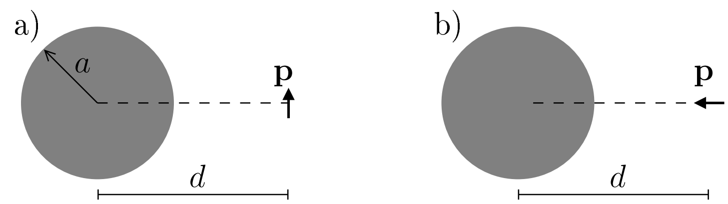
Dipoles and Spheres

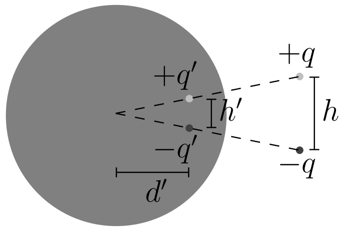
An electric dipole $\mathbf{p}$ is at a distance d from the center of a conducting sphere of radius a. Find the solution for the electrostatic potential (both for a grounded and for an electrically isolated, charge neutral sphere) in the two cases of



a) $\mathbf{p}$ perpendicular to the direction from $\mathbf{p}$ to the center of the sphere,
b) $\mathbf{p}$ directed towards the center of the sphere.

Solution

a)

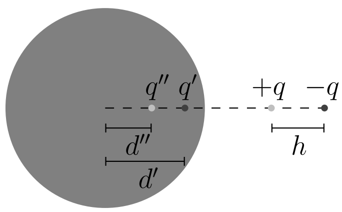


We use the method of image charges, starting from the known solution for a point charge in front of a conducting, grounded sphere. We replace the dipole by two charges $\pm q$ separated by a displacement $\mathbf{h}$, and we will later take the limit $h \to 0$ with $q\mathbf{h} \to \mathbf{p}$. The two charges induce two images $\pm q' = \mp(a/d)q$, respectively, which to first order in h/a are at a distance $d' \simeq a^2/d$ from the center. Thus the two images are displaced vertically by $h' = (d'/d)h = h(a/d)^2$. This corresponds to an image dipole

$$\mathbf{p}' = q\mathbf{h}' = -q\mathbf{h}(a/d)^3 = -\mathbf{p}(a/d)^3 \; . \tag{1}$$

Since the induced charge on the sphere (equal to the total charge of the images) is zero, there is no difference between the solution for a grounded and an isolated sphere.

b)



Following the same method as in **a)**, in this case $+q$ and $-q$ are at different distances from the center of the sphere (at d and $d+h$, respectively) and therefore they induce different image charges at different positions:

$$q' = -qa/d \; , \quad d' = a^2/d \; , \qquad q'' = +qa/(d+h) \; , \quad d'' = a^2/(d+h) \; . \tag{2}$$

Taking the limit $h \to 0$, the image q'' "falls" on q' and the net remaining charge is

$$q' + q'' = -qh\frac{a}{d(d+h)} \xrightarrow{h\to 0} -p\frac{a}{d^2} \; . \tag{3}$$

The electric dipole moment calculated with respect to the position of q' is

$$p' = q''(d' - d'') = qh\frac{a^3}{d(d+h)^2} \xrightarrow{h\to 0} p\frac{a^3}{d^3} \; . \tag{4}$$

Notice that $\mathbf{p}'$ is in the same direction as $\mathbf{p}$.

The above solution works for the grounded sphere. Since there is a net charge $q''' = -pa/d^2$ on the sphere, in the case of an isolated, charge neutral system we must add another image charge $-q'''$ in the center of the sphere.