Quantum calculation of nucleus-vortex interaction in the inner crust of neutron stars

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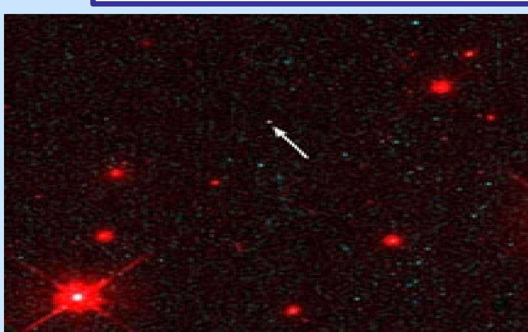
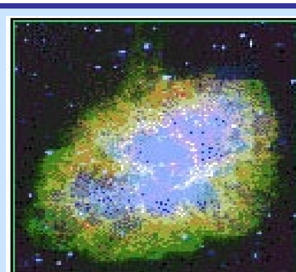
Milano University and INFN

F. Barranco

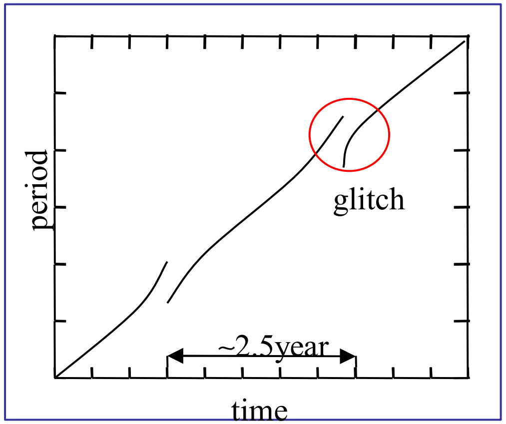
Sevilla University





Crab Nebula

Glitches



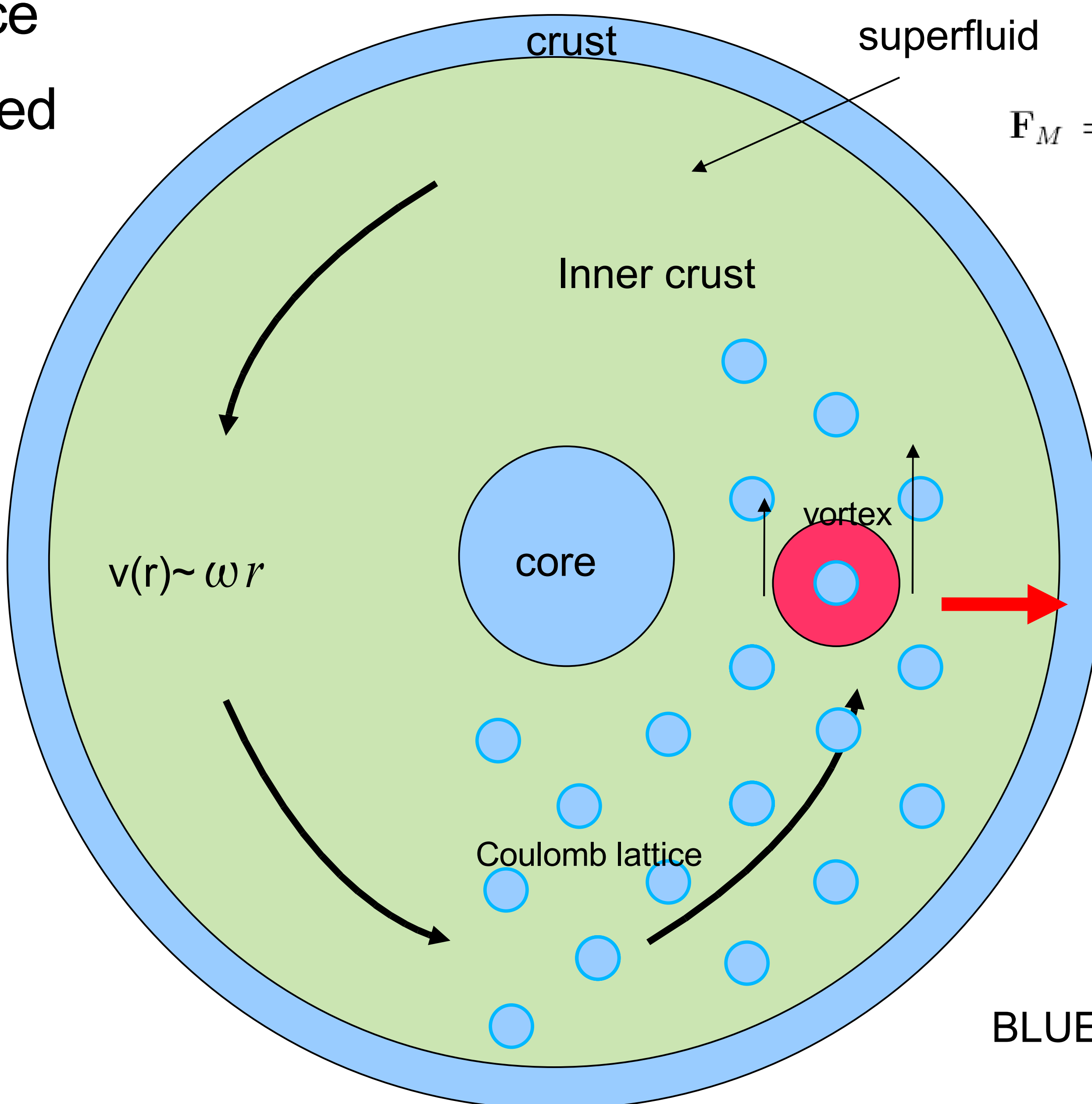
As a rule, rotational period of a neutron star slowly increases because the system loses energy emitting electromagnetic radiation.

Sudden spin ups are measured

One of the accredited explanations → Superfuid nature of nucleons in the inner crust

P.W. Anderson and N.Itoh, Nature 256(1975)25

Reference frame fixed with the nuclear lattice



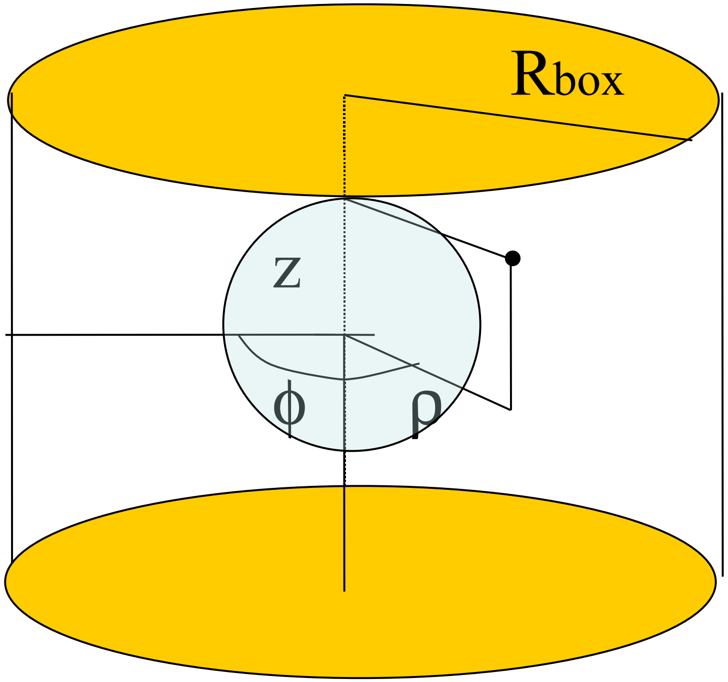
$$\mathbf{F}_M = \rho_s \mathbf{K} \times (\mathbf{v}_V - \mathbf{v}_s),$$

The Magnus force pushes the vortex outwards

BLUE MOVES AS A RIGID BODY!!

Yu.G.Mamaladze E.L.Andronikashvili. Review of modern Physics, 38,4:567-625, 1986.

We solve the HFB (De Gennes) equations expanding on a single-particle basis in cylindrical coordinates



- The equations are solved self-consistently
- we used a Skyrme interactions
- we constrained protons to have spherical symmetry
- we neglected spin-orbit interaction

$$\begin{pmatrix} \varepsilon_i - \lambda & \Delta \\ \Delta & -(\varepsilon_i - \lambda) \end{pmatrix} \begin{pmatrix} U_i \\ V_i \end{pmatrix} = E_i \begin{pmatrix} U_i \\ V_i \end{pmatrix}$$

R_{BOX} = 30 fm

$$U_\alpha(\rho, z, \theta) = \Sigma_{nk} u_{nk}^{\alpha m} J_{nm}(\rho) \sin(kz) e^{im\theta}$$

$$V_\alpha(\rho, z, \theta) = \Sigma_{nk} u_{nk}^{\alpha m} J_{n(m-\nu)}(\rho) \sin(kz) e^{i(m-\nu)\theta}$$

Microscopic quantum calculation of the vortex-nucleus system

The characteristic ansatz for the study of a vortex is

$$\Delta(\rho, z, \phi) = \Delta(\rho, z)\, e^{i\nu\phi}$$

where $\Delta(\rho, z)$ is a real function and ν=0,1,2,... is the vortex index. When ν=0 the no vortex standard HFB situation is recovered .

The parity of this function is given by the change in sign when

$$\phi \to \phi + \pi$$

$$z \to -z$$

For a system with mirror symmetry with respect to x-y plane,

$$\Delta(\rho, z) = \Delta(\rho, -z),$$

we have

$$\pi = (-1)^{\nu}$$

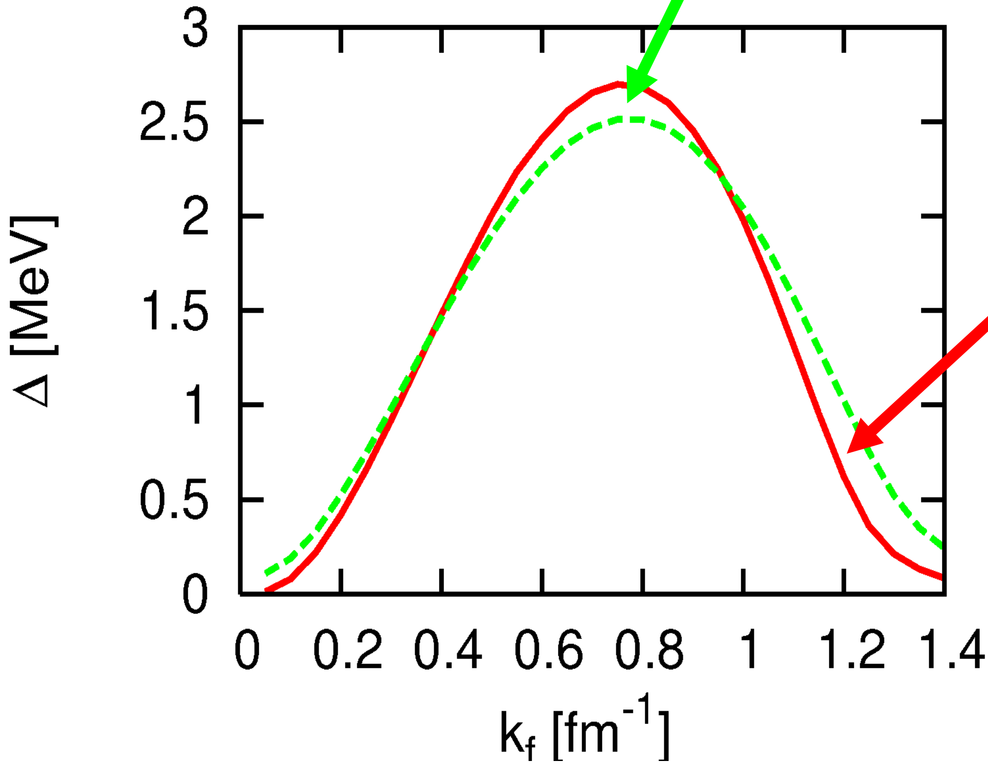
PAIRING INTERACTION

$$V = -481\left(1 - 0.7\left(\rho/\rho_0\right)^{0.45}\right)\delta\left(r_1 - r_2\right)\ MeV\ fm^3$$

With a HF field based on the Skyrme interaction

Ecut = 60 MeV

Results in neutron matter



Argonne force

E. Garrido et al. Phys. Rev. C60 (1999) 64312

For $\nu=1$ the vortex pairing field shows negative parity, and thus **the u-v amplitudes entering the microscopic expression for the pairing field (ZERO RANGE PAIRING INTERACTION ASSUMED)**

$$\Delta(\rho,z,\phi)=\frac{g}{2}\sum_{\alpha}u_{\alpha}(\rho,\phi,z)v_{\alpha}^{*}(\rho,\phi,z) \quad ,$$

must show also different parity

For an **infinitely long vortex** this leads to

$$u_{\alpha}(\rho,\phi,z)=u_{qmk}(\rho)\,e^{im\phi}e^{ikz}$$
$$v_{\alpha}(\rho,\phi,z)=v_{qmk}(\rho)\,e^{i(m-1)\phi}e^{ikz}$$

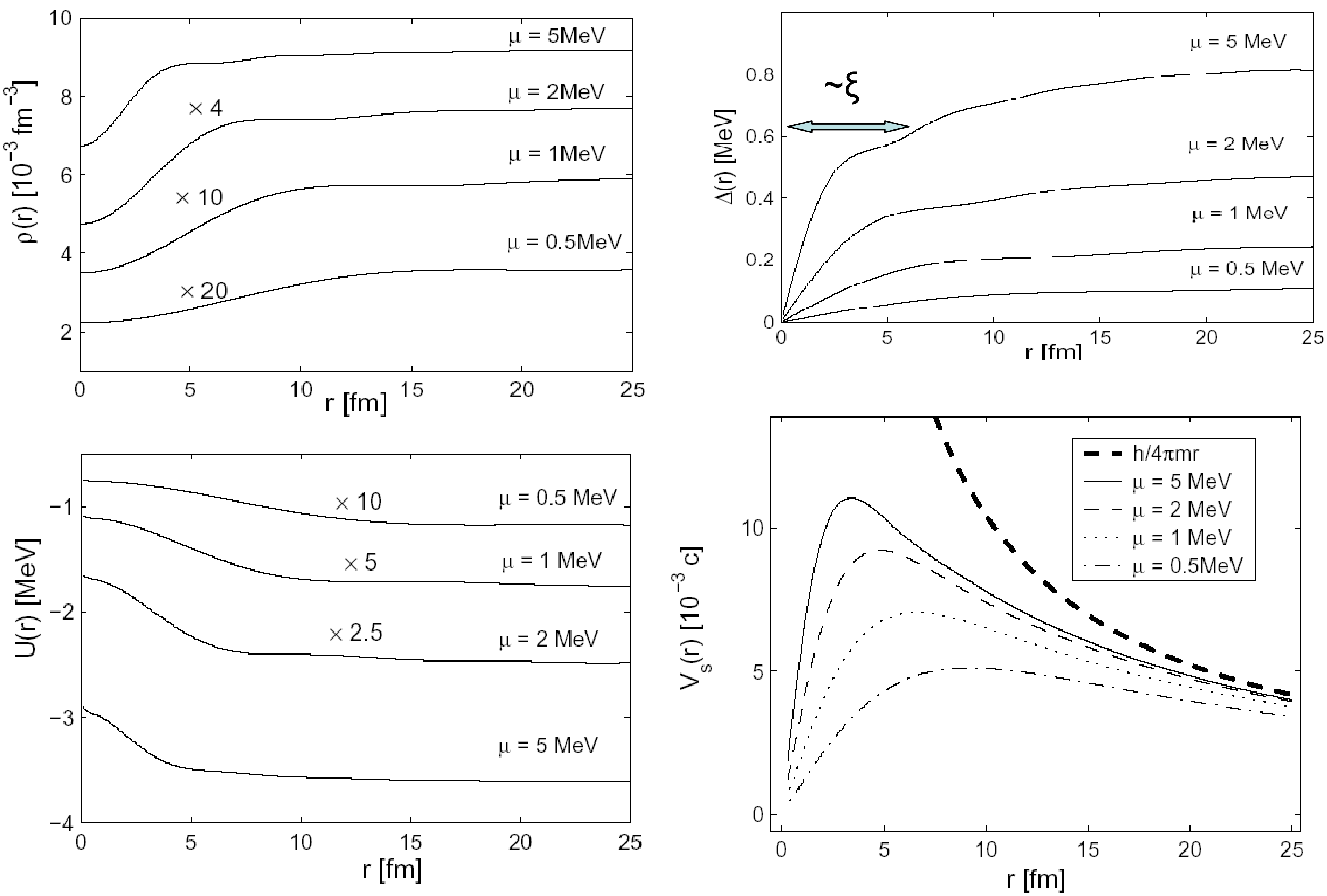
$$Vel_{vortex}(\rho,z)=-\frac{i\hbar}{m\rho n(\rho,z)}\sum_{\alpha}v_{\alpha}^{*}(\rho,z,\phi)\frac{\partial v_{\alpha}(\rho,z,\phi)}{\partial\phi}$$

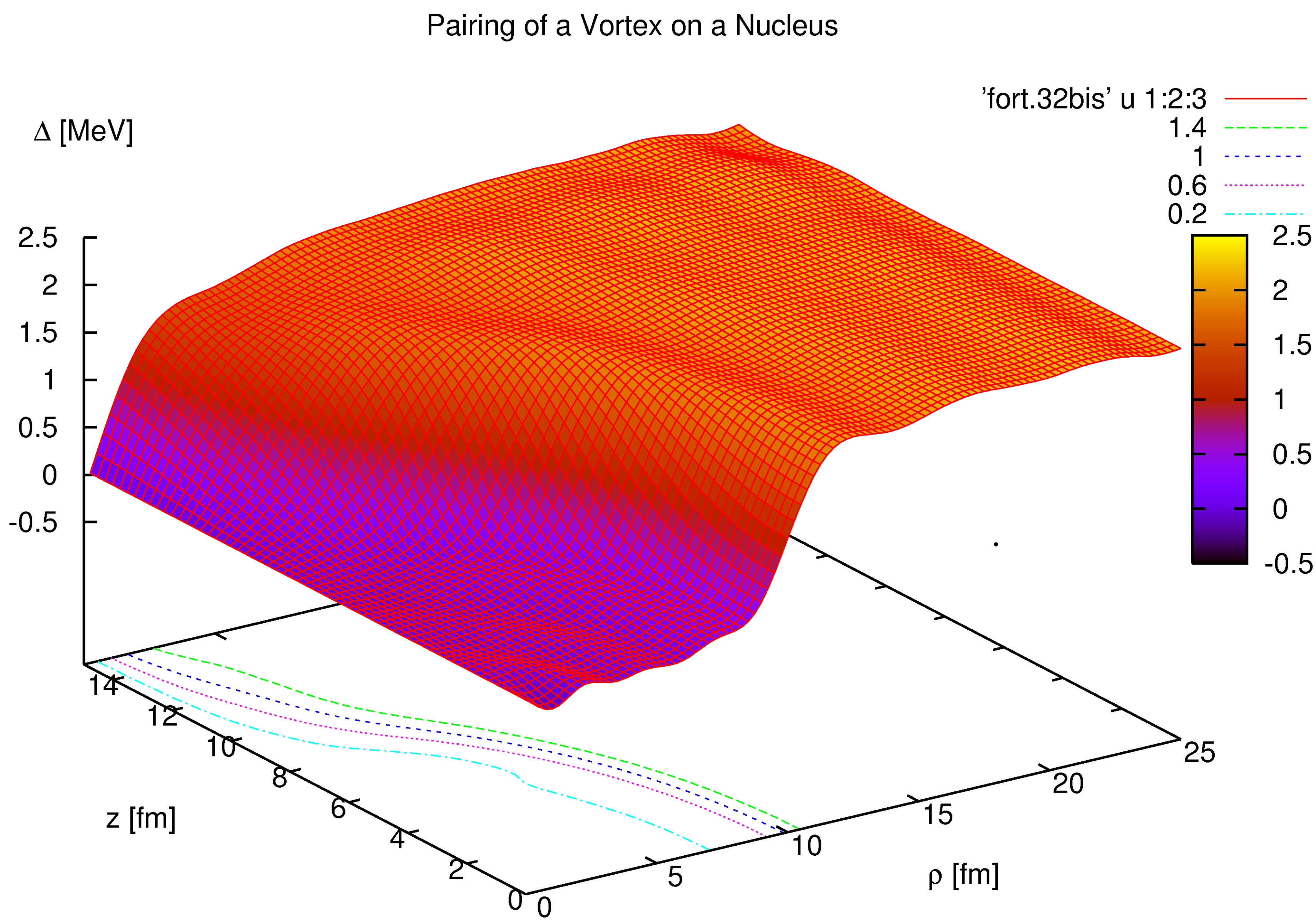
With the radial wave functions satisfying the Bogoliubov de Gennes equation

$$\left[-\frac{\hbar^2}{2\mu}\left(-k^2-\frac{m^2}{\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho}\right)+V(\rho)-\lambda\right]u_{qmk}+\Delta(\rho)\,v_{qmk}=E_{qmk}\,u_{qmk}$$

$$\Delta(\rho)\,u_{qmk}-\left[-\frac{\hbar^2}{2\mu}\left(-k^2-\frac{(m-1)^2}{\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho}\right)+V(\rho)-\lambda\right]v_{qmk}=E_{qmk}\,v_{qmk}$$

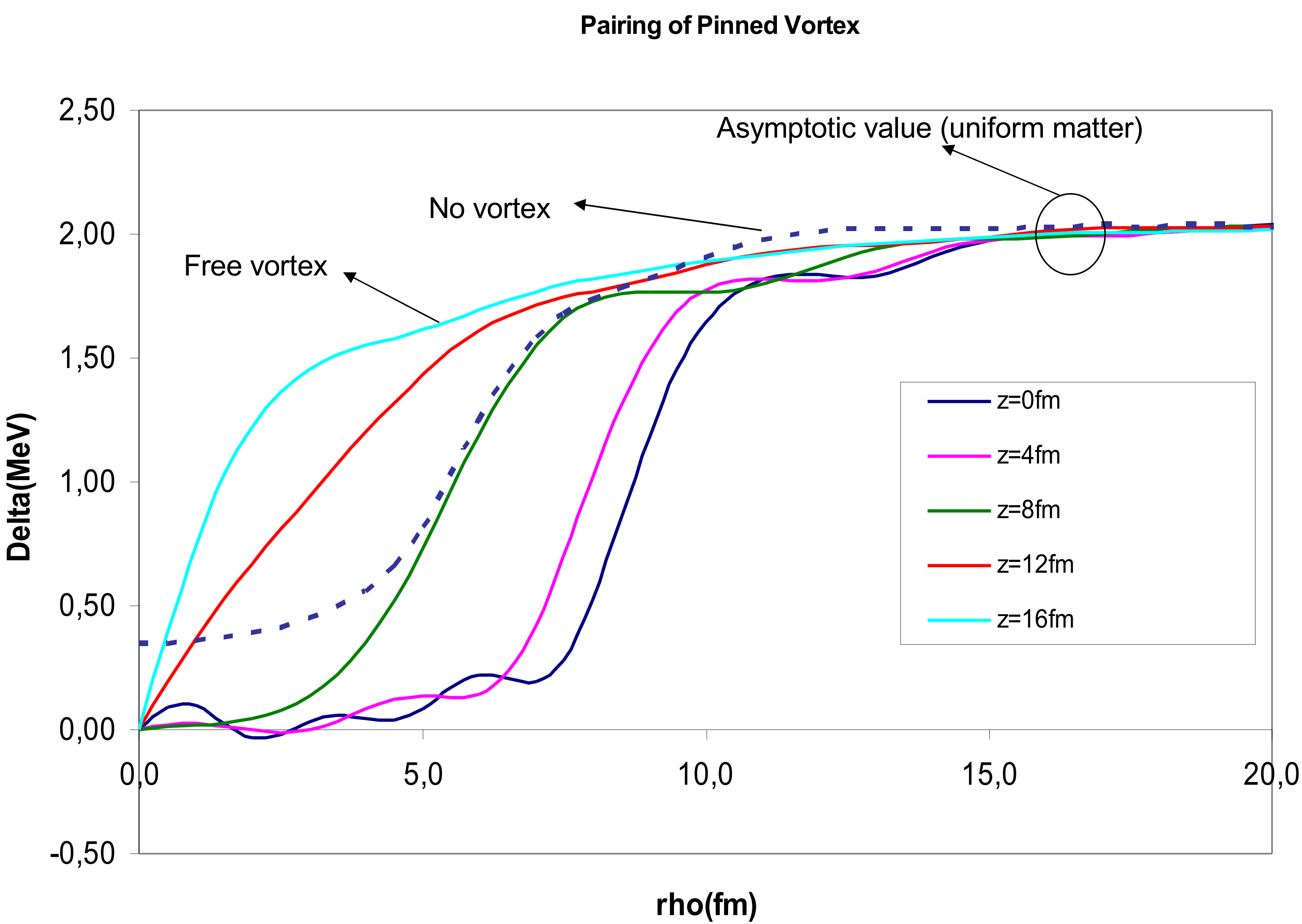
Vortex in uniform matter: Y. Yu and A. Bulgac, PRL 90, 161101 (2003)

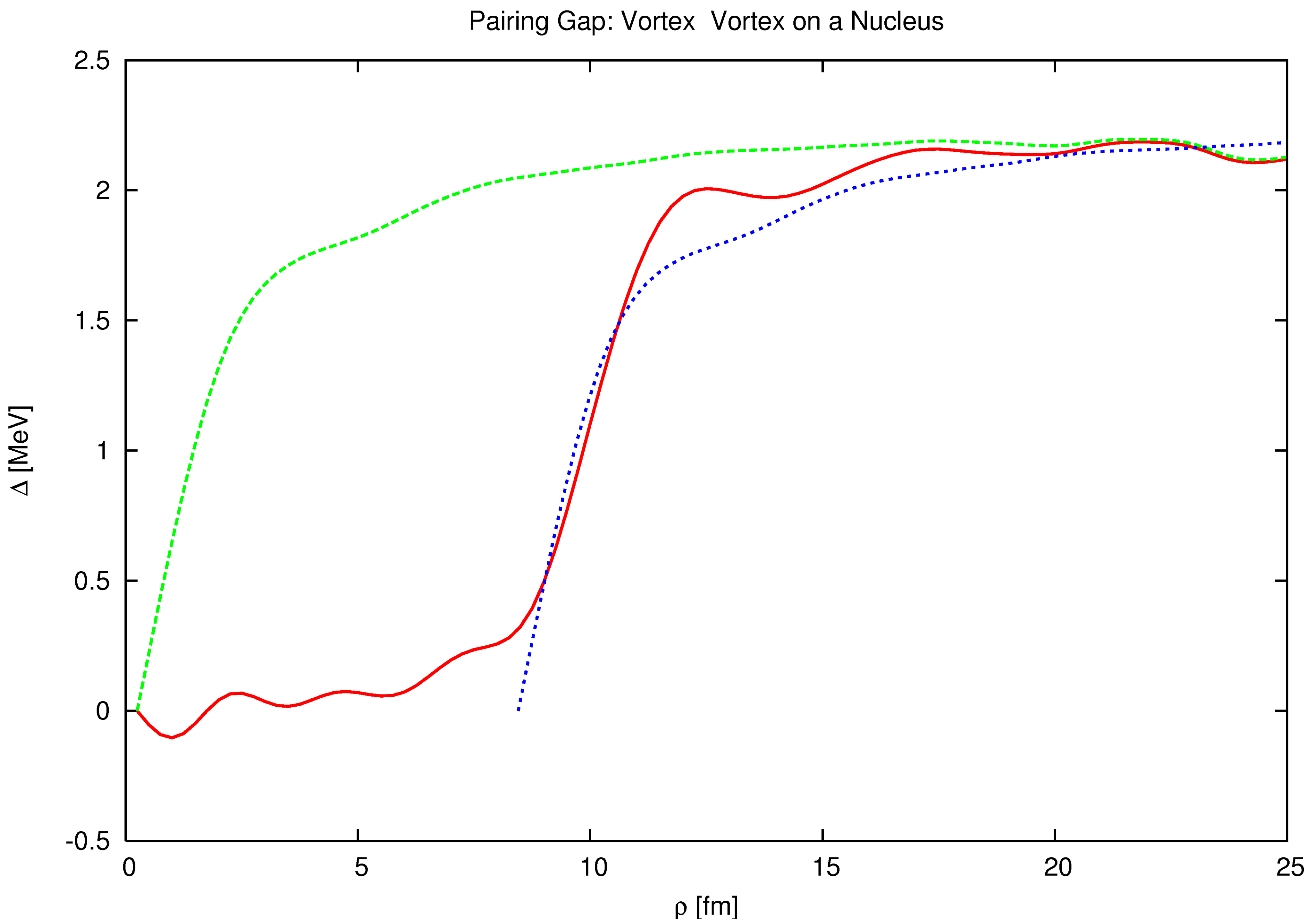




Pairing of a Vortex on a Nucleus
'fort.32bis' u 1:2:3
1.4
1
0.6
0.2
Δ [MeV]
2.5
2
1.5
1
0.5
0
-0.5
z [fm]
14
12
10
8
6
4
2
0
ρ [fm]
0
5
10
15
20
25

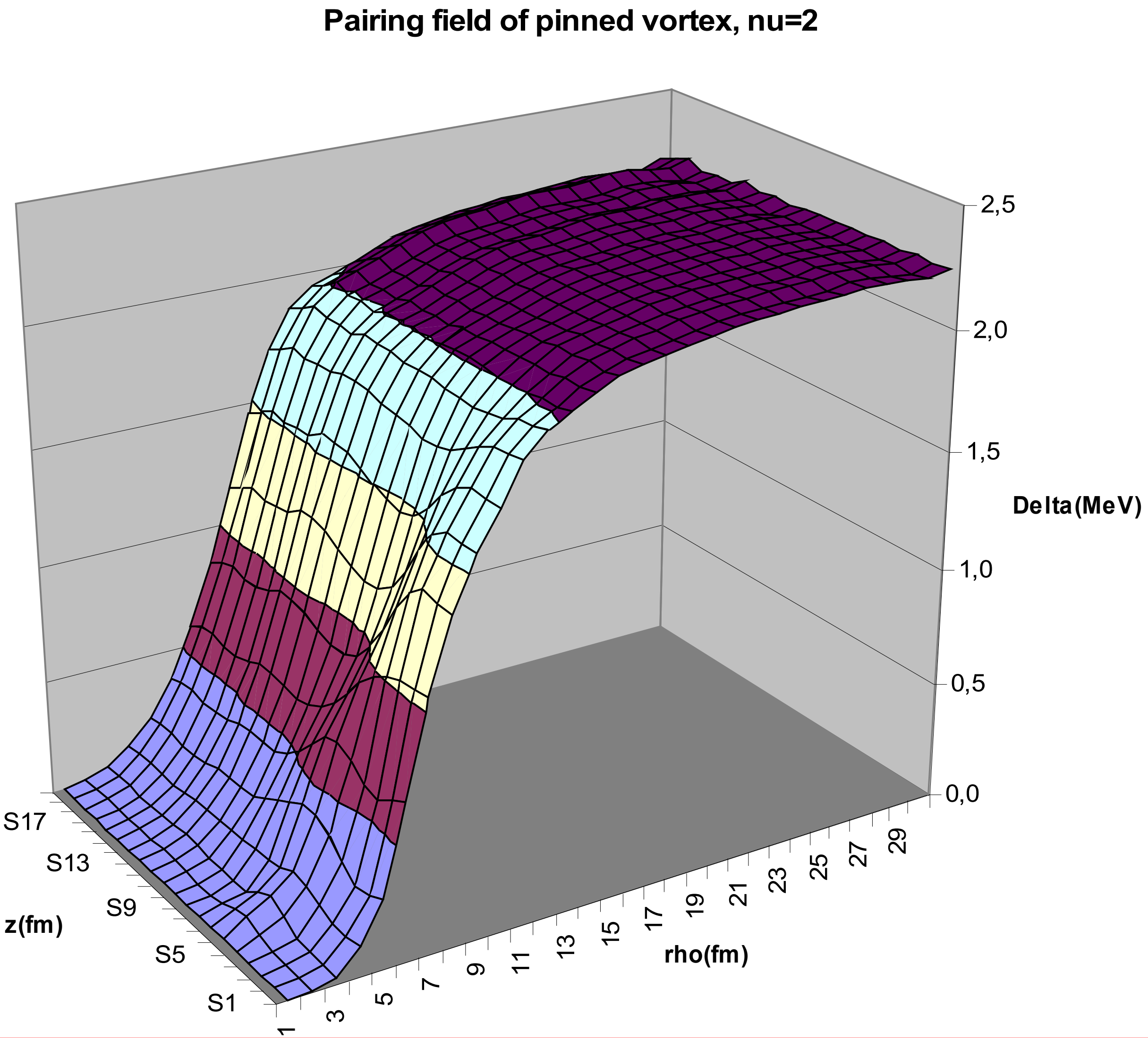
Pairing gap of pinned vortex



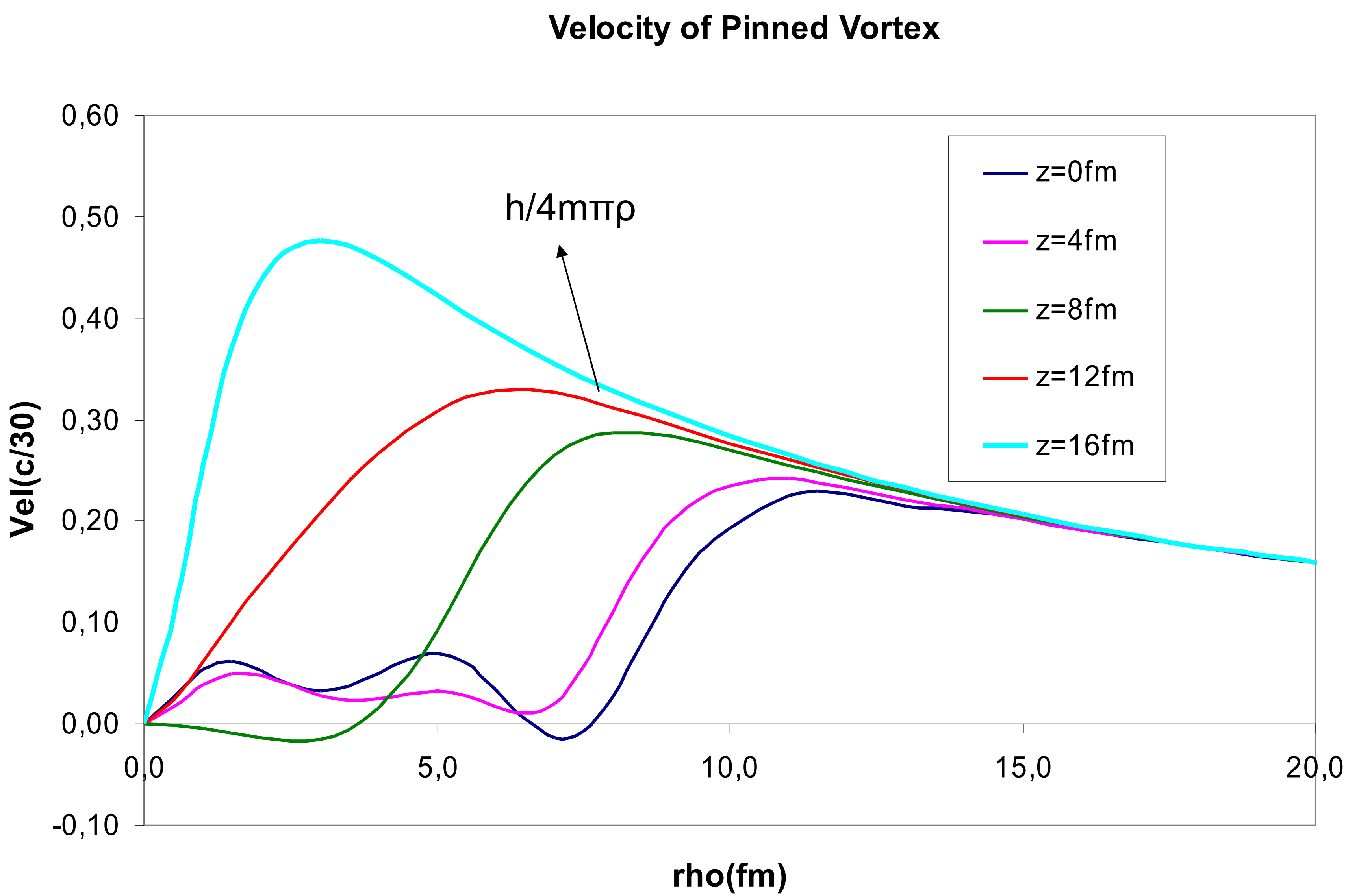


Pairing Gap: Vortex Vortex on a Nucleus
2.5
2
1.5
1
0.5
0
-0.5
0
5
10
15
20
25
Δ [MeV]
ρ [fm]

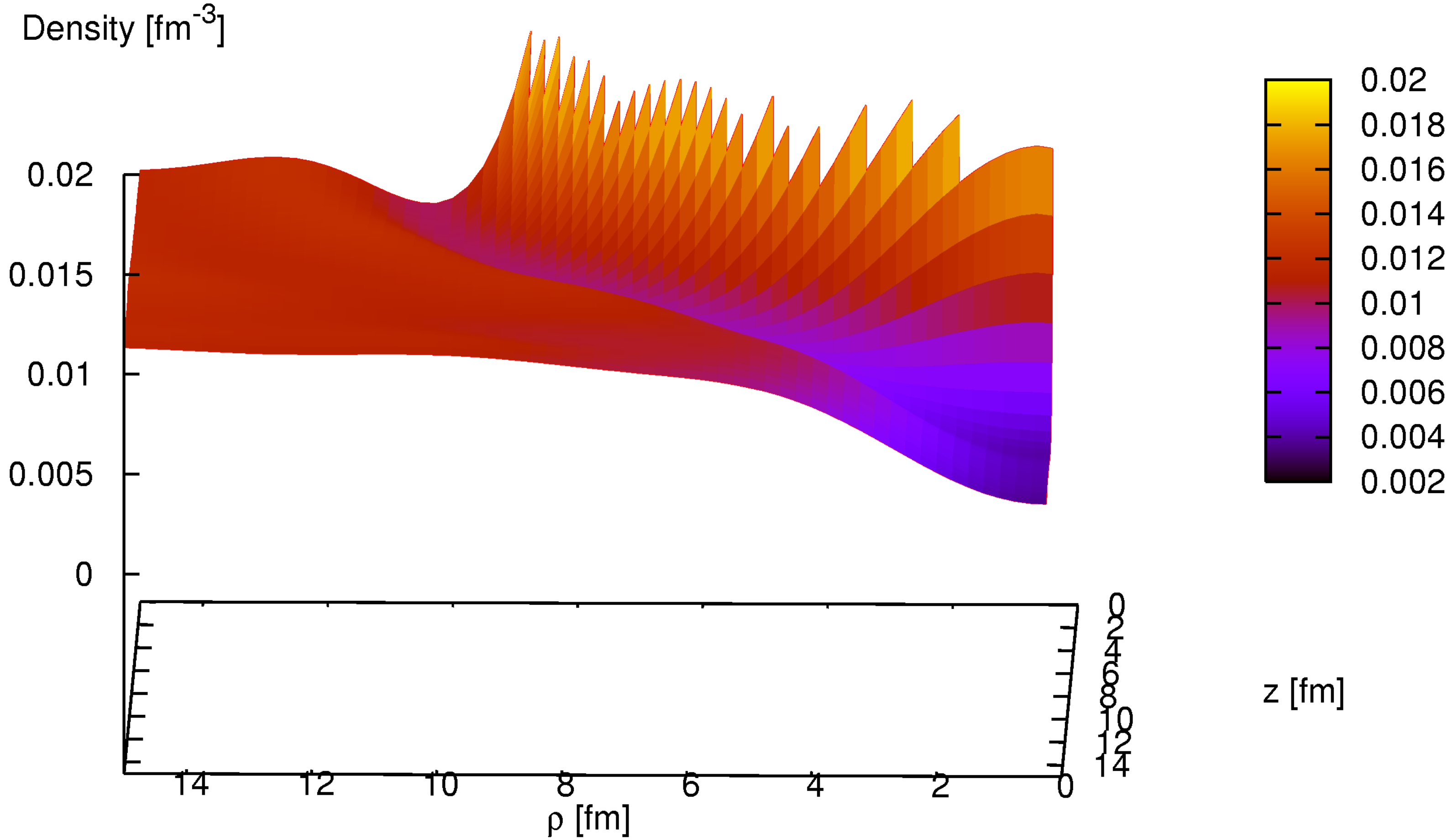
Pairing gap of pinned vortex, v = 2



In this case the vortex goes through the nuclear volume, essentially undisturbed, because the levels can satisfy the parity and angular momentum conditions.



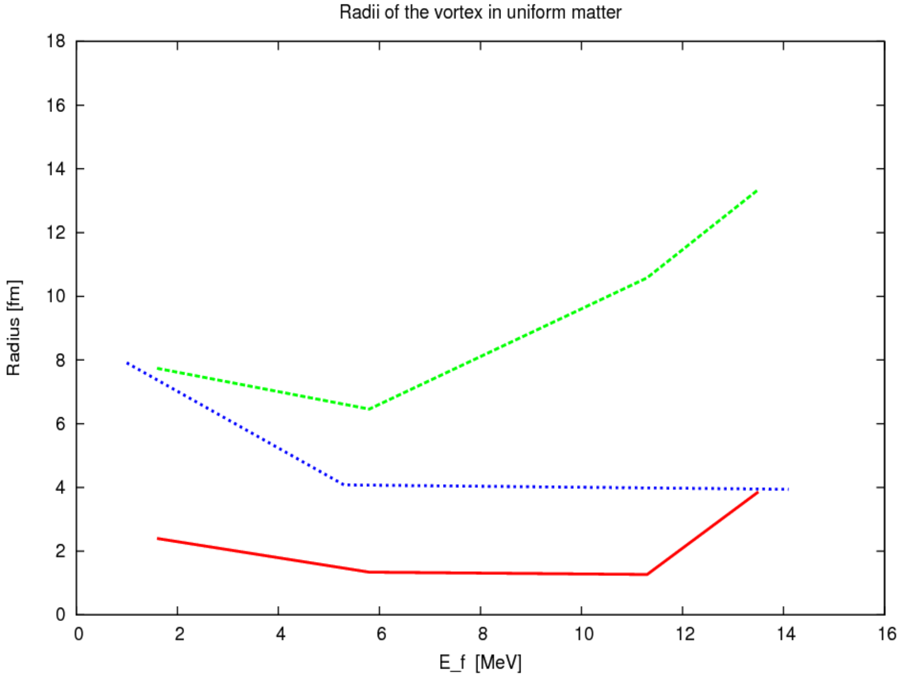
Velocity of Pinned Vortex
0,60
0,50
0,40
0,30
0,20
0,10
0,00
-0,10
0,0
5,0
10,0
15,0
20,0
Vel(c/30)
rho(fm)
h/4mπρ
z=0fm
z=4fm
z=8fm
z=12fm
z=16fm

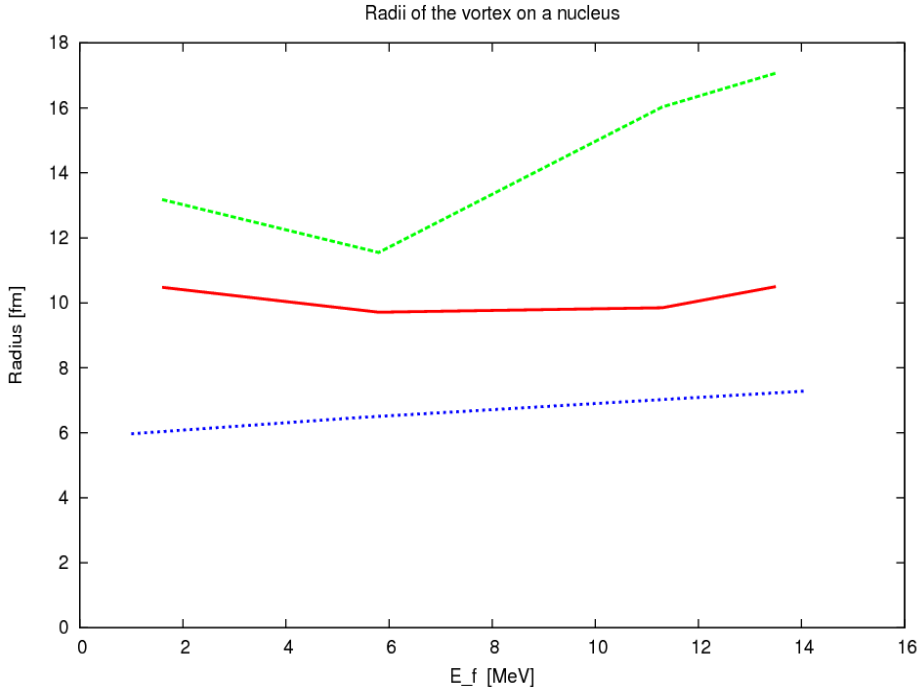

Density [fm^-3]
0.02
0.015
0.01
0.005
0
14
12
10
8
6
4
2
0
ρ [fm]
z [fm]
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002

Single Wigner cell assumption:

n (fm^{-3})	Δ (MeV)	R_{WS} (fm)	ξ (fm) $= \hbar^2 \tilde{k}_F / \pi m \Delta$	Ef	Density fm^-3
0.08	0.5	14	13		
0.04	2.0	20	7	13.5 MeV	0.045
0.02	2.6	28	4	11.3 MeV	0.035
0.005	1.0	36	7	5 .8 MeV	0.012
0.0015	0.6	42	8	1.6 MeV	0.0013

red line= r_50% green line= r_90% blue line= coherence length





The energy to create a vortex is obtained taking the difference between two calculations in the cylindrical box, one with and the other without the vortex, each with the same number of particles.
In each calculation, the energy has three different contributions:

- Kinetic energy
- Mean field potential energy
- Pairing field potential energy

	Uniform	Vortex	Diff.
Ekin	6841.9	6776.0	-65.9
Epot	-1735.3	-1737.5	-2.2
Epair	-1322.1	-1203.9	118.2
			50.1 MeV

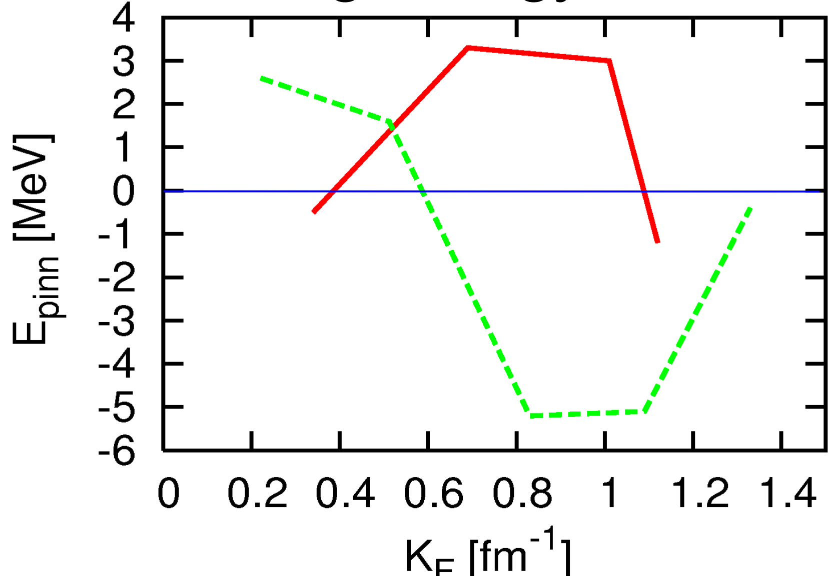
Energy cost for a vortex in uniform matter

	Nucleus	Vortex	Diff.
Ekin	9971.9	9893.4	-78.5
Epot	-5784.0	-5806.1	-22.1
Epair	-1274.5	-1120.5	154.0
			53.4 MeV

Energy cost for a vortex pinned to the nucleus

Pinning energy: 53.4 – 50.1 = 3.3 MeV   (antipinning)

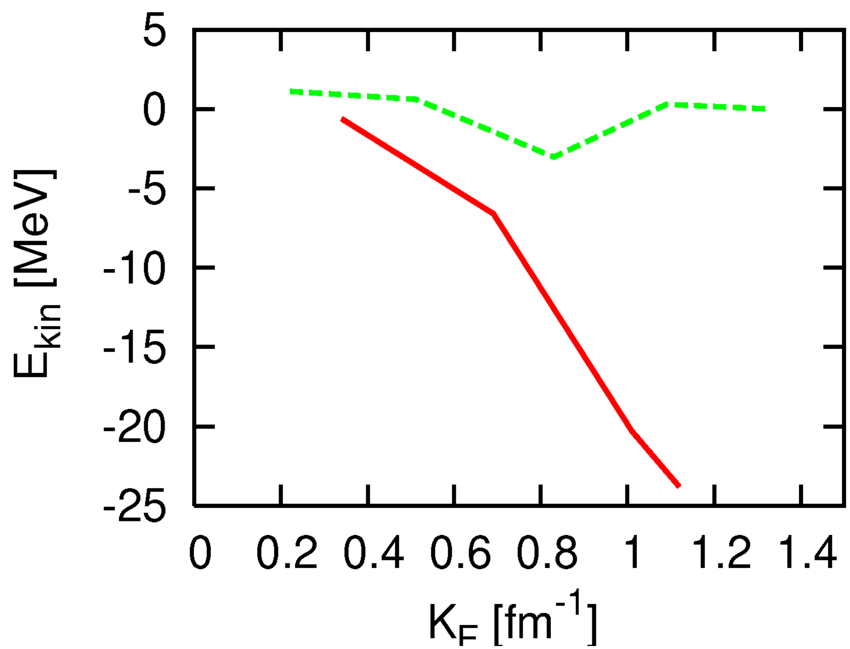
Pinning Energy: results

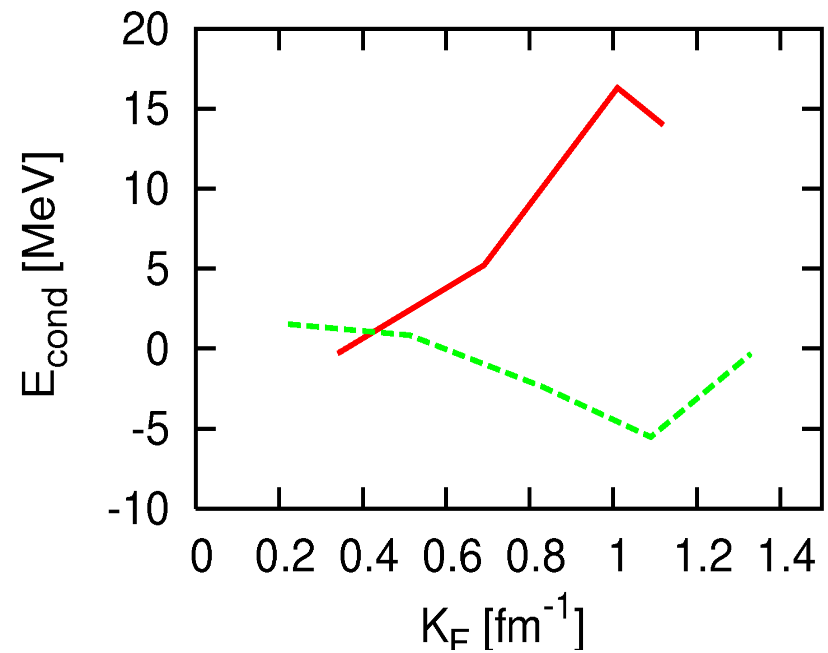


Our results : RED

Pizzochero & Donati: GREEN

P.M. Pizzochero and P. Donati, Nucl. Phys. A742,363(2004) Semiclassical model with spherical nuclei.





Conclusions and perspectives

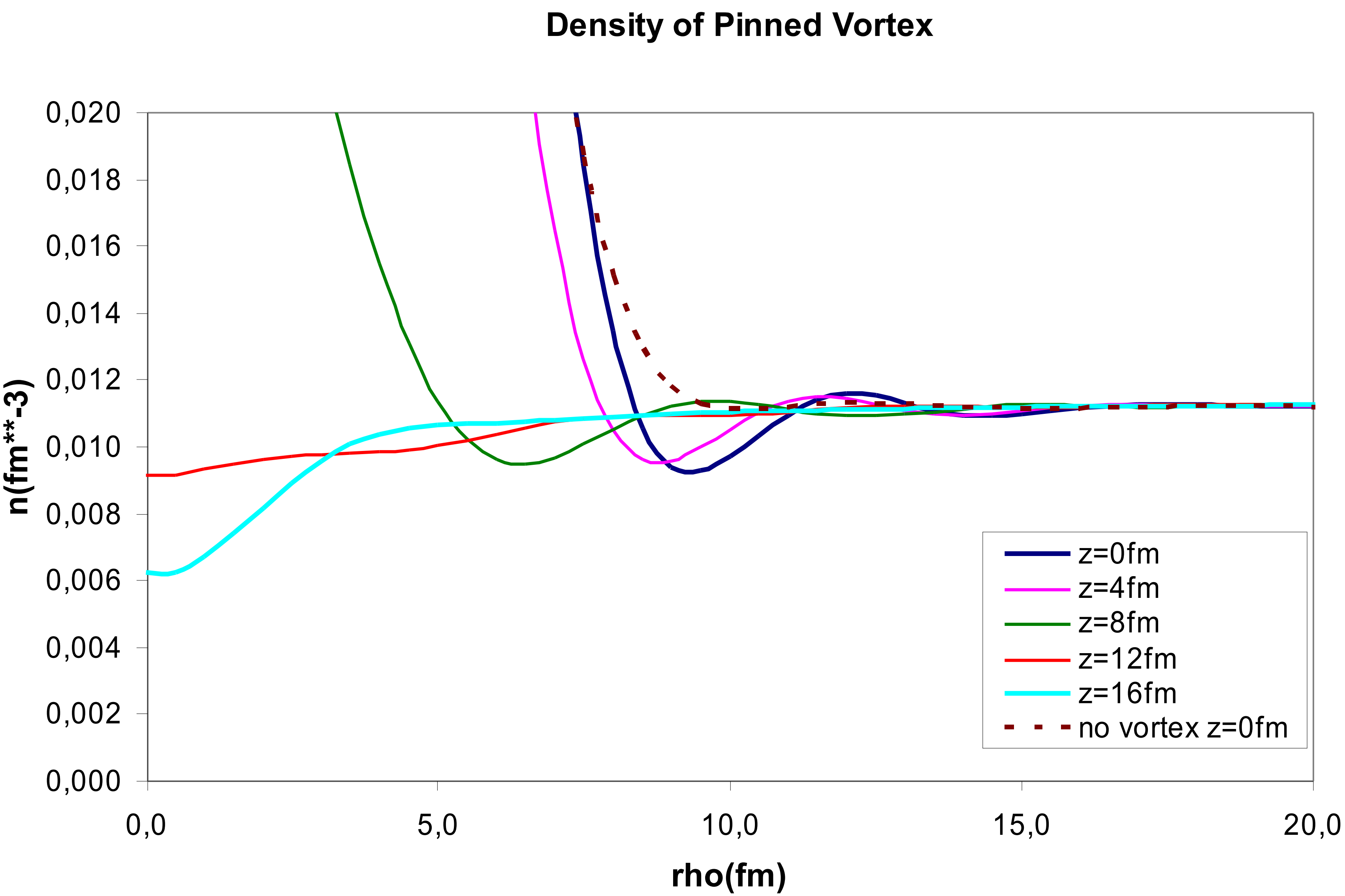
-We have solved the HFB equations for a single vortex in the crust of neutron stars, considering explicitly the presence of the nucleus, generalizing previous studies in uniform matter.

-We have found that finite size effects are important.
(v=1) vortex stay outside of the nuclear volume,
where the pairing goes to zero.

-Numerical results at different densities with SII interaction indicate that the pinning energy is very small and of the order of a few MeV.

Many open questions. Among them:

-Which interactions to adopt to describe the mean field
Include medium polarization effects
-Vortex dynamics

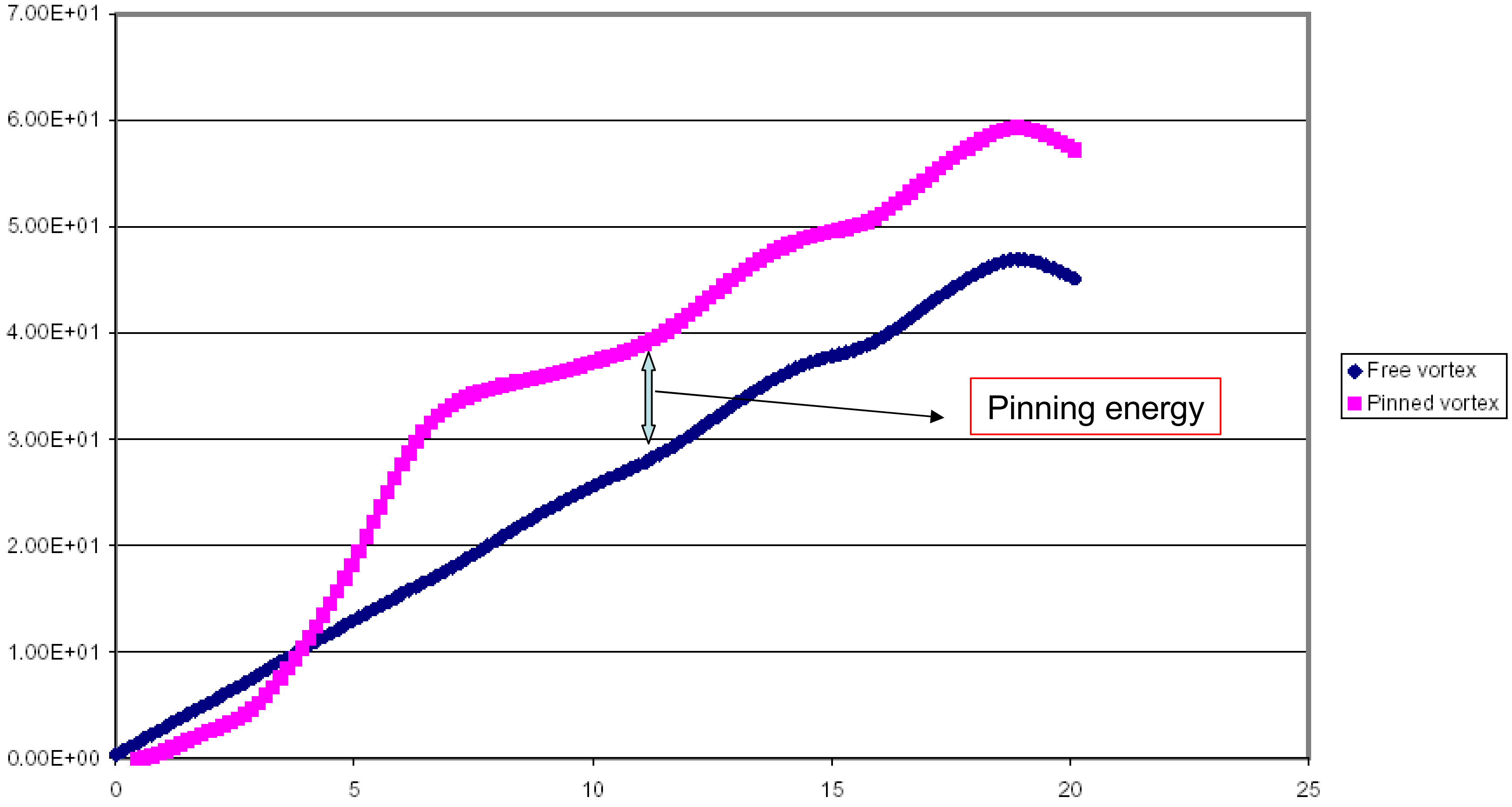


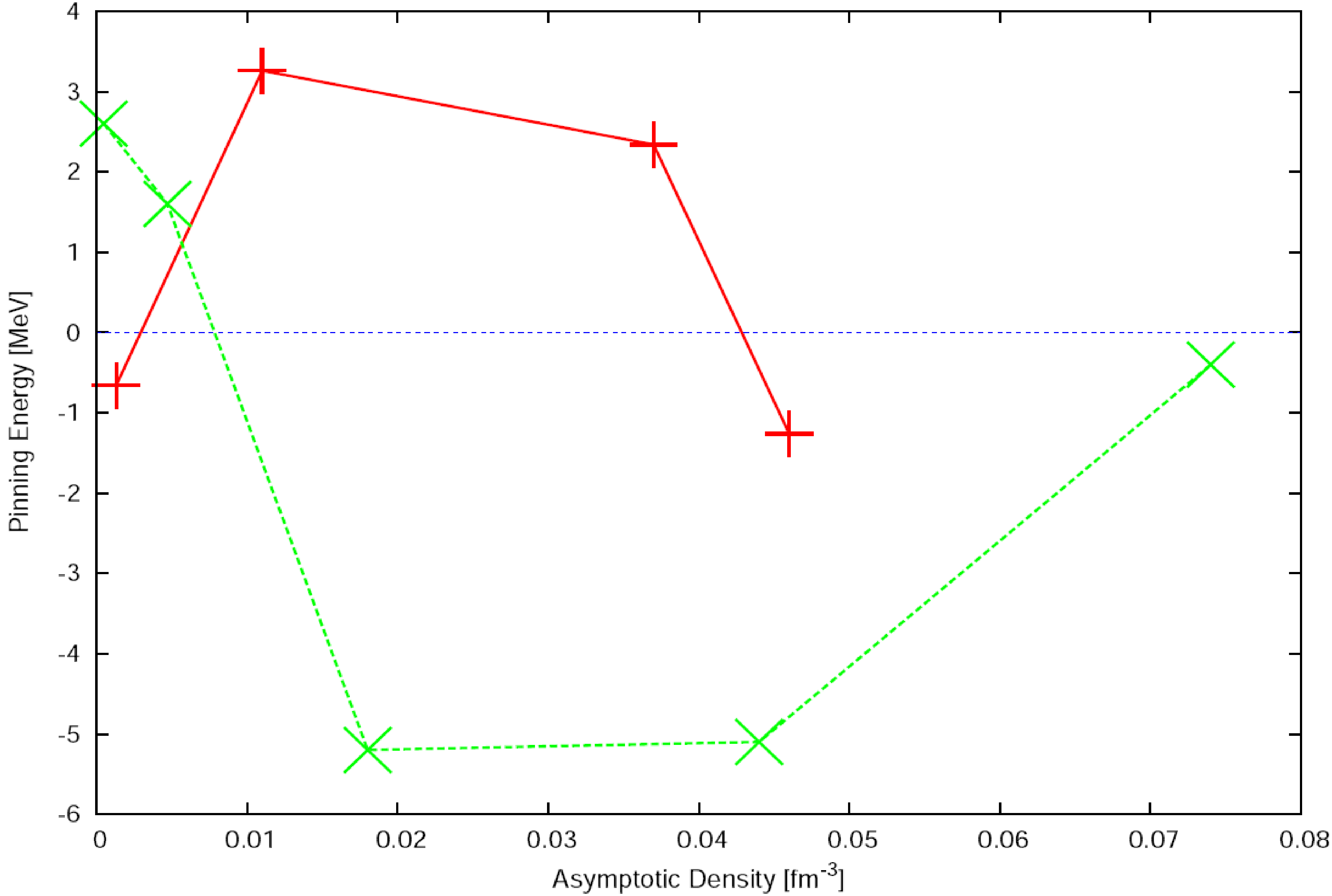
Density of Pinned Vortex
n(fm**-3)
0,020
0,018
0,016
0,014
0,012
0,010
0,008
0,006
0,004
0,002
0,000
0,0
5,0
10,0
15,0
20,0
rho(fm)
z=0fm
z=4fm
z=8fm
z=12fm
z=16fm
no vortex z=0fm

Pinning Energy

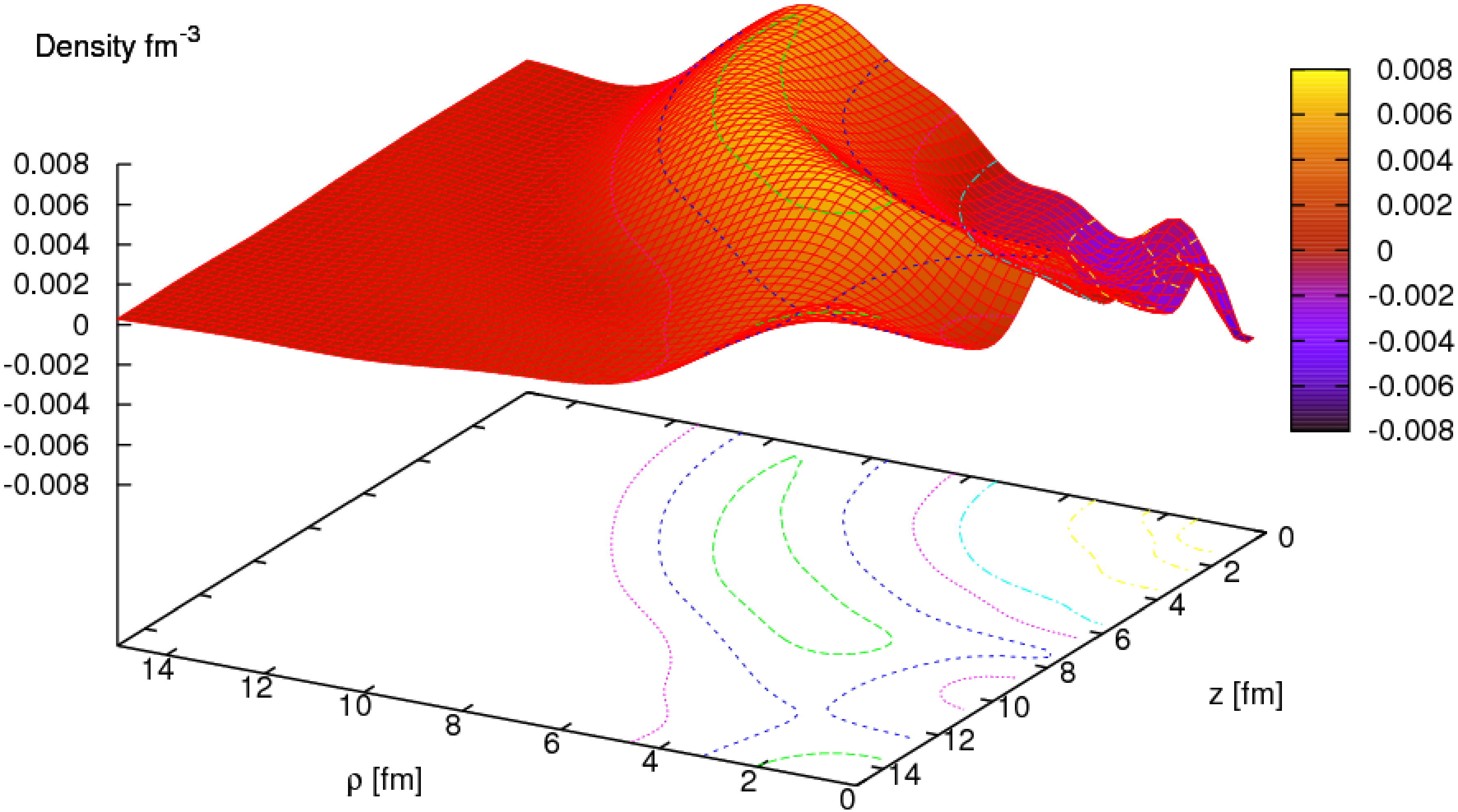
- E_p= Energy cost to create a vortex on the top of a nucleus (En)– Energy cost to create a vortex in the uniform neutron gas (Eu)
- En= total energy of a vortex on a nucleus -total energy of the nucleus
- Eu=total energy of a vortex on uniform neutron gas – total energy of the gas

Dependence of energy costs on vortex length





Red= SII,   Green= Pizzochero et al.

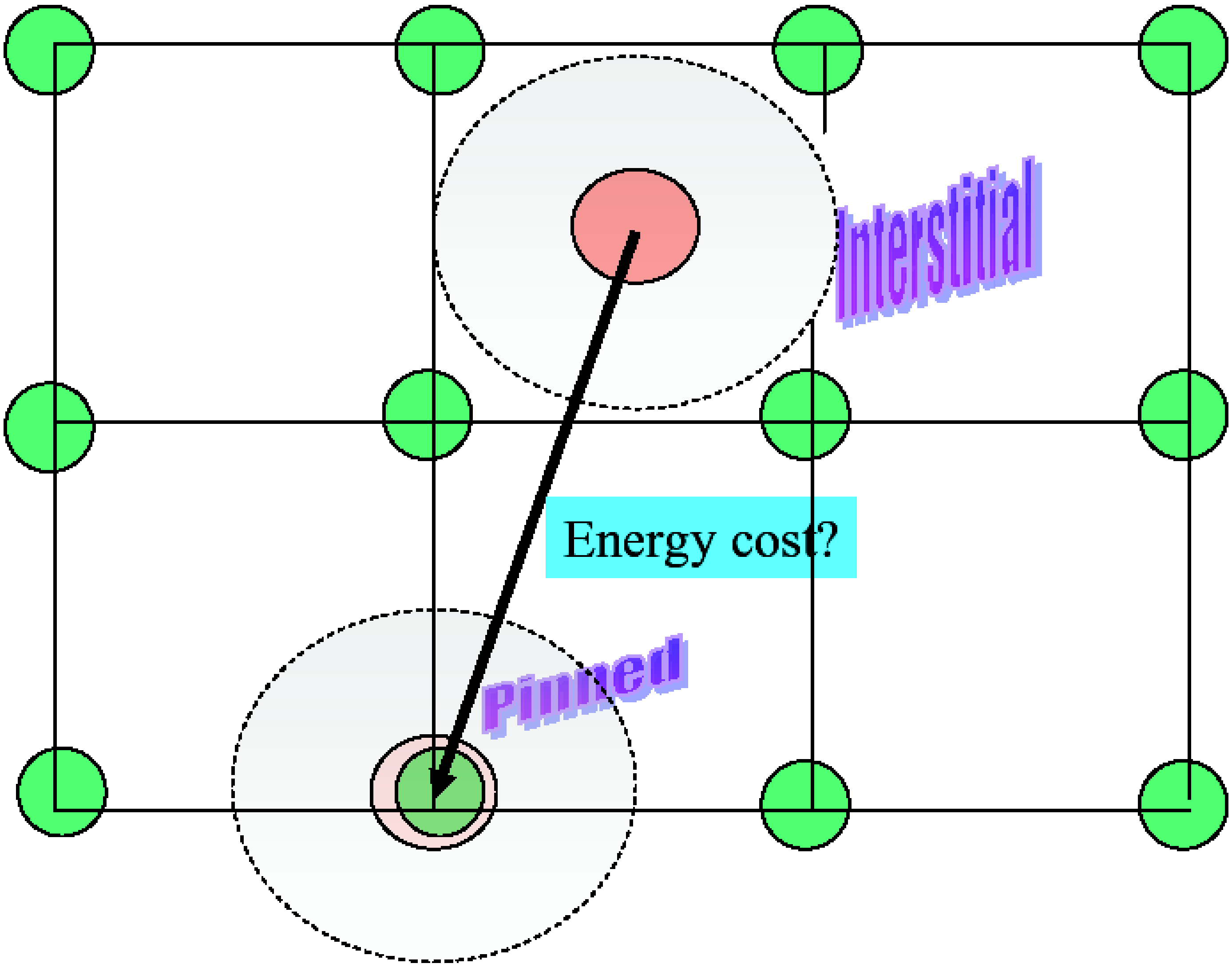


Density fm^{-3}
0.008
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
14
12
10
8
6
4
2
0
ρ [fm]
z [fm]

Pinning Energy: results SII

- E_f= 1.6 MeV pinning energy= -0.49 MeV
- E_f= 5.8 MeV pinning energy= +3.3 MeV
- E_f= 11.3 MeV pinning energy= +2.99 MeV
- E_f= 13.5 MeV pinning energy= -1.0 MeV

We assume that it is enough to consider the effect of the cylindrical region around the vortex axis (Wigner cell radius larger than the coherence length)



Previous calculations of pinned vortices in Neutron Stars:

-R. Epstein and G. Baym, Astrophys. J. 328(1988)680
Analytic treatment based on the Ginzburg-Landau equation

-F. De Blasio and O. Elgarøy, Astr. Astroph. 370,939(2001)
Numerical solution of De Gennes equations with a fixed nuclear mean field and imposing cylindrical symmetry (spaghetti phase)

-P.M. Pizzochero and P. Donati, Nucl. Phys. A742,363(2004)
Semiclassical model with spherical nuclei.

HFB calculation of vortex in uniform neutron matter:

-Y. Yu and A. Bulgac, PRL 90, 161101 (2003)

HFB calculation of superfluid trapped Fermi gases:

- M. Nygaard, G. M. Bruun, C.W. Clark, D.L. Feder, PRL 90, 210402 (2003)

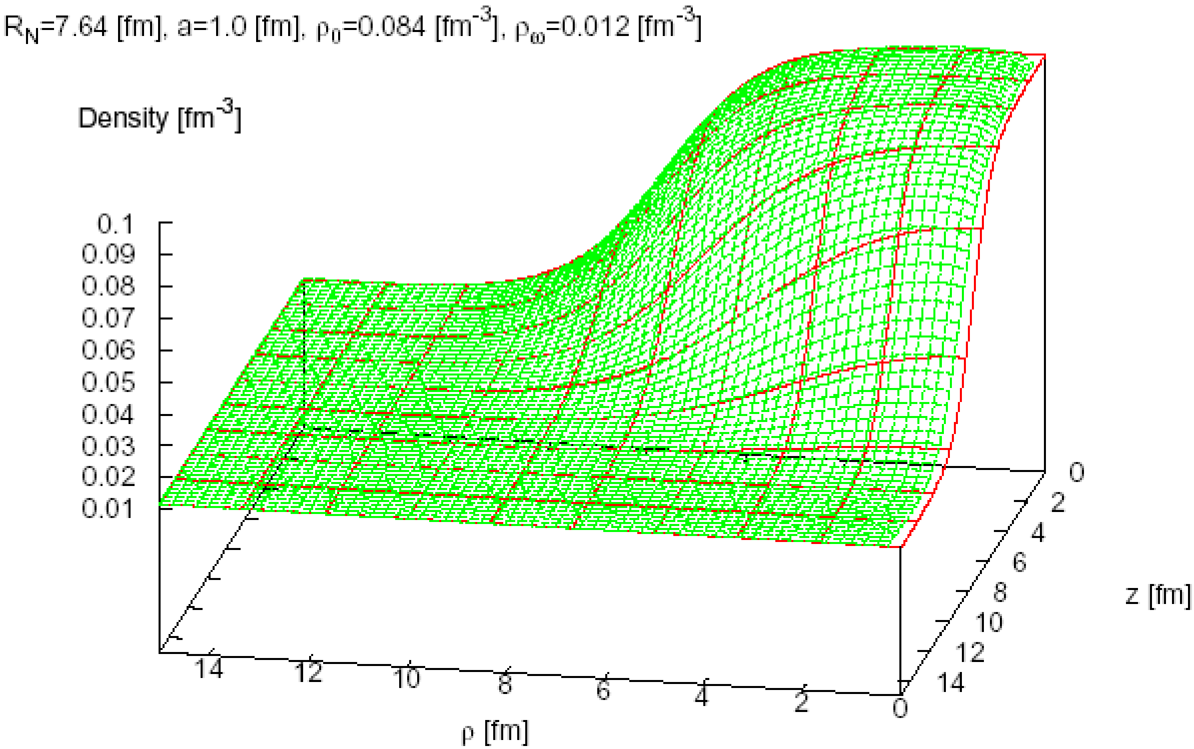
Example:

$k_F = 0.7\ fm^{-1}$ ($n = 0.012\ fm^{-3}$)

$N \sim 1000$ $R_{WS} \sim 30$ fm $\Delta \sim 2.5$ MeV $\xi \sim 5$fm

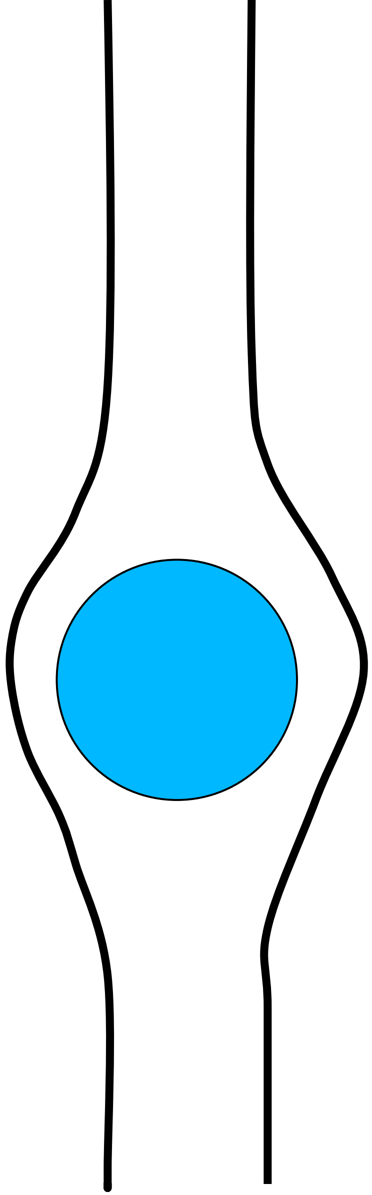
$\rho_{box} = 30$ fm $h_{box} = 40$ fm $d\rho = dh = 0.25$ fm

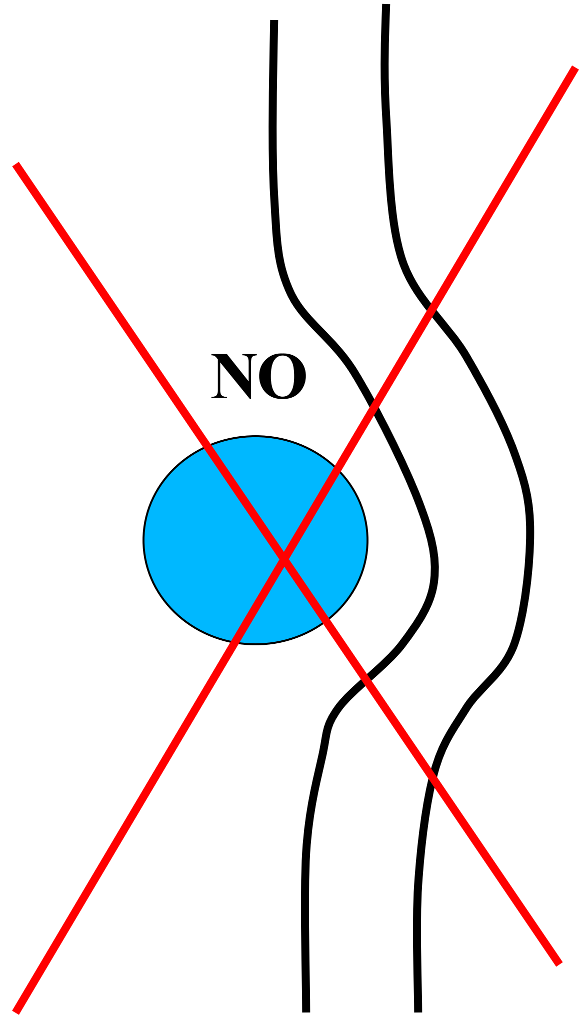
Nucleus in the Wigner cell (no vortex)

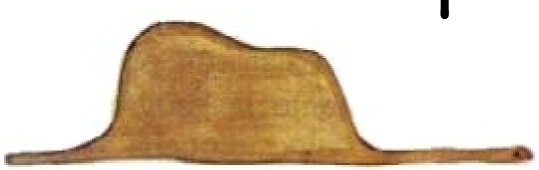


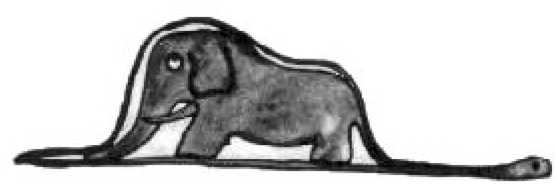
Symmetry is conserved!

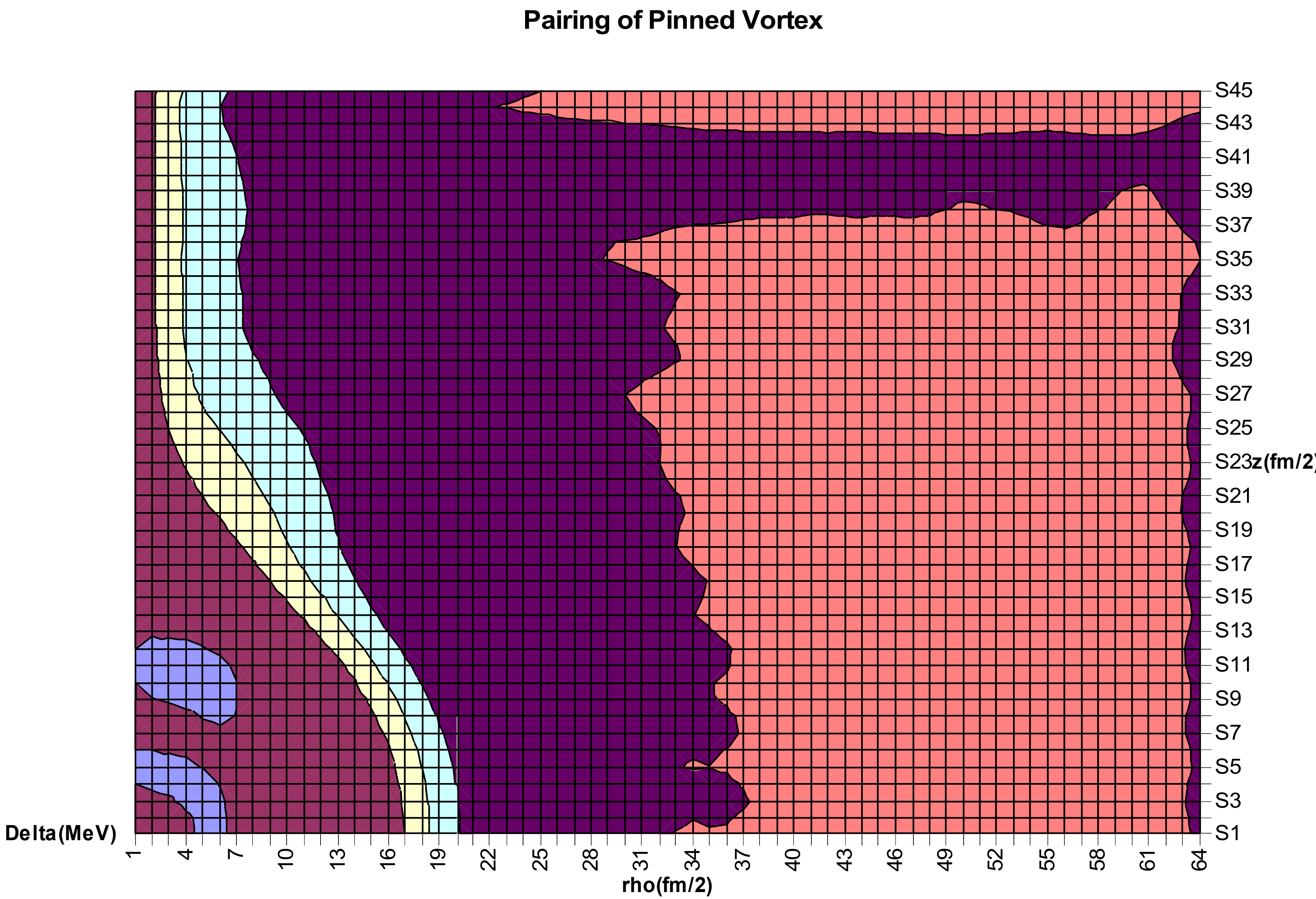
YES!










Pairing of Pinned Vortex
S45
S43
S41
S39
S37
S35
S33
S31
S29
S27
S25
S23
z(fm/2)
S21
S19
S17
S15
S13
S11
S9
S7
S5
S3
S1
Delta(MeV)
1 4 7 10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64
rho(fm/2)

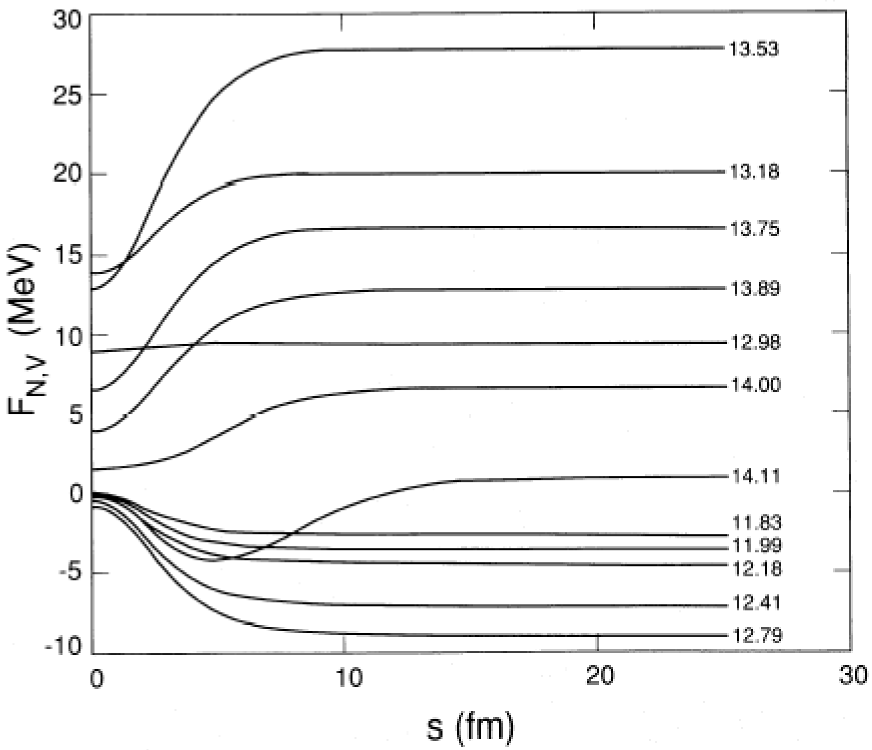
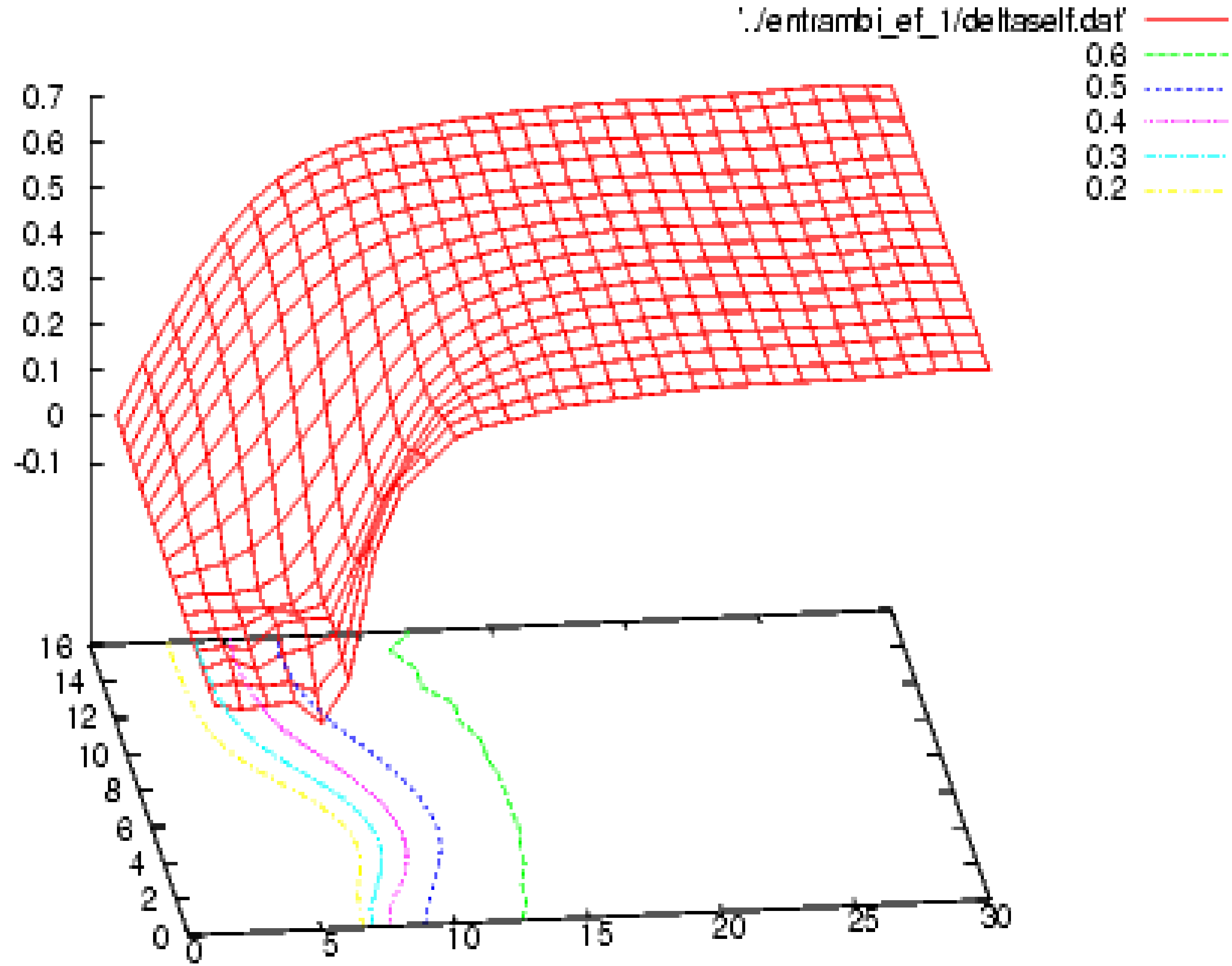
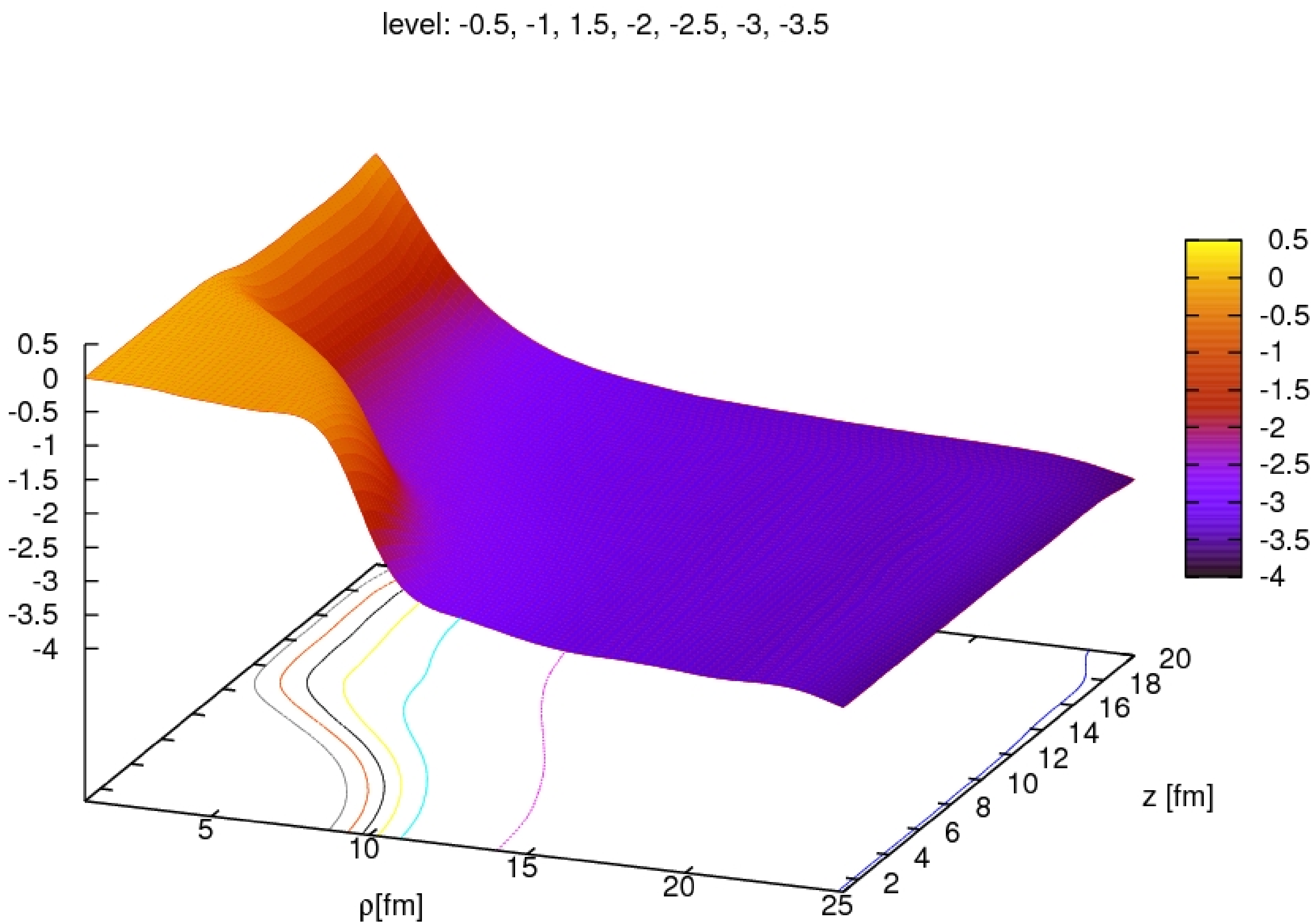


FIG. 3.—Free energy of a nucleus as a function of its distance from a vortex line for 12 mass densities. The curves are labeled by log ρ_*.

R. Epstein, G. Baym, ApJ 328 (1988)680

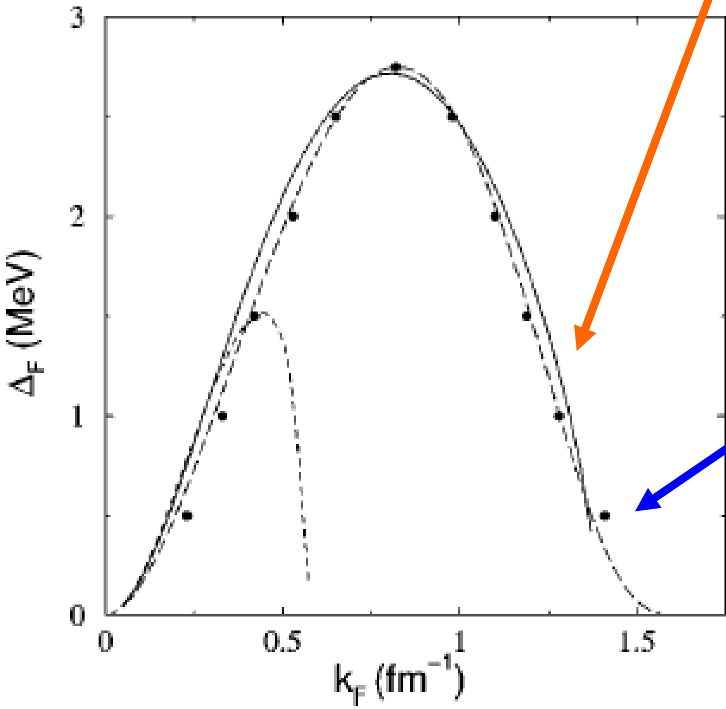


'./entrambi_ef_1/deltaself.dat'
0.6
0.5
0.4
0.3
0.2
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
16
14
12
10
8
6
4
2
0
0
5
10
15
20
25
30


level: -0.5, -1, 1.5, -2, -2.5, -3, -3.5
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
5
10
15
20
25
ρ[fm]
2
4
6
8
10
12
14
16
18
20
z [fm]

PAIRING INTERACTION

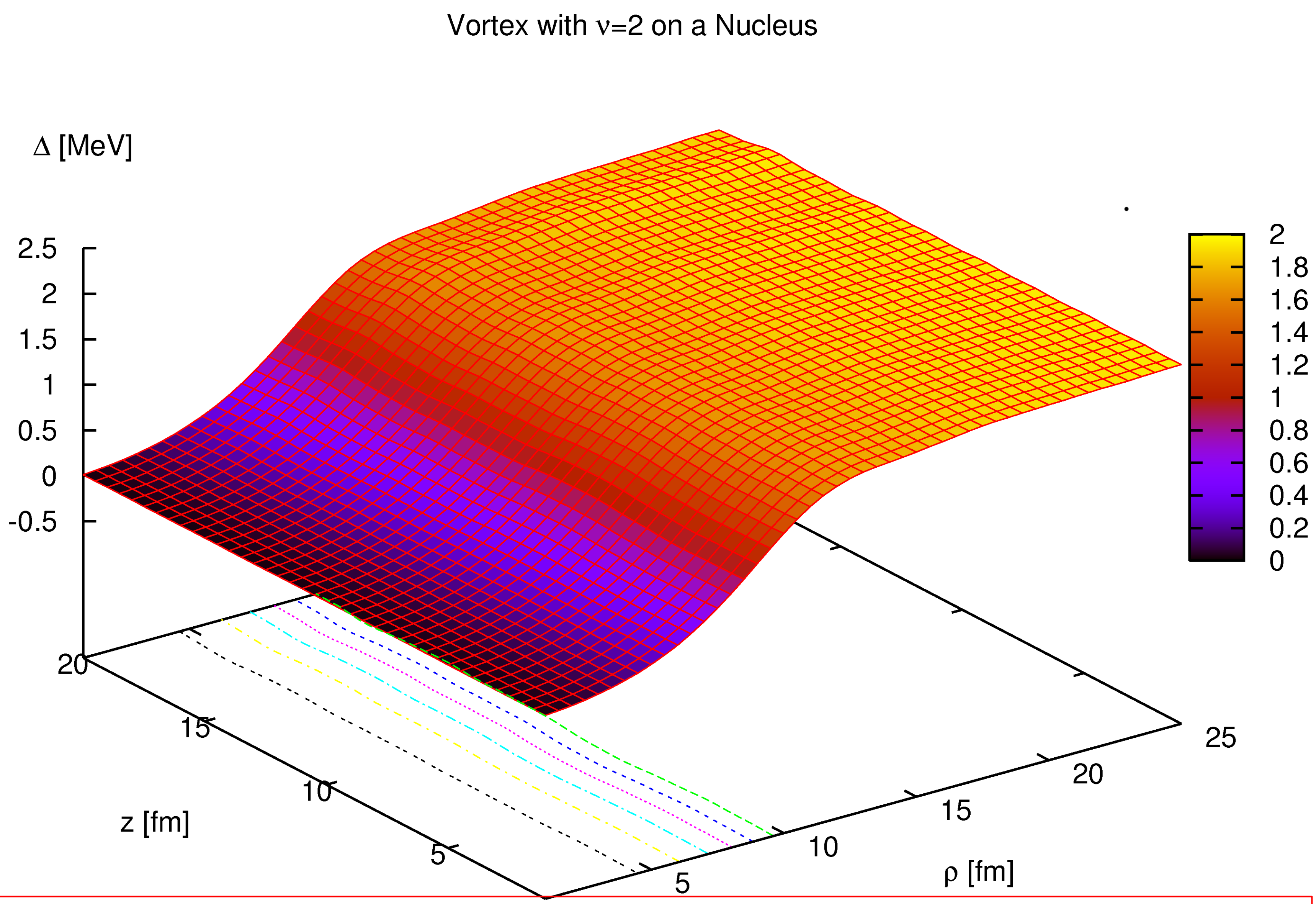
$$V = -480\,(1 - 0.7\,(\rho/\rho_0)^{0.45})\,\delta(r_1 - r_2)\ \mathrm{MeV\,fm}^3$$



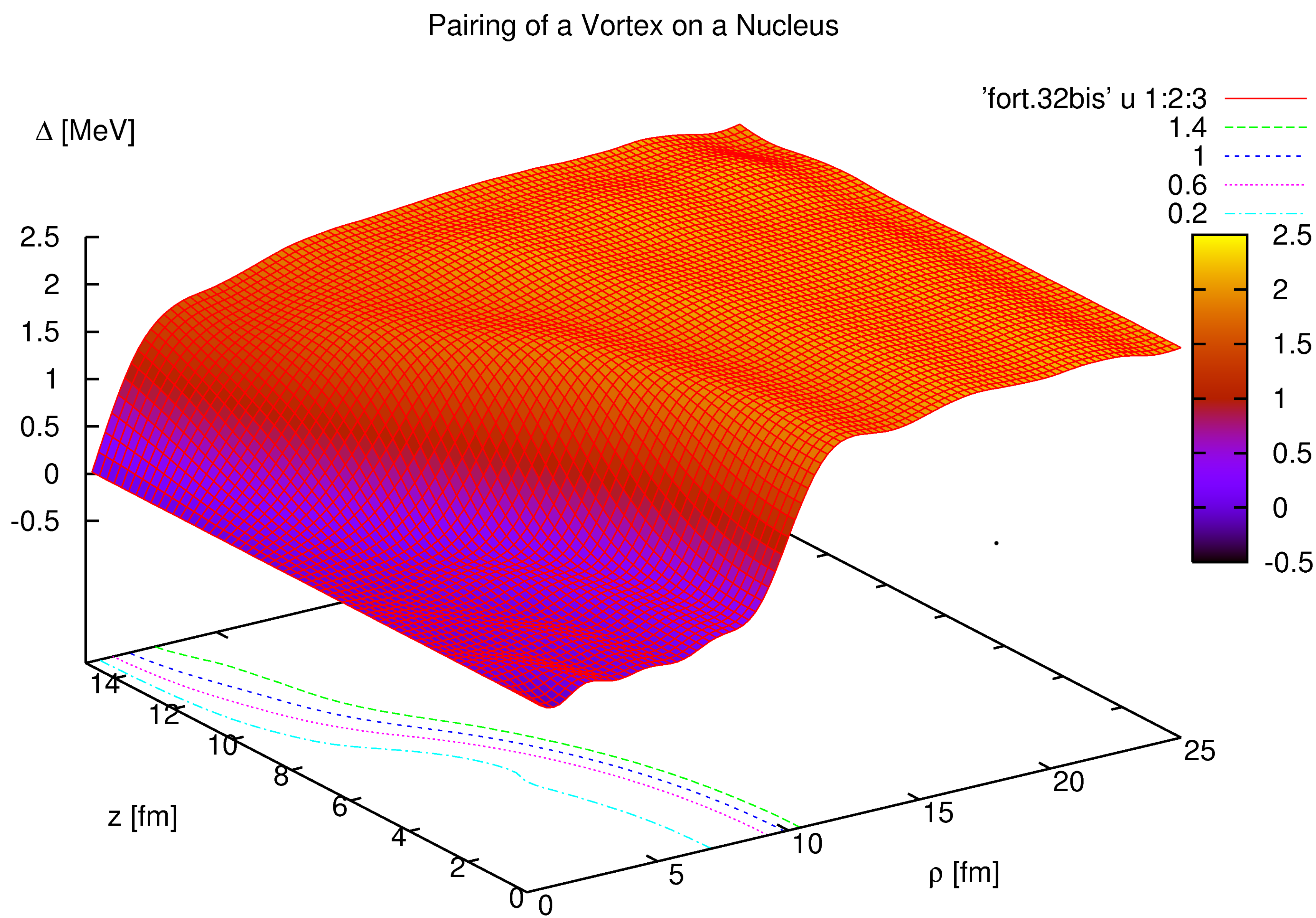
Ecut = 60 MeV

Gogny force

E. Garrido et al. Phys. Rev. C60 (1999) 64312



In this case the vortex goes through the nuclear volume, essentially undisturbed, because the levels can satisfy the parity and angular momentum conditions.


Pairing of a Vortex on a Nucleus
'fort.32bis' u 1:2:3
1.4
1
0.6
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Δ [MeV]
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2
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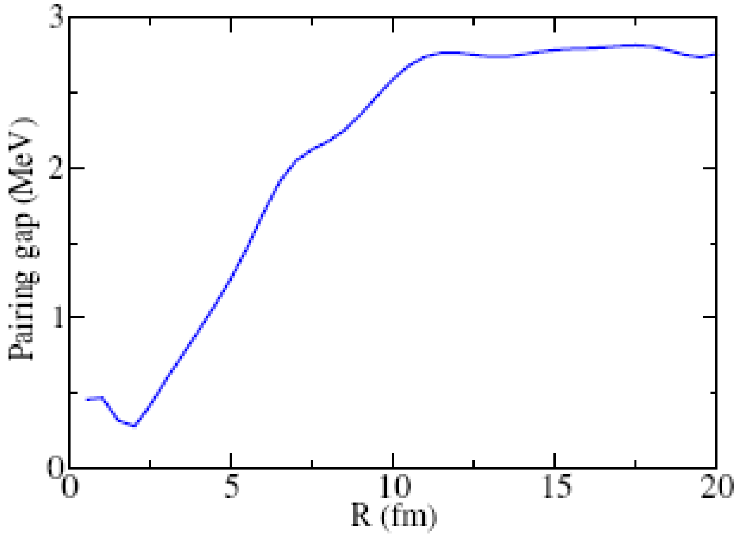
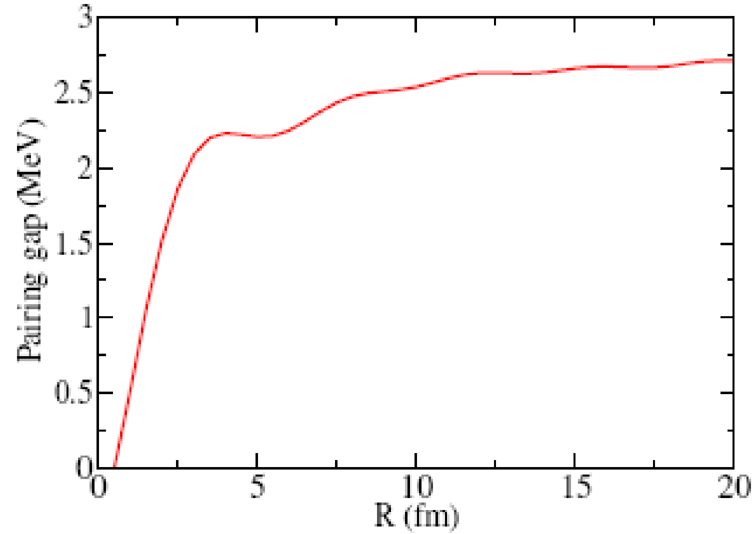
1
0.5
0
-0.5
z [fm]
14
12
10
8
6
4
2
0
ρ [fm]
0
5
10
15
20
25

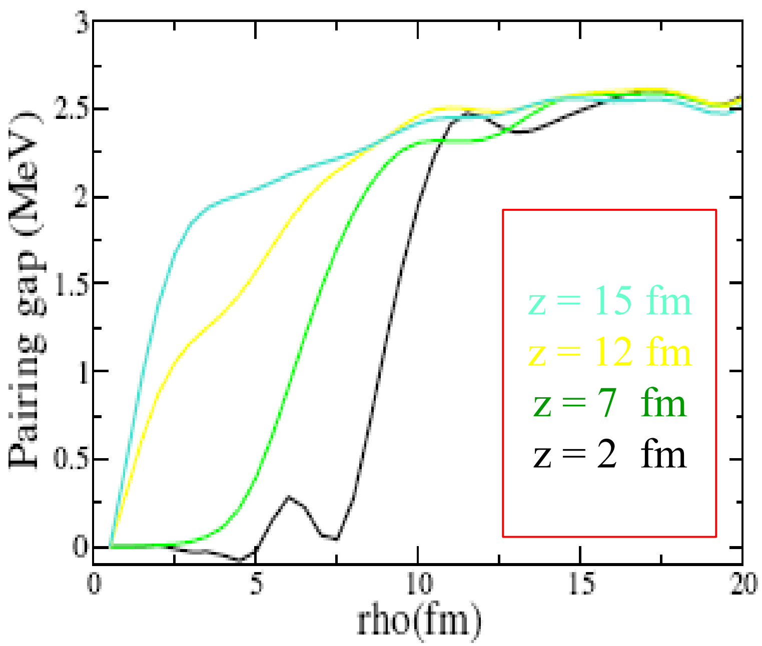
Pairing gap in the Wigner cell (no vortex)

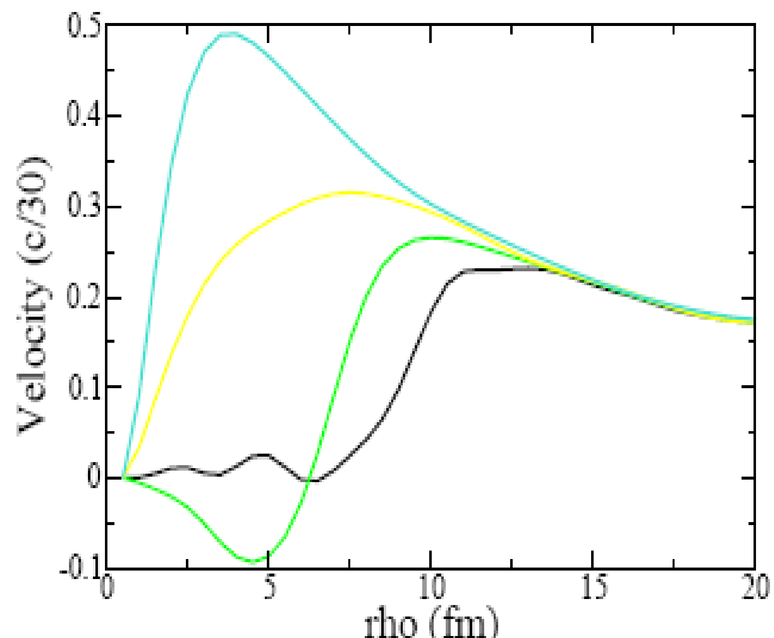


Pairing gap for a vortex in uniform matter



Pairing cap and velocity field for vortex in the Wigner cell





The energy cost to create a vortex is obtained taking the difference between two calculations in the cylindrical box, one with and the other without the vortex, each with the same number of particles.
The **pinning energy** is the difference between the cost to create a vortex pinned on a nucleus, and the cost to create a vortex far from the nucleus: it requires four independent calculations.

Single Wigner cell assumption:

$$\xi = \hbar^2 \tilde{k}_F / \pi m \Delta$$

n (fm^{-3})	Δ (MeV)	R_{WS} (fm)	ξ (fm)
0.08	0.5	14	13
0.04	2.0	20	7
0.02	2.6	28	4
0.005	1.0	36	7
0.0015	0.6	42	8

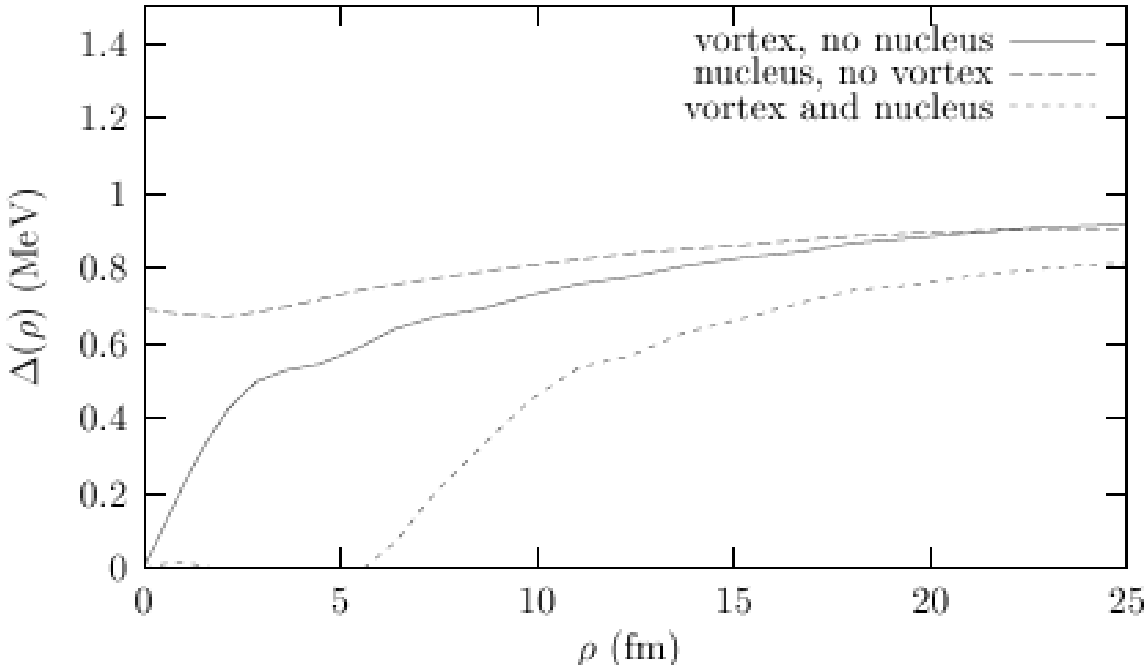


Table 9
Radii of the vortex core and coherence lengths calculated at $z = 0$ with Gogny pairing gap. See Table 8 for an explanation of the entries

	r_C (P)		r_C (M)		$\xi(0)$	
Zone	NP	IP	NP	IP	NP	IP
1	0.32	2.25	5.98	7.91	5.97	7.76
2	0.34	0.99	6.47	4.08	6.46	4.07
3	0.33	0.61	7.30	3.94	7.29	3.93
4	0.31	0.46	7.79	5.35	7.78	5.33
5	0.28	0.38	8.62	8.62	8.62	8.62

Boundary conditions

$$R_{nlj}(r = R_c) = 0$$

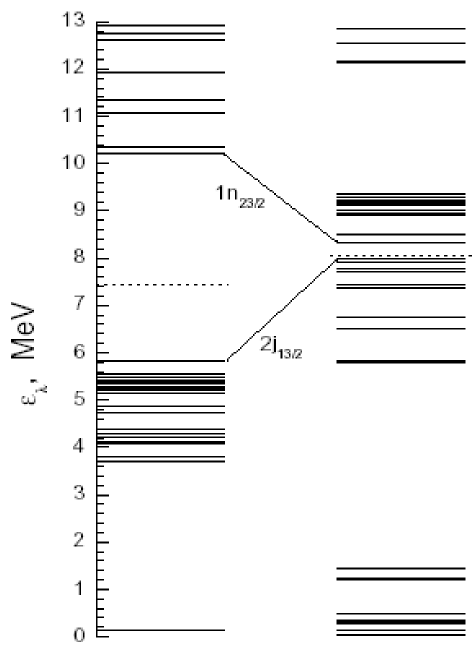
→ Adopted here. Density vanishes at the end of the box. Uniform matter values are reproduced

for $R_{nuc} < r < R_{box}$

$$R_{nlj}(r = R_c) = 0$$

$$\left(\frac{dR_{nlj}}{dr}\right)_{r=R_c} = 0,$$

→ Adopted by Negele and Vautherin. Different conditions for odd or even values. Constant value of the density at the end of the box. Strong influence on single-particle density.

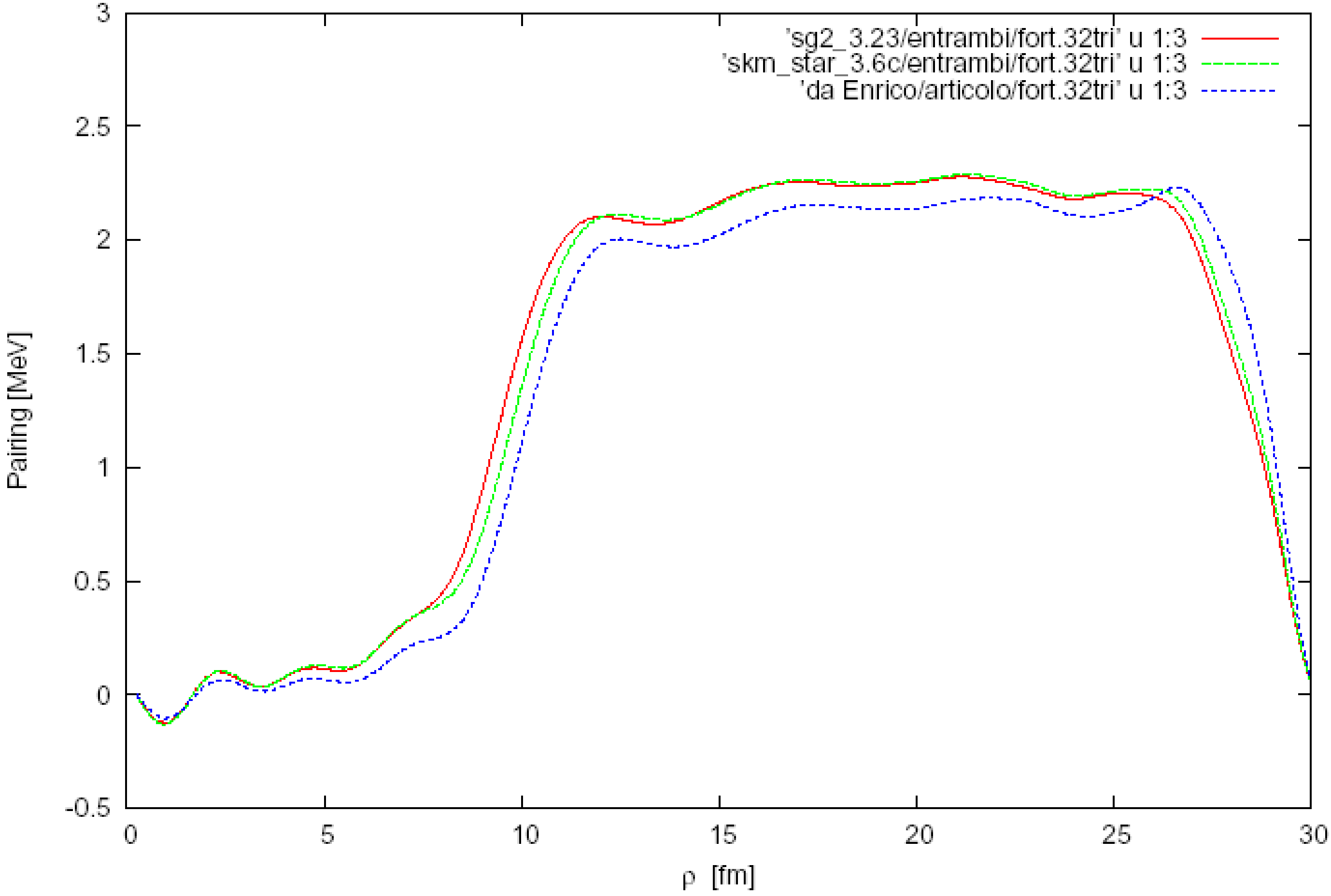


M. Baldo, E.E. Saperstein,S.V. Tolokonnikov, nucl/th 0601096

Dependence of energy costs on mesh size (drho) and on box dimensions

box	U-V	P-V	Epinn
drho=1			
26x38	87	117	30
32x44	116	147	30
38x38	136	167	30
drho=0.5			
21x30	51	54	3
26x38	74	76	2
32x44	93.6	95.0	1.4
drho=0.25			
21x30	46	59	13
26x38	70	81	11
drho=0.15			
21x30	45	58	13

Pairing gaps with different interactions



Using a zero-range pairing interaction,

only local quantities are needed

➔

$$\eta(\rho,z) = \sum_\alpha v_\alpha(\rho,\phi,z) v^*_\alpha(\rho,\phi,z)$$

$$V(\rho,z) = \textit{Skyrme Density Functional}$$

$$\kappa(\rho,\phi,z) = \sum_\alpha u_\alpha(\rho,\phi,z) v^*_\alpha(\rho,\phi,z)$$

$$\Delta(\rho,\phi,z) = \Delta(\rho,z)e^{i\nu\phi} = \frac{g}{2}\sum_\alpha u_\alpha(\rho,\phi,z) v^*_\alpha(\rho,\phi,z)$$

$$Vel_{vortex}(\rho,z) = -\frac{i\hbar}{m\rho n(\rho,z)}\sum_\alpha v^*_\alpha(\rho,z,\phi)\frac{\partial v_\alpha(\rho,z,\phi)}{\partial\phi}$$

- **The equations are solved self-consistently**
- **SII Skyrme interaction (Brink-Vautherin)**
- **Protons are constrained to have a spherical geometry**
- **No spin-orbit interaction**

$$\begin{bmatrix} \hat{K} + V(\rho,z) - \lambda & \Delta(\rho,\phi,z) \\ \Delta^*(\rho,\phi,z) & -(\hat{K} + V(\rho,z) - \lambda) \end{bmatrix} \begin{bmatrix} u_\alpha(\rho,\phi,z) \\ v_\alpha(\rho,\phi,z) \end{bmatrix} = E_\alpha \begin{bmatrix} u_\alpha(\rho,\phi,z) \\ v_\alpha(\rho,\phi,z) \end{bmatrix}$$

Potentials with different interactions

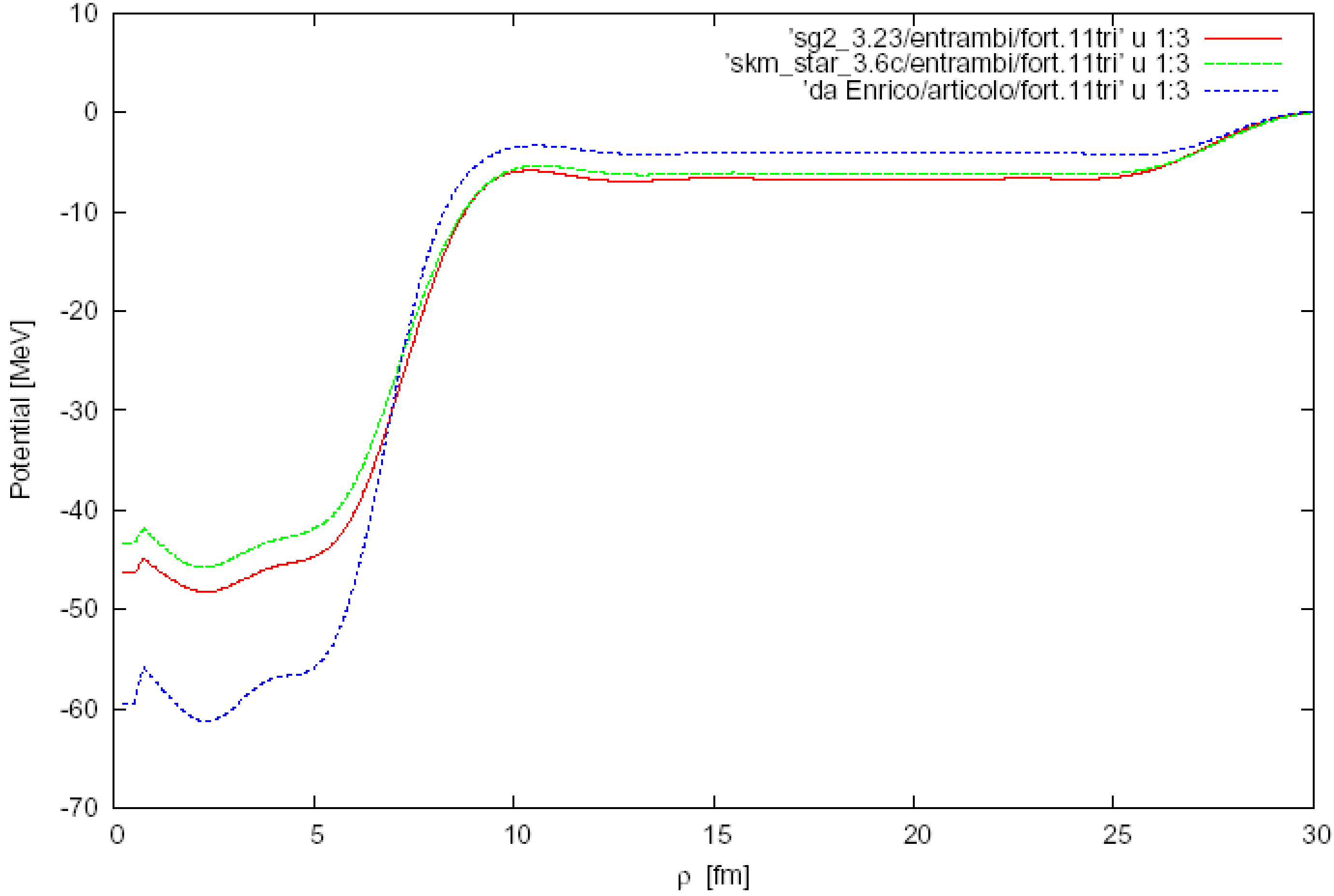
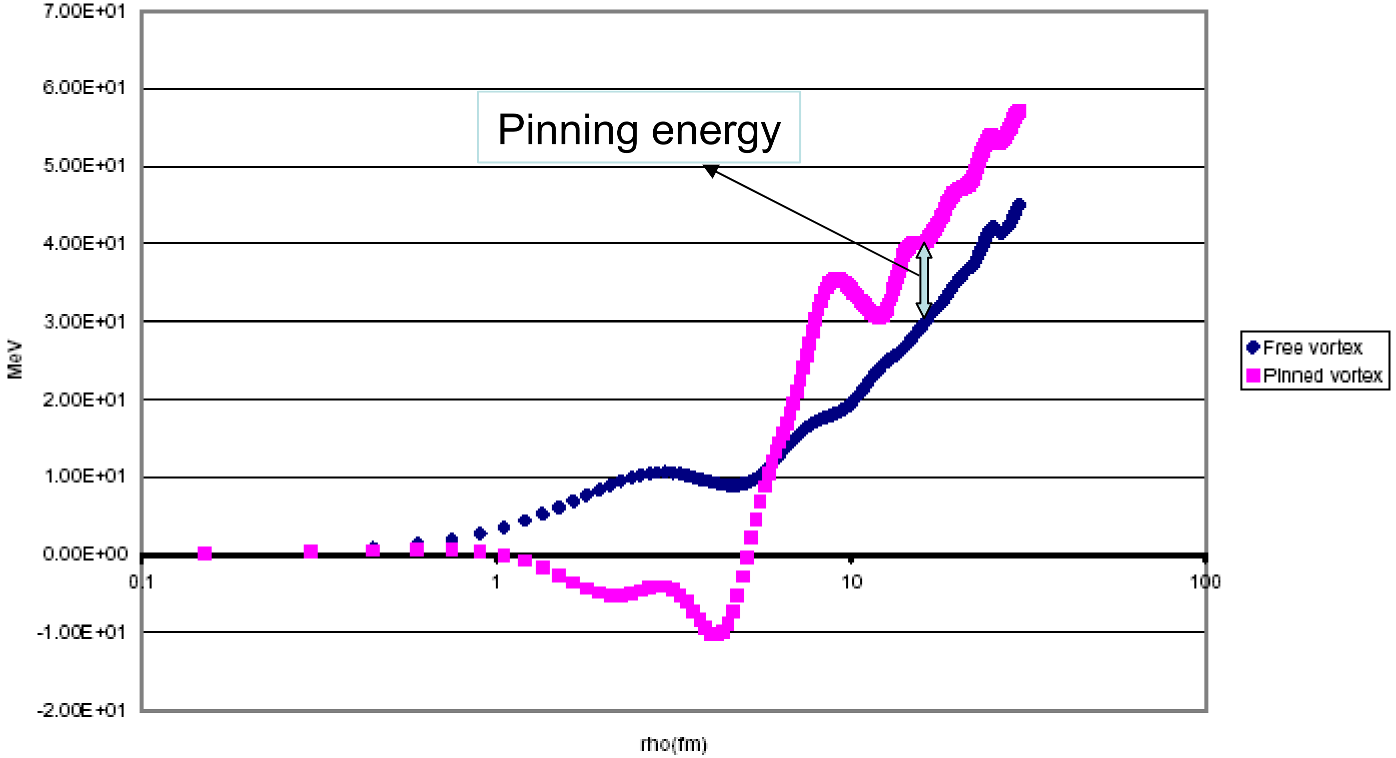
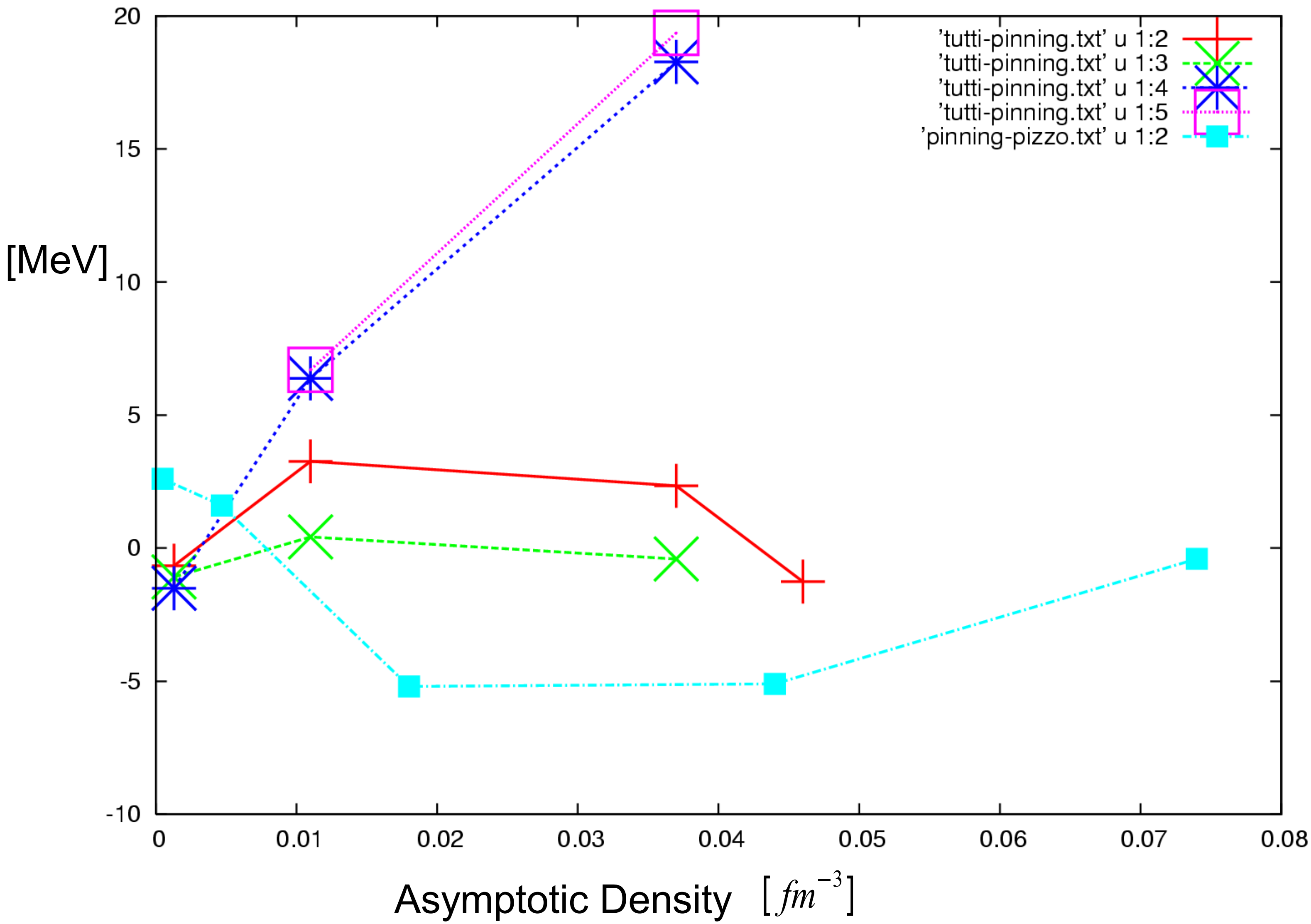


Fig. 4 Energy content as a function of the vortex considered radial size



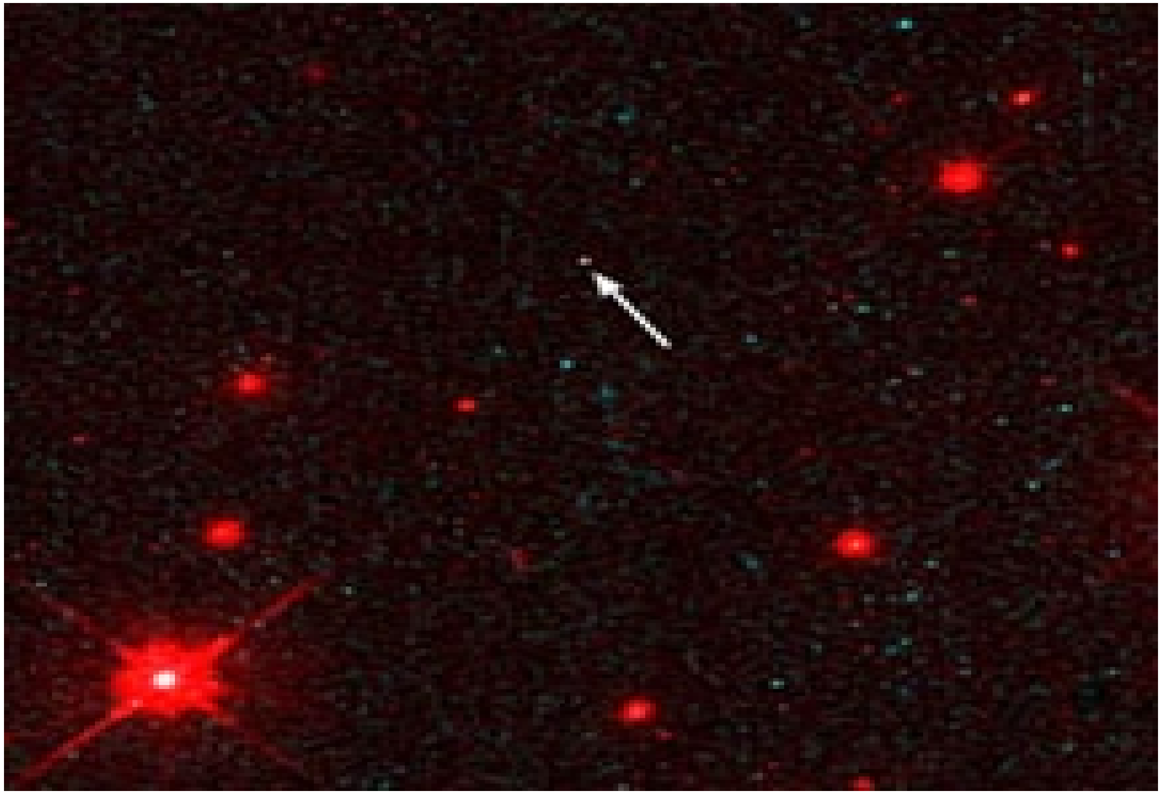
PINNING ENERGY

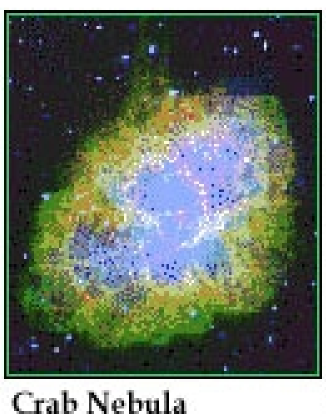


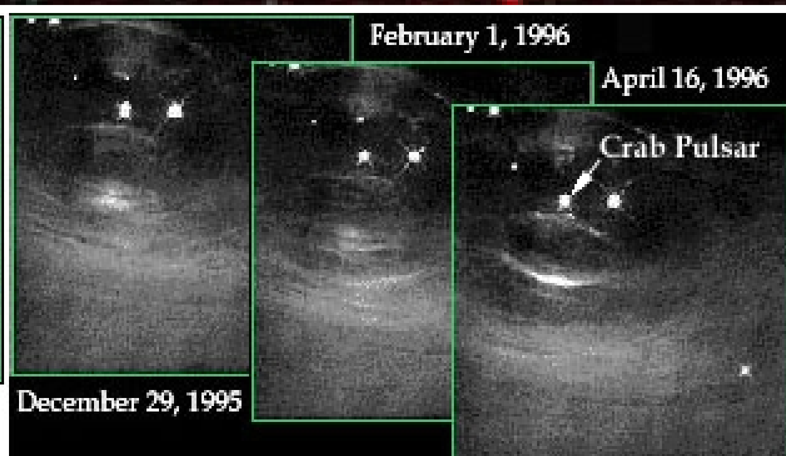
Green= Sly4; Red=SII; Blue= Skm*; Violet= Sg2; Azure = Pizzochero & al.

Interaction	chemical potential [MeV]	potself [MeV]	Fermi Energy[MeV]	K_f [fm^-1]
Skm*	1.2	-1.13	2.33	0.335
skm*	3.53	-6.23	9.76	0.685
skm*	7.5	-14.6	22.10	1.03
sg2	1.15	-1.15	2.30	0.312
sg2	3.17	-6.84	10.01	0.694
sg2	6.00	-16.12	22.12	1.03
sly4	1.6	-0.43	2.03	0.312
sly4	5.67	-4.38	10.05	0.695
sly4	10.05	-15.0	25.14	1.10
sII	1.6	-0.50	2.10	0.318
sII	5.8	-4.10	9.9	0.690
sII	11.3	-12.2	23.55	1.06

PULSARS

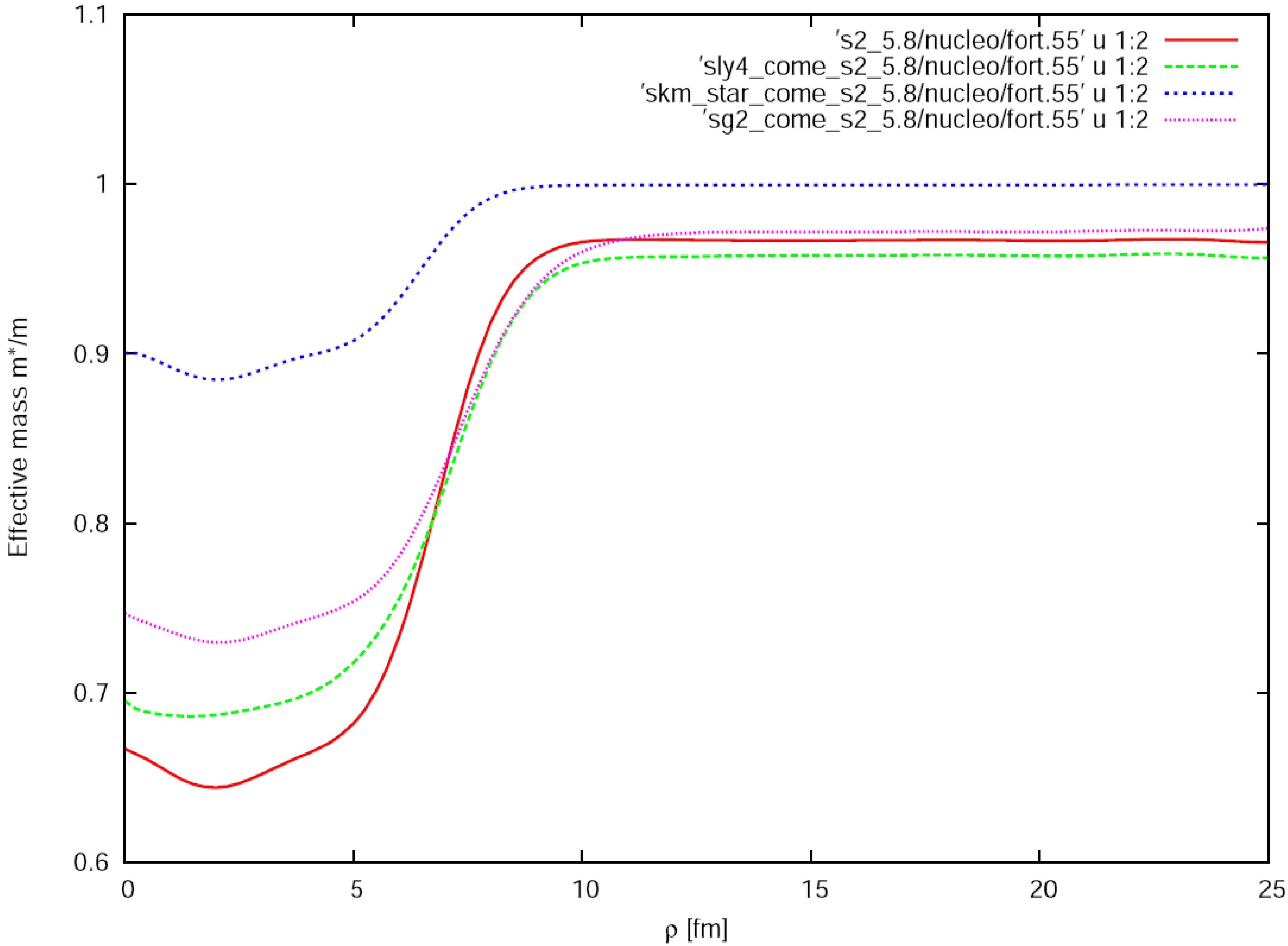






Hubble Space Telescope image of a lone neutron star (identified by the arrow) in the direction of the southern constellation Corona Australis. The star, which was originally discovered due to its copious emission of X-rays, has a surface temperature of 1.2 million degrees Fahrenheit, which is far hotter than that of ordinary stars.

Image Credit: Frederick M. Walter (State University of New York at Stony Brook) and NASA.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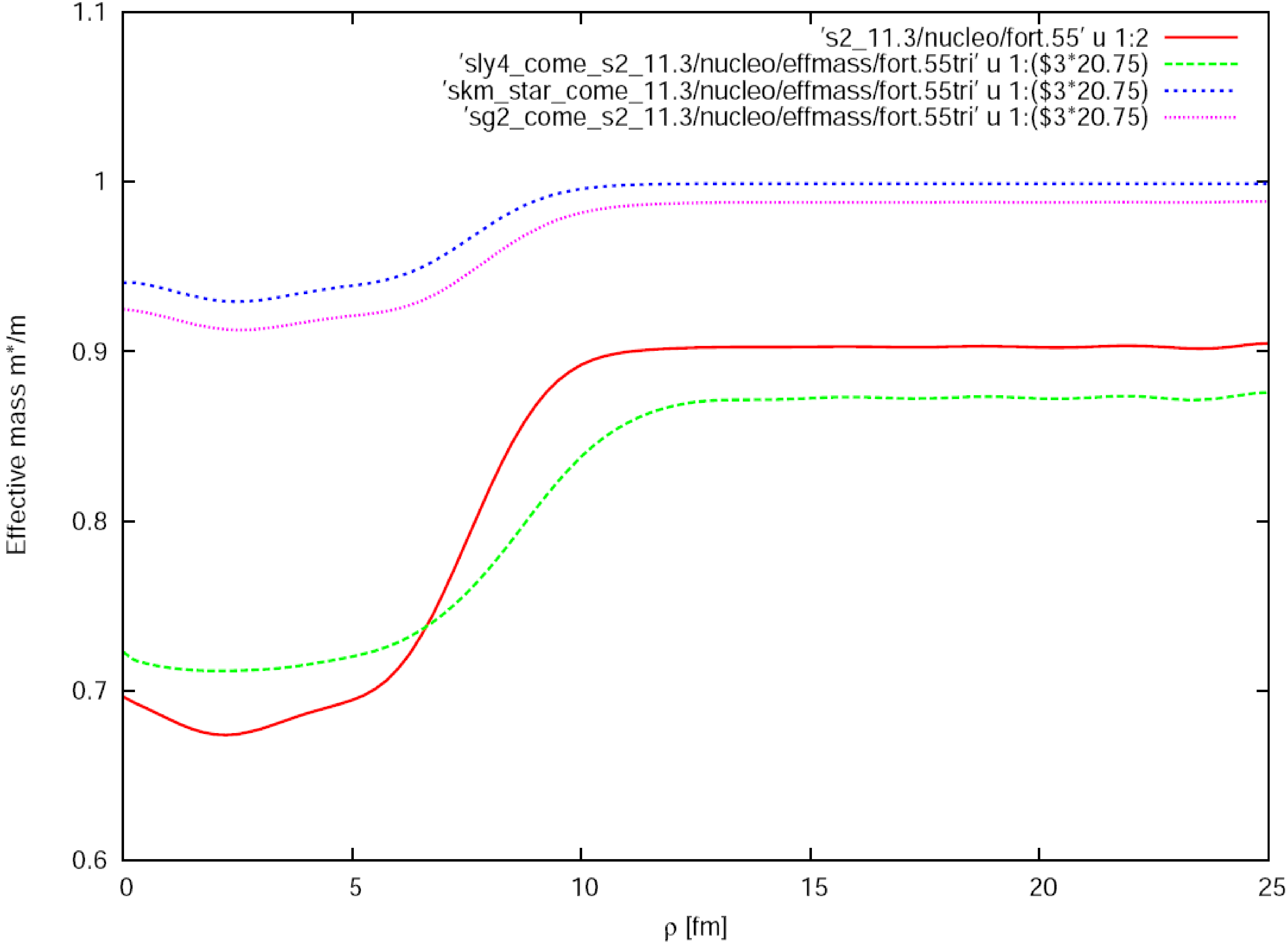
'sly4_come_s2_5.8/nucleo/fort.55' u 1:2
'skm_star_come_s2_5.8/nucleo/fort.55' u 1:2
'sg2_come_s2_5.8/nucleo/fort.55' u 1:2
Effective mass m*/m
ρ [fm]
1.1
1
0.9
0.8
0.7
0.6
0
5
10
15
20
25



's2_11.3/nucleo/fort.55' u 1:2
'sly4_come_s2_11.3/nucleo/effmass/fort.55tri' u 1:($3*20.75)
'skm_star_come_11.3/nucleo/effmass/fort.55tri' u 1:($3*20.75)
'sg2_come_s2_11.3/nucleo/effmass/fort.55tri' u 1:($3*20.75)
Effective mass m*/m
ρ [fm]
1.1
1
0.9
0.8
0.7
0.6
0
5
10
15
20
25