Correlations in Economic Time Series

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Abstract

The correlation function of a financial index of the New York stock exchange, the S&P 500, is analyzed at 1 min intervals over the 13-year period, Jan 84 – Dec 96. We quantify the correlations of the absolute values of the index increment. We find that these correlations can be described by two different power laws with a crossover time $t_\times \approx 600$ min. Detrended fluctuation analysis gives exponents $\alpha_1 = 0.66$ and $\alpha_2 = 0.93$ for $t < t_\times$ and $t > t_\times$ respectively. Power spectrum analysis gives corresponding exponents $\beta_1 = 0.31$ and $\beta_2 = 0.90$ for $f > f_\times$ and $f < f_\times$ respectively.

A topic of considerable recent interest to both the economics and physics communities is whether there are correlations in economic time series and, if so, how to best quantify these correlations [1,2,3,4]. Here we study the S&P 500 index of the New York stock exchange over a 13-year period (Fig. 1a). We calculate the logarithmic increments $g(t) \equiv \ln Z(t+1) - \ln Z(t)$ over a fixed time lag of 1 min, where $Z(t)$ denotes the index at time t (t counts the number of minutes during the opening hours of the stock market), and quantify the correlations as follows:

(i) We find that the correlation function of $g(t)$ decays exponentially with a characteristic time of the order of 1-10 min, but the absolute value $|g(t)|$ does not. This result is consistent with previous studies on several economic series [2,3,4].

(ii) We calculate the power spectrum of $|g(t)|$ (Fig. 2a), and find that the data fit not one but rather two separate power laws: for $f > f_\times$ the power law exponent is $\beta_1 = 0.31$, while for $f < f_\times$ the exponent $\beta_2 = 0.90$ is three times larger; here $f_\times$ is called the crossover frequency.

(iii) We confirm these results using the DFA (detrended fluctuation analysis) method (see Fig.2b), which allows accurate estimates of exponents *independent* of local trends [5]. From the behavior of the power spectrum, we expect that the DFA method will also predict two distinct regions of power law behavior, with exponents $\alpha_1 = 0.66$ and $\alpha_2 = 0.95$ for t less than or greater than a characteristic time scale $t_\times \equiv 1/f_\times$, where we have used the general mathematical result [6] that $\alpha = (1+\beta)/2$. The data of Fig. 2b yield $\alpha_1 = 0.66$, $\alpha_2 = 0.93$, thereby confirming the consistency of the power spectrum and DFA methods. Also the crossover time is very close to the result obtained from the power spectrum, with $t_\times \approx 1/f_\times \approx 600\,\text{min}$ (about 1.5 trading days).

We observed the crossover behavior noted above by considering the entire 13-year period studied, so it is natural to enquire whether it will still hold for periods smaller than 13 y. Therefore, we choose a sliding window (with size 1 y) and calculate both exponents α_1 and α_2 within this window as the window is dragged, down the data set. We find (Fig. 1b) that the value of α_1 is very "stable" (independent of the position of the window) fluctuating around the mean value 2/3. Surprisingly, however, the variation of α_2 is much greater, showing sudden jumps when very volatile periods enter or leave the time window.

We studied several standard mathematical models, such as fractional Brownian motion [6,7] and fractional ARIMA processes [8], commonly used to account for long-range correlation in a time series and found that none of them can reproduce the large fluctuation of α_2.

REFERENCES

[1] Mandelbrot, B. B., *J. Business* **36**, 393–413 (1963).

[2] Fama, E.-F., *J. Finance* **25**, 383–417 (1970).

[3] Ding, Z., Granger, C. W. J., and Engle, R. F., *J. Empirical Finance* **1**, 83–106 (1993).

[4] Dacorogna, M. M., Muller, U. A., Nagler, R. J., Olsen, R.B., and Pictet, O.V., *J. International Money and Finance* **12**, 413–438 (1993).

[5] Peng, C.-K. et al., *Phys. Rev.* E **49**, 1684–1688 (1994).

[6] Beran, J., *Statistics for Long-Memory Processes* (Chapman & Hall, NY, 1994).

[7] Mandelbrot, B. B., and van Ness, J. W., *SIAM Rev.* **10**, 422–437 (1968).

[8] Granger, C. W. J., and Ding, Z., *J. Econometrics* **73**, 61–77 (1996).

[9] Wood, R. A., McInish, T. H., and Ord, J. K., *J. of Finance* **40**, 723–741 (1985).

[10] Harris, L., *J. of Financial Economics* **16**, 99–117 (1986).

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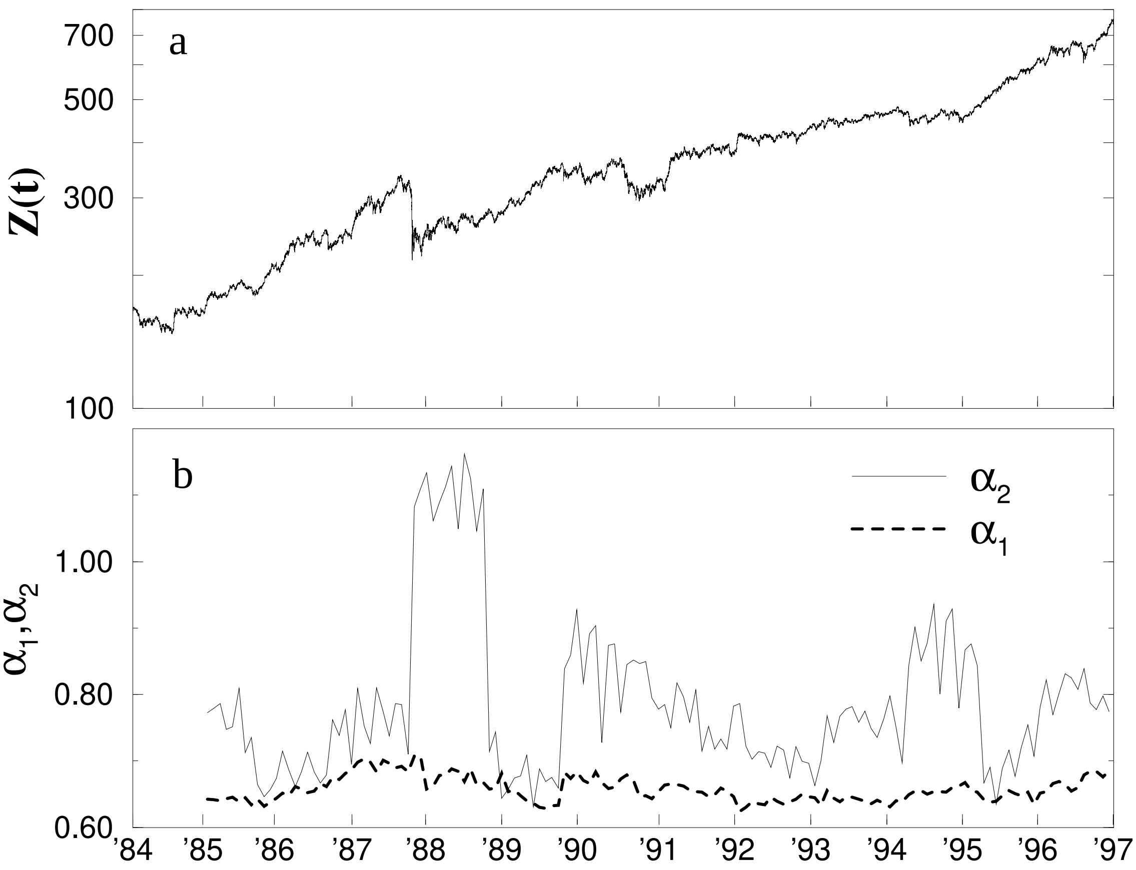
FIGURES

FIG. 1. (a) Raw data analyzed: The S&P 500 index $Z(t)$for the 13-year period 1 Jan 1984 – 31 Dec 1996 at intervals of 1 min. Note the large fluctuations, such as that on 19 Oct 1987 ("black Monday"). (b) Results of dragging a window of size 1 y down the same data base, one month at a time, and calculating the best fit exponent α_1 (dashed line) and α_2 (full line) for the time intervals $t < t_\times$ and $t > t_\times$ respectively.

FIG. 2. Plot of (a) the power spectrum $S(f)$ and (b) the detrended fluctuation analysis $F(t)$ of the absolute values of the 1 min increments. The lines show the best power law fits (r values are better than 0.99) to the data above and below the crossover frequency of $f_\times = (1/570)\ \mathrm{min}^{-1}$ in (a) and of the crossover time $t_\times = 600\,\mathrm{min}$ in (b). To remove artificial correlations resulting from the intra-day pattern of the market activity [9,10], we analyze normalized data $|g_n(t)| \equiv |g(t)|/A(t)$, where $A(t)$ is the activity at the same time of the day averaged over all days of the data set. For the DFA method, we integrate $|g_n(t)|$ once; then we determine the fluctuations $F(t)$ of the integrated signal around the best linear fit in a time window of size t.



a
Z(t)
700
500
300
100
b
α_2
α_1
α_1,α_2
1.00
0.80
0.60
'84
'85
'86
'87
'88
'89
'90
'91
'92
'93
'94
'95
'96
'97

Fig. 1

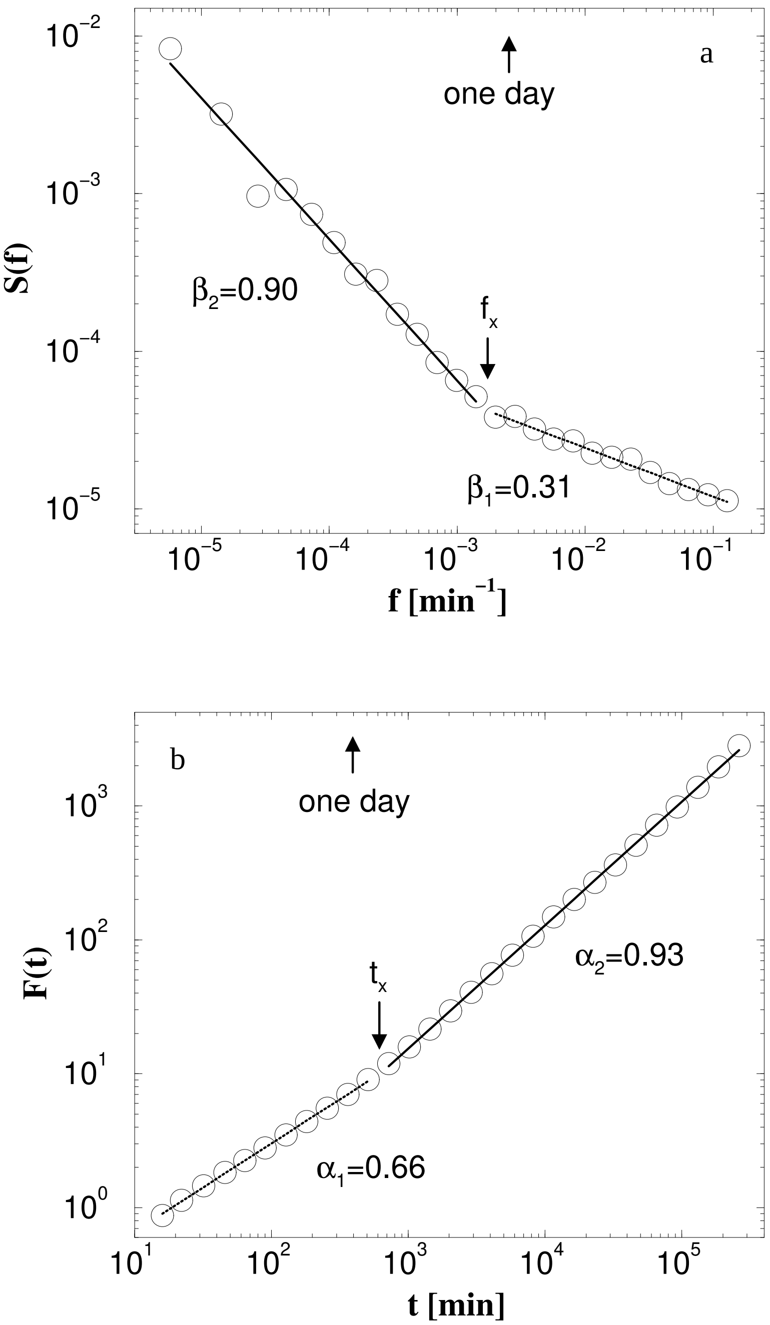

a
one day
S(f)
β_2=0.90
f_x
β_1=0.31
f [min^{-1}]
b
one day
F(t)
t_x
α_2=0.93
α_1=0.66
t [min]

Fig. 2