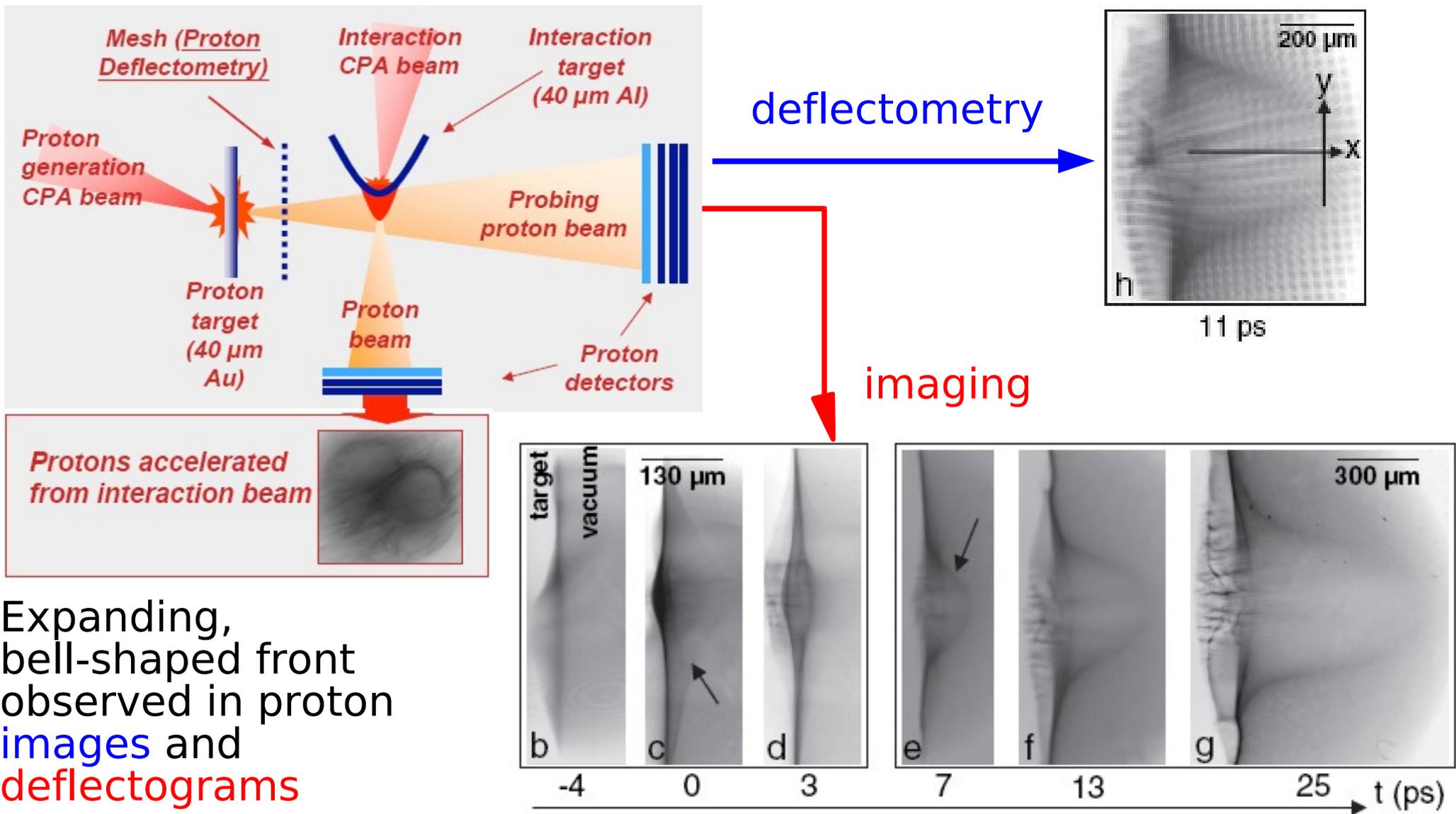
# Detection of Proton-Accelerating Sheath Fields



**Goal:** study of TNSA mechanism for ion acceleration by the direct detection of related space-charge electric field

**Technique:** use a second proton beam as a transverse probe

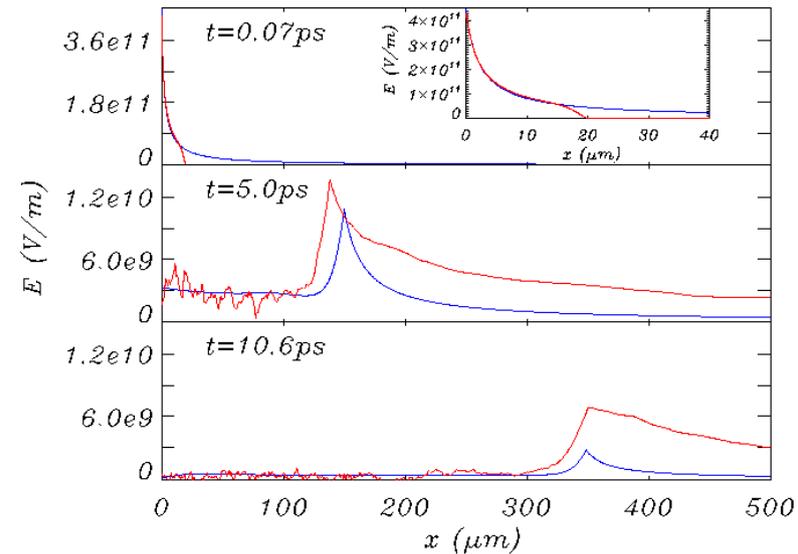
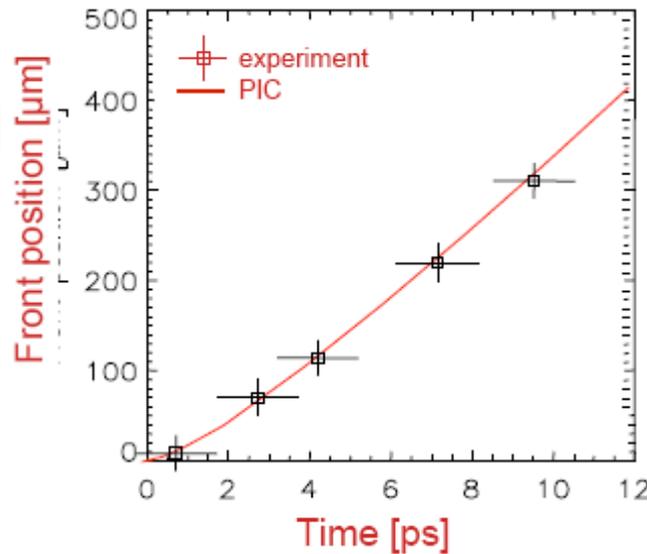
# Detection of Proton-Accelerating Sheath Fields



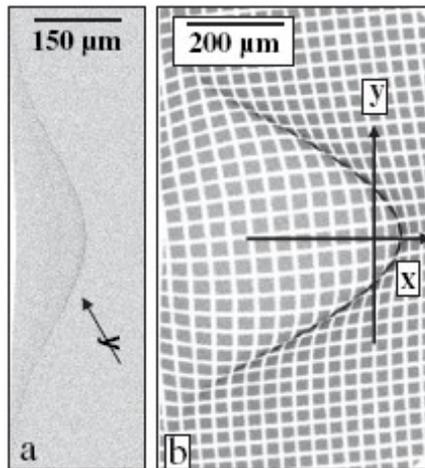
L. Romagnani, J. Fuchs, M. Borghesi, P. Antici, P. Audebert, F. Ceccherini, T. Cowan, T. Grismayer, S. Kar, A. Macchi, P. Mora, G. Pretzler, A. Schiavi, T. Toncian, O. Willi, Phys. Rev. Lett. **95** (2005) 195001

# Comparison with simulations

Experimental results have been compared with **PIC simulations** using the **plasma expansion model** (classic problem of plasma physics!)



Particle tracing simulations of proton deflection in the **PIC fields** (plus an “heuristic” modeling of the 2D expansion) fit well experimental images and deflectograms



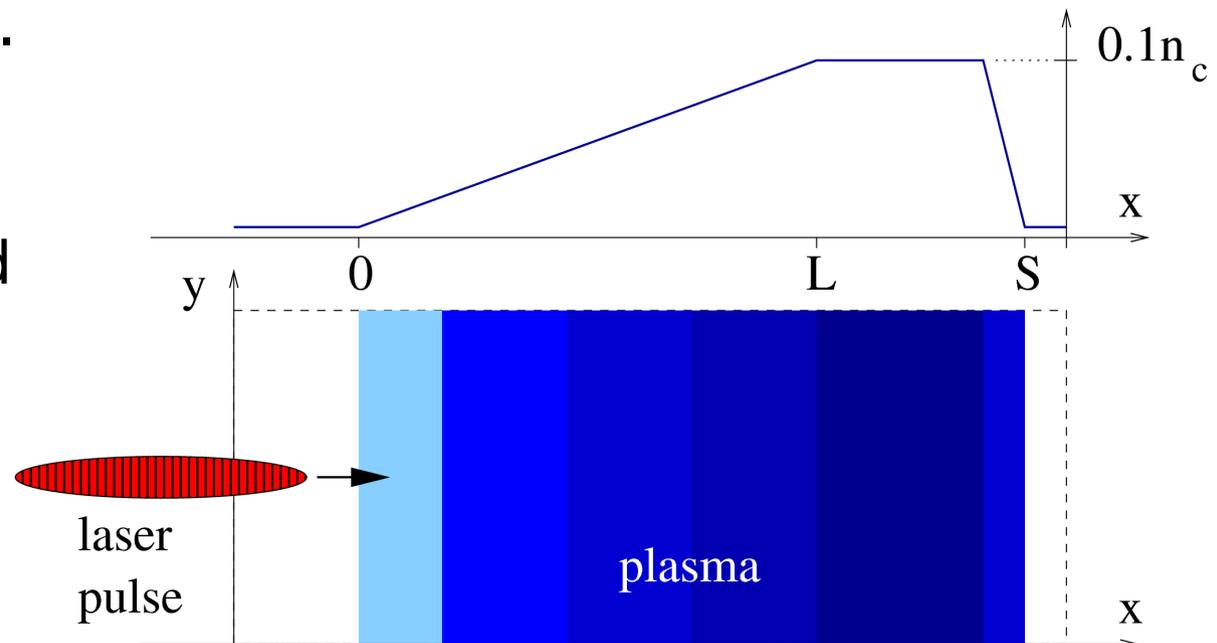
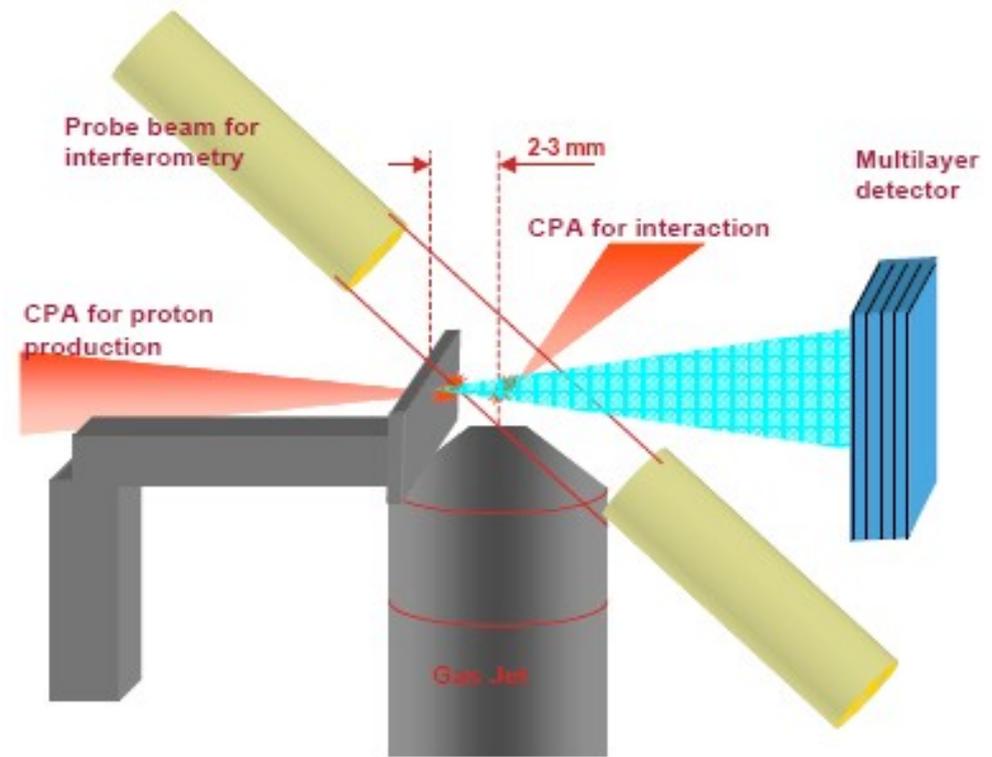
Comparison of **fluid** and **kinetic (PIC)** results show the importance of **non-equilibrium, kinetic** and **non-Maxwellian** effects in the plasma expansion

# Study of charge-displacement self-channeling

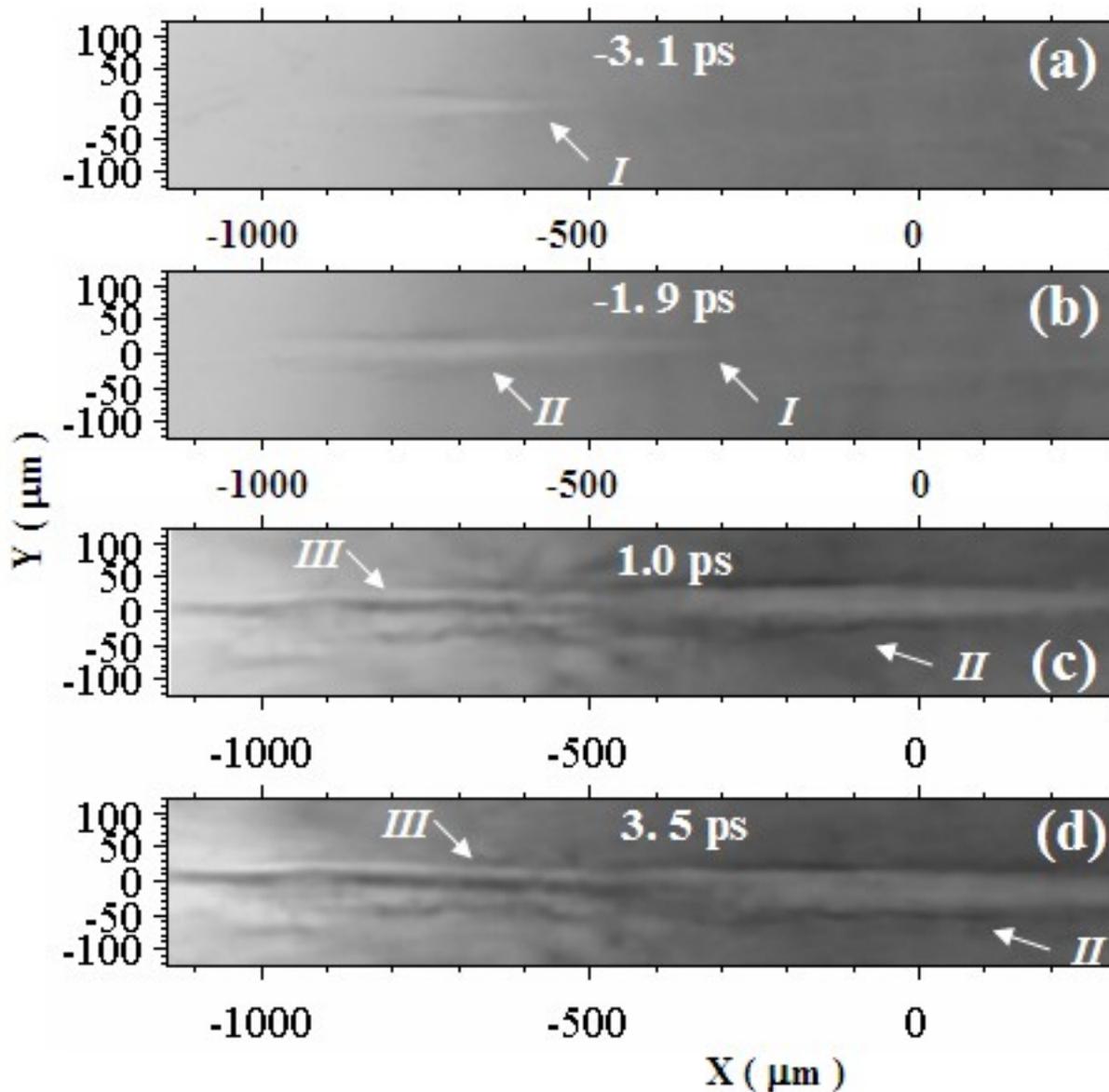
A superintense laser pulse propagating in a **low-density plasma** (produced in a gas jet) undergoes **self-focusing** and **channeling** due to both **relativistic effects** and radial plasma expulsion by **radiation pressure**.

For a transient stage the channel is **charged** since electrons are expelled first.

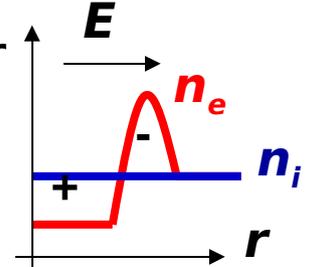
**Proton probing** along the direction **perpendicular** to propagation has been used to study this effect



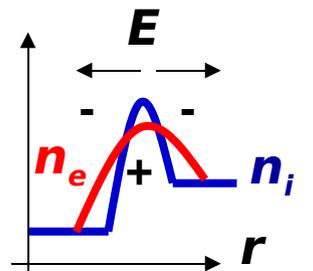
# Proton images of charged channel evolution



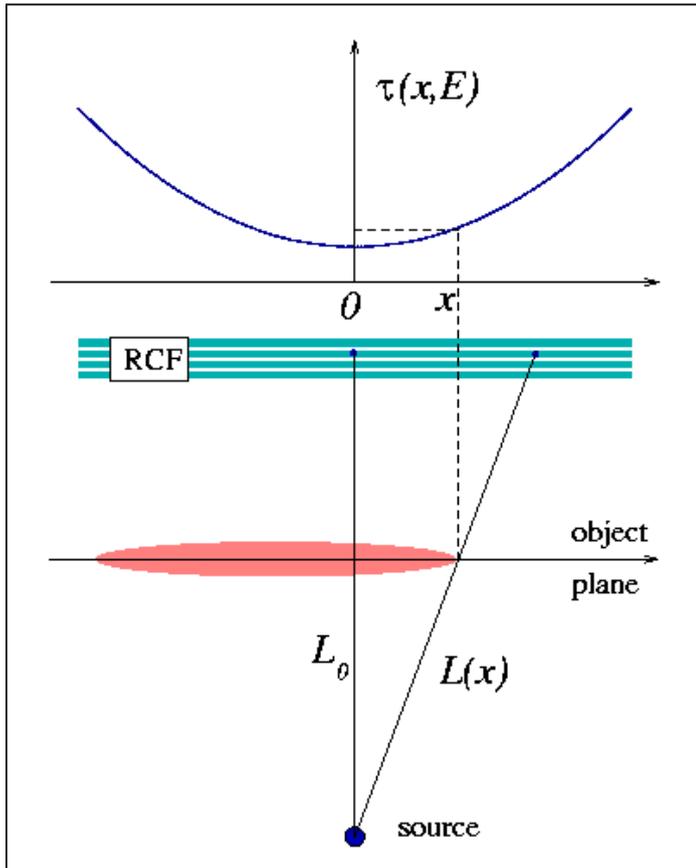
Early times  
(during the laser pulse)  
propagating  
"white" channel  
front indicates  
**electron expulsion**  
from the axis



Late times  
(after the laser pulse)  
"black" line on  
axis indicates  
**field inversion**  
at some location



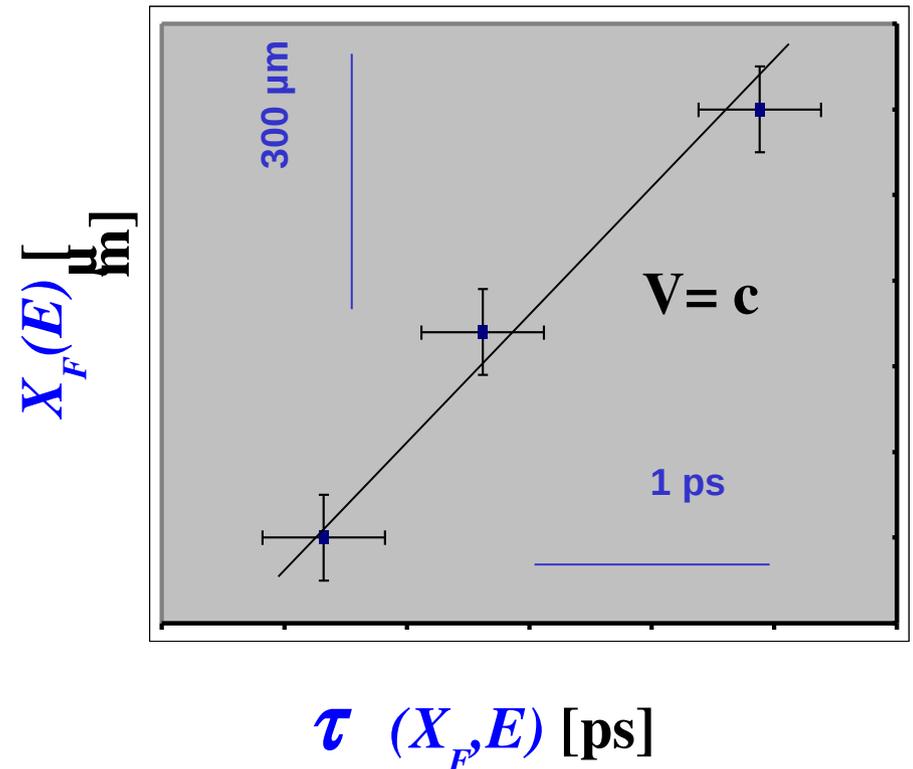
# Channel front propagation speed



Due to the divergence of the proton beam the “probing time” depends on angle (i.e. on the position on the object plane)

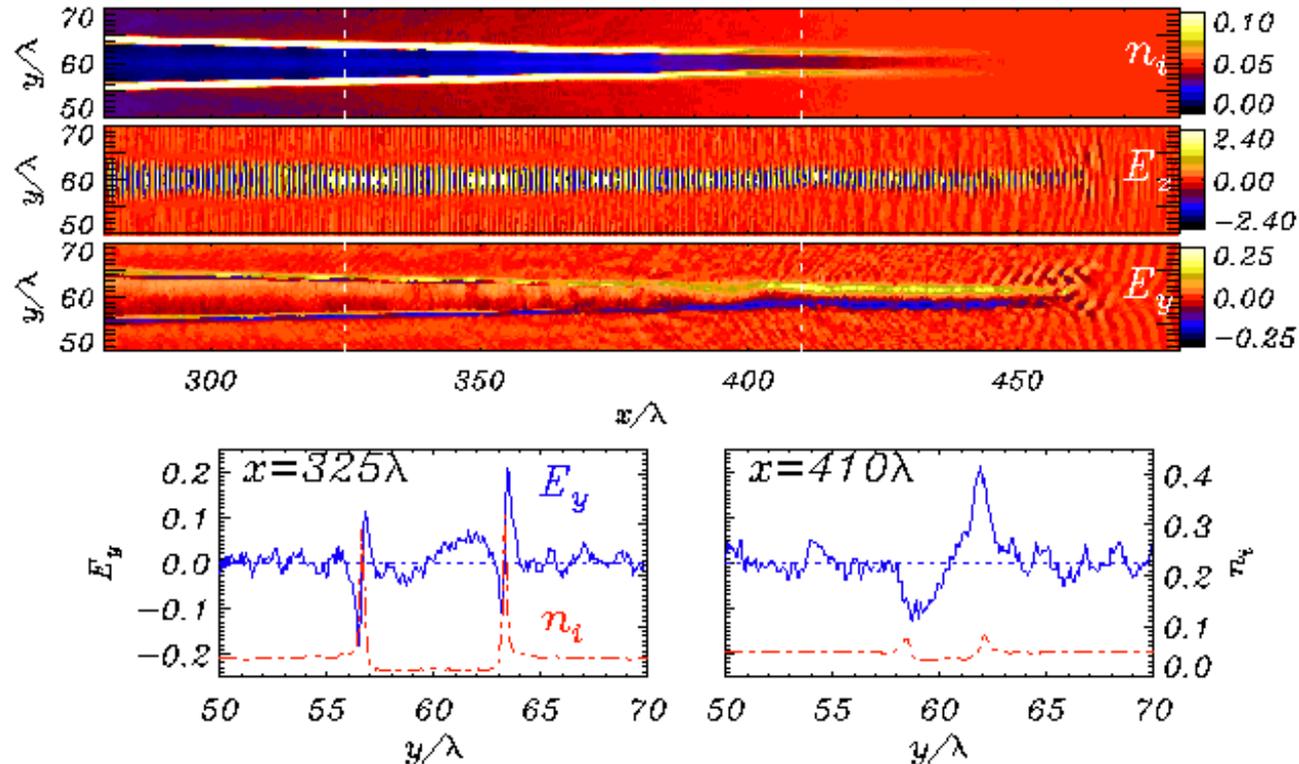
$$\tau(x, E) = t_0(E) + \frac{L_0}{\sqrt{2E/m_p}} (\sqrt{1 + x^2/L_0^2} - 1)$$

Plotting the channel front displacement  $X_F(E)$  vs. the probing time  $\tau(X_F, E)$  we obtain the front propagation speed  $V \sim c$



# 2D PIC simulations show “radial” field dynamics

➔  
Laser



Two ambipolar fronts of  $E_y$  appear in the trailing edge of the channel; “negative” part can produce “black line” in proton images

Outward-directed radial field  $E_y$  due to electron expulsion from axis  
EM component  $E_z$  reveals self-focusing

# Ponderomotive model of self-channeling

Assumptions:

- cylindrical symmetry
- non-evolving laser pulse
- electrostatic approximation

Solution based on kinetic PIC model

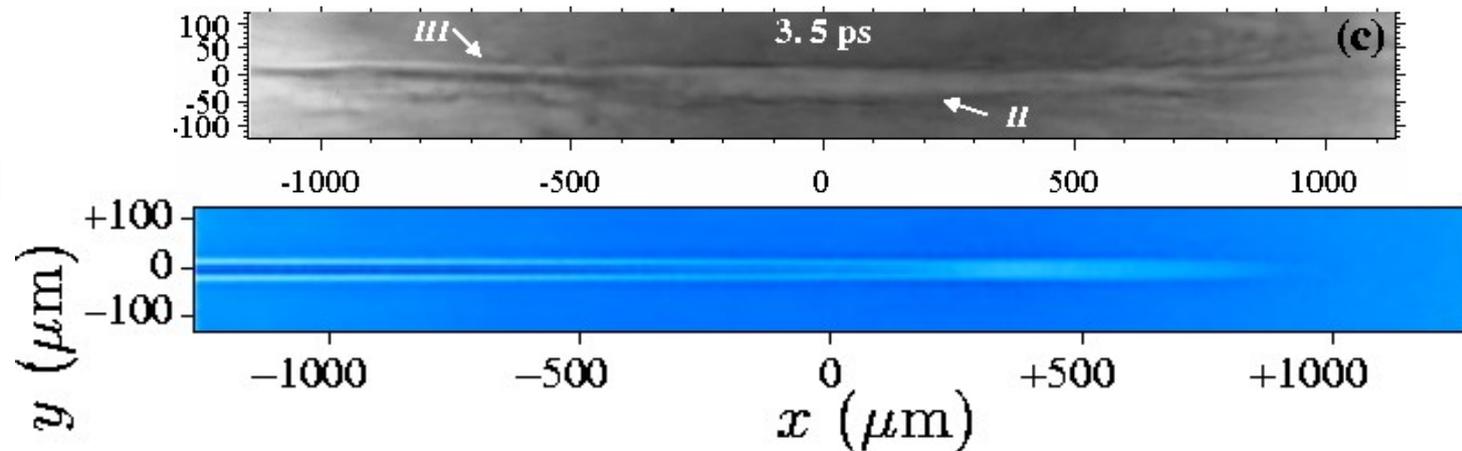
$$m_e dv_e/dt = -eE_r - m_e c^2 \partial_r \sqrt{1 + a^2}$$

$$a = a(x, r, t) = a_0 e^{-r^2/r_0^2 - (x-ct)^2/c^2\tau^2}$$

$$m_i dv_i/dt = ZeE_r$$

$$\frac{1}{r} \partial_r (r \cdot E_r) = 4\pi e (Zn_i - n_e)$$

Particle tracing simulations using  $\mathbf{E}$  from PIC model well reproduce experimental features



# Ponderomotive model of self-channeling

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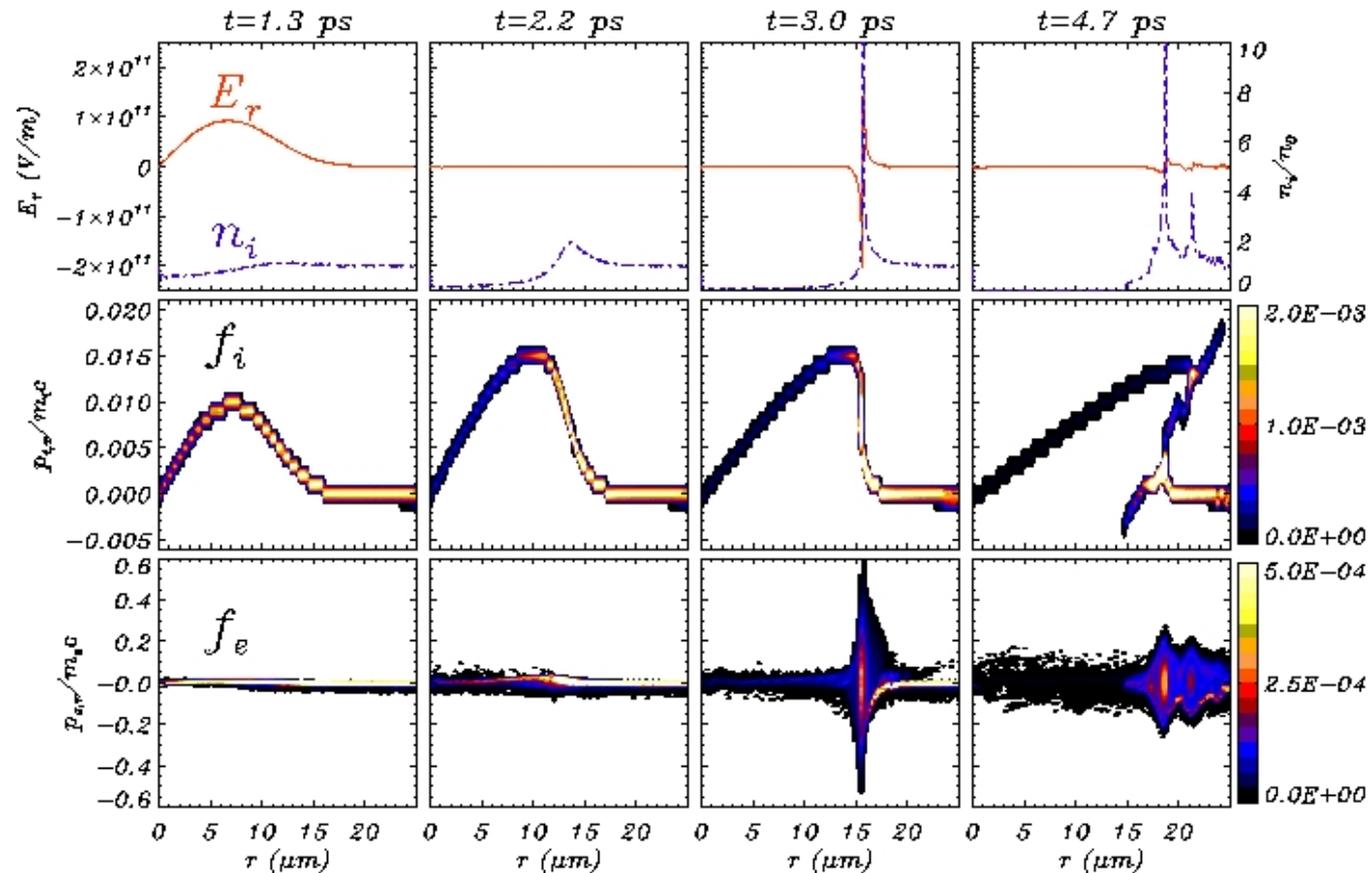
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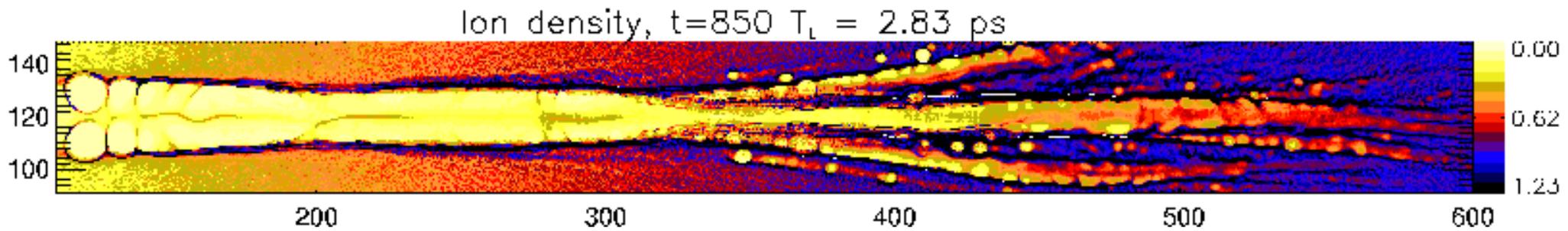
$$m_i dv_i/dt = ZeE_r$$

$$\frac{1}{r} \partial_r (r \cdot E_r) = 4\pi e (Zn_i - n_e)$$

The late ambipolar field appears after the vanishing of the early field (“echo” effect) due to **hydrodynamical breaking** in the ion density profile causing **strong electron heating**



# Coherent field structures in 2D PIC simulation



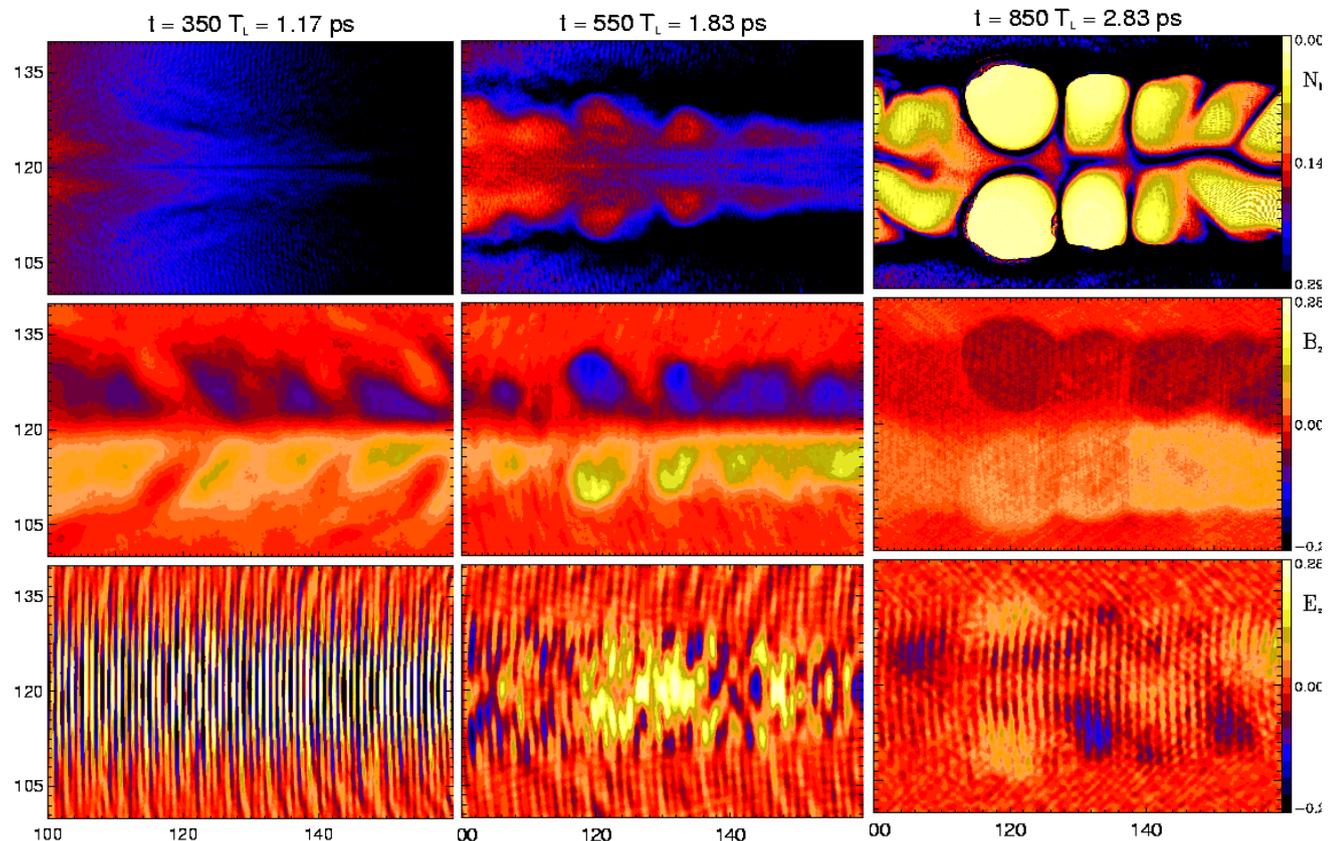
Pulse front: beam  
breakup  
and EM **cavitons**

Left channel side:  
“hybrid” quasi-  
periodic  
structures,  
“part **soliton**,  
part **vortex**” ...

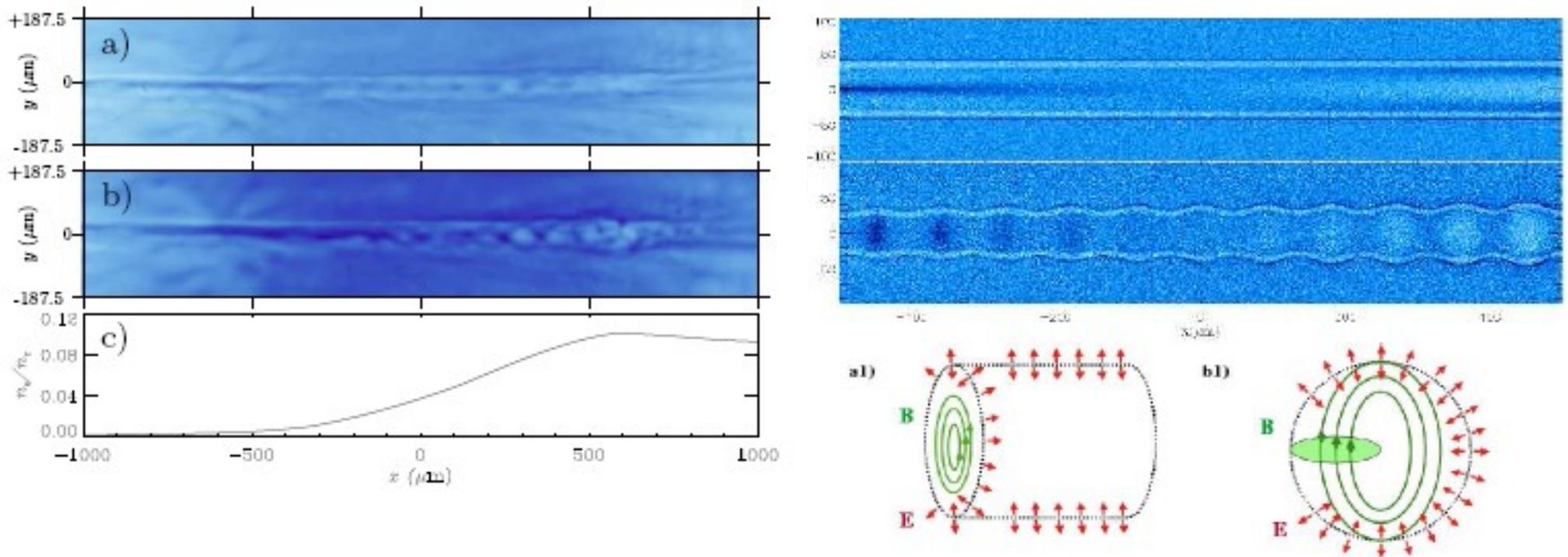
Lyseikina et al,  
[arXiv:physics/0701139](https://arxiv.org/abs/physics/0701139)

Macchi et al,  
*PPCF* **49** (2007) B71

Romagnani et al,  
in preparation



# Simulation of proton images data: magnetic vortices?



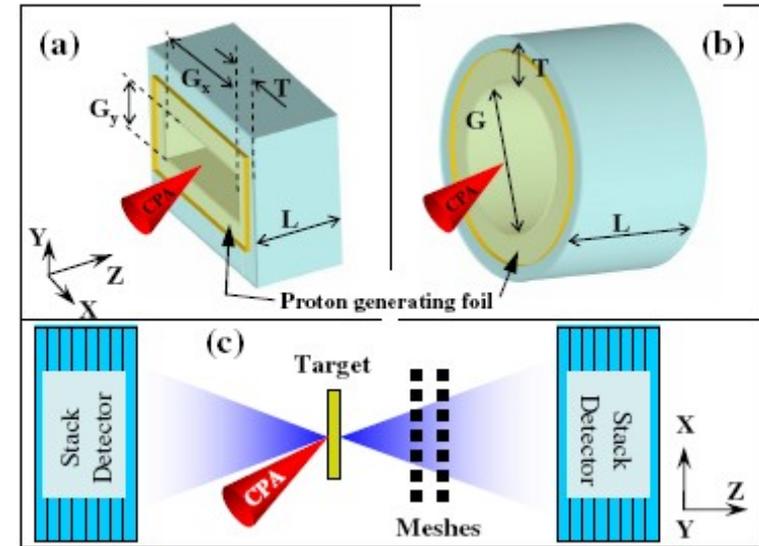
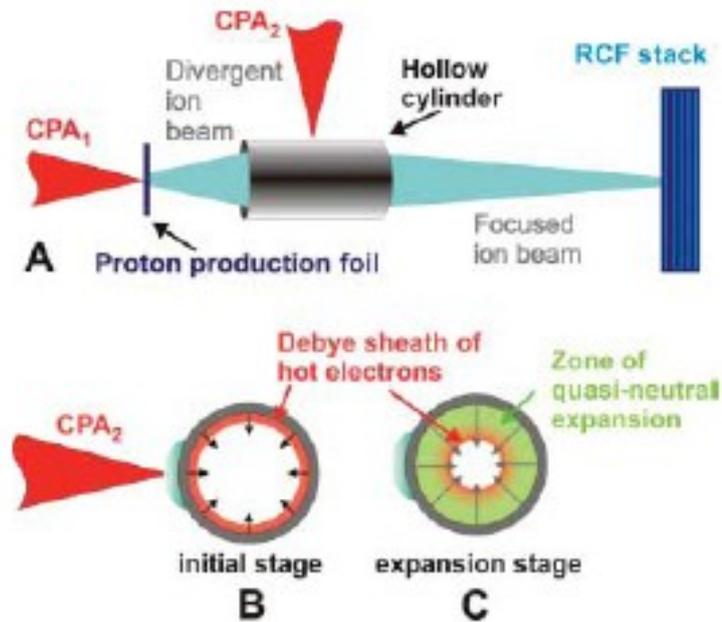
The **3D topology** of the “coherent”, slowly evolving structures was inferred heuristically from 2D PIC simulations and used as an input for the particle tracing code producing **synthetic proton images**.

The comparison suggests that image formation is dominated by **magnetic field deflections** and suggests the formation of patterns of “**magnetized vortex rings**” along the channel

Romagnani et al, [in preparation](#)

# Dynamic control of proton beam properties

Concept: achieve **focusing** and **energy selection** of the proton beam by “external” devices or by “target engineering”



## Laser-driven cylindrical microlens

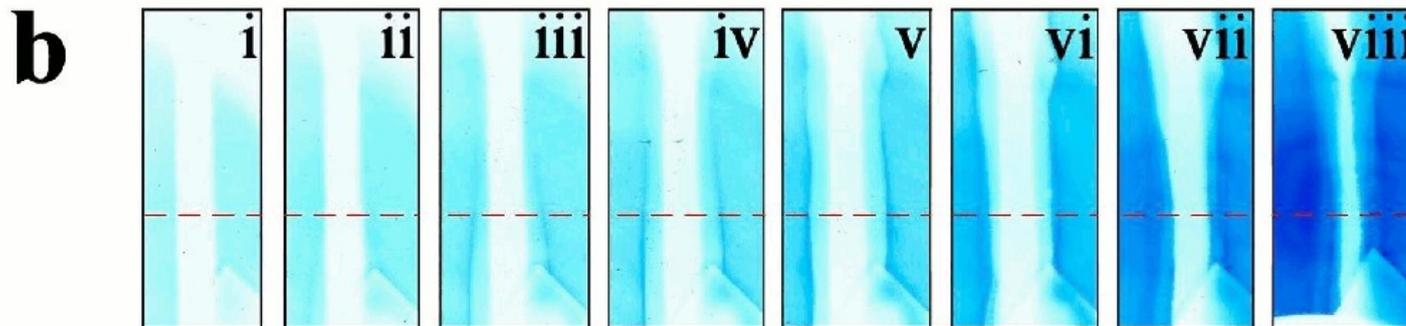
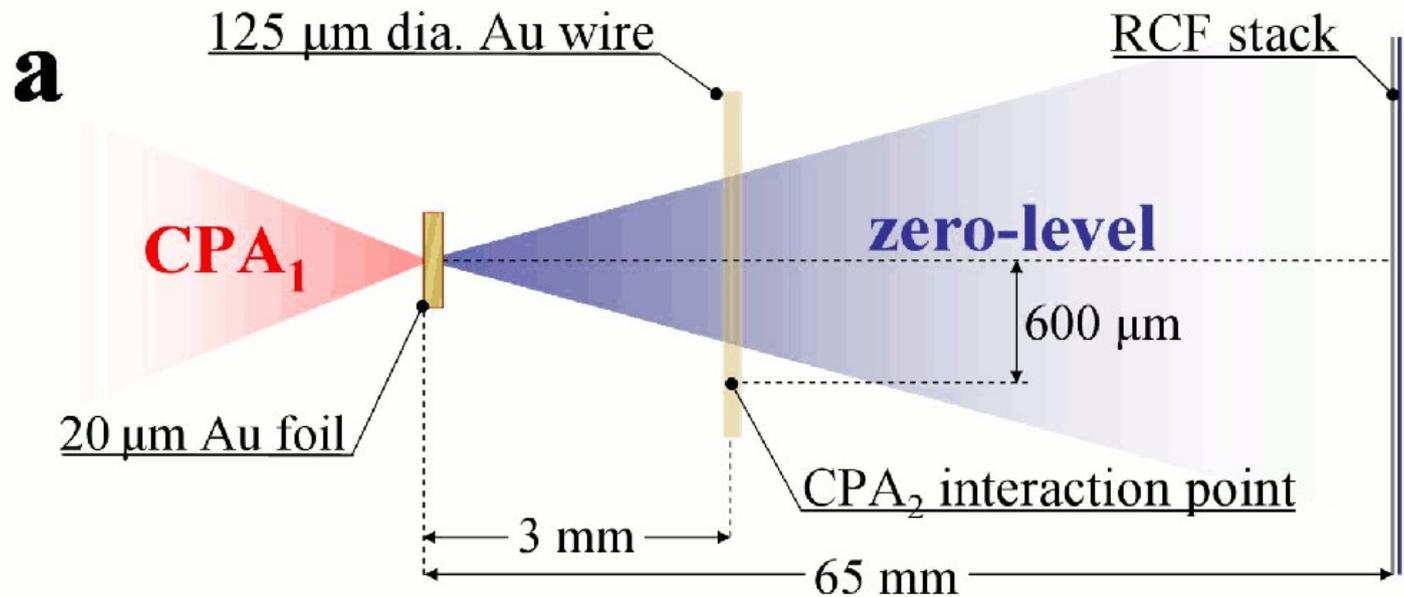
Toncian et al., Science **312** (2006) 410

## Shaped targets designed as electrostatic (?) lenses

Kar et al., PRL **100** (2008) 105004

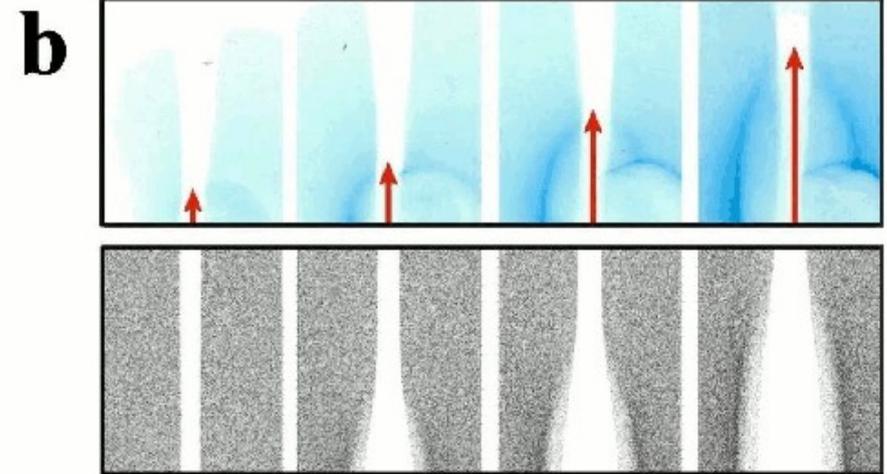
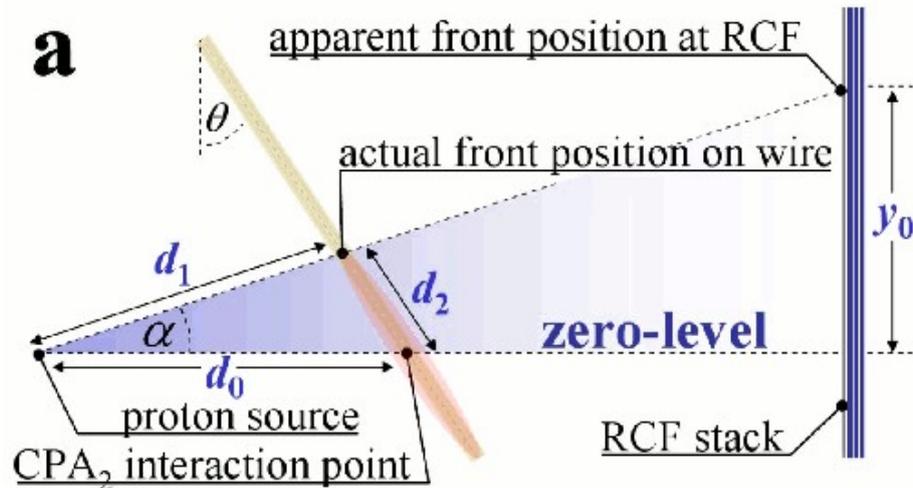
Both approaches pose the question on **how rapidly** the electric field created by escaping electrons propagates on the surface of the target

# Field propagation on the rear surface of solid targets



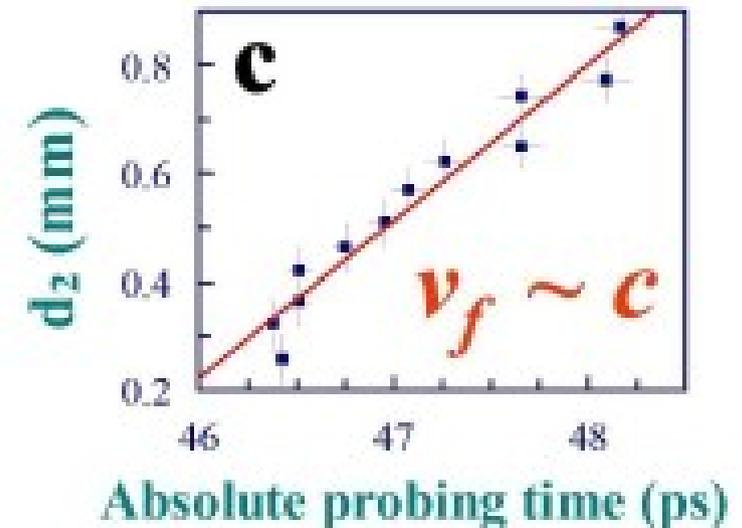
- In the interaction with a **wire target** a **fast positive charging** followed by later **discharging** is observed:  
escape of fast electrons and return neutralizing current?
- The propagation of the field out of the interaction region is **not** resolved with a “vertical” wire

# Field propagation on the rear surface of solid targets



By inclining the wire to an angle  $\theta$  with respect to the vertical axis the propagation of the field is resolved now ;

the speed  $v_f = 0.96 \pm 0.04c$



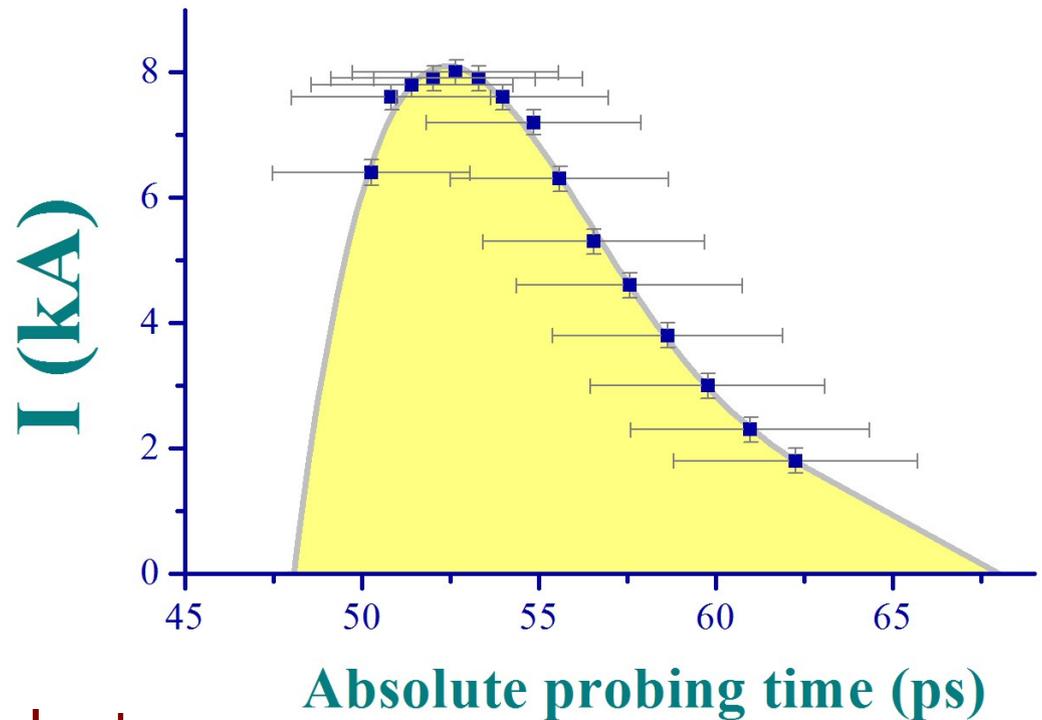
K.E.Quinn, P.A.Wilson, C.A.Cecchetti, B.Ramakrishna, L.Romagnani, G.Sarri, L. Lancia, J. Fuchs, A. Pipahl, T. Toncian, O.Willi, R.J.Clark, D.Neely, M.Notley, P.Gallegos, D.C.Carroll, M.N.Quinn, X.H.Yuan, P.McKenna, T. V. Liseykina, A. Macchi, M. Borghesi,  
PRL 102, 194801 (2009)



# Flowing current and loss of electrons from the wire

From the measurement of the radial field  $E_r$  and the propagation velocity  $v_f$  it is possible to reconstruct the history of the **total current**  $I$  flowing through the wire

$$J(t) = \frac{1}{2} r_w v_f E_s(t)$$



The estimate of the **fraction of electrons escaped in vacuum**  $f_{esc}$  thus obtained is roughly consistent with a simple estimate based on the charging of an “hot” plasma sphere of radius  $r_0$  with  $N_e$  electrons in Boltzmann equilibrium

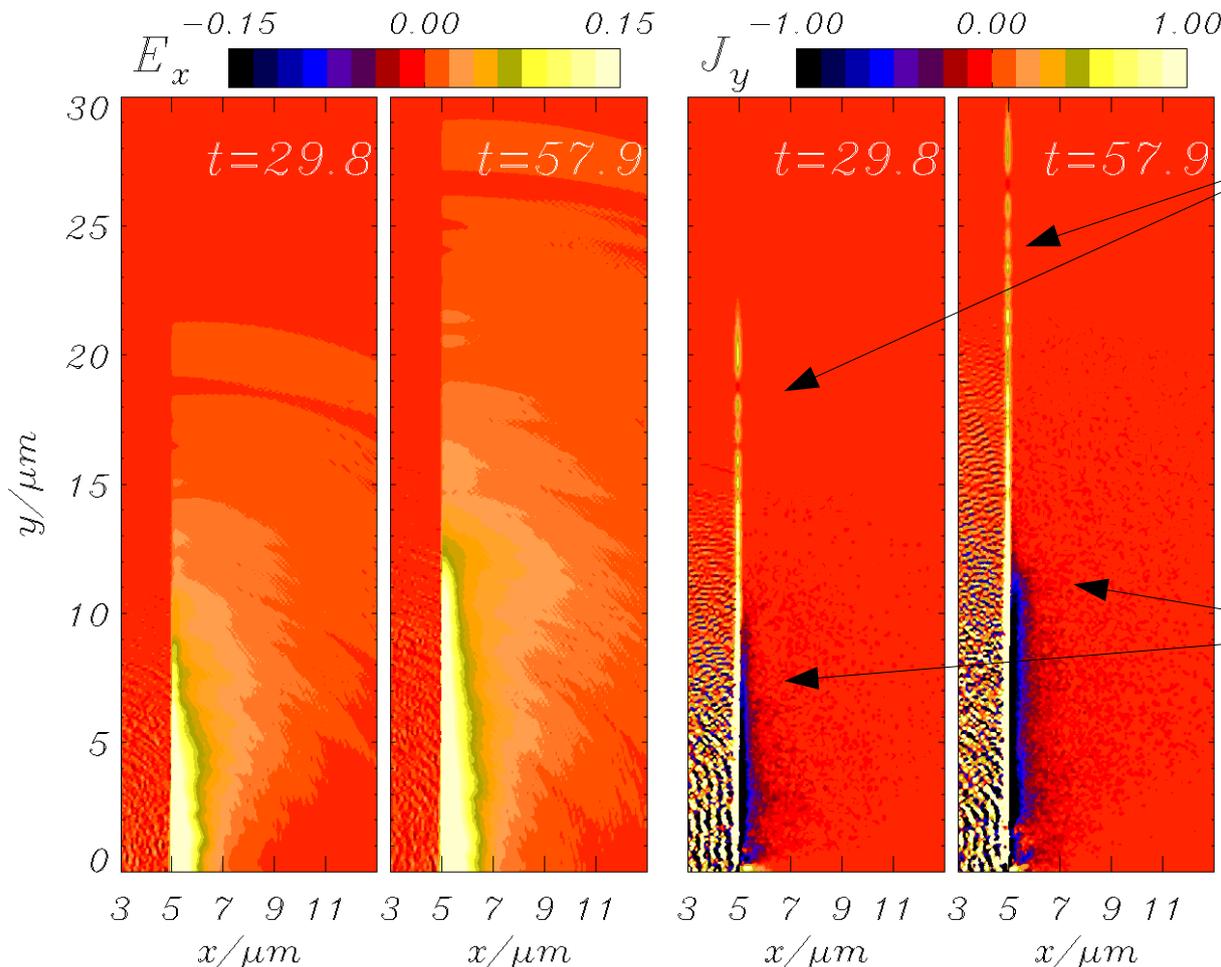
$$\frac{\ln f_{esc}}{f_{esc}} = -\frac{r_c m_e c^2}{r_0 k_B T_e} N_e$$

K.E.Quinn, P.A.Wilson, C.A.Cecchetti, B.Ramakrishna, L.Romagnani, G.Sarri, L. Lancia, J. Fuchs, A. Pipahl, T. Toncian, O.Willi, R.J.Clarke, D.Neely, M.Notley, P.Gallegos, D.C.Carroll, M.N.Quinn, X.H.Yuan, P.McKenna, T. V. Liseykina, A. Macchi, M. Borghesi,  
PRL 102, 194801 (2009)



# Simulation of field propagation on the rear surface

PIC simulations of a “model problem” show a “double front” structure of the current at the rear surface:



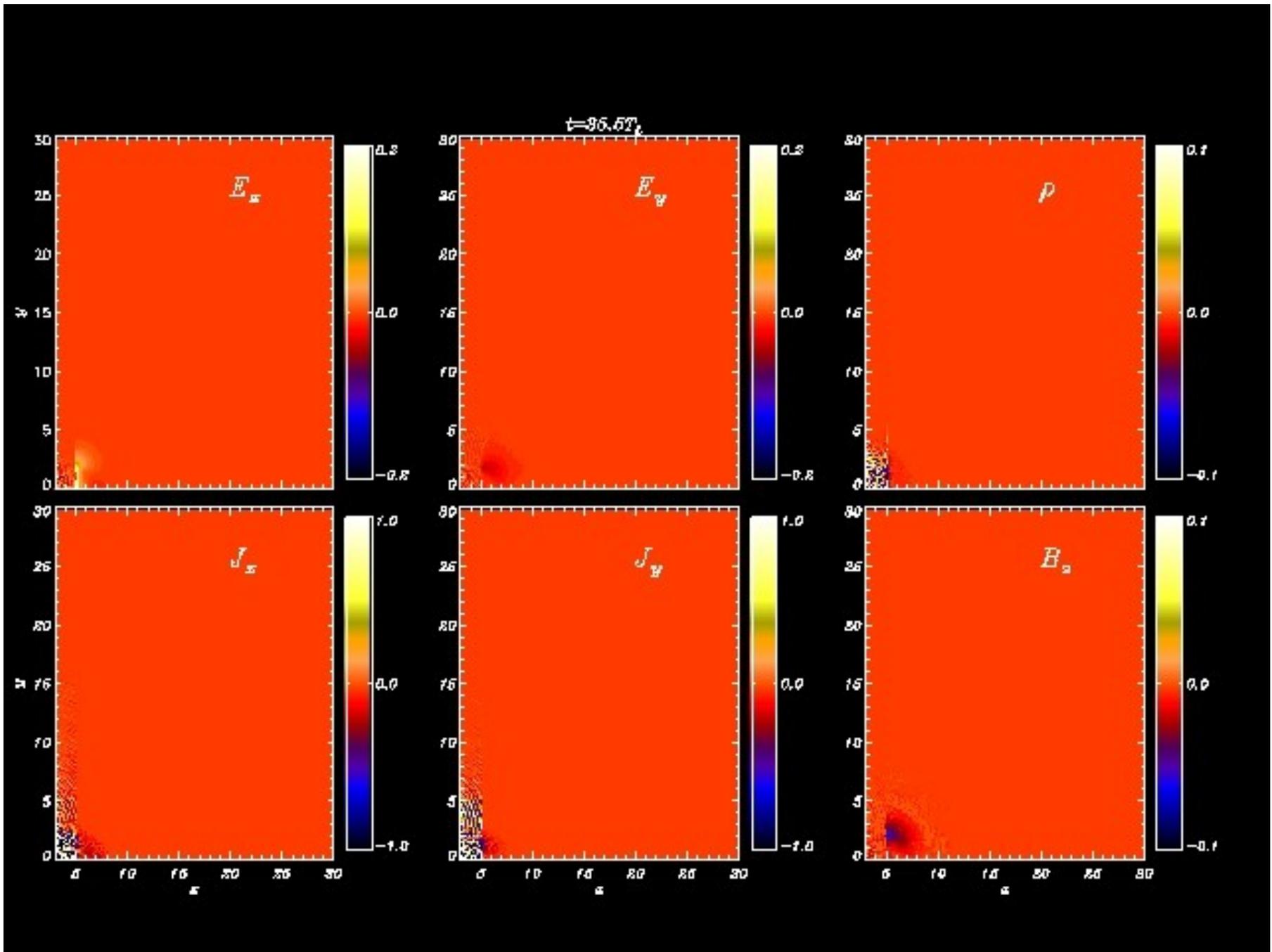
- thin current layer on the surface, propagating at speed  $c$
- > driven by EM fields by the transient “antenna” created by escaping electrons

- bulk current of “fast” electrons, propagating slower than  $c$
- > due to the spread of “fast” electrons in the transverse direction

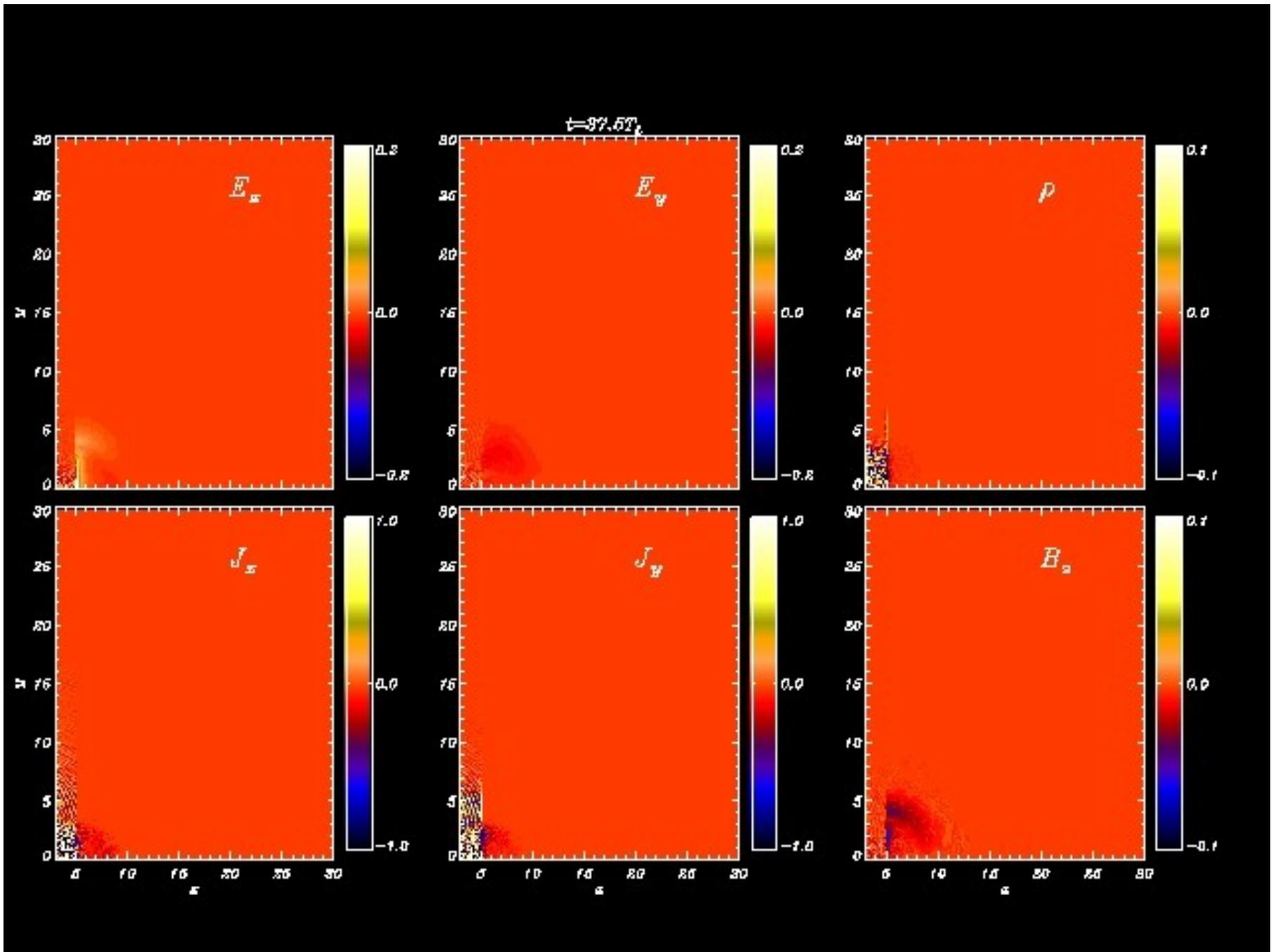
K.E.Quinn, P.A.Wilson, C.A.Cecchetti, B.Ramakrishna, L.Romagnani, G.Sarri, L. Lancia, J. Fuchs, A. Pipahl, T. Toncian, O.Willi, R.J.Clarke, D.Neely, M.Notley, P.Gallegos, D.C.Carroll, M.N.Quinn, X.H.Yuan, P.McKenna, T. V. Liseykina, A. Macchi, M. Borghesi,  
[PRL 102, 194801 \(2009\)](#)



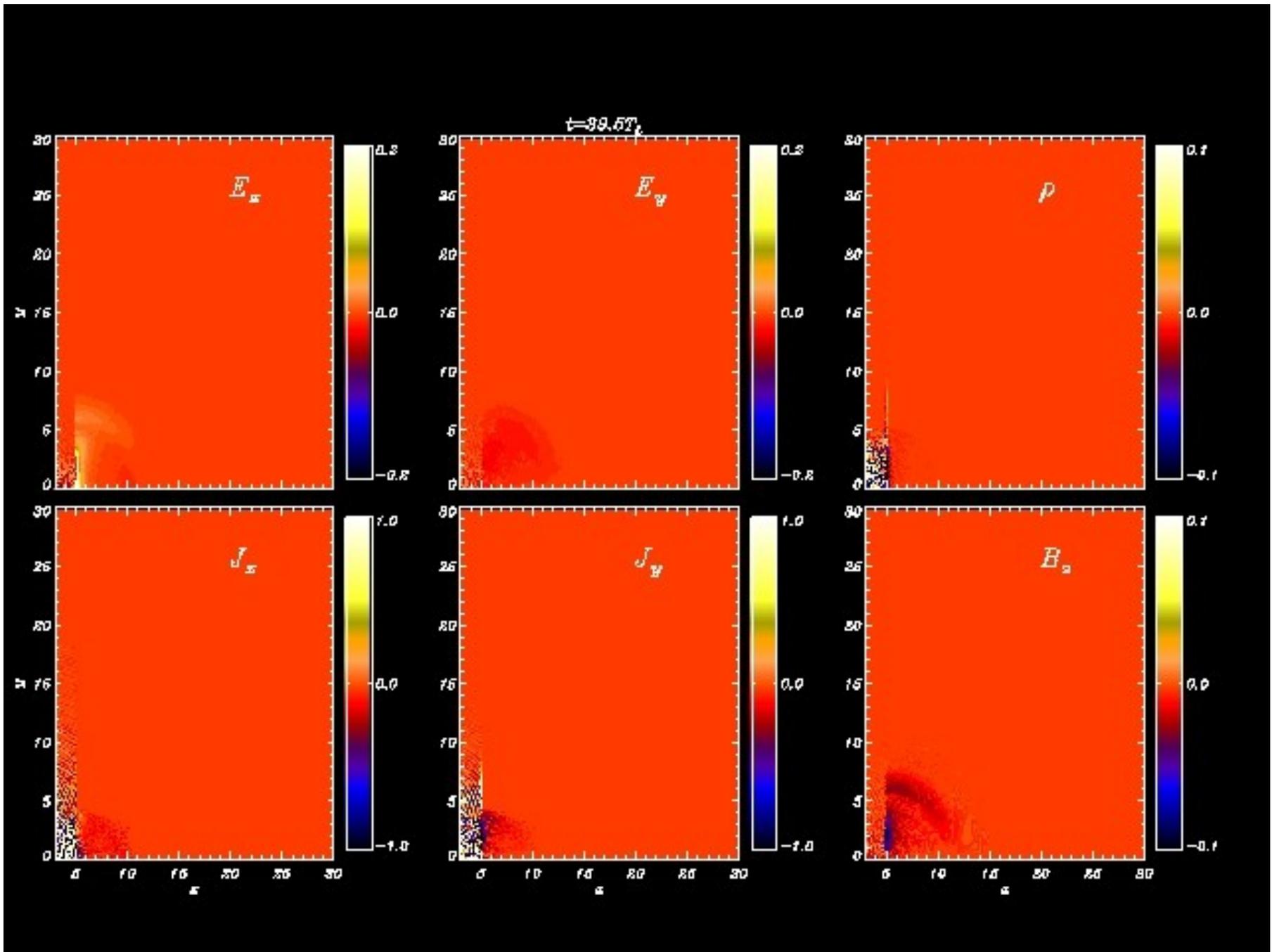
# Field propagation on the rear surface of solid targets



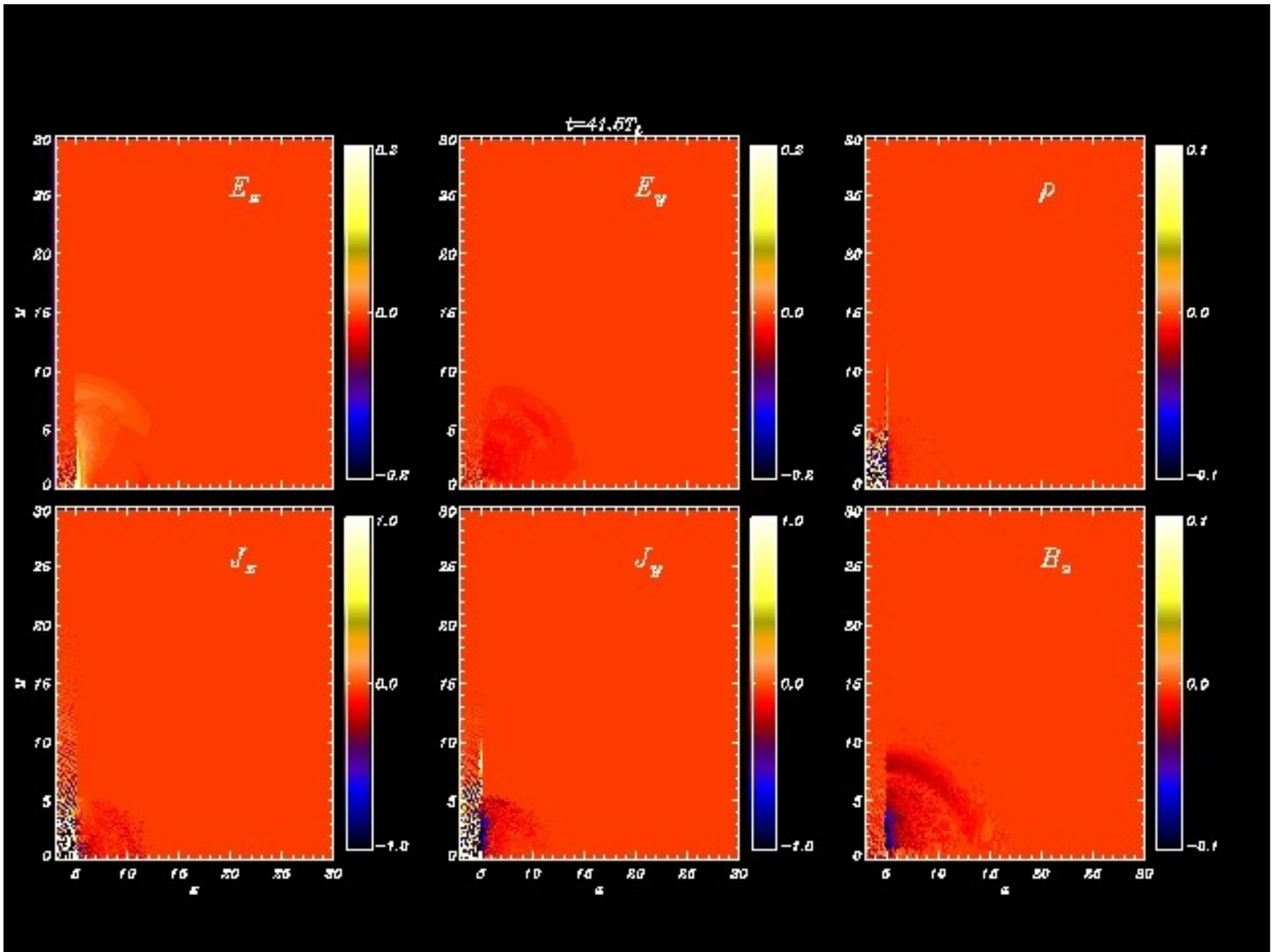
# Field propagation on the rear surface of solid targets



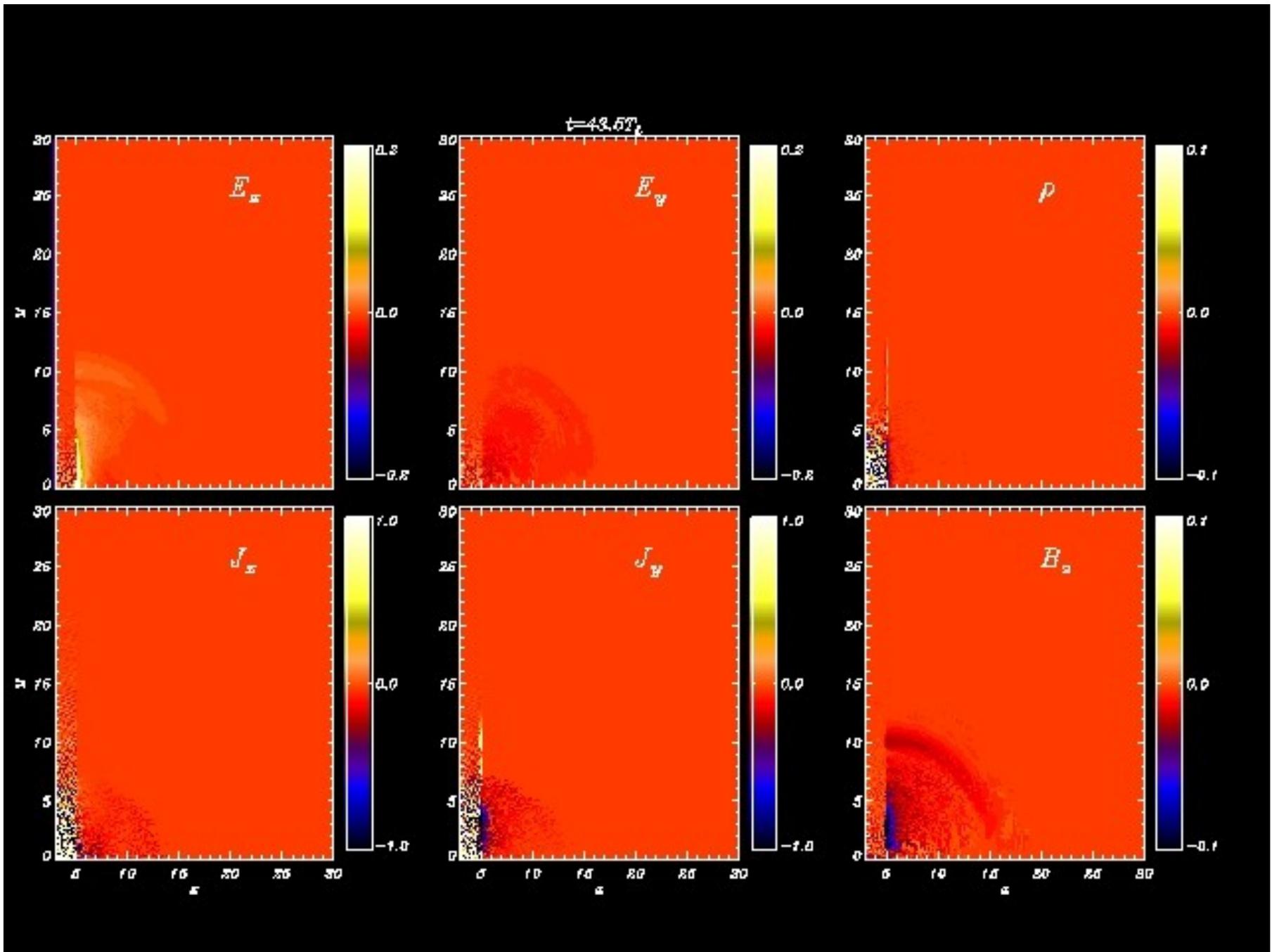
# Field propagation on the rear surface of solid targets



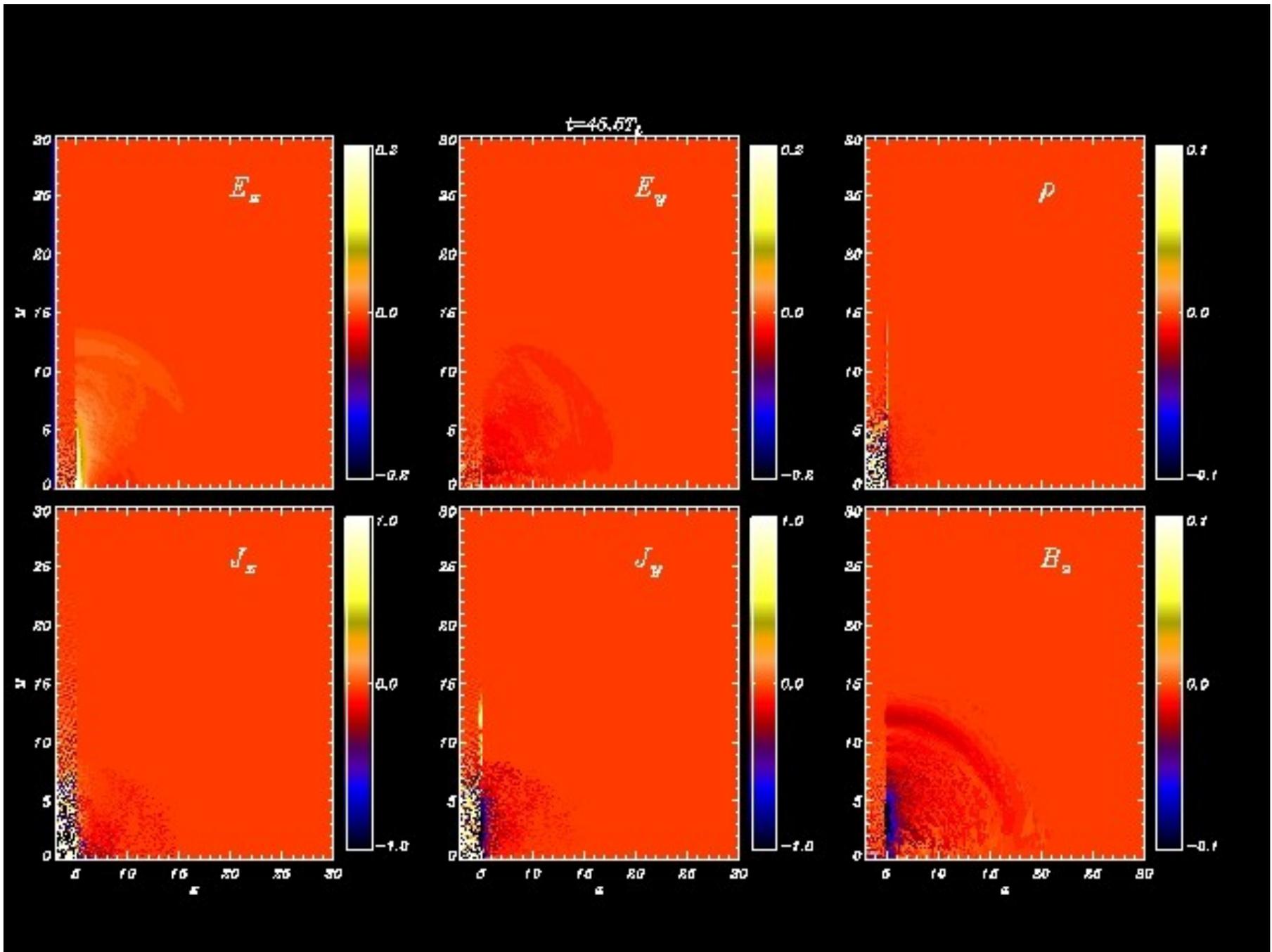
# Field propagation on the rear surface of solid targets



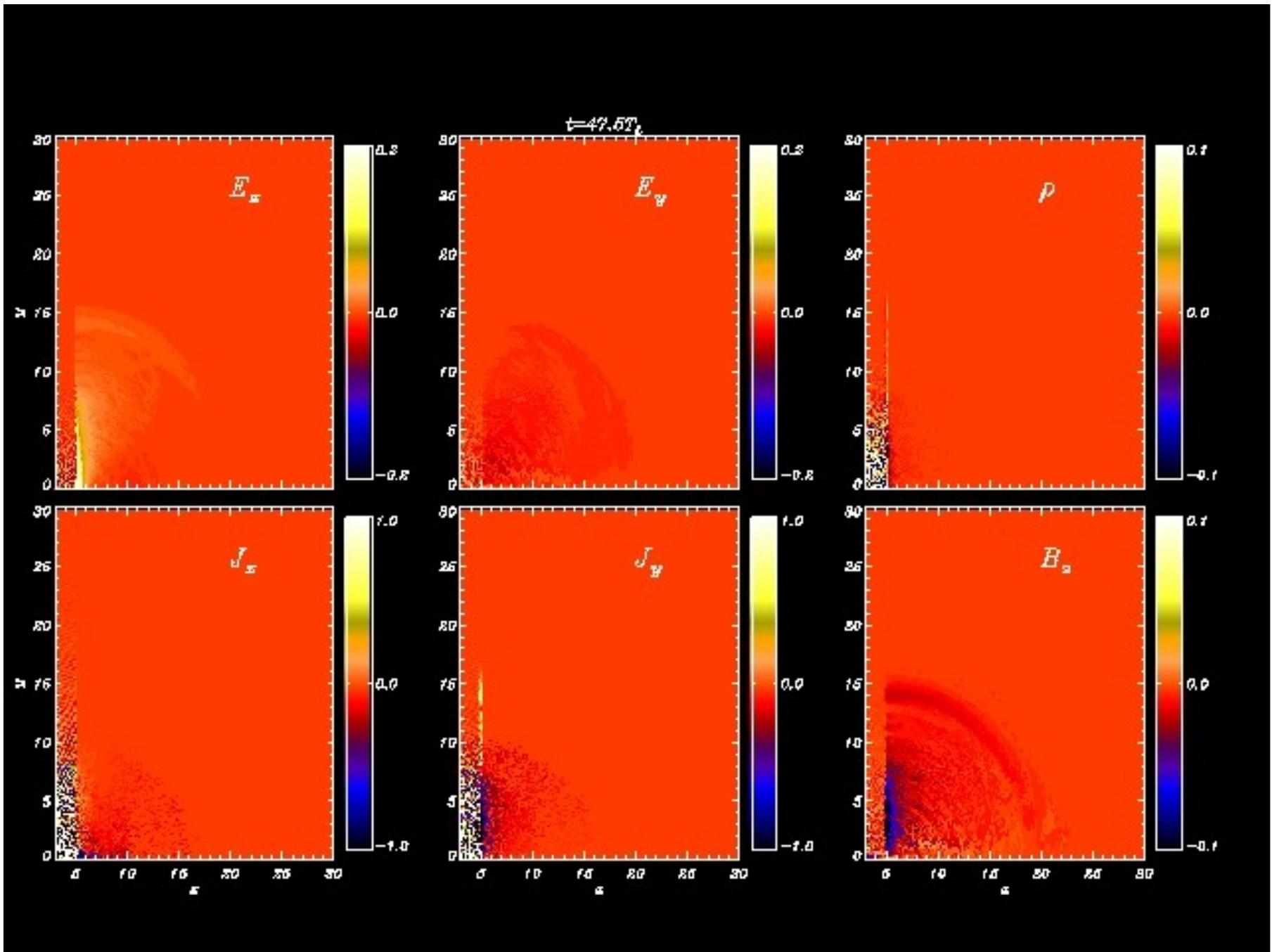
# Field propagation on the rear surface of solid targets



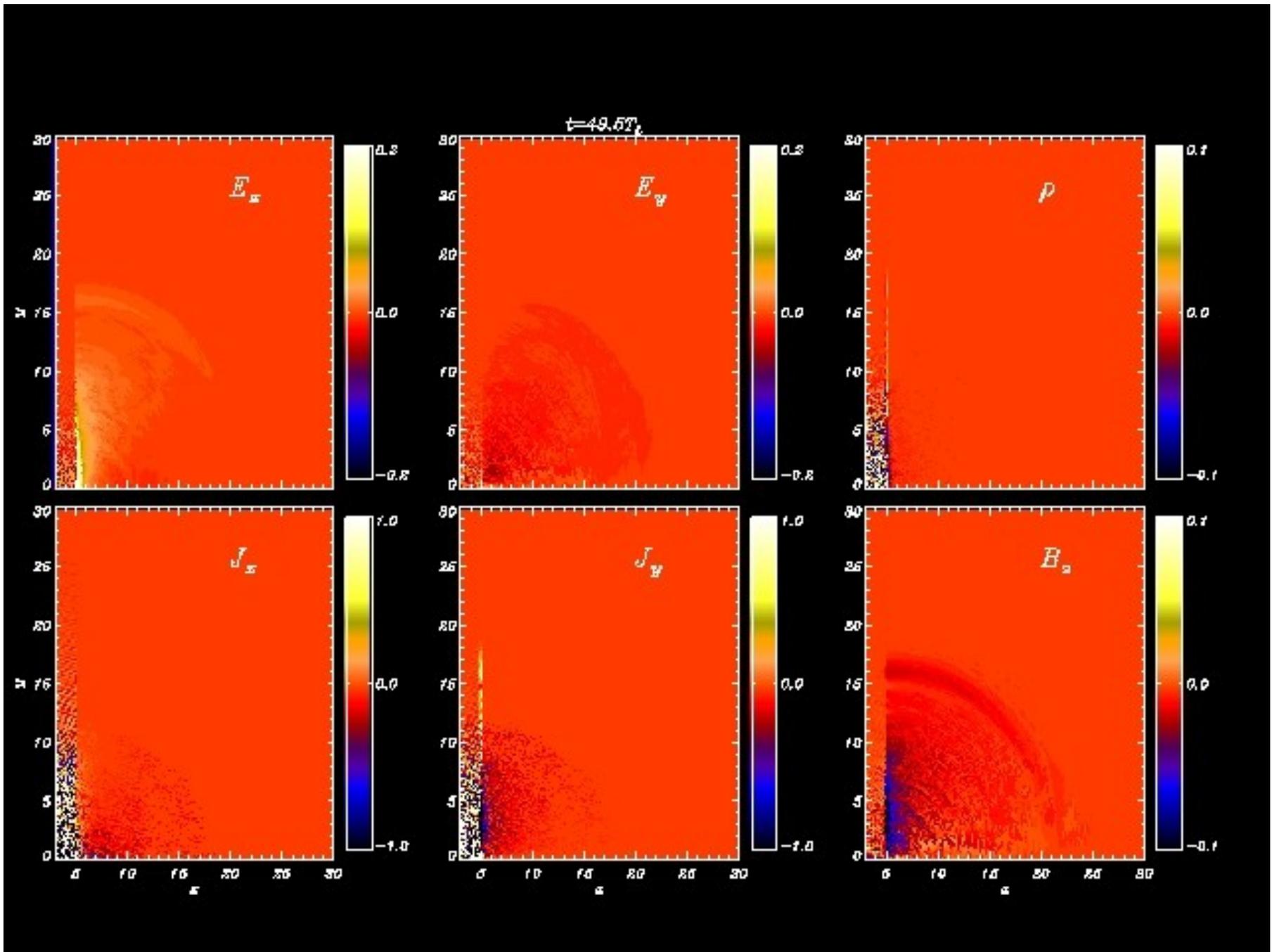
# Field propagation on the rear surface of solid targets



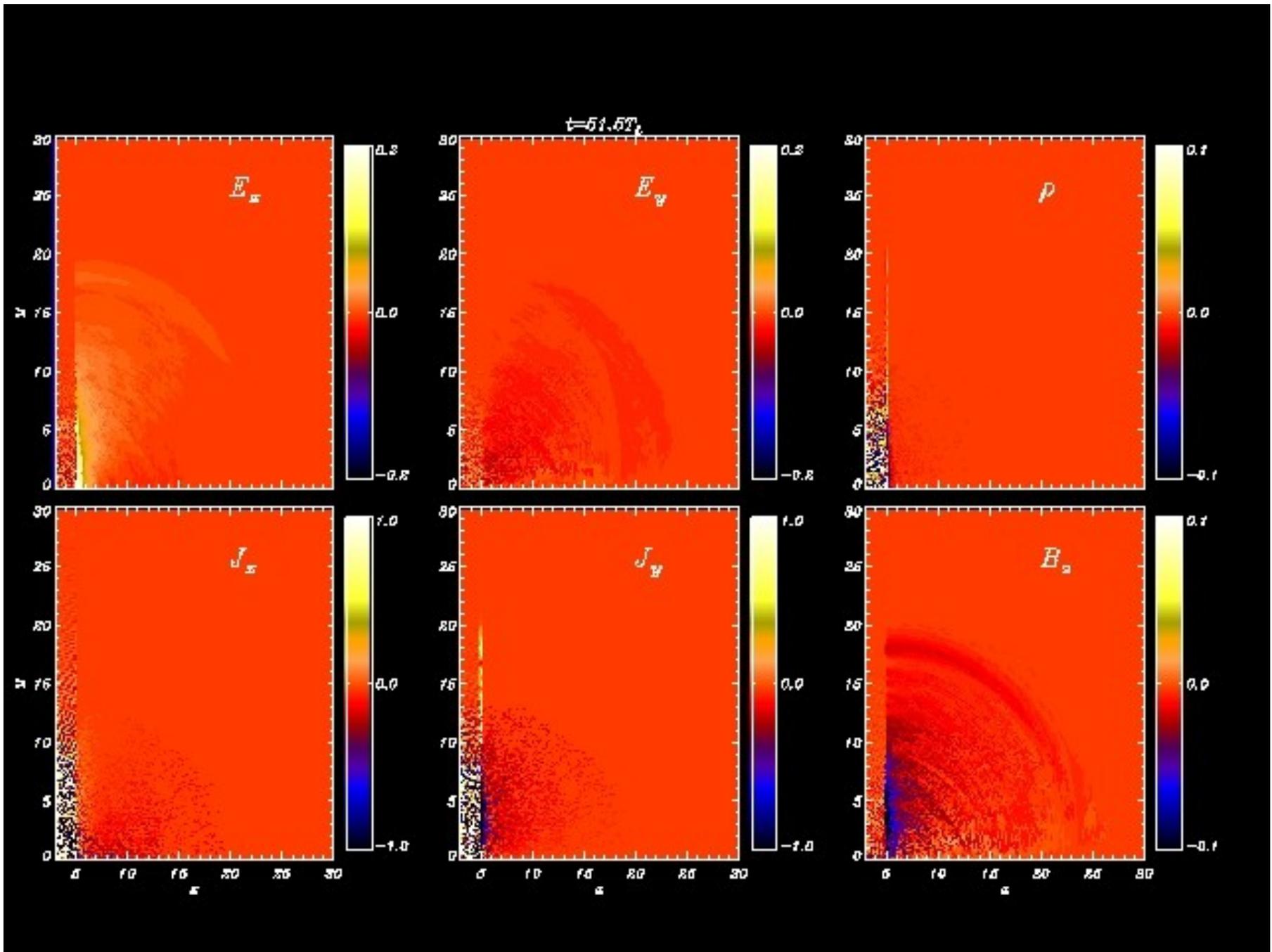
# Field propagation on the rear surface of solid targets



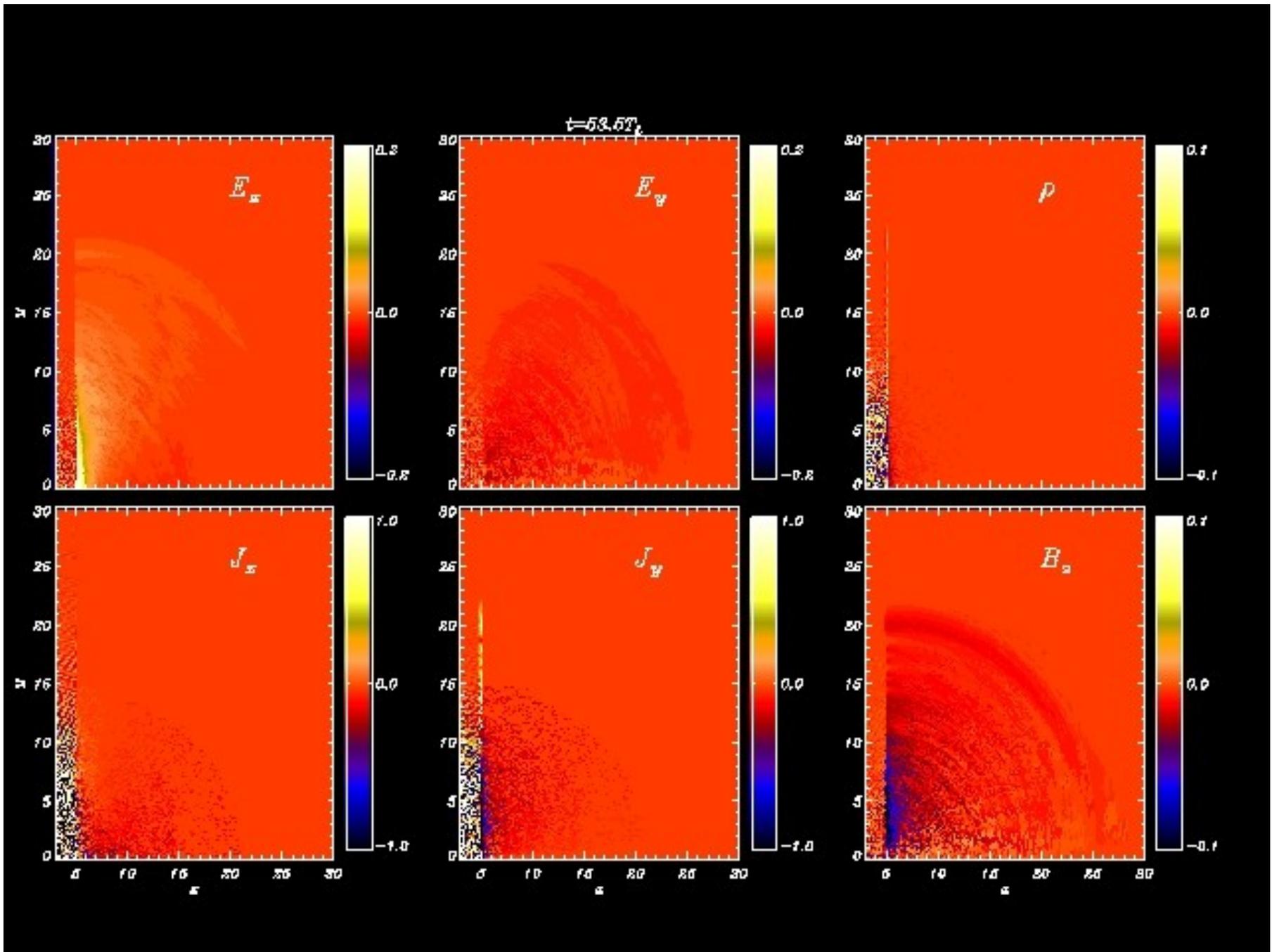
# Field propagation on the rear surface of solid targets



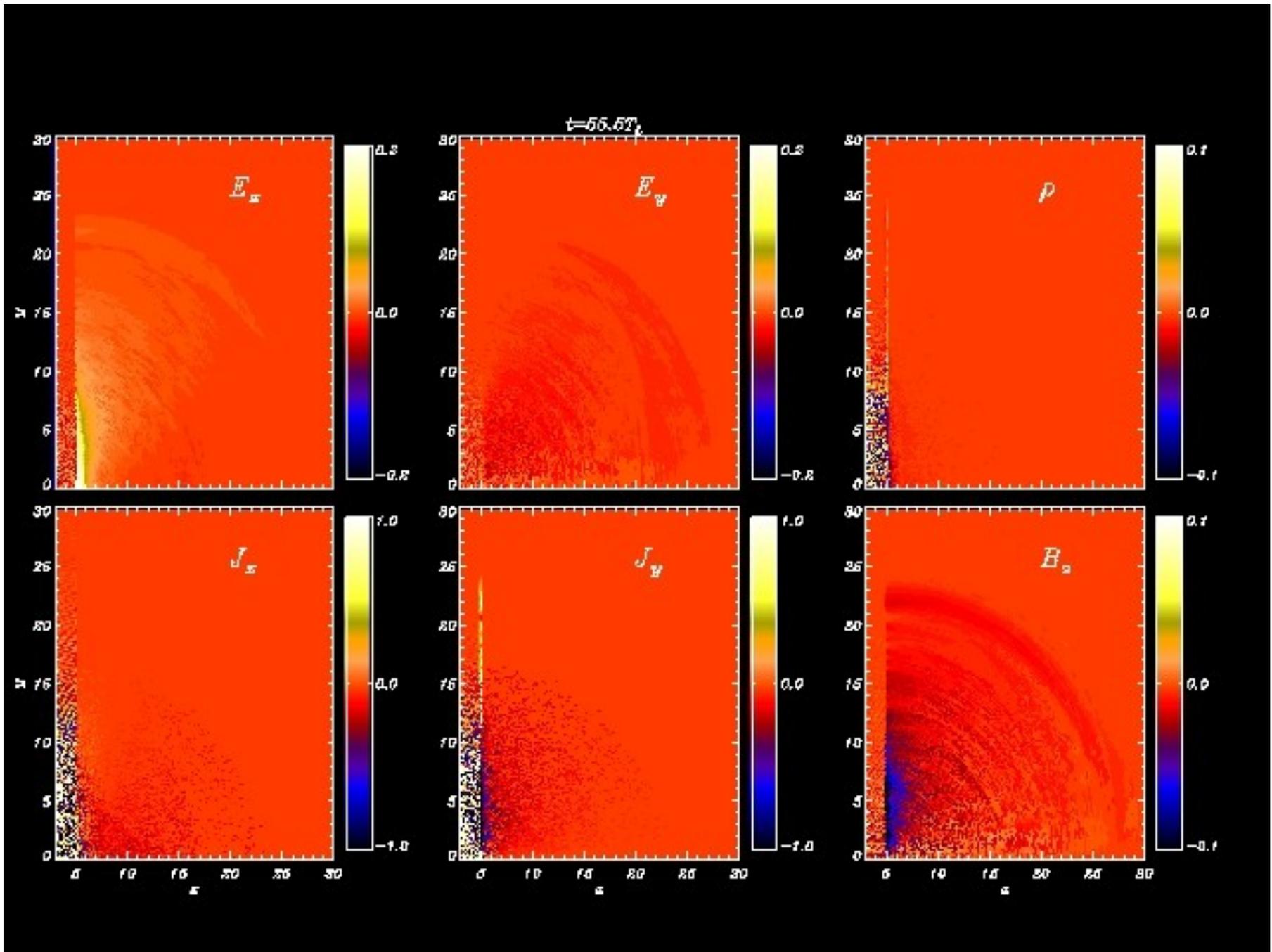
# Field propagation on the rear surface of solid targets



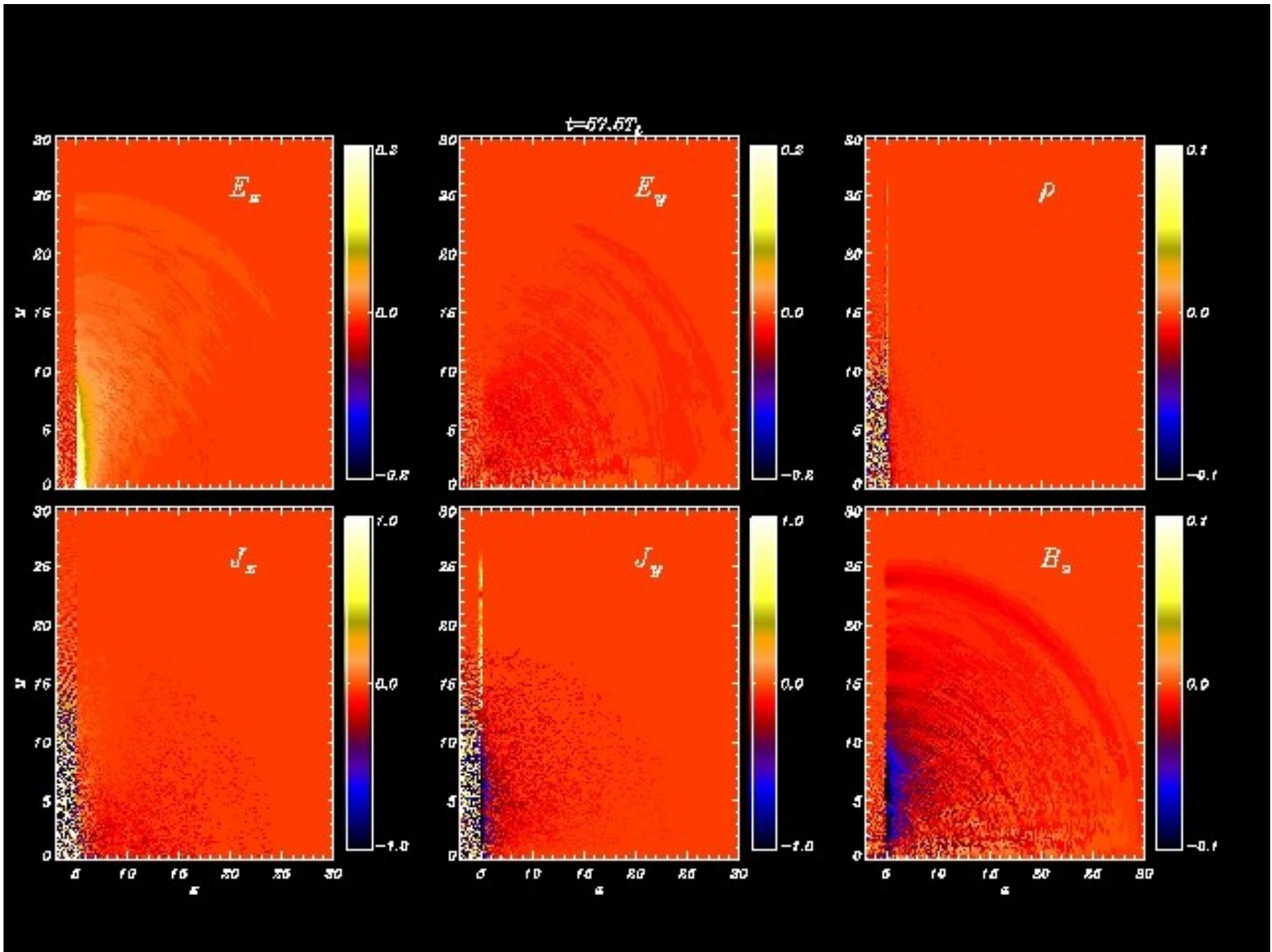
# Field propagation on the rear surface of solid targets



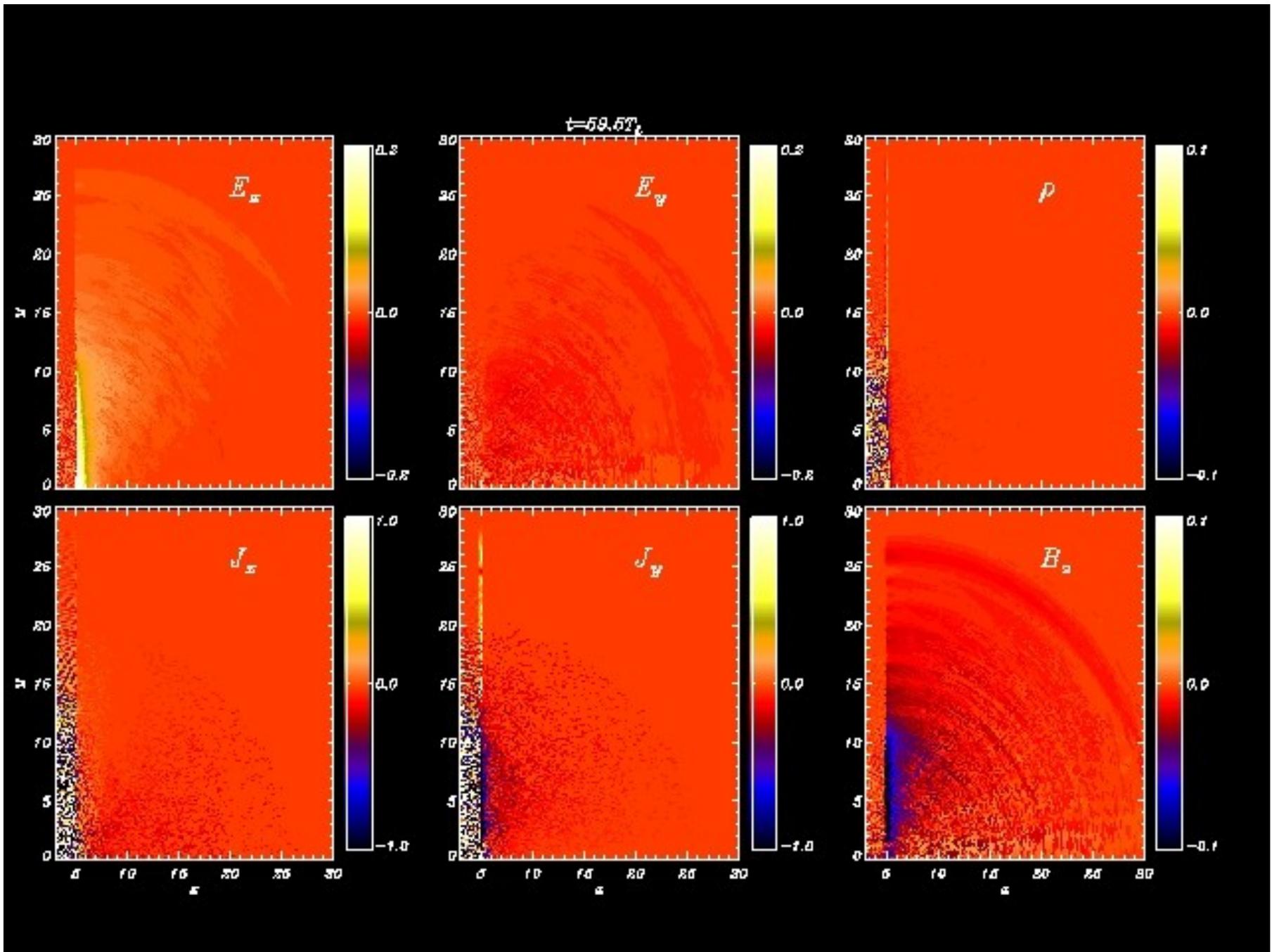
# Field propagation on the rear surface of solid targets



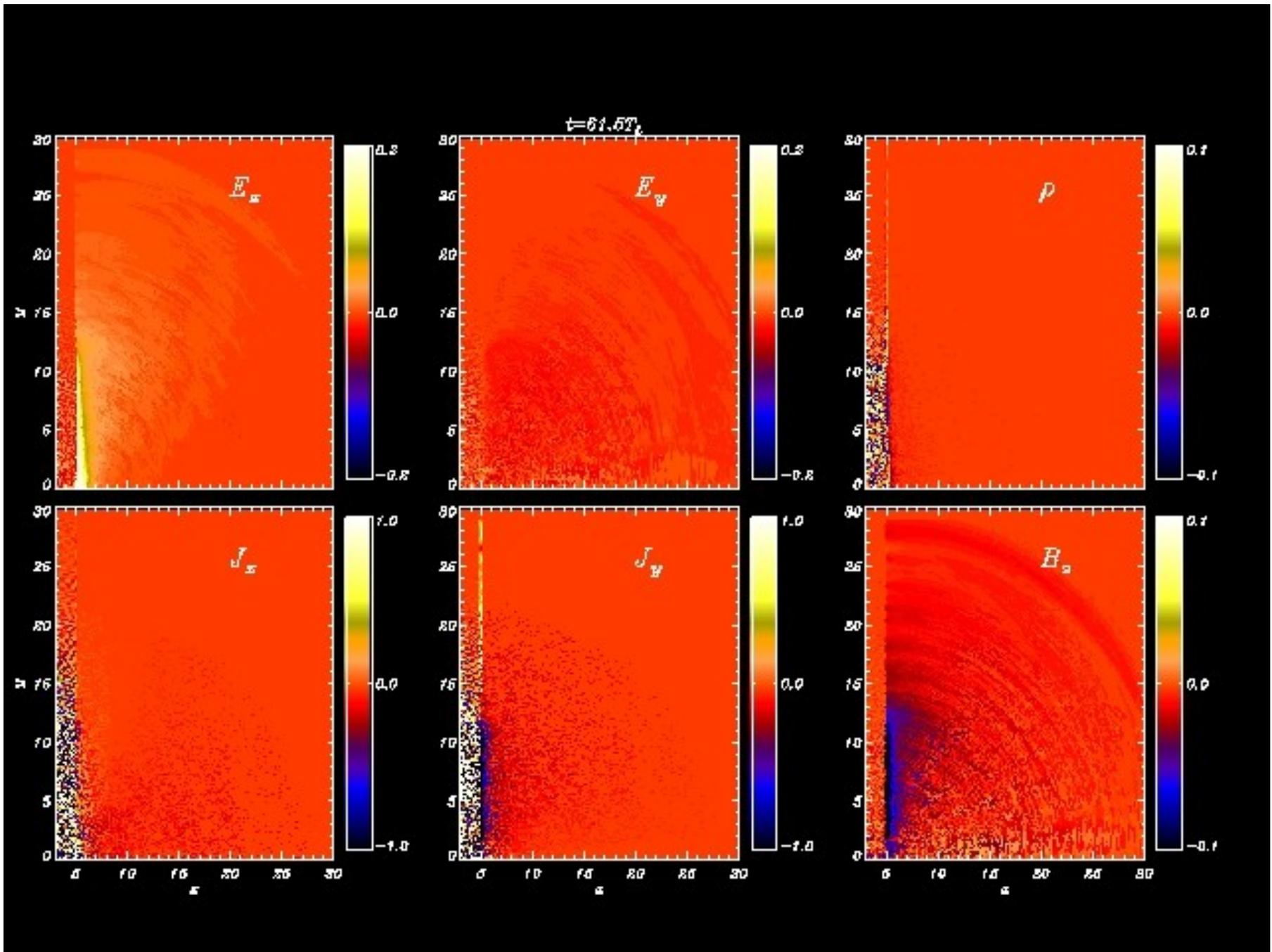
# Field propagation on the rear surface of solid targets



# Field propagation on the rear surface of solid targets



# Field propagation on the rear surface of solid targets



# Conclusions

- The proton probing technique (PPT) with picosecond resolution allowed detailed studies of many relevant ultrafast phenomena in laser-plasma interactions for the first time, stimulating and challenging theoretical and computational work
- Noticeably, PPT improved our understanding of the physics of proton or ion acceleration itself
- Several additional phenomena may be investigated in the future, especially with laser systems allowing
  - multiple beam probing
  - higher ion energies (study of faster phenomena, probing of very dense plasmas)

This talk may be downloaded from

[www.df.unipi.it/~macchi/talks.html](http://www.df.unipi.it/~macchi/talks.html)

# Electron probing of “colder” plasmas

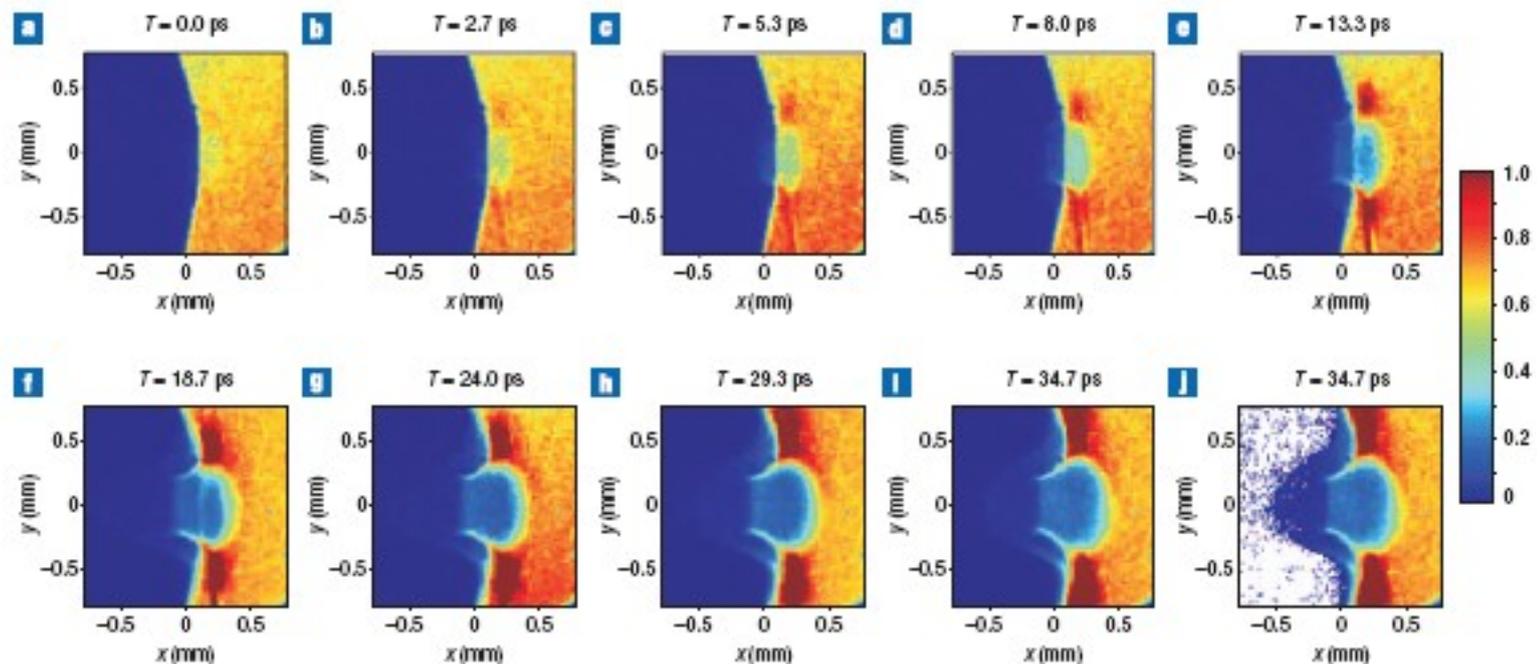
## Picosecond electron deflectometry of optical-field ionized plasmas

MARTIN CENTURION<sup>1\*†</sup>, PETER RECKENTHAELER<sup>1,2†</sup>, SERGEI A. TRUSHIN<sup>1</sup>, FERENC KRAUSZ<sup>1,2</sup>  
AND ERNST E. FILL<sup>1</sup>

<sup>1</sup>Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, D-85748 Garching, Germany

<sup>2</sup>Ludwig-Maximilians-Universität München, Am Coulombwall 1, D-85748 Garching, Germany

Nature  
Photonics  
2 (2008) 315



20 keV electrons suitable for measurements of  
 $E \sim 10^7$ - $10^8$  V/cm and  $B \sim 10^4$  Gauss

# Study of “coherent”, long-lived field structures

**Theory** and **numerical simulations** show that a variety of slowly varying structures (**solitons**, **vortices**, **cavitons** ...) is generated during laser-plasma interactions.

Bubble-like structures interpreted as remnants of **relativistic solitons** (“post-solitons”)

[Borghesi et al., Phys. Rev. Lett. **88** (2002) 135002]

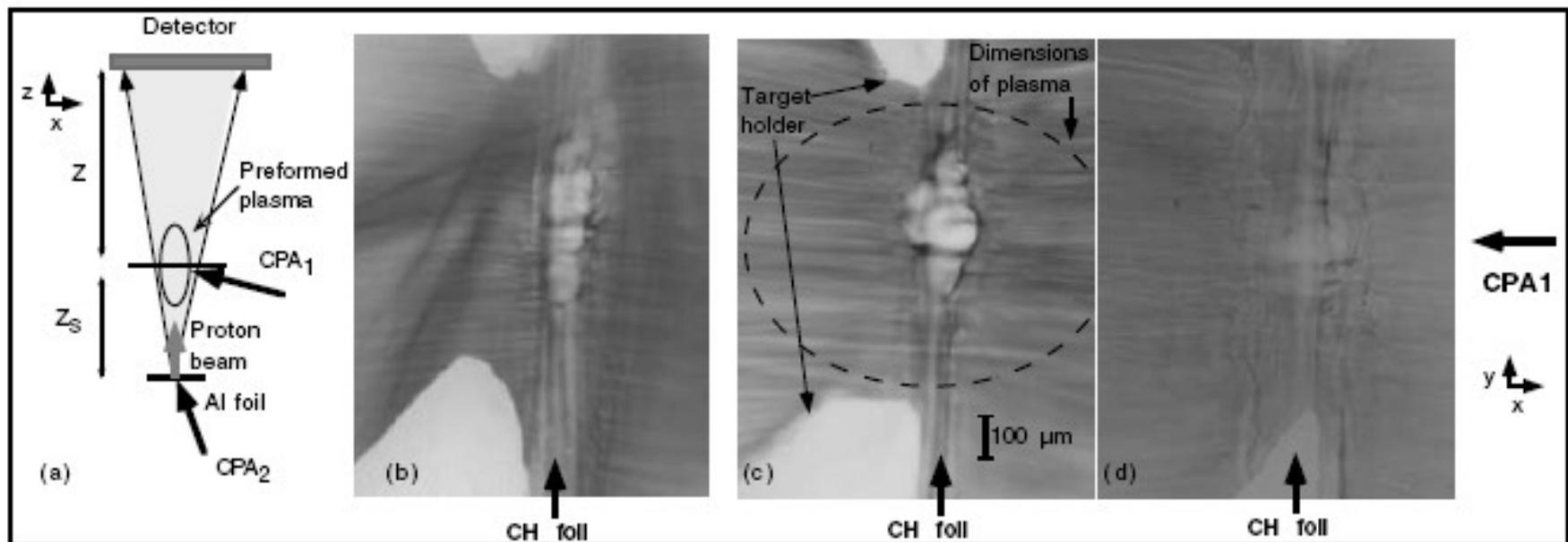


FIG. 1. (a) Experimental arrangement. (b), (c), (d) Proton images of the preformed plasma taken with 6–7 MeV protons, respectively: (b) 25 ps; (c) 45 ps; (d) 95 ps after the CPA<sub>1</sub> interaction. The scale refers to dimensions in the object plane. The dashed line indicates the dimensions of the preformed plasma defined by  $n \approx 0.01n_{cr}$  (at  $\lambda = 1 \mu\text{m}$ ).

# Study of “coherent”, long-lived field structures

**Theory** and **numerical simulations** show that a variety of slowly varying structures (**solitons**, **vortices**, **cavitons** ...) is generated during laser-plasma interactions.

Ion modulations resulting from onset and evolution of **Buneman instability** in the late evolution of a plasma wake

[Borghesi et al., Phys. Rev. Lett. **94** (2005) 195003]

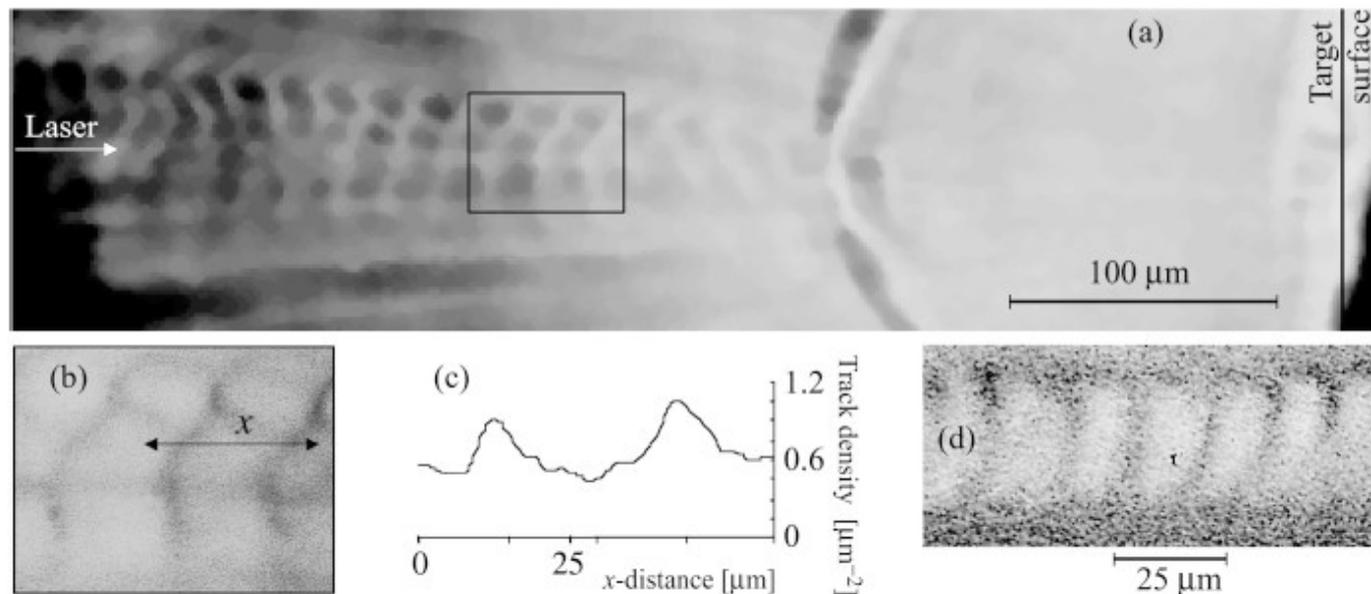


FIG. 1. (a) Proton projection image of the region in front of the laser-irradiated target, taken 20 ps after the interaction. The picture is a reflection scan of the exposed CR 39; (b) Detail of the image in frame (a); (c) Profile of the proton track density along the direction indicated by the arrow in (a); (d) Detail of the pattern observed at the back of a 0.9 μm Mylar target 20 ps after the interaction. The detail shown was located at a distance of about 200 μm from the original target plane.

# Study of “coherent”, long-lived field structures

**Theory** and **numerical simulations** show that a variety of slowly varying structures (**solitons**, **vortices**, **cavitons** ...) is generated during laser-plasma interactions.

Regular, quasi-periodic structures observed inside or near the charge-displacement channel at late times

[T.V.Lyseikina, F.Ceccherini, F. Cornolti, E.Yu.Echkina, A.Macchi, F.Pegoraro, M.Borghesi, S.Kar, L.Romagnani, S.V.Bulanov, O.Willi, M.Galimberti, [arXiv:physics/0701139](https://arxiv.org/abs/physics/0701139)]

