

Gesellschaft für Schwerionenforschung, Darmstadt, Theory Division

⁴He photodisintegration with realistic nuclear forces

S. Bacca in collaboration with:

W. Leidemann and G. Orlandini

Dipartimento di Fisica, Università di Trento and INFN

D. Gazit and N. Barnea

The Racah Institute of Physics, The Hebrew University, Jerusalem







Gesellschaft für Schwerionenforschung, Darmstadt, Theory Division

⁴He photodisintegration with realistic nuclear forces

S. Bacca

Outline

Motivation



Gesellschaft für Schwerionenforschung, Darmstadt, <u>Theory Division</u>

⁴He photodisintegration with realistic nuclear forces

S. Bacca

Outline

- Motivation
- Theoretical Tools



Gesellschaft für Schwerionenforschung, Darmstadt, Theory Division

⁴He photodisintegration with realistic nuclear forces

S. Bacca

Outline

- Motivation
- Theoretical Tools
- Realistic Photoabsorption



Gesellschaft für Schwerionenforschung, Darmstadt, Theory Division

⁴He photodisintegration with realistic nuclear forces

S. Bacca

Outline

- Motivation
- Theoretical Tools
- Realistic Photoabsorption
- Conclusions





Probing Few Nucleon Systems with Photons



Probing Few Nucleon Systems with Photons

- Exact description of the wave function
- Perturbative approach + consistent current operator
- Study structure of light nuclei and reactions



Probing Few Nucleon Systems with Photons

- Exact description of the wave function
- Perturbative approach + consistent current operator
- Study structure of light nuclei and reactions
 - • "Theoretical laboratory" to test nuclear force models:

 two-body and three-body interactions

Response Function

$$R(\omega) = \sum_{f} \left| \langle \Psi_{f} | \hat{O} | \Psi_{0} \rangle \right|^{2} \delta(E_{f} - E_{0} - \omega)$$

Response Function
$$R(\omega) = \sum_{f} \left| \langle \Psi_{f} | \hat{O} | \Psi_{0} \rangle \right|^{2} \delta(E_{f} - E_{0} - \omega)$$

Challange: treat the continuum \rightarrow Lorentz Integral Transform Approach

$$\begin{aligned} \mathcal{L}(\sigma) &= \int d\omega R(\omega) \frac{1}{(\omega - \sigma_R)^2 + \sigma_I^2} \\ \mathcal{L}(\sigma) &= \sum_f \left\langle \psi_0 \left| \hat{O}^{\dagger} \frac{1}{E_f - E_0 - \sigma^*} \right| \psi_f \right\rangle \left\langle \psi_f \left| \frac{1}{E_f - E_0 - \sigma} \hat{O} \right| \psi_0 \right\rangle \\ &= \sum_f \left\langle \psi_0 \left| \hat{O}^{\dagger} \frac{1}{\hat{H} - E_0 - \sigma^*} \right| \psi_f \right\rangle \left\langle \psi_f \left| \frac{1}{\hat{H} - E_0 - \sigma} \hat{O} \right| \psi_0 \right\rangle \end{aligned}$$

Response Function
$$R(\omega) = \sum_{f} \left| \langle \Psi_{f} | \hat{O} | \Psi_{0} \rangle \right|^{2} \delta(E_{f} - E_{0} - \omega)$$

Challange: treat the continuum \rightarrow Lorentz Integral Transform Approach

$$\begin{aligned} \mathcal{L}(\sigma) &= \int d\omega R(\omega) \frac{1}{(\omega - \sigma_R)^2 + \sigma_I^2} \\ \mathcal{L}(\sigma) &= \sum_{f} \left\langle \psi_0 \left| \hat{O}^{\dagger} \frac{1}{E_f - E_0 - \sigma^*} \right| \psi_f \right\rangle \left\langle \psi_f \left| \frac{1}{E_f - E_0 - \sigma} \hat{O} \right| \psi_0 \right\rangle \\ &= \left[\sum_{f} \left\langle \psi_0 \left| \hat{O}^{\dagger} \frac{1}{\hat{H} - E_0 - \sigma^*} \right| \psi_f \right\rangle \left\langle \psi_f \right| \frac{1}{\hat{H} - E_0 - \sigma} \hat{O} \right| \psi_0 \right\rangle \\ &= \left\langle \psi_0 \left| \hat{O}^{\dagger} \frac{1}{\hat{H} - E_0 - \sigma^*} \frac{1}{\hat{H} - E_0 - \sigma} \hat{O} \right| \psi_0 \right\rangle = \left\langle \widetilde{\Psi} \right| \widetilde{\Psi} \right\rangle \\ (\hat{H} - E_0 - \sigma_R + i\sigma_I) \left| \widetilde{\Psi} \right\rangle = \hat{O} \left| \Psi_0 \right\rangle \quad \text{Bound-state-like equation} \end{aligned}$$

Response Function
$$R(\omega) = \sum_{f} \left| \langle \Psi_{f} | \hat{O} | \Psi_{0} \rangle \right|^{2} \delta(E_{f} - E_{0} - \omega)$$

Challange: treat the continuum \rightarrow Lorentz Integral Transform Approach

$$\mathcal{L}(\sigma) = \int d\omega R(\omega) \frac{1}{(\omega - \sigma_R)^2 + \sigma_I^2} \mathcal{L}(\sigma) = \sum_{f} \left\langle \psi_0 \left| \hat{O}^{\dagger} \frac{1}{E_f - E_0 - \sigma^*} \right| \psi_f \right\rangle \left\langle \psi_f \left| \frac{1}{E_f - E_0 - \sigma} \hat{O} \right| \psi_0 \right\rangle \right. \\ = \left. \left[\sum_{f} \left\langle \psi_0 \left| \hat{O}^{\dagger} \frac{1}{\hat{H} - E_0 - \sigma^*} \right| \psi_f \right\rangle \left\langle \psi_f \right| \frac{1}{\hat{H} - E_0 - \sigma} \hat{O} \right| \psi_0 \right\rangle \\ = \left\langle \psi_0 \left| \hat{O}^{\dagger} \frac{1}{\hat{H} - E_0 - \sigma^*} \frac{1}{\hat{H} - E_0 - \sigma} \hat{O} \right| \psi_0 \right\rangle = \left\langle \tilde{\Psi} \right| \tilde{\Psi} \right\rangle \\ \left. \left(\hat{H} - E_0 - \sigma_R + i\sigma_I \right) \left| \tilde{\Psi} \right\rangle = \hat{O} \left| \Psi_0 \right\rangle \quad \text{Bound-state-like equation} \\ \left. \mathcal{L}(\sigma) \xrightarrow{\text{Inversion}} R(\omega) \text{ with full FSI} \qquad \text{Efros, Leidemann and Orlandini} \\ \text{PL B338 (1994) 130-133} \end{aligned}$$

Response Function
$$R(\omega) = \sum_{f} \left| \langle \Psi_{f} | \hat{O} | \Psi_{0} \rangle \right|^{2} \delta(E_{f} - E_{0} - \omega)$$

Challange: treat the continuum \rightarrow Lorentz Integral Transform Approach

$$\mathcal{L}(\sigma) = \int d\omega R(\omega) \frac{1}{(\omega - \sigma_R)^2 + \sigma_I^2} \mathcal{L}(\sigma) = \sum_{f} \left\langle \psi_0 \left| \hat{O}^{\dagger} \frac{1}{E_f - E_0 - \sigma^*} \right| \psi_f \right\rangle \left\langle \psi_f \left| \frac{1}{E_f - E_0 - \sigma} \hat{O} \right| \psi_0 \right\rangle \right. \\ = \left. \left\{ \sum_{f} \left\langle \psi_0 \left| \hat{O}^{\dagger} \frac{1}{\hat{H} - E_0 - \sigma^*} \right| \psi_f \right\rangle \left\langle \psi_f \right| \frac{1}{\hat{H} - E_0 - \sigma} \hat{O} \right| \psi_0 \right\rangle \\ = \left\langle \psi_0 \left| \hat{O}^{\dagger} \frac{1}{\hat{H} - E_0 - \sigma^*} \frac{1}{\hat{H} - E_0 - \sigma} \hat{O} \right| \psi_0 \right\rangle = \left\langle \tilde{\Psi} \right| \tilde{\Psi} \right\rangle \\ \left(\hat{H} - E_0 - \sigma_R + i\sigma_I \right) \left| \tilde{\Psi} \right\rangle = \hat{O} \left| \Psi_0 \right\rangle \quad \text{Bound-state-like equation} \\ \mathcal{L}(\sigma) \xrightarrow{\text{Inversion}} R(\omega) \text{ with full FSI} \qquad \text{Efros, Leidemann and Orlandini} \\ \text{PL B338 (1994) 130-133} \end{aligned}$$

Good bound state techniques are needed!

The Hyperspherical Harmonics Expansion

 $\Phi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) = \Phi(\vec{R}_{CM})\Psi(\vec{\eta}_1, \vec{\eta}_2, \vec{\eta}_3)$



Center of mass removed form the beginning

$$H = \frac{1}{2m} \frac{\hat{K}^2}{\rho^2} + \sum_{i < j} V_{i,j}$$
$$\hat{K}^2 \mathcal{Y}^{\mu}_{[\mathrm{K}]}(\Omega) = K(K+7) \mathcal{Y}^{\mu}_{[\mathrm{K}]}(\Omega)$$

The Hyperspherical Harmonics Expansion

 $\Phi(\vec{r_1}, \vec{r_2}, \vec{r_3}, \vec{r_4}) = \Phi(\vec{R}_{CM}) \Psi(\vec{\eta_1}, \vec{\eta_2}, \vec{\eta_3})$



Center of mass removed form the beginning

$$H = \frac{1}{2m} \frac{\hat{K}^2}{\rho^2} + \sum_{i < j} V_{i,j}$$
$$\hat{K}^2 \mathcal{Y}^{\mu}_{[\mathrm{K}]}(\Omega) = K(K+7) \mathcal{Y}^{\mu}_{[\mathrm{K}]}(\Omega)$$

$$\Psi = \sum_{n \ [K]}^{n_{\max} \ K_{\max}} c_n^{[K]} e^{-\frac{\rho}{2}} \rho^{\frac{\nu}{2}} \ L_n^{\nu}(\rho) \ [\mathcal{Y}_{[K]}^{\mu}(\Omega) \chi_{ST}^{\bar{\mu}}]_{JT}^a$$

The Hyperspherical Harmonics Expansion

 $\Phi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) = \Phi(\vec{R}_{CM})\Psi(\vec{\eta}_1, \vec{\eta}_2, \vec{\eta}_3)$



Center of mass removed form the beginning

$$H = \frac{1}{2m} \frac{\hat{K}^2}{\rho^2} + \sum_{i < j} V_{i,j}$$
$$\hat{K}^2 \mathcal{Y}^{\mu}_{[\mathrm{K}]}(\Omega) = K(K+7) \mathcal{Y}^{\mu}_{[\mathrm{K}]}(\Omega)$$

1 hyper–radius (3A–4) hyperangles

$$\Psi = \sum_{n \ [K]}^{n_{\max} \ K_{\max}} c_n^{[K]} e^{-\frac{\rho}{2}} \rho^{\frac{\nu}{2}} \ L_n^{\nu}(\rho) \ [\mathcal{Y}_{[K]}^{\mu}(\Omega) \chi_{ST}^{\bar{\mu}}]_{JT}^a$$

The Hyperspherical Harmonics Expansion

 $\Phi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) = \Phi(\vec{R}_{CM}) \Psi(\vec{\eta}_1, \vec{\eta}_2, \vec{\eta}_3)$



To accelerate the convergence of the expansion in an effective interaction is constructed via the Lee-Suzuki method, similarly as in the NCSM

→ Effective Interaction with the Hyperspherical Harmonics: EIHH Barnea, Leidemann, Orlandini, PRC 61, 054001 (2000)

two-body forces







What is the effect of three-body forces in a scattering observable?



What is the effect of three-body forces in a scattering observable? Total photodisintegration cross section as interesting observable $\sigma(\omega) = 4\pi^2 \alpha \omega R(\omega)$ $R(\omega) = \sum_f |\langle \Psi_f | O | \Psi_0 \rangle|^2 \delta(E_f - E_0 - \omega)$ O = E1 via Siegert Theorem Dominant part of Meson Exchange Currents is included

Semirealistic vs Realistic Potential



Efros, Leidemann, Orlandini, Tomusiak PLB 484 (2000) 223

Semirealistic vs Realistic Potential



- Semirelistic \rightarrow peak overestimation < 10% + strong tail underestimation
- Three-body forces \rightarrow peak damping 10% + tail enhancing 15%

Convergence of the Lorentz Integral Transform



Convergence of the Lorentz Integral Transform



Semirealistic vs Realistic Potential



 ω [MeV]

Semirealistic vs Realistic Potential



- Semirelistic \rightarrow peak overestimation 10-15% + strong tail underestimation
- Three-body forces \rightarrow peak damping 6% + tail enhancing 35%

Comparison with experiments



Comparison with experiments



Comparison with experiments



Conclusions

 Big step forward in the theory: we can treat simultaneously the different four-body disintegration channels with exact treatment of initial and final state interaction using modern potentials

Conclusions

- Big step forward in the theory: we can treat simultaneously the different four-body disintegration channels with exact treatment of initial and final state interaction using modern potentials
- The experimental situation is not settled and does not allow for a conclusive judgment on the reliability of modern two and three-body forces in the description of photodisintegration of ⁴He

Inversion of the LIT $\mathcal{L}(\mathbf{q}, \sigma_R, \sigma_I) \Rightarrow \text{Inversion} \Rightarrow R(\mathbf{q}, \omega)$



Inversion of the LIT $\mathcal{L}(\mathbf{q}, \sigma_R, \sigma_I) \Rightarrow \text{Inversion} \Rightarrow R(\mathbf{q}, \omega)$



- Fix σ_I to a reasonable value: $\sigma_I \sim$ of the order of width of Response
- Calculate $\mathcal{L}(\mathbf{q}, \sigma_R)$ for a grid of M values of σ_R

Inversion of the LIT

• Expand the response over a set of functions whose LIT is well known

$$R(\mathbf{q},\omega) = \sum_{n=1}^{N} c_n \chi_n(\mathbf{q},\omega) \text{ with } \chi_n(\mathbf{q},\omega) \xrightarrow{\text{LIT}} \tilde{\chi}_n(\mathbf{q},\sigma)$$

The LIT is then $\mathcal{L}(\mathbf{q},\sigma) = \sum_{n=1}^{N} c_n \tilde{\chi}_n(\mathbf{q},\sigma)$

Inversion of the LIT

• Expand the response over a set of functions whose LIT is well known

$$R(\mathbf{q},\omega) = \sum_{n=1}^{N} c_n \chi_n(\mathbf{q},\omega) \text{ with } \chi_n(\mathbf{q},\omega) \xrightarrow{\text{LIT}} \tilde{\chi}_n(\mathbf{q},\sigma)$$

The LIT is then $\mathcal{L}(\mathbf{q},\sigma) = \sum_{n=1}^{N} c_n \tilde{\chi}_n(\mathbf{q},\sigma)$

• Make best fit to calculate c_n with M > N

$$\sum_{k=1}^{M} \left| \mathcal{L}\left(\sigma_{R}^{k}, \sigma_{I}, \mathbf{q}\right) - \sum_{n=1}^{N} c_{n} \mathcal{L}_{n}\left(\sigma_{R}^{k}, \sigma_{I}, \mathbf{q}, \alpha\right) \right|^{2} = \min,$$

• Check stability by increasing N to N + i with (N + i < M)