Few-Body Systems

A Survey (2005-2006)

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The European Few-Body Conference

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An Overview & Some Remarks

- New technical ideas allows one to extend the accuracy of the description of Final-State-Interaction (FSI) effects for A=3,4 nuclei and together with the detailed study of ground state properties open a golden doorway to understand fundamental issues still under discussion, like the role of the three-nucleon forces.
- Relativity is an emerging issue, driven by the accuracy reached by present-day experiments at momentum transfer $\geq$ 300 MeV/c (e.g., asymmetries for $^3\vec{\mathrm{He}}(e, e'))$. It could open the possibility to gain new insights in the many-body interaction and in the short range correlations.
- Relevant outcomes will be discussed in other Sessions as well, and this will be a plain evidence of great efforts carried on in the phenomenological arena by the Few-Body Community.
- Wide and sound International Collaborations !

Relativistic Hamiltonian Dynamics

Some groups are becoming involved in applying the RHD approach to few-body systems, therefore it could be useful to highlight some of the issues characteristic of this framework, proposed by Dirac in 1949.

RHD as an intermediate step between the non-relativistic approach and the full-glory field theory. Within RHD, one can embed the successful few-nucleon phenomenology, developed within non relativistic approaches, in a Poincaré covariant theory.

Dirac Aim : to merge the principles from Poincaré Covariance and the dynamical description of an interacting system.

Tool: the Hamiltonian point of view. In non relativistic Quantum Mechanics, once the quantum state at some instant $t = t_0$ is known, (i.e. one knows $\psi(x, t_0)$), then the time (dynamical) evolution can be obtained through the application of the proper operator (i.e. $\psi(x, t) = exp[-\imath H(t - t_0)]\psi(x, t_0)$, H is the generator of the time translations).

Results: due to the finite limit of the light speed, in a relativistic framework there are different choices for the initial hypersurface, not only the hypersurface defined by $t = 0$ and any values for $\vec{r}$ (Instant form), but also $z + t = 0$ (Front form), and $t^2 - r^2 = a > 0$ (Point form). The three ”initial” hypersurfaces investigated by Dirac have each very distinctive features. Note that the evolution variable, or ”time” in the standard language, assumes a different expression for each RHD.

The basic properties are related to the generators of the Poincaré Algebra (i.e. the four-momentum, P^μ, the 3 boosts, B^i, and the three rotations, J^i) that allow one to accomplish all the possible transformations of a quantum state in the Minkowski space.

Different "initial" hypersurfaces lead to different combinations of the generators, as dictated by the symmetry properties of the same hypersurfaces. For an interacting system, some generators necessarily contain the interaction. In non relativistic QM, the Hamiltonian (generator of the time translation) contains the interaction, while the three-momentum and the rotations do not (given the translational and rotational invariance of the 3D Euclidean space, a hypersurface in the 4D Minkowski space)

- Instant form: space translations, $\vec{P}$, and space rotations, $\vec{J}$, do not contain interaction, while $P^0 = H$ and the boosts must contain interaction, in order to allow the evolution of the quantum state.
- Front form (or Light-front form): $\vec{P}_\perp$, $P^0 + P_z$, J_z, B_z and the other 2 LF boosts do not contain interaction, while $P^0 - P_z$ and 2 LF rotations do.
- Point form: boost $\vec{B}$ and rotations $\vec{J}$ do not contain interaction, while P^μ do.

A simple consequence for an interacting system (two-body case for the sake of simplicity, note that in the two-body case $k^2/m \to (4E^2 - 4m^2)/4m!$)

$$p_1^\mu + p_2^\mu \neq P^\mu$$

at least one component of P^μ, the 4-momentum of the interacting system, must contain interaction. Note: p_i^μ are the momenta of free constituents

- Instant form (the standard t is the evolution variable):

$$\vec{p}_{1\perp} + \vec{p}_{1\perp} = \vec{P}_\perp \quad p_{1z} + p_{1z} = \vec{P}_z \quad p_1^0 + p_2^0 \neq P^0$$

- Front form, (the combination $x^+ = t + z$ plays the role of evolution variable):

$$\vec{p}_{1\perp} + \vec{p}_{1\perp} = \vec{P}_\perp \quad p_1^+ + p_2^+ = P^+ \quad p_1^- + p_2^- \neq P^-$$

- Point form (t can be used as evolution variable, but x^+ as well, since P^μ contains the interaction):

$$P^\mu = M_{int} \frac{p_1^\mu + p_2^\mu}{M_{free}} \qquad M_{int} \neq M_{free} = |p_1^\mu + p_2^\mu|$$

Padova

Relativistic approaches for few-body systems

Physical Motivations: To include in a consistent way, from the field theoretical point of view, the pion in elementary processes like $NN \to NN$, $\pi N \to \pi N$ and $NN \to \pi NN$.

Some hints: i)the necessity to deal with the absorption/emission of particles enlarges the energy range and forces toward a relativistic treatment, ii) in the π-NN vertex all the particles cannot stay on their-own mass shell, i.e. conservation of the 3-momentum, at each vertex, but not the energy ($\rightarrow$ Instant form will be adopted).

Aim of the Pd-Karkhov coll.: To rearrange the Hamiltonian, by introducing a suitable Unitary Transformation, in order to have, at a given order of the coupling constant g, a $\mathbf{H}(g)$ such that

$$\mathbf{H}(g)\,|Vacuum\rangle = \mathbf{H}_{free}\,|Vacuum\rangle$$

$$\mathbf{H}(g)\,|1\rangle = \mathbf{H}_{free}\,|1\rangle$$

In this way, at a given order of the coupling constant g, no virtual processes, dressing the vacuum and the one-particle state, are present, namely they are summarized in the creation/destruction operators clothed by the Unitary Transformation .

For the sake of concreteness the clothing procedure has been applied to an interacting Hamiltonian, that contains a simple Yukawa model, with a pseudoscalar meson-nucleon coupling, $V = ig\bar{\psi}\gamma_5\psi\phi$.

After applying a proper mass-changing unitary transformation to the Hamiltonian, in order to dress the bare masses of the particles, one has to look for a Unitary Clothing Transformation, that makes the eigenstates of the free Hamiltonian, with 0 and 1 particle, be eigenstates of the total one viz

$$
\begin{aligned}
H &= H(\alpha) = H\left(W\left(\alpha_c\right)\alpha_c W^{\dagger}\left(\alpha_c\right)\right) = \\
&= W\left(\alpha_c\right)H\left(\alpha_c\right)W^{\dagger}\left(\alpha_c\right) = K\left(\alpha_c\right) = \\
&= K_F(\alpha_c) + K_I(\alpha_c)
\end{aligned}
$$

where $W(\alpha_c) = \exp R(\alpha_c)$ is a unitary transformation ($R(\alpha_c)$ is the operator to be determined), α_c are the new, clothed creation/destruction operators and the operator K_I to be found must possess the property: $K_I|1\rangle_c = 0$.

It is found that the explicit expression obtained for R eliminates all the terms in the Hamiltonian that prevent the vacuum and the one-particle state be eigenstate of the total Hamiltonian at g^1 order, where g is the meson-nucleon coupling. Some bad terms of order g^2 can be eliminated with one more transformation $R(2)$. In this way, one restricts to g^3 order the bad terms.

Along this guidelines, we arrive at the decomposition:

$$
\begin{aligned}
&K(\alpha_c) = K_F(\alpha_c) \\
&+K(NN \to NN) + K(\bar{N}\bar{N} \to \bar{N}\bar{N}) + K(N\bar{N} \to N\bar{N}) \\
&+K(\pi N \to \pi N) + K(\pi\bar{N} \to \pi\bar{N}) + K(\pi\pi \leftrightarrow N\bar{N}) \\
&+K(NN \leftrightarrow \pi NN) + K(\bar{N}\bar{N} \leftrightarrow \pi\bar{N}\bar{N}) + K(N\bar{N} \leftrightarrow \pi N\bar{N}) \\
&+K(N\bar{N} \leftrightarrow \pi\pi\pi) + K(\pi N \leftrightarrow \pi\pi N) + K(\pi\bar{N} \leftrightarrow \pi\pi\bar{N}) + \ldots
\end{aligned}
$$

where the interactions between the clothed nucleons (N), antinucleons ($\bar{N}$) and pions (π) have been separated out.

As an example: $NN \to NN$ interaction operator

$$
\begin{aligned}
&K(NN \to NN) = \\
&= \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}'_1 d\,\mathbf{p}'_2 V_{NN}(\mathbf{p}'_1, \mathbf{p}'_2; \mathbf{p}_1, \mathbf{p}_2) b_c^\dagger(\mathbf{p}'_1) b_c^\dagger(\mathbf{p}'_2) b_c(\mathbf{p}_1) b_c(\mathbf{p}_2)
\end{aligned}
$$

where

$$
\begin{aligned}
&V_{NN}(\mathbf{p}'_1, \mathbf{p}'_2; \mathbf{p}_1, \mathbf{p}_2) = -\frac{1}{2}\frac{g^2}{(2\pi)^3}\frac{1}{2\omega_{\mathbf{p}'_1 - \mathbf{p}_1}} \times \\
&\frac{m^2}{\sqrt{E_{\mathbf{p}_1} E_{\mathbf{p}_2} E_{\mathbf{p}'_1} E_{\mathbf{p}'_2}}} \delta(\mathbf{p}'_1 + \mathbf{p}'_2 - \mathbf{p}_1 - \mathbf{p}_2) \bar{u}(\mathbf{p}'_1)\gamma_5 u(\mathbf{p}_1) \times \\
&\left\{ \frac{1}{E_{\mathbf{p}_1} - E_{\mathbf{p}'_1} - \omega_{\mathbf{p}'_1 - \mathbf{p}_1}} + \frac{1}{E_{\mathbf{p}'_1} - E_{\mathbf{p}_1} - \omega_{\mathbf{p}'_1 - \mathbf{p}_1}} \right\} \bar{u}(\mathbf{p}'_2)\gamma_5 u(\mathbf{p}_2)
\end{aligned}
$$

The Point Form of the Relativistic Hamiltonian Dynamics (one out three introduced by Dirac in 1949) has been investigated in detail for baryonic systems (seen as bound qqq system) (EPJ A25 (2005) 97).

Aim: the ambiguity related to the choice of the normalization factor of the baryon wave function has been elucidated within the so-called Point form Spectator model.

The 3-momentum conservation for an electromagnetic interaction reads

$$\vec{P}\,' - \vec{P} = \vec{q} = M' \frac{\vec{p}_1\,' + \vec{p}_2\,' + \vec{p}_3\,'}{M'_{free}} - M \frac{\vec{p}_1 + \vec{p}_2 + \vec{p}_3}{M_{free}}$$

where $P'(P)$ is the final (initial) baryon 3-momenta, $\vec{q}$ the 3-momentum transfer and it is different from the 3-momentum absorbed by the kicked quark (labeled by 1, for concreteness), namely $\vec{q} \neq \vec{p}_1\,' - \vec{p}_1$

This quantity has to be determined in order to perform actual calculations !

Initial baryon CM: $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$,

$$\vec{q} = M' \frac{\vec{p}_1{}' + \vec{p}_2{}' + \vec{p}_3{}'}{M'_{free}}$$

but **if** we assume that the spectator quarks do not change their-own 3-momenta one has

$$\vec{q} = M' \frac{\vec{p}_1{}' + \vec{p}_2 + \vec{p}_3}{M'_{free}} = M' \frac{\vec{p}_1{}' - \vec{p}_1}{M'_{free}}$$

The normalization factor to be used is

$$\mathcal{N}_{in} = \left(\frac{M'}{M'_{free}} \right)^3$$

Final baryon CM: one has the following normalization factor

$$\mathcal{N}_{fin} = \left(\frac{M}{M_{free}} \right)^3$$

Results: the mesonic decays of baryon resonances, calculated with baryon wave-functions obtained within both Goldstone-boson exchange model, have been analyzed by varying the normalization factor between the two extrema

Note that the normalization factor, given the dependence upon the mass of the interacting system, introduces in the evaluation of matrix elements of one-body operators many-body effects.

See also the contribution by T. Melde on Friday.

Pisa

A=3,4 and the Hyperspherical Harmonics approach

The solution of the Schrödinger equation for the four-nucleon ground states has been undertaken within the Hyperspherical Harmonics approach (PRC 71 (2005) 024006), namely by expanding the ^4He wave function on a basis, constructed from the Hyperspherical Harmonics functions (that depend upon the polar and azimuthal angles for each of the 3 Jacobi coordinates, and the two hyperspherical angles) and the spin-isospin states. The remaining dependence upon the hyperradius is given by the set of the expansion coefficients, or ,better, functions depending upon the hyperradius.

The solution has been obtained both by using model interactions (like the MT-V, MT-I/III, Volkov etc.) both realistic two- (AV18, Nijm II) and three-body (Urbana IX, Tucson Melbourne) nuclear potentials.

A great care has been devoted to the analysis of the convergence of different classes of HH functions, since, in presence of a huge amount of basis functions, some criteria have to be introduced in order to reduce the complexity of the numerical task. The outcome of this analysis, essential for sound theoretical evaluations of the ground-state properties, was successful.

The α–particle binding energies B (MeV), the rms radii (fm), the expectation values of the kinetic energy operator $\langle K \rangle$ (MeV), and the P and D probabilities (%) for various realistic interaction models as computed by means of the HH expansion are compared with the results obtained by other techniques. The binding energies obtained by using an extrapolation technique are enclosed in parentheses.

Interaction	Method	B	$\langle r^2 \rangle^{1/2}$	$\langle K \rangle$	P_P	P_D
AV18	HH	24.210(24.222)	97.84	1.512	0.347	13.74
	FY	24.25	97.80		0.35	13.78
	FY	24.223	97.77	1.516		
Nijm II	HH	24.419(24.432)	100.27	1.504	0.334	13.37
	F	24.56	100.31			
AV18+UIX	HH	28.462(28.474)	113.30	1.428	0.73	16.03
	FY	28.50	113.21		0.75	16.03
	GFMC	28.34(4)	110.7(7)	1.44		
AV18+TM’	HH	28.301(28.313)	110.27	1.435	0.73	15.63
	FY	28.36	110.14		0.75	15.67

The $T > 0$ components in ^{4}He are very small, but nonetheless important for the key-issue of the parity violation in experiments involving ^{4}He. The presence of a $T = 1$ component could be relevant in extracting information on the $s\bar{s}$ pair in the nucleon.

Percentages of the total isospin components $T = 1$ and 2 in the α-particle ground states for various interaction models.

interaction	method	$P_{T=1}$ [%]	$P_{T=2}$ [%]
AV18	HH	$2.8\ 10^{-3}$	$5.2\ 10^{-3}$
AV18	FY	$3\ 10^{-3}$	$5\ 10^{-3}$
Nijm-II	HH	$1.6\ 10^{-3}$	$7.4\ 10^{-3}$
AV18+UIX	HH	$2.5\ 10^{-4}$	$5.0\ 10^{-3}$

See the talk by Michele Viviani, this afternoon.

For the proton-deuteron elastic scattering, in the energy region between 3 MeV and 65 MeV, a benchmark calculation has been carried on (PRC71 (2005) 064003) for comparing the predictions obtained by using Correlated HH (coordinate space) and AGS (momentum space) approaches.

In this calculation the key-point is represented by the calculation of the 3N scattering states. In HH methods The fully-interacting state for a three-nucleon system in the continuum can be decomposed as follows

$$\Phi^{LXjj_zTT_z} = \Psi_A^{LXjj_zTT_z} + \Psi_c^{LXjj_zTT_z} =$$

$$= \sum_{i=1}^{3} \left[\psi_A^{LXjj_zTT_z}(i) + \psi_c^{LXjj_zTT_z}(i) \right]$$

where $\psi_A^{LXjj_zTT_z}(i)$ and $\psi_c^{LXjj_zTT_z}(i)$ are three Faddeev-like amplitudes, corresponding to the three permutations of the intrinsic coordinates ($\equiv \{\mathbf{r_1},\ \mathbf{r_2},\ \mathbf{r_3}\}$).

- $\Psi_c^{LXjj_zTT_z}$ describes the system when the three nucleons are close each other. For large interparticle distances and energies below the Deuteron breakup threshold $\Psi_c^{LXjj_zTT_z}$ goes to zero, while for higher energies, $\Psi_c^{LXjj_zTT_z}$ must reproduce an outgoing three-particle state .
- $\Psi_A^{LXjj_zTT_z}$ is the solution of the Schödinger eqn. in the asymptotic region.

★ $\Psi_A^{LXjj_zTT_z}$ can be recast in a different way, in order to emphasize its physical content.

1. the first one, contains an interacting pair times a free particle (for the sake of concreteness the particle 1),
2. the second term describes the rescattering between the interacting pair and the particle 1
3. the third term ($\psi_A^{LXjj_zTT_z}(2) + \psi_A^{LXjj_zTT_z}(3)$) takes care of the correct antisymmetrization of $\Psi_A^{LXjj_zTT_z}$ itself.

As an example $\psi_A^{LXjj_zTT_z}(1)$ can be written

$$\psi_A^{LXjj_zTT_z}(1) = \Omega^R_{LXJ}(\mathbf{x}_1, \mathbf{y}_1) + \sum_{L'X'} {}^J\mathcal{L}^{XX'}_{LL'} \left[\imath\, \Omega^R_{L'X'J}(\mathbf{x}_1, \mathbf{y}_1) + \Omega^I_{L'X'J}(\mathbf{x}_1, \mathbf{y}_1) \right]$$

- $\mathbf{x}_1 = \mathbf{r}_2 - \mathbf{r}_3 \qquad \mathbf{y}_1 = [\mathbf{r}_2 + \mathbf{r}_3 - 2\mathbf{r}_1]/\sqrt{3}$, L is the angular orbital momentum of the third particle and X an intermediate momentum coupling between the spin of the third particle and the total momentum of the pair.
- $\Omega^{R(I)}_{LSJ}$ is the regular ("irregular") solution describing the free scattering of a nucleon by an interacting pair ;

- The matrix

$$\mathcal{L} = \frac{S-1}{2\imath} = -\pi T$$

  is related to the Scattering matrix and represents a key ingredient of the variational approach for obtaining the whole wave function.

★ $\Psi_c^{LXjj_zTT_z}$ is explicitly expanded on the complete set of Hyperspherical Harmonics functions, with the inclusion of proper pair-correlation function

★★ The wave function, (i.e. the matrix elements of the S-matrix and the correlation functions), comes out from a variational approach applied to a functional constructed from the expectation value of the S-matrix (complex Köhn variational principle).

Results: both the HH method and the AGS one include the repulsive Coulomb interaction for describing the p-d state, even if in a different way, and this substantial difference lead to say that the present techniques are able to master Coulomb effects, clearly identifying the kinematical region where they become negligible. In general, the methods agree at 1% level on all the calculated observables, and it should be pointed out that at energies below the 3B breakup threshold the CHH works better, while for energies higher than 65 MeV, the AGS method seems more efficient.

The extended theoretical analysis of the polarization observables extracted from the reactions $d(\vec{p},\vec{p})d$ and $d(\vec{p},\vec{d})p$ at $E_p^{lab} = 22.7$ MeV (PRC73 (2006) 044004), has yielded another piece of evidence of the relevance of 3N forces, when the energy of the three-nucleon system become larger and larger.

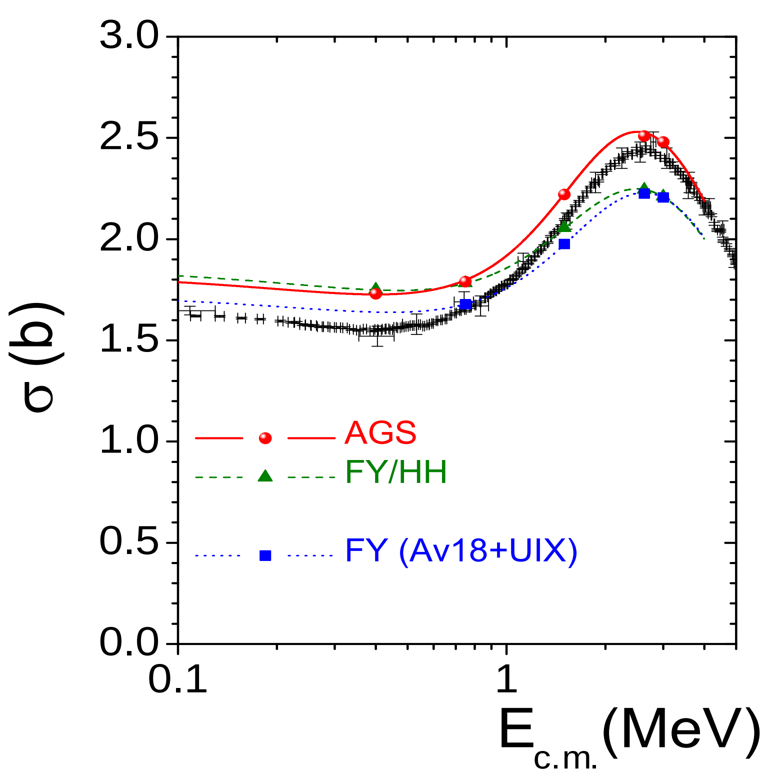
This immediately becomes a practical suggestion for calling new accurate experiments at higher energies.

The theoretical analysis was based on solutions of the scattering states, obtained by using both the Correlated HH method and the Faddeev equations in momentum space. Furthermore, i) realistic NN interactions (AV18, CD Bonn, Nijm I and Nijm II) and 3N forces (TucsonMelboune 99 and Urbana IX) and ii) NN interaction based on a chiral perturbation theory (NLO and NNLO) have been adopted.

The low energy elastic n-^{3}H scattering appears one one side a simple reaction channel (no Coulomb interaction, and to a very good approximation a pure T=1 state), on the other side has a rich dynamics, since a resonance structure is present around $E_{cm} \sim 3\ MeV$. Its investigation (PRC71 (2005) 034004) can be considered highly complementary for studying of 4N ground state, and allows one to more deeply test both the existing and new models for NN and 3N interactions. 4N scattering states !

Aim: to test NN interactions (in this work, local) through the comparison of the outcomes of present-day techniques (Alt-Gassberger-Sandhas,Faddeev-Yakubosky and Hyperspherical Harmonics), with a particular care to the issue of the convergence in angular momenta (FY and HH).

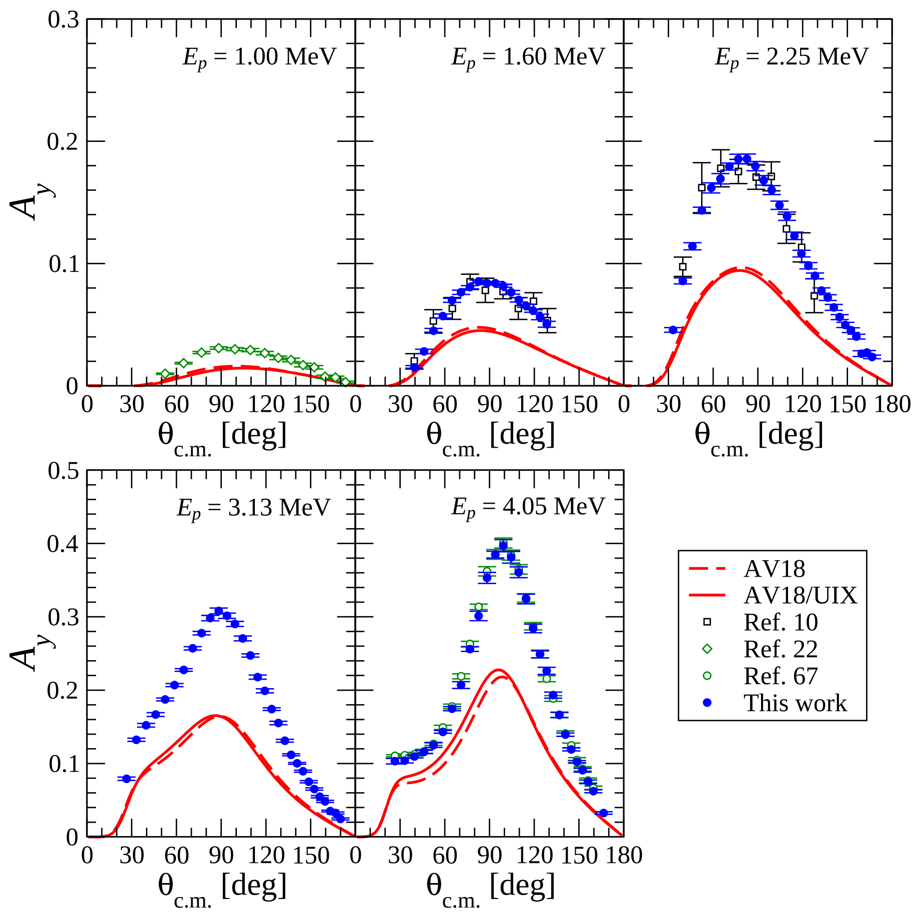
Results: very nice agreement between FY and HH, but underestimate of the data at the peak. AGS yields a better description of the peak but it is based on a rank-one approximation of the two-body t-matrix !



The detailed analysis (PRC 74 (2006) 034001), in collaboration with the experimental group, of the proton-^{3}He elastic scattering, at low energy ($1.6\ MeV \leq\ E_p\ \leq 4.05\ MeV$) has shown the analogous of the "A_y puzzle" known for the past 20 years in the nucleon-deuteron elastic channel.

The unpolarized cross section is very well described by the variational wave functions for the 3N and 4N states, when the 3N interaction (Urbana IX) is included.

The puzzle arises when the comparison with the measured A_y is carried on. A discrepancy of about 50% at the maximum of the proton analyzing power for each values of E_p measured. Some possible failure in our understanding of 3N forces or a subtle role of higher angular momenta (P-waves)?



A new class of realistic NN interactions with a χ^2/datum $\simeq$ 1, like the Charge-Dependent Bonn 2000 or N3LO, based on chiral perturbation theory, are non local and given in momentum space.

Such interactions appear a profitable tool that could shed light on still-unclear topics, like the structure and the role played by 3-body forces.

Therefore accurate techniques which can solve the corresponding Schrödinger equation for A=3 and 4 are very important.

The Pisa group (FBS 39 (2006) 159) has applied their variational approach, based on the expansion in terms of Hyperspherical Harmonics (no correlation functions), to solve the Schrödinger equation with those momentum-dependent potential. The approach is very effective, since the matrix elements of the various terms of the interactions, can be evaluated in the coordinate space or in momentum space, depending upon the convenience (local or non local terms), in almost a straightforward way (but, yet, a non trivial numerical task!). The impressive accuracy that can be reached for the ground state properties of A=3 and A=4 nuclei is shown in the Tables.

^{3}H

The triton binding energies B (MeV), the mean square radii $\sqrt{\langle r^2 \rangle}$ (fm), the expectation values of the kinetic energy operator $\langle T \rangle$ (MeV), and the mixed-symmetry S', P, D, and isospin T=3/2 (largely dominated by the charge symmetry breaking!) probabilities (all in %) , calculated with the CD Bonn 2000 and N3LO potentials, are compared with the results obtained within the Faddeev equations approach (FE) and within the No Core Shell Model (NCSM) approach. The results within the HH, correlated-hyperspherical-harmonics (CHH) and FE approaches for the AV18 potential have been also reported for sake of comparison. These last HH results do not include the T=3/2 states.

Inter.	Meth.	B	$\langle T \rangle$	$\sqrt{\langle r^2 \rangle}$	$P_{S'}$	P_P	P_D	$P_{T=3/2}$
CDB	HH	7.998	37.630	1.721	1.31	0.047	7.02	0.0049
2000	FE	7.997	37.620	-	1.31	0.047	7.02	0.0048
	FE	7.998	37.627	-	1.31	0.047	7.02	0.0048
	NCSM	7.99(1)	-	-	-	-	-	-
N3LO	HH	7.854	34.555	1.758	1.36	0.037	6.31	0.0009
	FE	7.854	34.546	-	1.37	0.037	6.32	0.0009
	FE	7.854	34.547	-	1.37	0.037	6.32	0.0009
	NCSM	7.85(1)	-	-	-	-	-	-
AV18	CHH	7.624	46.727	-	1.293	0.066	8.510	0.0025
	HH	7.618	46.707	1.770	1.294	0.066	8.511	-
	FE	7.621	46.73	-	1.291	0.066	8.510	0.0025

^{4}He

The α–particle binding energies B (MeV), the mean square radii $\sqrt{\langle r^2 \rangle}$ (fm), the expectation values of the kinetic energy operator $\langle T \rangle$ (MeV), and the P, D, $T = 1$ and $T = 2$ probabilities (%) for the two non-local potentials considered in this paper. The results obtained for the AV18 potential have been also reported for sake of comparison. The results obtained by other techniques are also listed.

Inter.	Meth.	B	$\langle T \rangle$	$\sqrt{\langle r^2 \rangle}$	P_P	P_D	$P_{T=1}$	$P_{T=2}$
CDB	HH	26.13	77.58	1.454	0.223	10.74	0.0029	0.0108
2000	FY	26.16	77.59	-	0.225	10.77	0.0030	0.0108
N3LO	HH	25.38	69.24	1.516	0.172	9.289	0.0035	0.0024
	FY	25.37	69.20	-	0.172	9.293	0.0033	0.0024
	NCSM	25.36(4)	-	-	-	-	-	-
AV18	HH	24.210	97.84	1.512	0.347	13.74	0.0028	0.0052
	FY	24.25	97.80	-	0.35	13.78	0.003	0.005
	FY	24.223	97.77	1.516	-	-	-	-

Trento

A=3,4 Nuclei and the Lorentz Integral Transform

The Lorentz Integral Transform approach allows microscopic calculations of electromagnetic reaction cross-sections without an explicit knowledge of the final-state wave functions, namely one can fully include the FSI effects, without solving an infinite set of eqns.

The inversion of the Lorentz Transform is a delicate step, since it constitutes a so-called ill-posed problem (i.e. it is a possible source of instabilities in the solution, for given interval of the variables involved).

The Group has devoted a careful analysis of this issue (EPJ A24 (2005)361) , introducing consequently new inversion techniques, that contain suitable regularization schemes, necessary to overcome the ill-posed problem.

Furthermore, the new techniques enlarge the range of applicability of LIT

A resumé ⇒

The LIT method is a two-step method. Firstly, one calculates an integral transform of the nuclear response, $R(\omega)$, with a Lorentzian-shape kernel

$$L(\sigma_R, \sigma_I) = \int d\omega \, \frac{1}{(\omega - \sigma_R)^2 + \sigma_I^2} \, R(\omega) \qquad \sigma_R, \sigma_I > 0$$

Then, one inverts the obtained integral transform. The inversion, is in principle an ill-posed problem, since it is unstable with respect to high frequency oscillations. Unfortunately, unphysical oscillations cannot be simply separated from the solution. In order to avoid such problems, a regularization scheme for the inversion is necessary.

In the actual case $R(\omega)$ represents the response of the nucleus to some probe, viz

$$R(\omega) = \int d\Psi_f \; |\langle \Psi_f | \mathbf{O} | \Psi_0 \rangle|^2 \; \delta(E_f - E_0 - \omega)$$

where $|\Psi_0\rangle$ is the ground state of the target nucleus ($\mathbf{H}|\Psi_0\rangle = E_0|\Psi_0\rangle$), and $|\Psi_f\rangle$ one of the possible final states reachable after interacting with the probe, ($\mathbf{H}|\Psi_f\rangle = E_f|\Psi_f\rangle$). Then the Lorentz Transform becomes

$$L(\sigma_R, \sigma_I) = \langle \Psi_0 | \mathbf{O}^\dagger \, \frac{1}{(\mathbf{H} - E_0 - \sigma_R)^2 + \sigma_I^2} \, \mathbf{O} | \Psi_0 \rangle = \langle \tilde{\Psi} | \tilde{\Psi} \rangle$$

where $(\mathbf{H} - E_0 - \sigma_R - \imath\sigma_I) \, |\tilde{\Psi}\rangle = \mathbf{O}|\Psi_0\rangle$

The standard inversion method was based on the expansion of $R(\omega)$ in terms of a given basis, and the coefficients were determined through a fit of the Lorentz Transform, previously calculated by solving the differential equation for $|\tilde{\Psi}\rangle$.

The number of functions in the expansion plays the role of regularization parameter.

This procedure turns out to be a very reliable one and quickly converging for $R(\omega)$, without too much structure (in this case an extra attention should be paid in finding the suitable σ_I).

New Inversion Methods

★ Fridman approach: the inversion of the Lorentz transform is based on the iterative procedure for solving a Fredholm integral equation of first order

$$L_I(\sigma_R) = \int_a^b d\omega\; K_I(\sigma_R, \omega)\; R(\omega)$$

$$R_{n+1}(x_j) = R_n(x_j) + \lambda \left[w_j L_I(x_j) - \sum_{\ell=0}^{N} K_I(x_j, x_\ell) R_n(x_\ell) \right]$$

where w_j is the proper weight for the chosen quadrature rule for integration.

The regularization scheme is provided by the grid on x_j, suitably chosen.

★★ Banach fix-point approach: the Lorentz transform can be seen as the matrix elements of a Lorentz operator $\mathbf{L}(\sigma_I)$

$$\langle \omega' | \mathbf{L}(\sigma_I) | R \rangle \ = \int d\omega \ \frac{1}{(\omega' - \omega)^2 + \sigma_I^2} \ R(\omega)$$

where $|R\rangle \in$ to the space of the square-integrable functions.

Then, one introduces a mapping acting on $R(\omega)$,

$\mathbf{T}(\sigma_I, L) \, [R(\omega')]$, viz

$$\mathbf{T}(\sigma_I, L) \left[R(\omega') \right] = \frac{\sigma_I}{\pi} \ L(\omega', \sigma_I - \frac{\sigma_I}{\pi} \ \int d\omega \ \frac{R(\omega') - R(\omega)}{(\omega' - \omega)^2 + \sigma_I^2}$$

that has a fixpoint, namely

$$\mathbf{T}(\sigma_I, L) \left[R(\omega') \right] = R(\omega')$$

By using the *Banach's fixpoint theorem*, one demonstrate the convergence of the iteration procedure, viz

$$R_{n+1}(\omega') \equiv \mathbf{T}(\sigma_I, L) \left[R_n(\omega') \right]$$
$$\lim_{n \to \infty} R_n(\omega') = R_{fix}(\omega') \equiv R(\omega') \quad \text{for } \ 0 < \sigma_I < \epsilon$$

The regularization scheme involves the mesh points and the number of iterations.

The reliability of LIT method is well established!

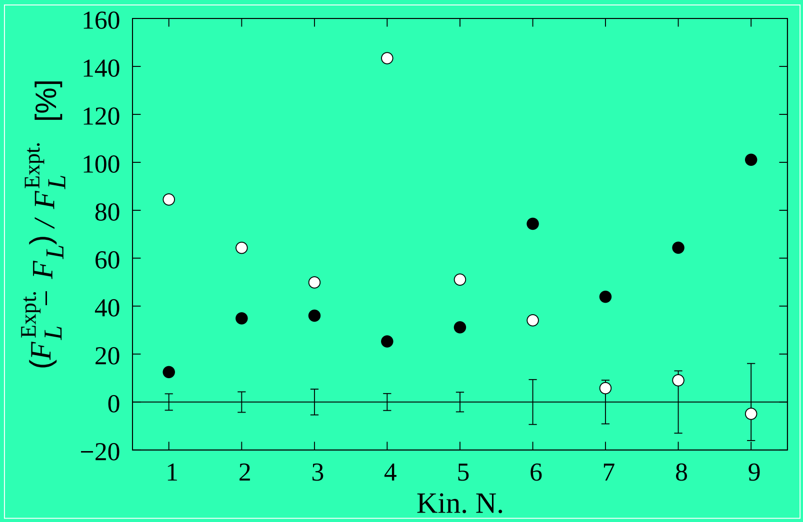
An interesting reaction channel investigated by the Trento group (PRC 72 (2005)064002) was:

$$^{4}\mathrm{He} + e \rightarrow ^{3}\mathrm{H} + p + e$$

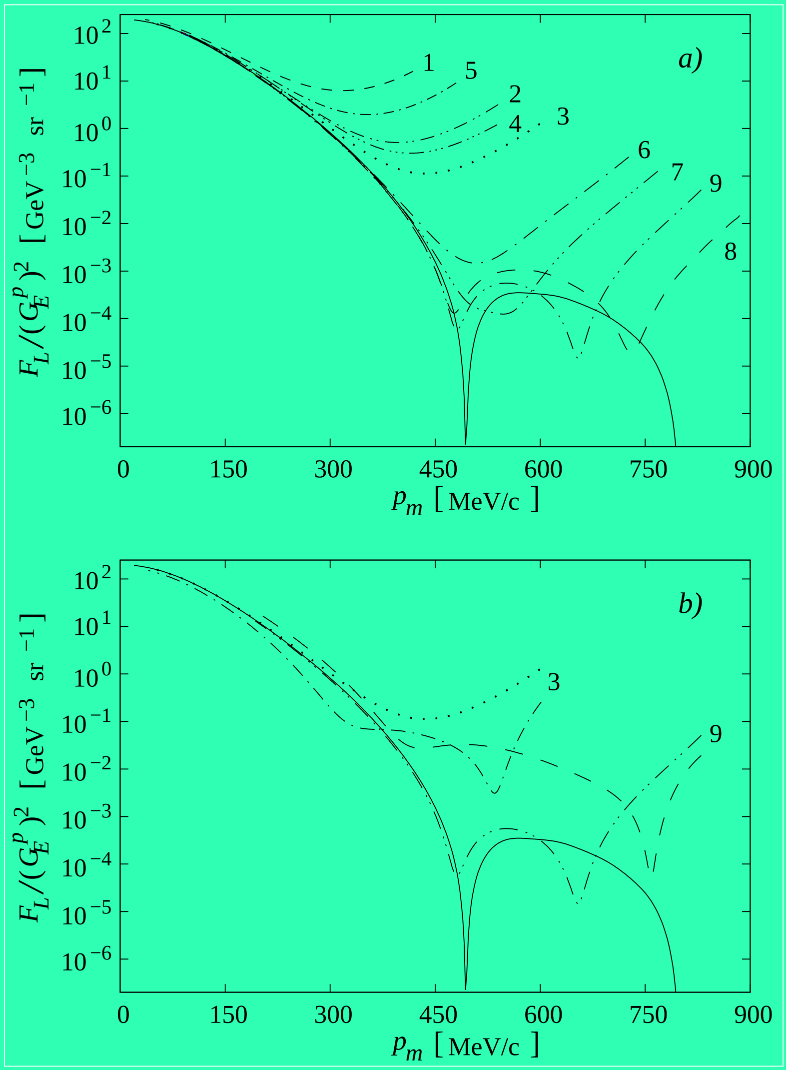
Longitudinal responses (extracted through a Rosenbluth separation method) have been calculated by using LIT and the semirealistic MT-I-III potential. Full calculations (FSI + proper antisymmetrization in the 4-nucleon final states) are compared with calculations in Plane Wave Impulse approximation, where the final state is: a plane wave (describing the struck nucleon) $\times$ the fully interacting wave function for the spectator 3-nucleon system.

Aim: to investigate the limits of the PWIA (and the fully symmetrized PWIA), in extracting relevant physical quantities, like the nucleon distribution inside the target nucleus and the spectroscopic factor for the shell (a shell model quantity, ...$A = 4 \sim A = \infty$). Within PWIA the cross section assumes a factorized form, and one can extract the above mentioned quantities. This should be reliable in a parallel kinematics, where a direct proton knock-out mechanism is highly likely.

Results: The PWIA might be a reasonable approximation for small missing momenta (below 100 MeV/c) and higher momentum transfer (above 400 MeV/c). Furthermore, the antiparallel kinematics drastically enhances (as expected) both FSI and antisymmetrization effects. This finding could be used in a reverse mode. This kinematical region could be the most profitable one for studying different approaches for including FSI.

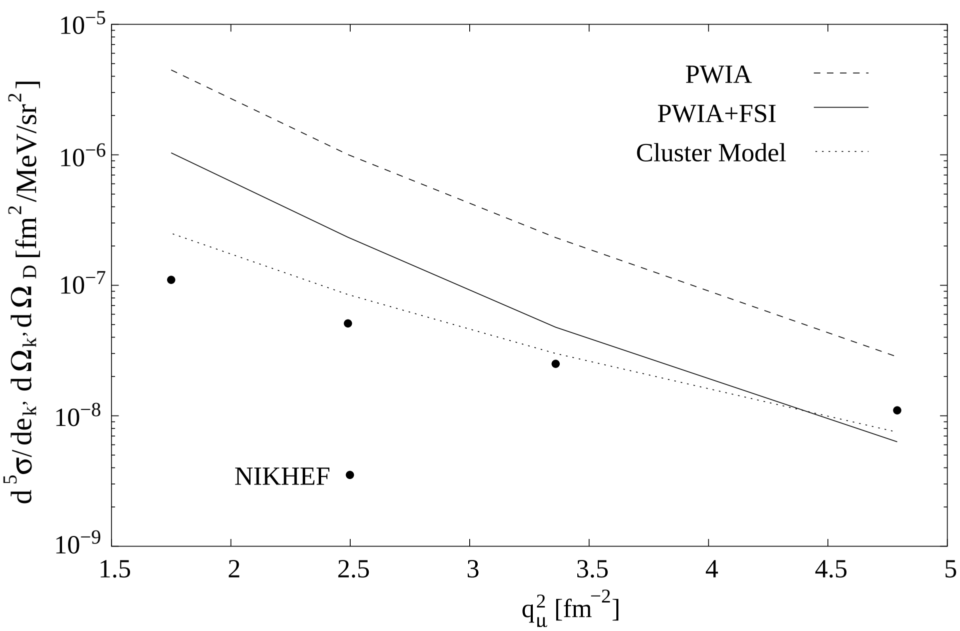


Percentage deviation from the experimental values: PWIA (open circles), FULL results (full circles). Kin. 1:(q = 299 MeV, ω = 57.68 MeV),$\rightarrow$ Kin. 9:(q = 680 MeV, ω = 146.48 MeV)



The $^4\mathrm{He}(e, e'p)^3\mathrm{H}$ longitudinal response function as function of p_m: *a)* PWIAS for the different (ω, q) values labeled with the corresponding numbers; *b)* PWIAS (dotted and dot–dot–dashed line) and FULL (dashed and dot–dashed line) results for the (ω, q) values of Kin. N. 3 and 9 of the Saclay exp. The solid line represents the PWIA.

Another reaction was $^4He(e,e'd)d$. This investigation has given clear hints about the possible improvements for FSI, beyond the central interaction like MT -I/III (EPJ A27(2006) 47). The comparison with NIKHEF data excludes the necessity of a substantial improvement in the description of the ^4He ground state, while the remarkable overestimate of the experimental cross-section at low momentum transfer and the comparison with other models for the ground state suggest to include tensor and other realistic force in the description of the final states.



A substantial step forward (PRL 96 (2006) 112301) in accurately calculating the total photo-absorption cross section for ^4He within the LIT, has been done by considering for the first time a realistic NN interaction, like Argonne V18, and an up-to date NNN force, like Urbana IX

See the Contribution, devoted to this topic, by Sonia Bacca, on Friday.

The ^3He longitudinal electromagnetic response is studied for different values of the momentum transfer $q = 500\ MeV/c,\ 600\ MeV/c,\ 700\ MeV/c$ at the quasi-elastic peak, in order to elucidate the frame dependence, and therefore the relevance of the relativity, of the calculation (PRC 72 (2005) 01100R). The response in the Lab frame can be put in relation to the responses calculated in other frames (all related by boosts along the direction of $\vec{q}$) through a pure kinematical factor, without considering the effect of the boosts on the 3-nucleon W. F.'s

Three frames are considered;

- Anitilab frame : $\vec{P}_i = -\vec{q}$ and $\vec{P}_f = 0$. Roughly speaking, $\vec{p}_i \sim -\vec{q}/3$, and $\vec{p}_f \sim 2\vec{q}/3$ (where $\vec{p}_{i(f)}$ is the nucleon momentum in the initial (final) nuclear state).
- Breit frame: $\vec{P}_i + \vec{P}_f = 0 \rightarrow \vec{P}_i = -\vec{q}/2$ and $\vec{P}_f = \vec{q}/2$. Roughly speaking, $\vec{p}_i \sim --\vec{q}/6$, and $\vec{p}_f \sim 5\vec{q}/6$.
- Active nucleon Breit frame: $\vec{P}_i = -3\vec{q}/2$ and $\vec{P}_f = -3\vec{q}/2 + \vec{q} = -\vec{q}/2$. Roughly speaking, $\vec{p}_i \sim --\vec{q}/2$, and $\vec{p}_f \sim \vec{q}/2$.

The nuclear potential adopted was the Argonne V18 (NN)+ Urbana IX.(NNN). The best results can be obtained by using the Active nucleon Breit frame, since as the Authors pointed out, nucleons with moderate momenta are present in this frame.

Finally, it has to be quoted the impressive taxonomy of responses of a polarized deuteron target in inclusive and exclusive electrodisintegration processes (EPJ A23 (2005) 147).

Roma
Trinucleon em form factors
in the LF Hamiltonian Dynamics

Ingredients

★ A Breit frame where

$$\boxed{\mathbf{q}_\perp = 0} \Rightarrow \boxed{q^+ \neq 0}$$

★★ A Poincaré covariant current

$$j^\mu(K\vec{e}_z) = \frac{\mathcal{J}^\mu(K\vec{e}_z)}{2} + L^\mu_\nu[r_x(-\pi)]\, e^{\imath\pi S_x} \frac{\mathcal{J}^\nu(K\vec{e}_z)^*}{2} e^{-\imath\pi S_x}$$

with

$$\mathcal{J}^+(K\vec{e}_z) = \mathcal{J}^-(K\vec{e}_z) = \langle \vec{P}_\perp = 0, P'^+ | \Pi J^+_{free}(0) \Pi | \vec{P}_\perp = 0, P^+\rangle$$

$$\vec{\mathcal{J}}_\perp(K\vec{e}_z) = \langle \vec{P}_\perp = 0, P'^+ | \Pi \vec{J}_{free\perp}(0) \Pi | \vec{P}_\perp = 0, P^+\rangle$$

$\Pi \equiv$ projector onto the subspace of a trinucleon bound state $|\chi_{\frac{1}{2}}\rangle$ of mass M_T and spin $1/2$,
$J^\mu_{free}(0) = \sum_i J^\mu_{pi}(0)(1+\tau_3)/2 + J^\mu_{ni}(0)(1+\tau_3)/2$ with
$J^\mu_N = -F_{2N}(p^\mu + p'^\mu)/2M + \gamma^\mu(F_{1N} + F_{2N})$.

★★★ A trinucleon bound state, obtained through a variational technique by Kievsky, Rosati, Viviani (NPA 577 (1994) 511) with AV18.

Preliminary calculation with only S+S' waves:

$\mathcal{P}_S(Av18) = \ 90.1\% \ \mathcal{P}_{S'}(Av18) = \ 1.29\%$

Trinucleon magnetic moments

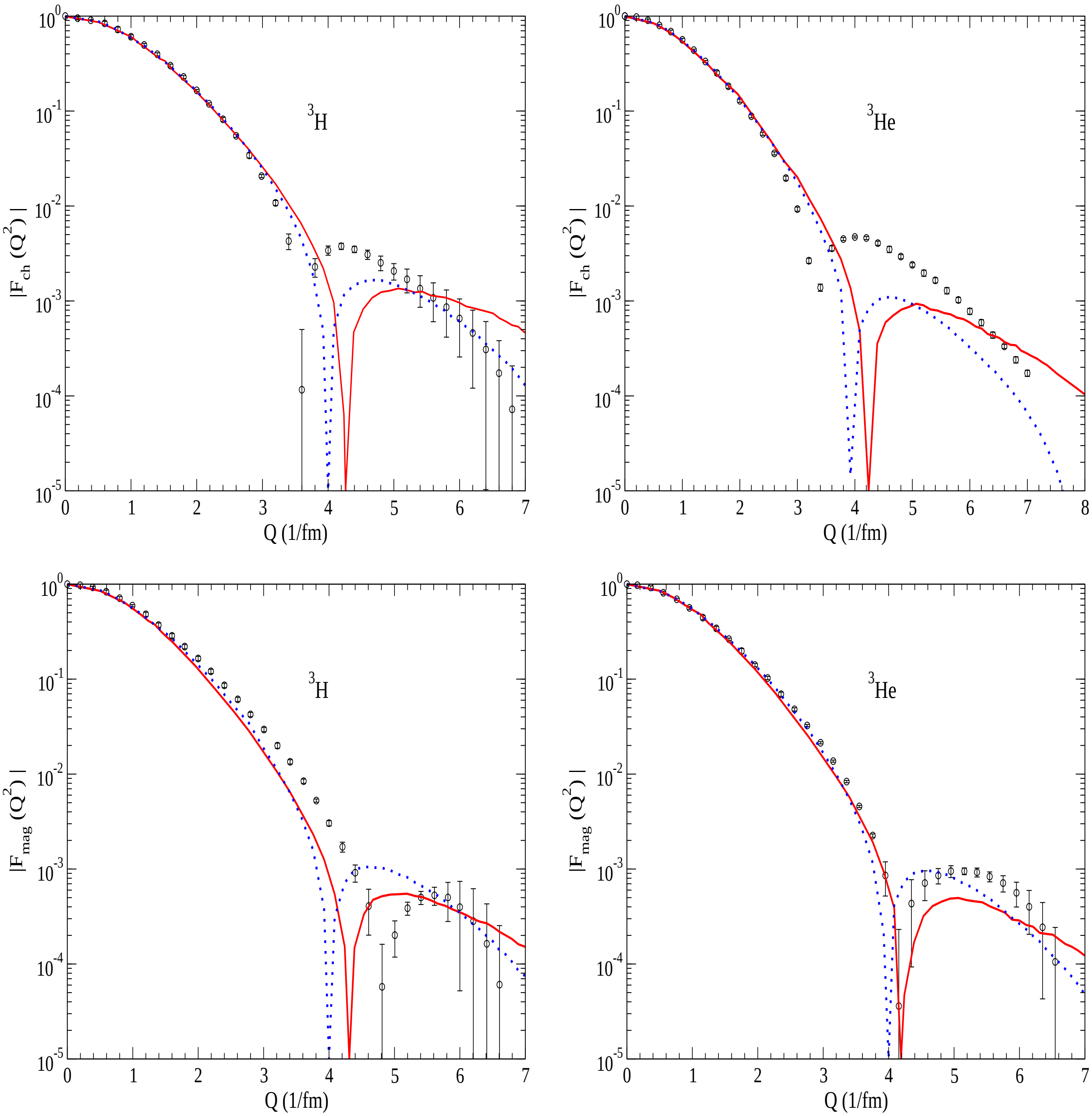
Theory	^{3}He	^{3}H
NR(S)	-1.723	2.515
LFD(S)	-1.778	2.597
NR(S+S')	-1.707	2.518
LFD(S+S')	-1.759	2.652
Exp.	-2.1276	2.9789

Charge and Magnetic form factors of ^3H and ^3He

For the first time in LFD !

Only S+S' waves

AV18 + Gari-Krümpelmann nucleon form factors



Solid line: LFD calculation in a Breit frame where $\boxed{\mathbf{q}_\perp = 0}$

Dotted line: non relativistic calculation.In cartesian coordinates, the chosen frame corresponds to a Breit frame where $\hat{q}_z = \hat{e}_z$.

Conclusions & Perspectives

- A=3,4 ground state properties can be investigated with a great accuracy, constructing a sound basis for addressing new issues, like the application of new NN interactions based on the chiral perturbation theory or the role of 3N forces.
- Different methods for dealing with the FSI effects allows us to get rid of uncertainties related to the model dependence, when relevant physical quantities are extracted from reactions involving few-nucleon systems. Furthermore, the subtle effects of 3N forces can be investigated in a more exclusive way ,when scattering states are involved, and therefore FSI must be under control.
- Relativity: an open question
- Few-Hadron Systems

Have a profitable year!

In view of the next

European Few-Body Conference in Pisa, (Sept. 10-14, 2007)