Testing superscaling on nuclei at low q with different nuclear models and predictions on electroweak cross section

Marco Martini - Università di Lecce and INFN Sez. di Lecce

In collaboration with: M.Anguiano, G.Co' and A.M. Lallena

Topics

* Scaling Procedure

* Nuclear Models
- RFG
- SM
- RPA
- MEC
- FSI

* Electron and Neutrino Cross Sections
- Scaling approach $\Longleftrightarrow$ full calculations
- Comparison with experiments (electron)
- Prediction on cross sections (neutrino)

Scaling

General procedure

*Divide the experimental cross section (response) by an appropriate single-nucleon cross section (response), having contributions from Z protons and N neutrons with their corresponding form factors, to obtain a reduced cross section (response).

*Plot the reduced cross section (response) versus one or more appropriately chosen variables: if the results do not depend on some of these variables and a universal behavior is found one says that the results scale.

Scaling of the first kind $\Longleftrightarrow$ Independence of the momentum transfer q
Scaling of the second kind $\Longleftrightarrow$ Independence of k_F or A
Superscaling $\Longleftrightarrow$ Both 1st- and 2nd- kind scaling
Scaling of zeroth- kind $\Longleftrightarrow$ Independence of type of external operator

Relativistic Fermi Gas Model

Universal Scaling Function $f^{RFG}(\Psi) = \frac{3}{4}\left(1 - \Psi^2\right)\theta\left(1 - \Psi^2\right)$

Scaling variable $\Psi = \Psi_0 \left(1 + \Psi_0 \frac{k_F}{2q}\sqrt{\frac{q^2}{m^2} + 1}\right)$

$$\Psi_0 = \frac{2m}{k_F}\left[\sqrt{\left(\frac{\omega - E_{shift}}{2m}\right)\left(1 + \frac{\omega - E_{shift}}{2m}\right)} - \frac{q}{2m}\right]$$

Some effects not considered in the RFG model:

*Finite size of the system
*Collective excitations
*Meson Exchange Currents (MEC)
*Final State Interactions (FSI)

Scaling ?

Electron scattering

$$\frac{d^2\sigma}{d\theta\, d\omega} = \sigma_M \left\{ \frac{(\omega^2 - q^2)^2}{q^4} R_L(\omega, q) + \left[\tan^2\left(\frac{\theta}{2}\right) - \frac{\omega^2 - q^2}{2q^2} \right] R_T(\omega, q) \right\}$$

Longitudinal Response $R_L(\omega, q) = 4\pi \sum_{j=0} |\langle J_f || \mathcal{C}_J || J_i \rangle|^2$

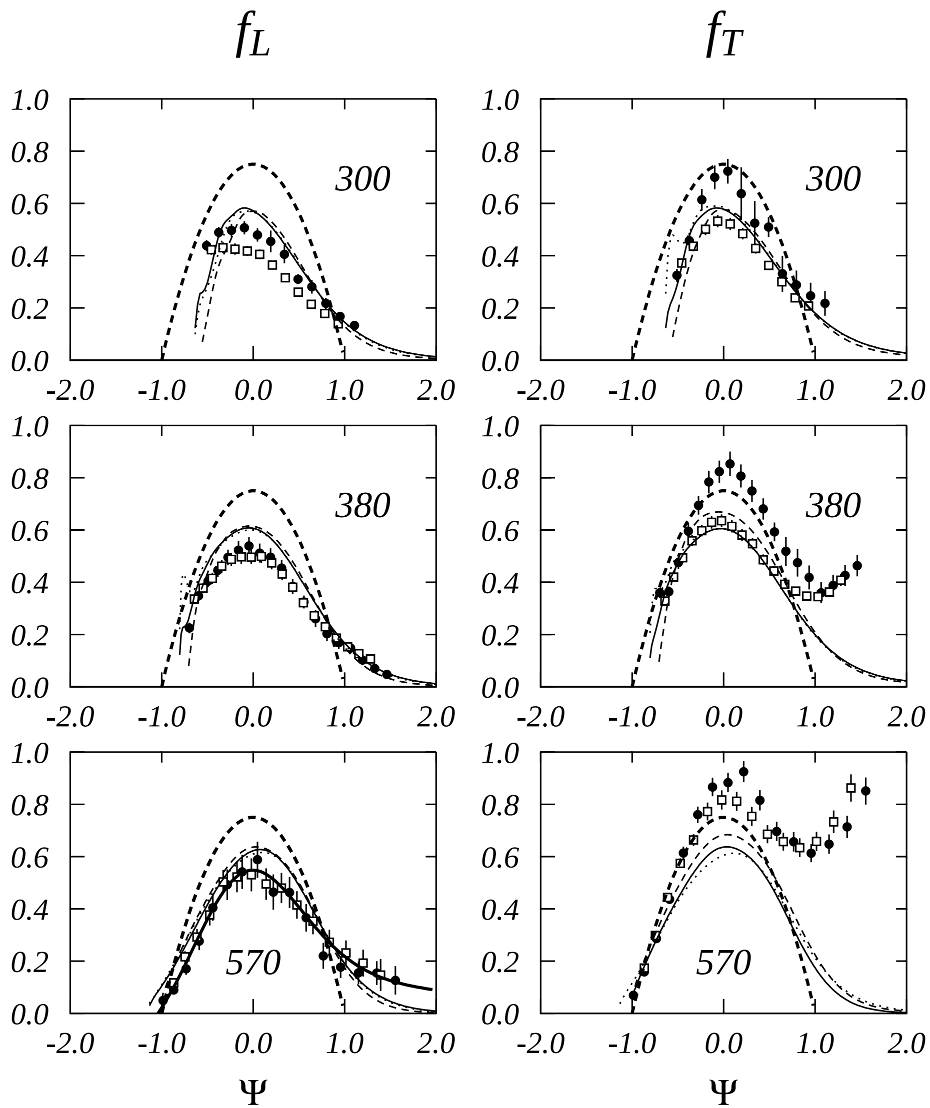
Transverse Response $R_T(\omega, q) = 4\pi \sum_{j=1} (|\langle J_f || \mathcal{E}_J || J_i \rangle|^2 + |\langle J_f || \mathcal{M}_J || J_i \rangle|^2)$

Scaling functions

$$\begin{aligned} f_L(\Psi) &= k_F \frac{q^2 - \omega^2}{q\, m} \frac{R_L(\omega, q)}{Z(G_E^p)^2 + N(G_E^n)^2} \\ f_T(\Psi) &= 2\, k_F \frac{q\, m}{q^2 - \omega^2} \frac{R_T(\omega, q)}{Z(G_M^p)^2 + N(G_M^n)^2} \end{aligned}$$

Semi-relativistic corrections

$$\epsilon_p \to \epsilon_p \left(1 + \frac{\epsilon_p}{2m}\right); R_L(q, \omega) \to \frac{q^2}{q^2 - \omega^2} R_L(q, \omega); R_T(q, \omega) \to \frac{q^2 - \omega^2}{q^2} R_T(q, \omega)$$



Full circles: ^{12}C; Empty squares: ^{40}Ca; - -RFG; —— Empirical Scaling Function;
— ^{12}C, · · · ^{16}O and - - ^{40}Ca with all the effects beyond RFG

Superscaling beyond RFG model

Numerical comparison of the various scaling functions

M scaling functions f_α; $\alpha = 1, M$

each of them known for

N values of the scaling variable Ψ_i; $i = 1, N$

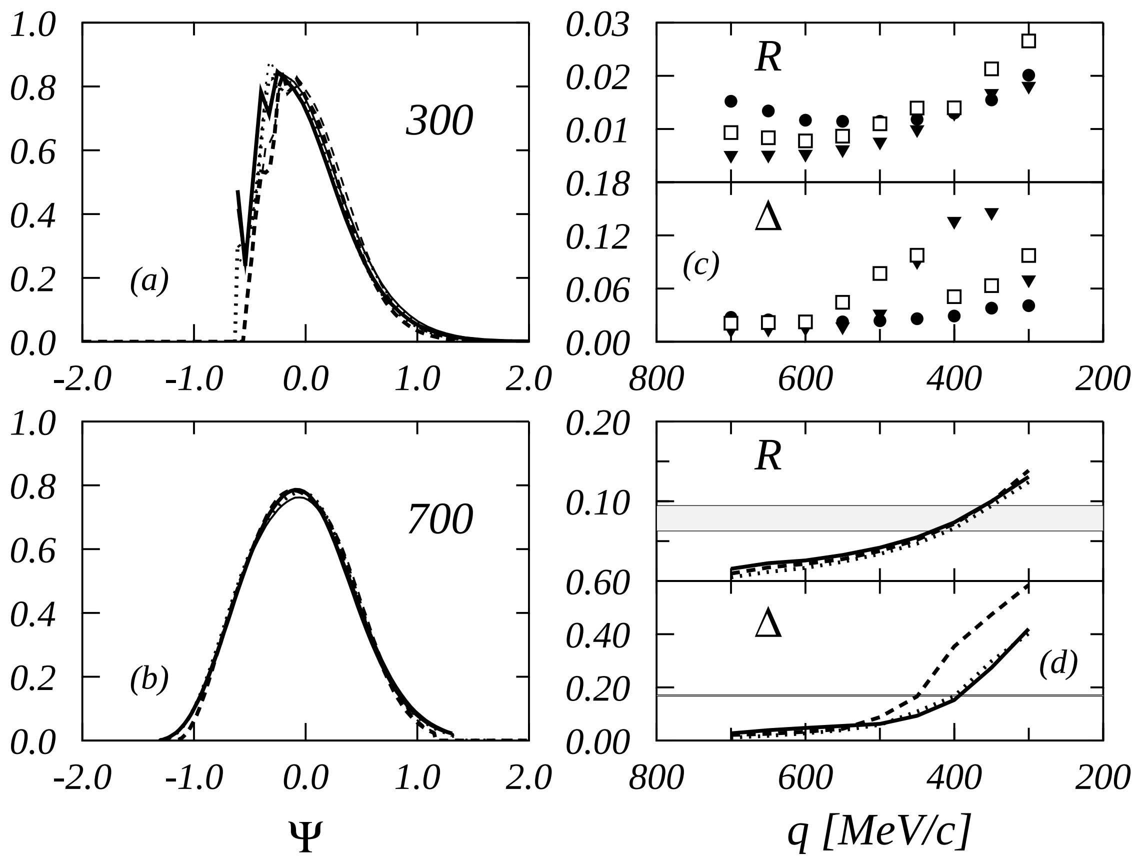
$$\Delta = \max_{i=1,N} \left[f^{max}(\Psi_i) - f^{min}(\Psi_i) \right]$$ **maximum distance**

$$\mathcal{R} = \frac{1}{N f^M} \sum_{i=1,N} \left[f^{max}(\Psi_i) - f^{min}(\Psi_i) \right]$$ **global properties**

*$\mathcal{R}$=0, Δ=0 $\Longleftrightarrow$ Perfect Scaling - Only in the RFG

*$\mathcal{R}$= 0.078 $\pm$ 0.016, Δ = 0.169 $\pm$ 0.002 $\Longleftrightarrow$ 2nd- kind scaling for f_L data at 570 MeV/c

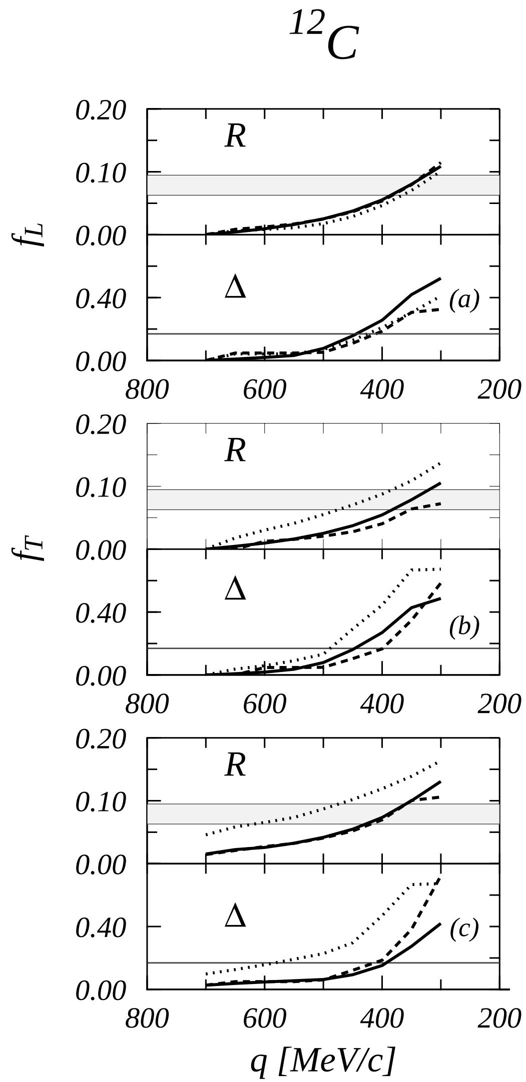
Continuum shell model result



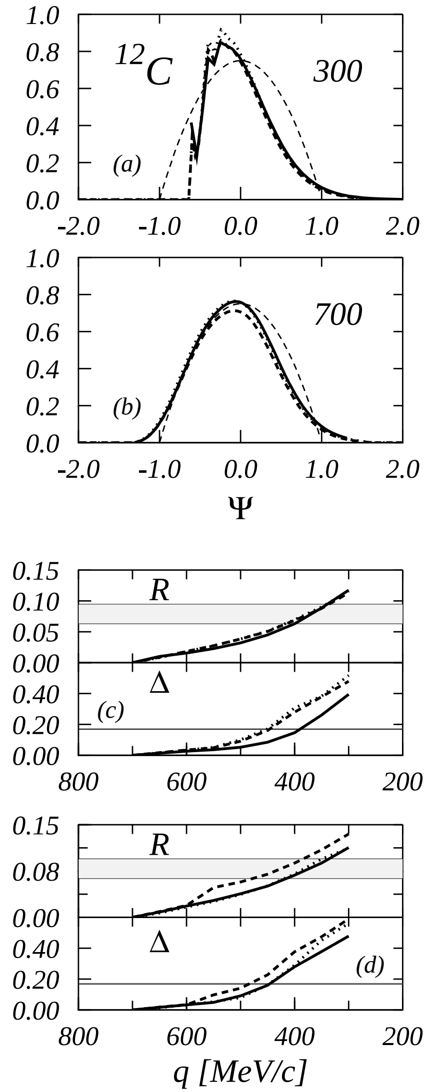
— ^{12}C, · · · ^{16}O and - - ^{40}Ca;   thicker lines: f_L, thinner lines: f_T
circles ^{12}C, squares ^{16}O and triangles ^{40}Ca;   horizontal bands: empirical $\mathcal{R}$, Δ

RPA

MEC



^{12}C
R
Δ
(a)
(b)
(c)
f_L
f_T
0.20
0.10
0.00
0.40
800
600
400
200
q [MeV/c]



^{12}C
300
700
(a)
(b)
(c)
(d)
R
Δ
Ψ
1.0
0.8
0.6
0.4
0.2
0.0
-2.0
-1.0
0.0
1.0
2.0
0.15
0.10
0.05
0.08
0.00
0.40
0.20
800
600
400
200
q [MeV/c]

Folding

Phenomenological model used to describe hadronic processes and electron scattering cross section in order to take into account effects beyond the RPA

G. Co', K.Q. Quader, R. Smith, J. Wambach, NPA 485 (1988) 61

J.E. Amaro, G. Co', A.M. Lallena, NPA 578 (1994) 365

known nuclear response $S^0(q,E)$

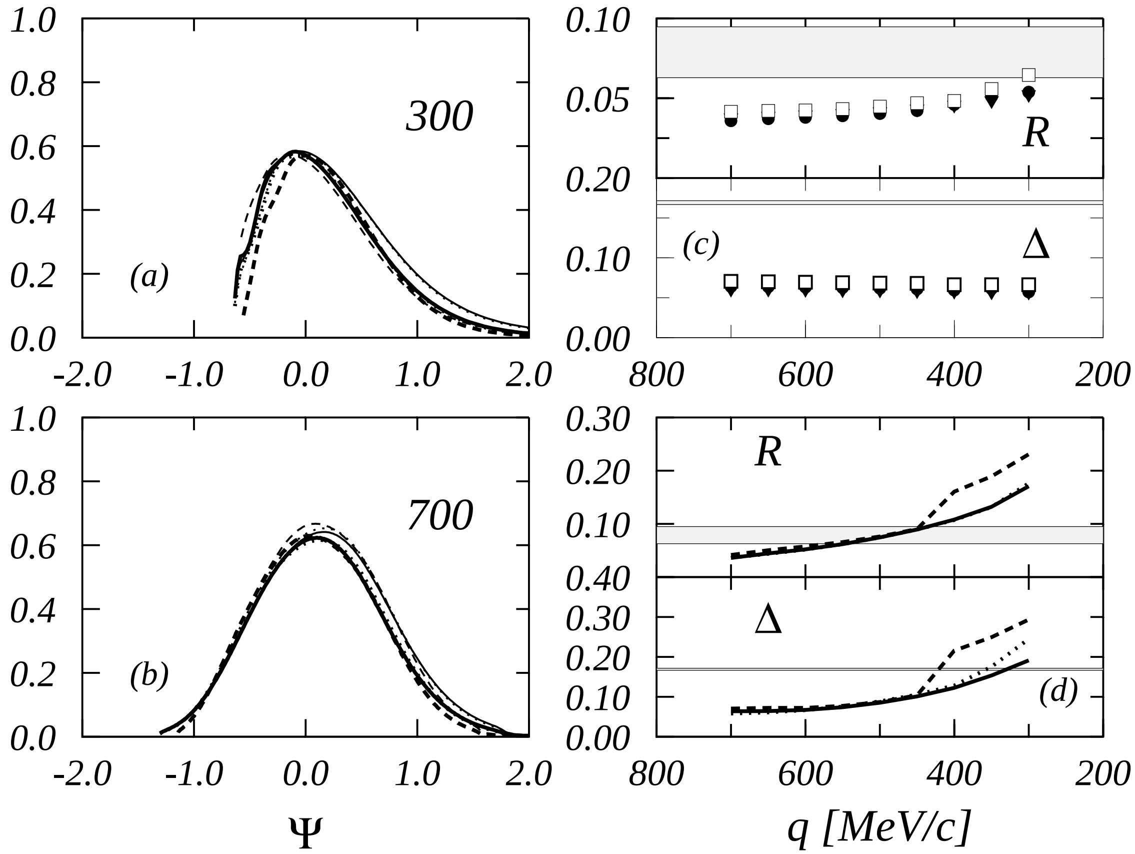
folding integral $S^{\rm FSI}(q,\omega) = \int_0^\infty dE S^0(q,E)[h(E,\omega)+h(E,-\omega)]$

folding function $h(E,\omega) = \frac{1}{2\pi}\frac{\Gamma(\omega)}{[E-\omega-\Delta(\omega)]^2+[\Gamma(\omega)/2]^2}$

$$\Gamma(\omega) = \frac{1}{\omega}\int_0^\omega d\epsilon[\gamma(\epsilon+\omega)+\gamma(\epsilon-\omega)]$$

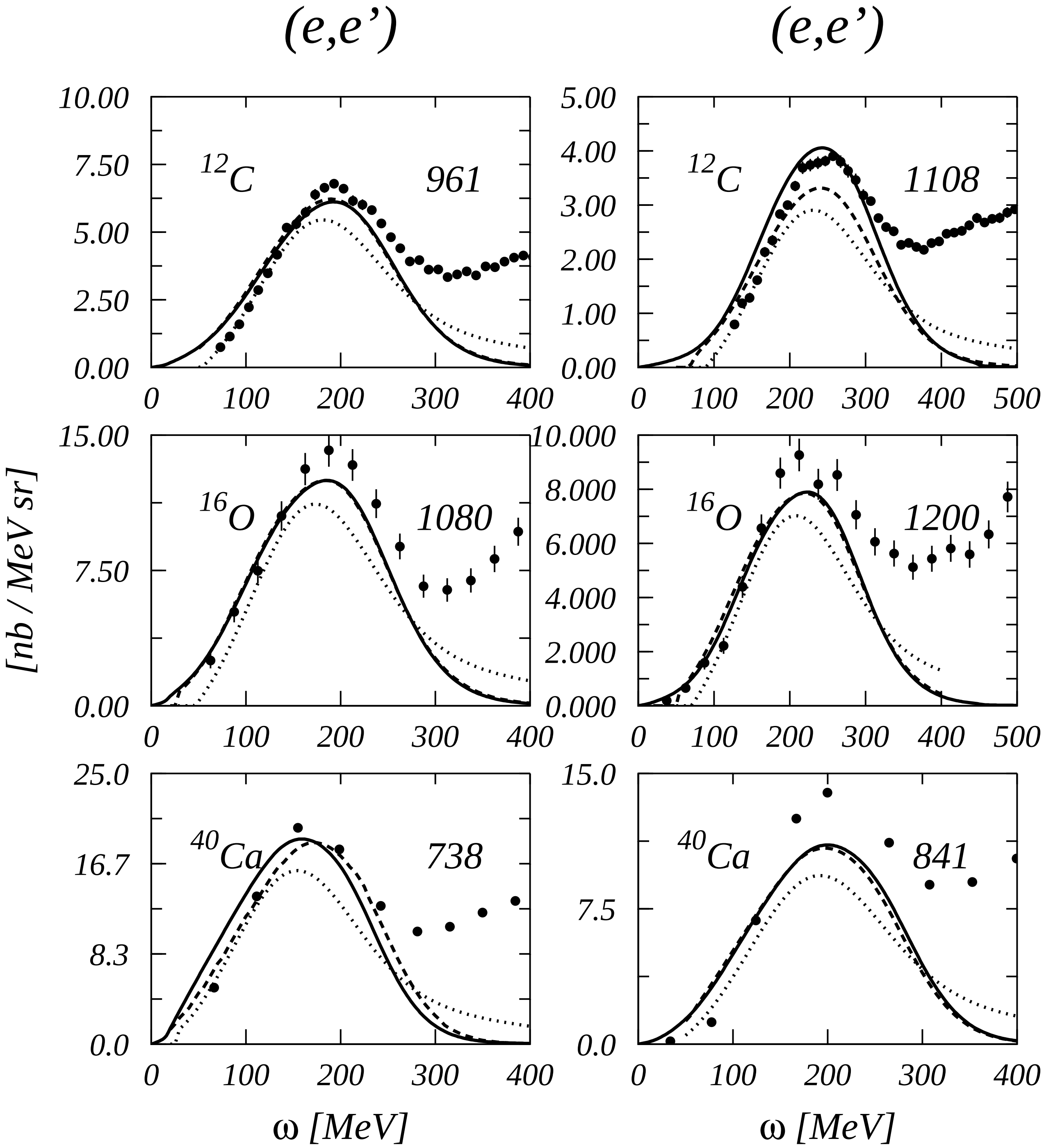
single particle width $\gamma(\epsilon) = A\left(\frac{\epsilon^2}{\epsilon^2+B^2}\right)\left(\frac{C^2}{\epsilon^2+C^2}\right)$

Scaling functions with the FSI



— ^{12}C, . . . ^{16}O and - - ^{40}Ca; thicker lines: f_L, thinner lines: f_T

circles ^{12}C, squares ^{16}O and triangles ^{40}Ca; horizontal bands: empirical $\mathcal{R}$, Δ


(e,e')
(e,e')
^{12}C
961
^{12}C
1108
^{16}O
1080
^{16}O
1200
^{40}Ca
738
^{40}Ca
841
[nb / MeV sr]
ω [MeV]
ω [MeV]

Inclusive electron scattering cross section - Figure Caption

The ^{12}C data, measured at 37.5^o, are from R.M. Sealock et al. Phys. Rev. Lett. 62, 1350 (1989), those of ^{16}O, measured at 32.0^o, are from M. Anguiano, A.M. Lallena and G. Co', Phys. Rev. C 53, 3155 (1996) and those of ^{40}Ca, measured at 45.5^o, from C. Williamson et al., Phys. Rev. C 56, 3152 (1997). The full lines show the results of the complete calculations. The cross sections obtained with our f_L are shown by the dashed lines, and those obtained with the empirical scaling function are given by the dotted lines.

(ν_e, e^-) Reaction

Differential Cross Section

$$\frac{d^2\sigma}{d\Omega d\omega} = \frac{G^2\cos^2\theta_C}{(2\pi)^2}|\mathbf{k}'|\epsilon' F(Z',\epsilon')\left\{\left(l_0 l_0^\star + \frac{\omega^2}{q^2} l_3 l_3^\star - \frac{\omega}{q} l_3 l_0^\star\right) R^V_{CC}(\omega,q)\right.$$

$$+ \; l_0 l_0^\star R^A_{CC}(\omega,q) + l_3 l_3^\star R^A_{LL}(\omega,q) + 2 l_3 l_0^\star R^A_{CL}(\omega,q)$$

$$\left. + \; \left[\frac{1}{2}(\vec{l}\cdot\vec{l}^\star - l_3 l_3^\star)\right]\left[R^V_T(\omega,q) + R^A_T(\omega,q)\right] + 2\left[-\frac{i}{2}(\vec{l}\times\vec{l}^\star)_3\right] R_{T'}(\omega,q)\right\}$$

Vector (V) and Axial-vector (A) contributions

in the Nuclear Response Functions

$$R^V_{CC}(\omega,q) = 4\pi \sum_{j=0} |\langle J_f||\mathcal{C}^V_J||J_i\rangle|^2 \quad ; \quad R^A_{CC}(\omega,q) = 4\pi \sum_{j=0} |\langle J_f||\mathcal{C}^A_J||J_i\rangle|^2$$

$$R^A_{CL}(\omega,q) = 2\pi \sum_{j=0} (\langle J_f||\mathcal{C}^A_J||J_i\rangle^\star \langle J_f||\mathcal{L}^A_J||J_i\rangle + \langle J_f||\mathcal{C}^A_J||J_i\rangle \langle J_f||\mathcal{L}^A_J||J_i\rangle^\star)$$

$$R^A_{LL}(\omega,q) = 4\pi \sum_{j=0} |\langle J_f||\mathcal{L}^A_J||J_i\rangle|^2$$

$$R^V_T(\omega,q) = 4\pi \sum_{j=1} (|\langle J_f||\mathcal{E}^V_J||J_i\rangle|^2 + |\langle J_f||\mathcal{M}^V_J||J_i\rangle|^2)$$

$$R^A_T(\omega,q) = 4\pi \sum_{j=1} (|\langle J_f||\mathcal{E}^A_J||J_i\rangle|^2 + |\langle J_f||\mathcal{M}^A_J||J_i\rangle|^2)$$

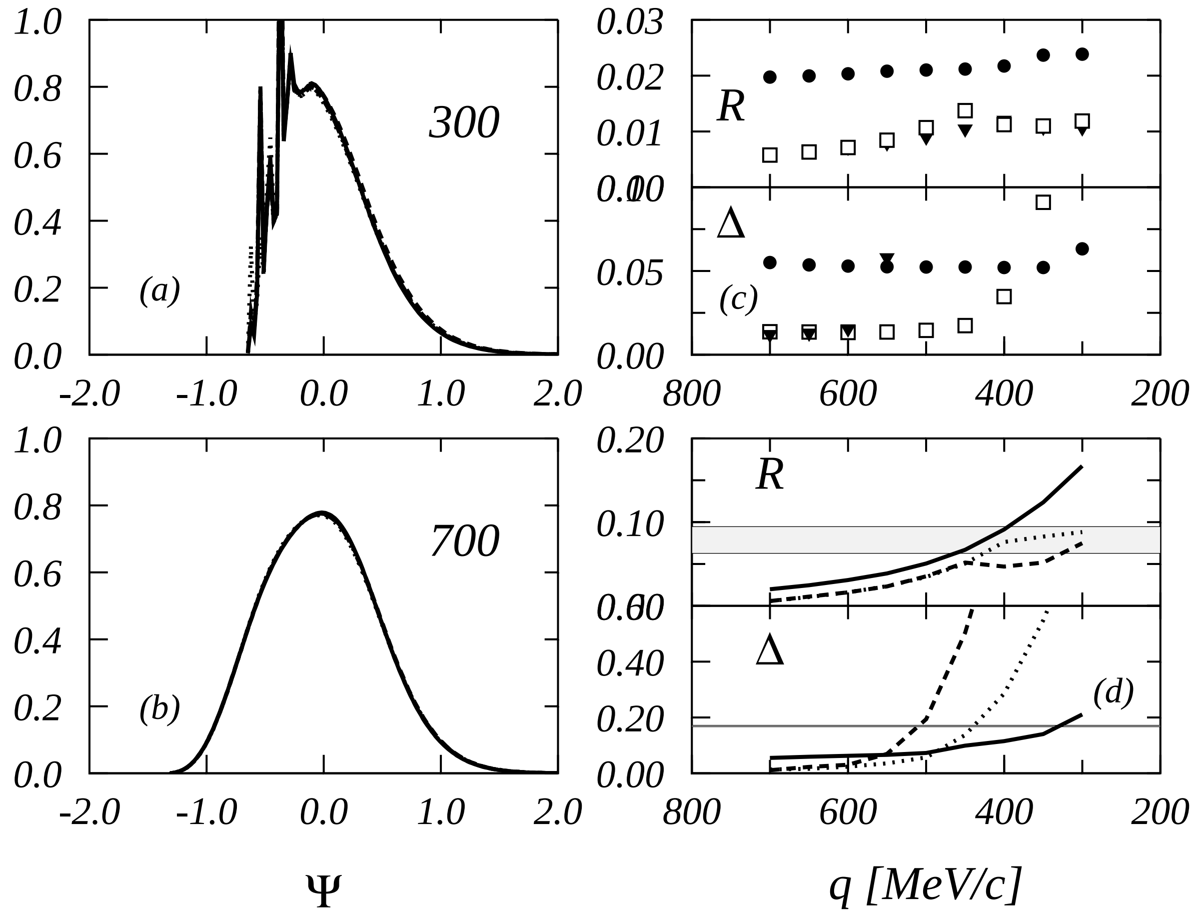
$$R^{VA}_{T'}(\omega,q) = 2\pi \sum_{j=1} (\langle J_f||\mathcal{E}^V_J||J_i\rangle^\star \langle J_f||\mathcal{M}^A_J||J_i\rangle + \langle J_f||\mathcal{E}^V_J||J_i\rangle \langle J_f||\mathcal{M}^A_J||J_i\rangle^\star +$$

$$\langle J_f||\mathcal{E}^A_J||J_i\rangle^\star \langle J_f||\mathcal{M}^V_J||J_i\rangle + \langle J_f||\mathcal{E}^A_J||J_i\rangle \langle J_f||\mathcal{M}^V_J||J_i\rangle^\star)$$

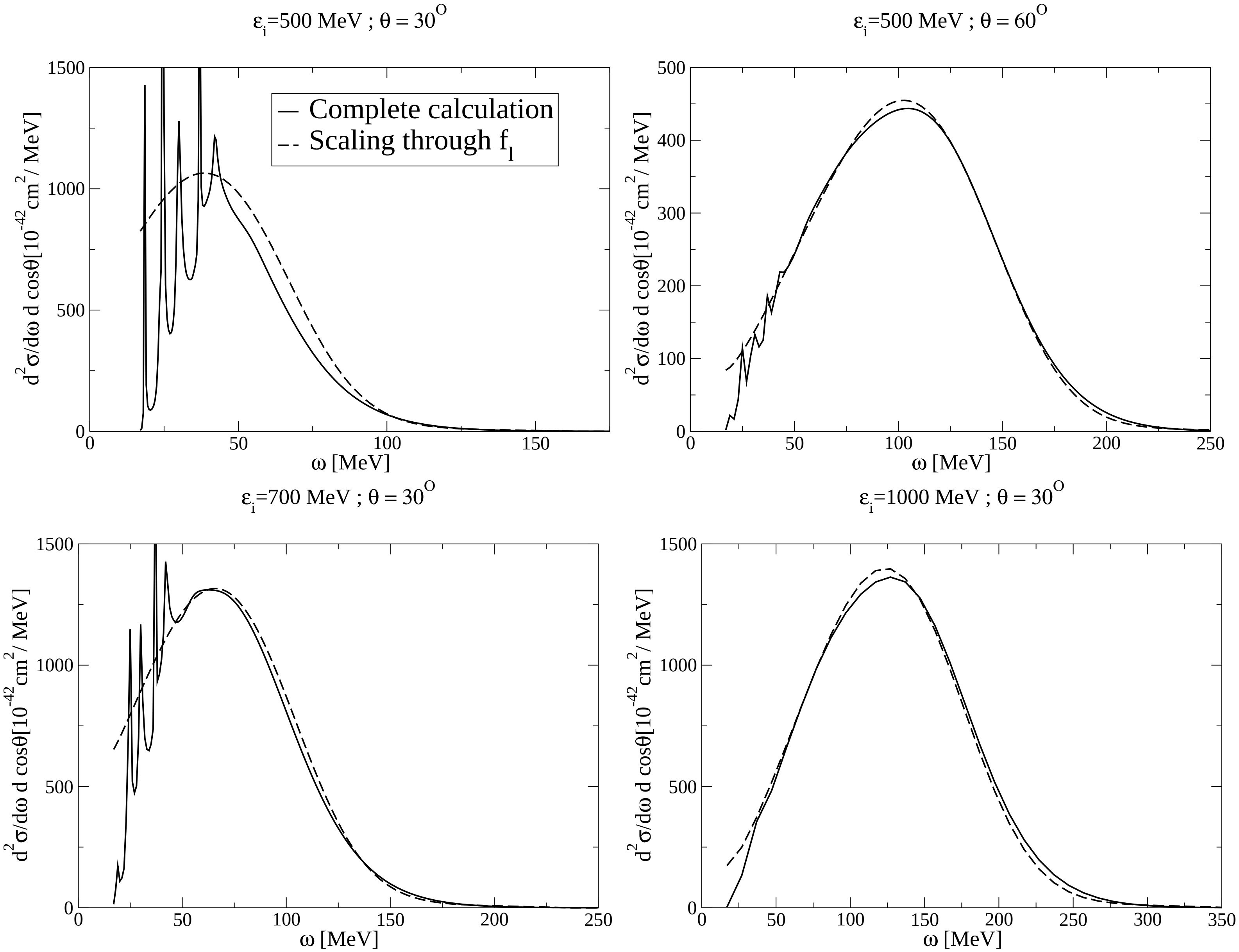
Scaling Functions

$$
\begin{aligned}
f_{CC}^{V}(\Psi) &= k_F \frac{q^2-\omega^2}{q\,m_N} \frac{R_{CC}^{V}(\omega,q)}{N(G_E^{(1)})^2} \\
f_{LL}^{A}(\Psi) &= 4\,k_F \frac{q\,m_N}{4m_N^2+q^2-\omega^2} \frac{R_{LL}^{A}(\omega,q)}{N(G_A)^2} \\
f_{T}^{V}(\Psi) &= 2\,k_F \frac{q\,m_N}{q^2-\omega^2} \frac{R_{T}^{V}(\omega,q)}{N(G_M^{(1)})^2} \\
f_{T}^{A}(\Psi) &= 2\,k_F \frac{q\,m_N}{4m_N^2+q^2-\omega^2} \frac{R_{T}^{A}(\omega,q)}{N(G_A)^2} \\
f_{T'}^{VA}(\Psi) &= 2\,k_F \frac{q\,m_N}{\sqrt{q^2-\omega^2}\sqrt{4m_N^2+q^2-\omega^2}} \frac{R_{T'}^{VA}(\omega,q)}{NG_M^{(1)}G_A}
\end{aligned}
$$

Scaling analysis

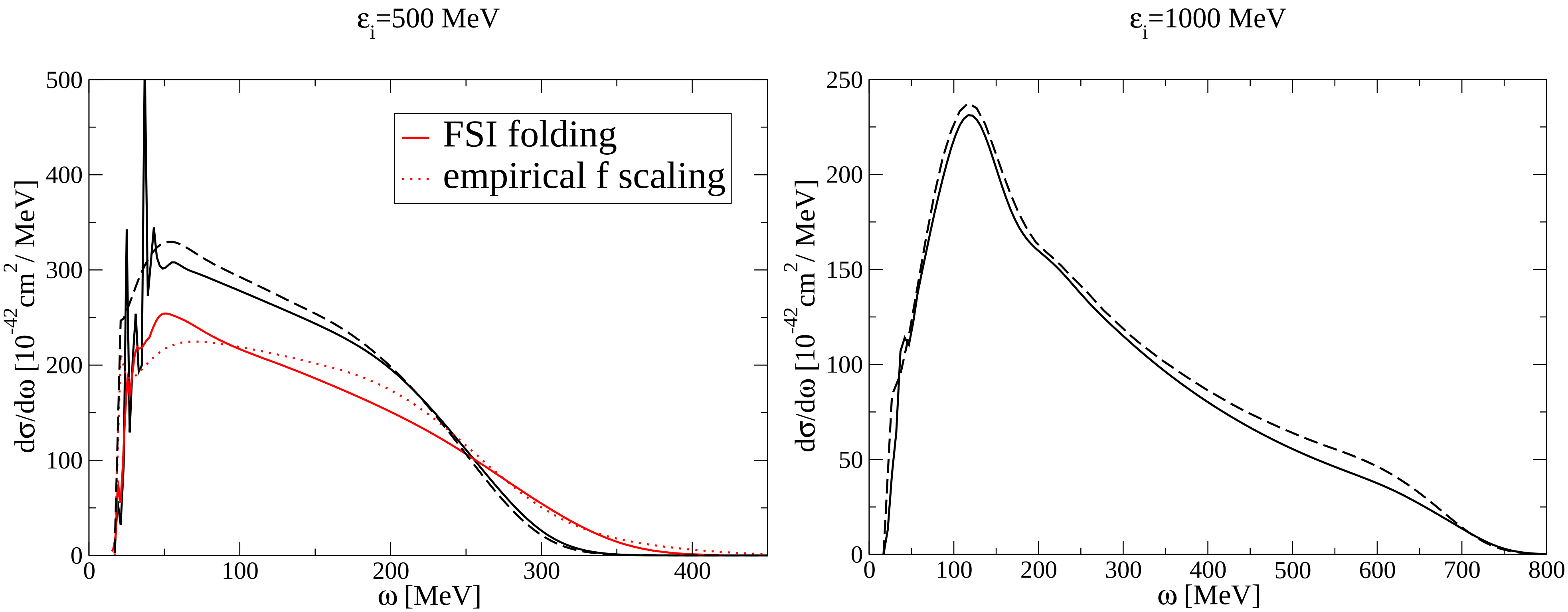


(a), (b): f_{CC}^{V} f_{LL}^{A} f_{T}^{V} f_{T}^{A} $f_{T'}^{VA}$ for ^{16}O; (c): circles ^{12}C, squares ^{16}O, triangles ^{40}Ca
(d): — ^{12}C, · · · ^{16}O and - - ^{40}Ca; horizontal bands: empirical $\mathcal{R}$, Δ

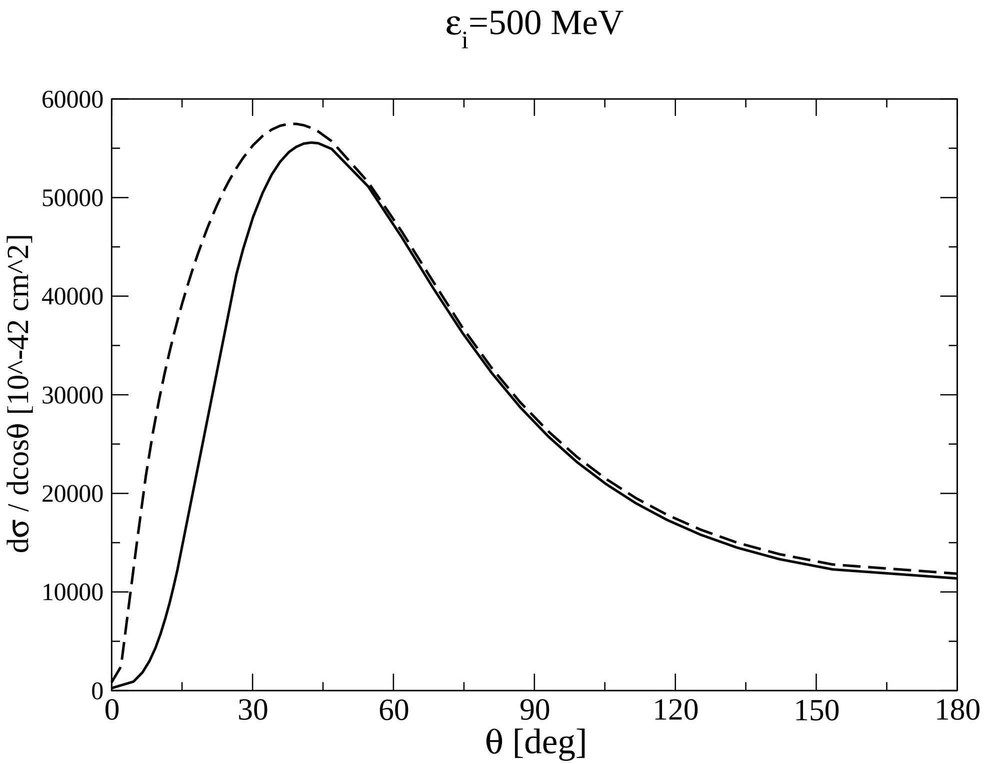


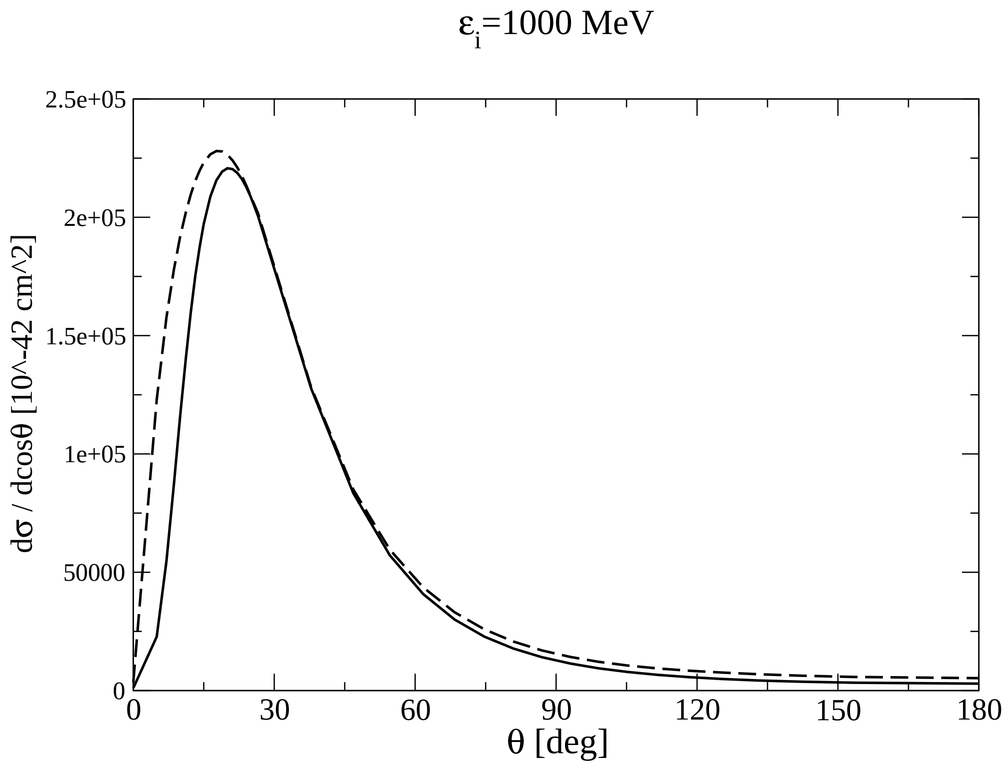
ε_i=500 MeV ; θ = 30^O
ε_i=500 MeV ; θ = 60^O
ε_i=700 MeV ; θ = 30^O
ε_i=1000 MeV ; θ = 30^O
Complete calculation
Scaling through f_l
$d^2\sigma/d\omega\, d\cos\theta[10^{-42}cm^2/$ MeV]
ω [MeV]

$\frac{d\sigma}{d\omega}$ for ^{16}O

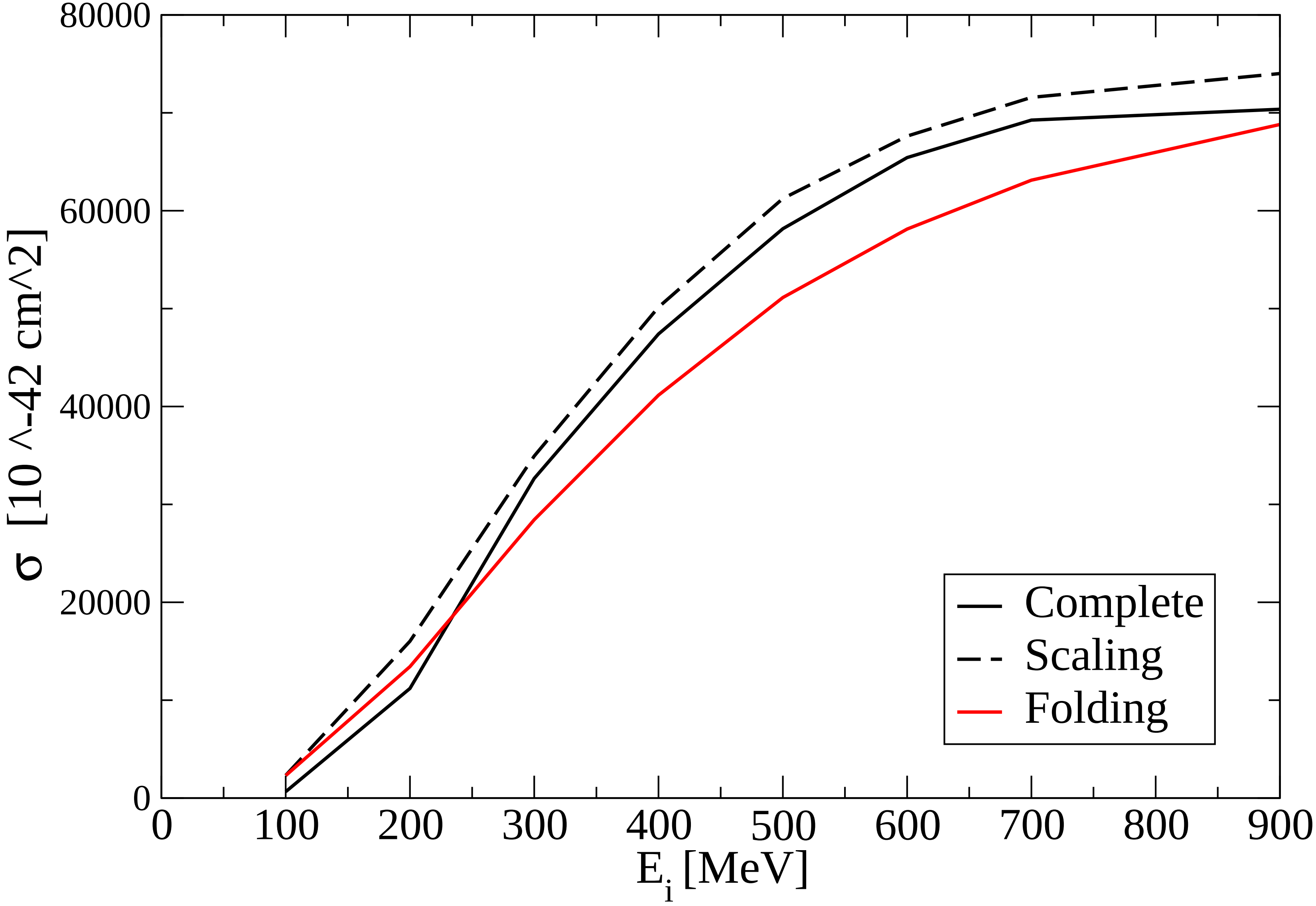


$$\frac{d\sigma}{d\cos\theta} \text{ for } {}^{16}O$$

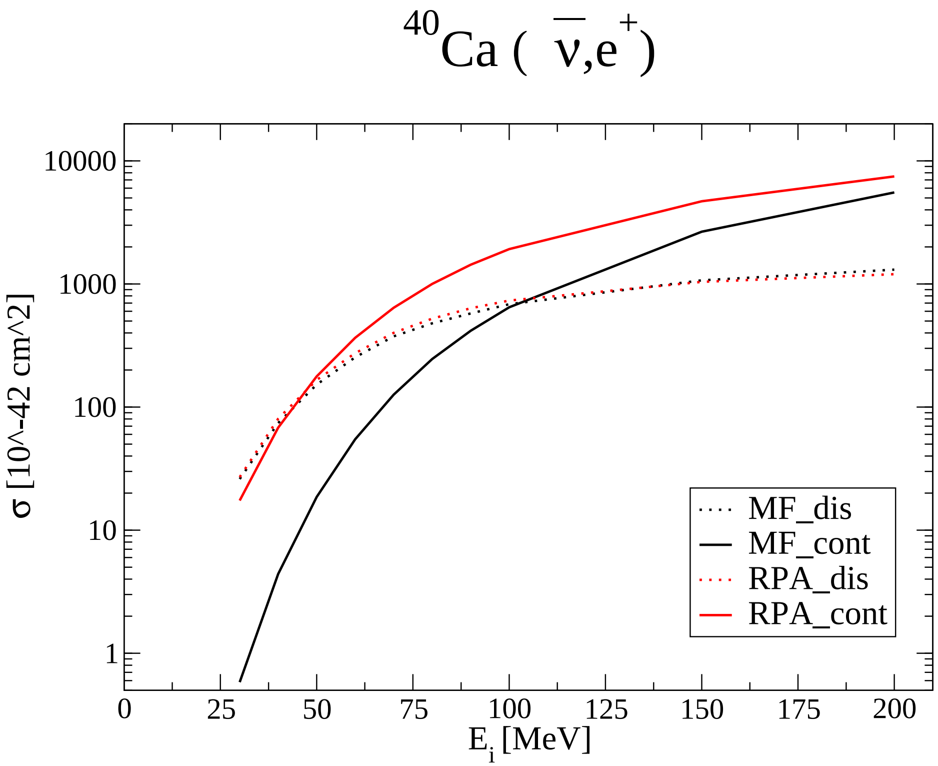


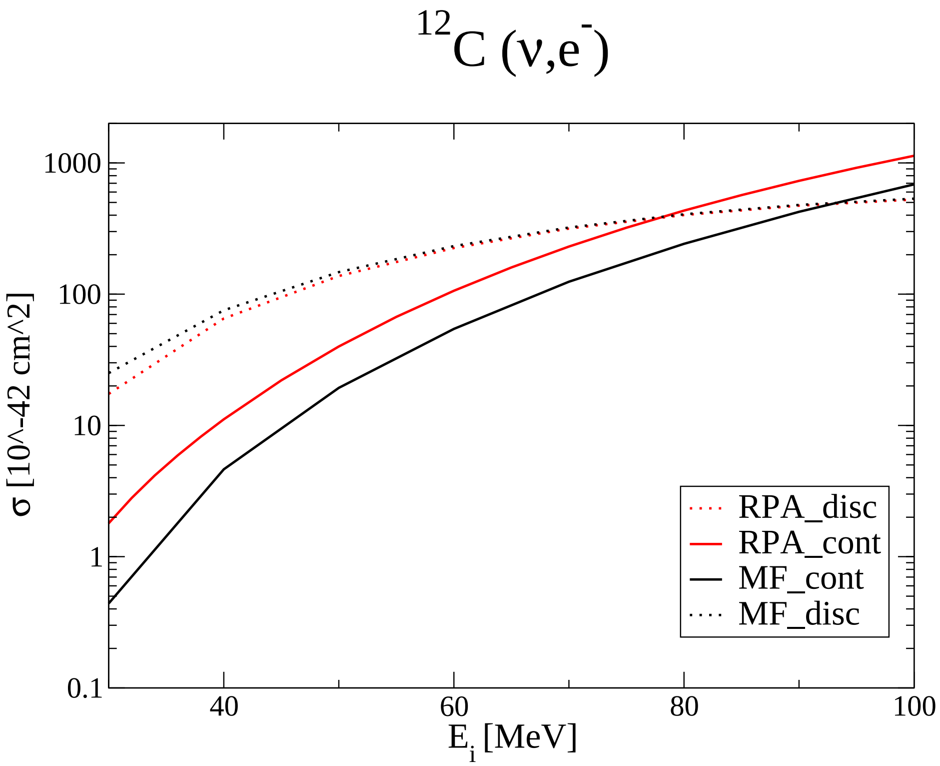


Total Cross Section



Warning: low energy





Discrete vs Continuum

MF vs RPA

Summary

*Scaling

SM: 1st- kind: $q \gtrsim$ 400 MeV/c;

2nd-kind: q=700 MeV/c yes, q=300 MeV/c no; 0-kind: good

RPA: for $q \gtrsim$ 500 MeV/c effects negligible; 1st- kind: $q \gtrsim$ 400 MeV/c;

worsening with zero range interaction; 0-kind: slightly ruined

MEC:1st- kind: $q \gtrsim$ 400 MeV/c;

inclusion of the Δ currents slightly decreases f_T

FSI: responsible for the largest modifications of the SM results;

do not strongly change scaling properties

*Electron and Neutrino Cross Sections

theoretical scaling functions

empirical scaling function

full calculations

Appendix

Semi-relativistic corrections in neutrino case

$$
\begin{aligned}
R^V_{CC}(q,\omega) &\rightarrow \frac{q^2}{q^2-\omega^2} R^V_{CC}(q,\omega) \\
R^A_{LL}(q,\omega) &\rightarrow \left(1+\frac{q^2-\omega^2}{4m_N^2}\right) R^A_{LL}(q,\omega) \\
R^V_T(q,\omega) &\rightarrow \frac{q^2-\omega^2}{q^2} R^V_T(q,\omega) \\
R^A_T(q,\omega) &\rightarrow \left(1+\frac{q^2-\omega^2}{4m_N^2}\right) R^A_T(q,\omega) \\
R^{VA}_{T'}(q,\omega) &\rightarrow \sqrt{\frac{q^2-\omega^2}{q^2}}\sqrt{1+\frac{q^2-\omega^2}{4m_N^2}} R^{VA}_{T'}(q,\omega)
\end{aligned}
$$

Equivalent definition of scaling functions

$$
\begin{aligned}
f_L(\Psi) &= k_F \frac{q}{m} \frac{R_L(\omega,q)}{Z(G_E^p)^2 + N(G_E^n)^2} \\
f_T(\Psi) &= 2\, k_F \frac{m}{q} \frac{R_T(\omega,q)}{Z(G_M^p)^2 + N(G_M^n)^2}
\end{aligned}
$$

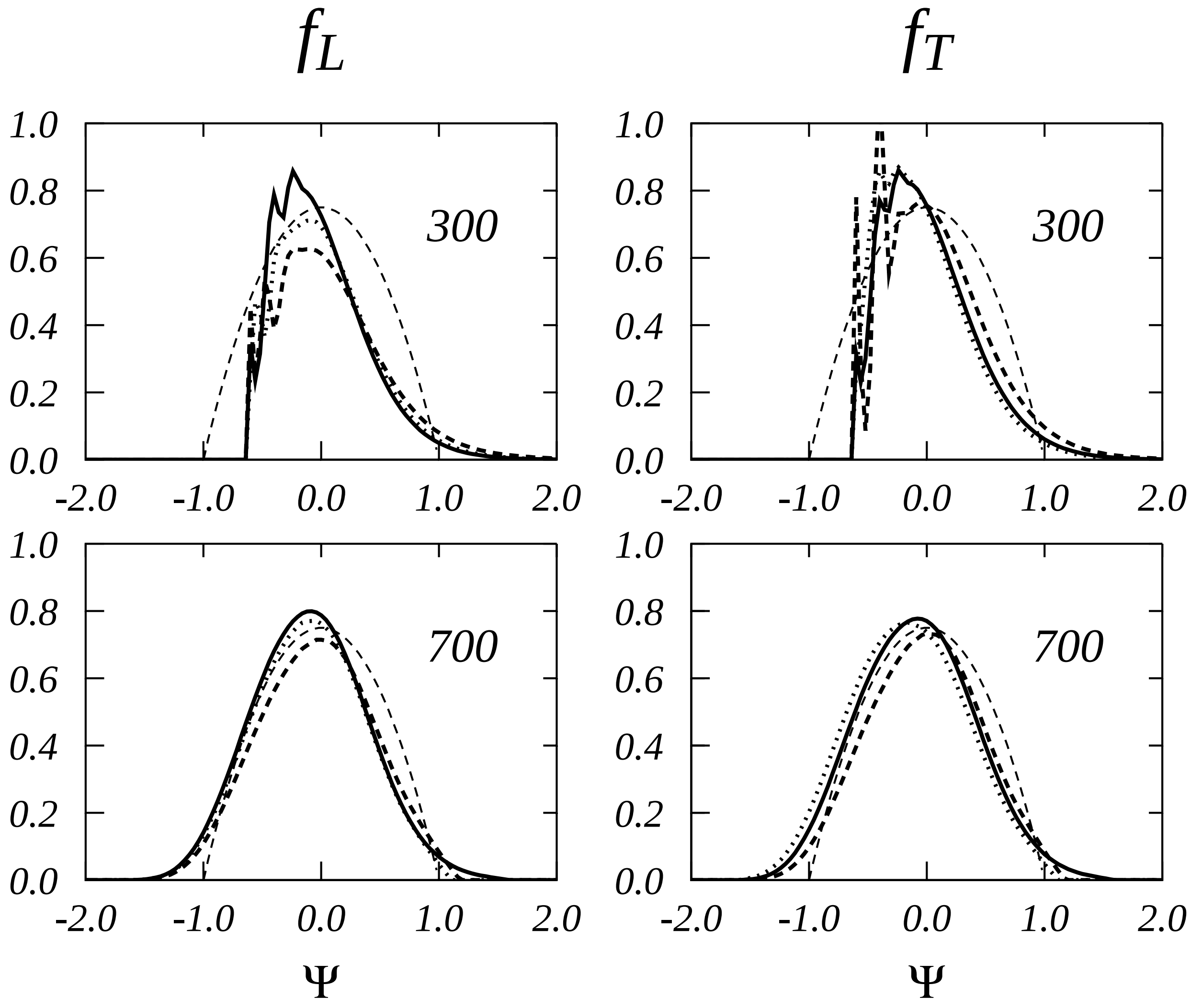
$$
\begin{aligned}
f_{CC}^V(\Psi) &= k_F \frac{q}{m} \frac{R_{CC}^V(\omega,q)}{N(G_E^{(1)})^2} \\
f_{LL}^A(\Psi) &= k_F \frac{q}{m} \frac{R_{LL}^A(\omega,q)}{N(G_A)^2} \\
f_T^V(\Psi) &= 2\, k_F \frac{m}{q} \frac{R_T^V(\omega,q)}{N(G_M^{(1)})^2} \\
f_T^A(\Psi) &= 1/2\, k_F \frac{q}{m} \frac{R_T^A(\omega,q)}{N(G_A)^2} \\
f_{T'}^{VA}(\Psi) &= k_F \frac{R_{T'}^{VA}(\omega,q)}{N G_M^{(1)} G_A}
\end{aligned}
$$

Empirical scaling function

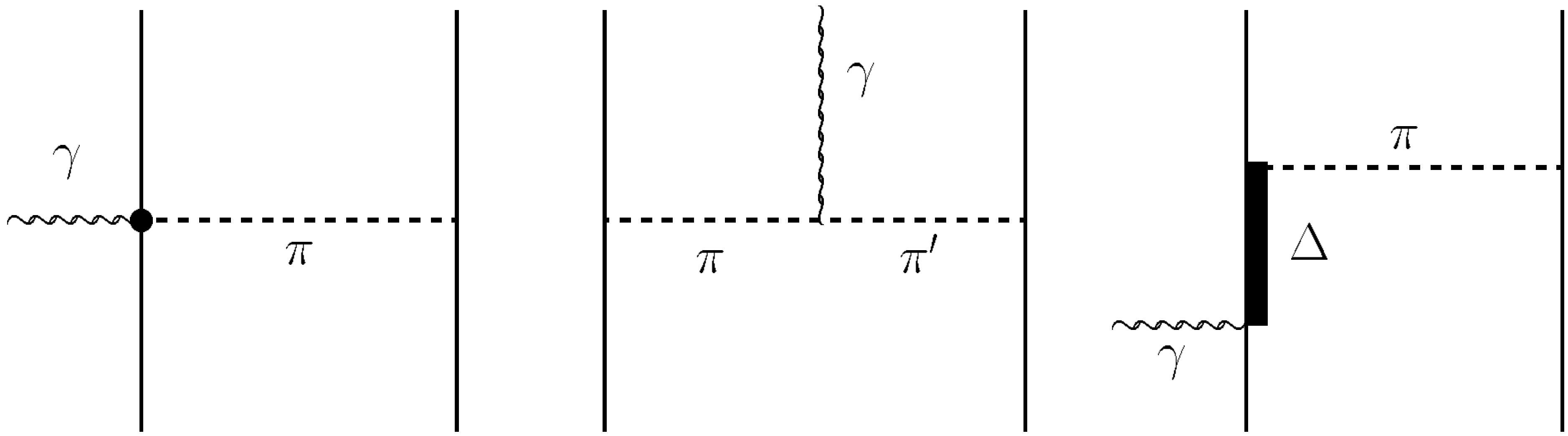
$$f^{\mathrm{emp}}(\Psi) = \frac{A\,\mathrm{exp}(-\Psi^2) + B\Psi^2 + C\Psi + D}{(\Psi + E)^2 + F^2}$$

A= 0.971, B=-0.067, C= 0.385, D= 0.145, E= 0.366, F= 1.900

RPA

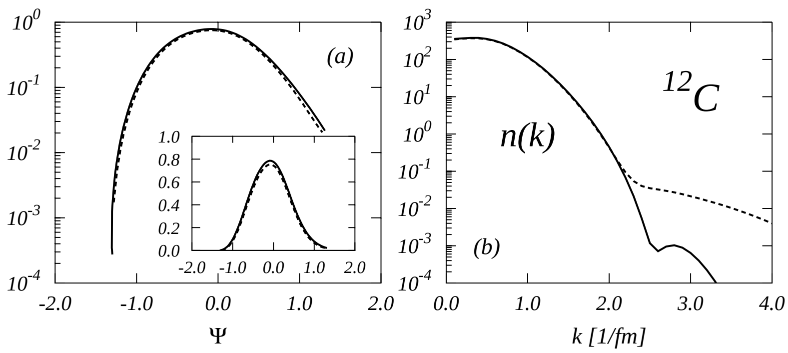


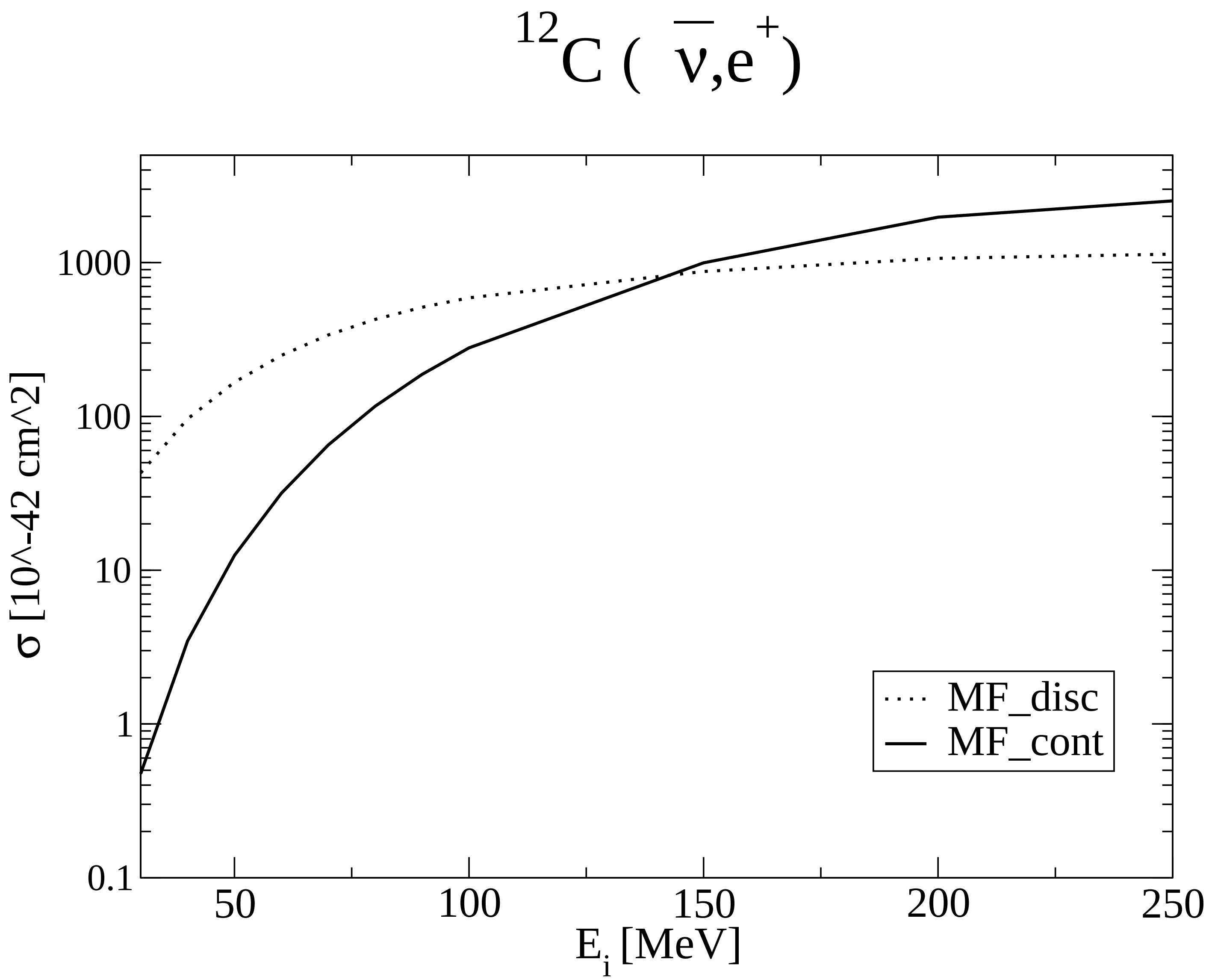
MEC



(a) (b) (c)

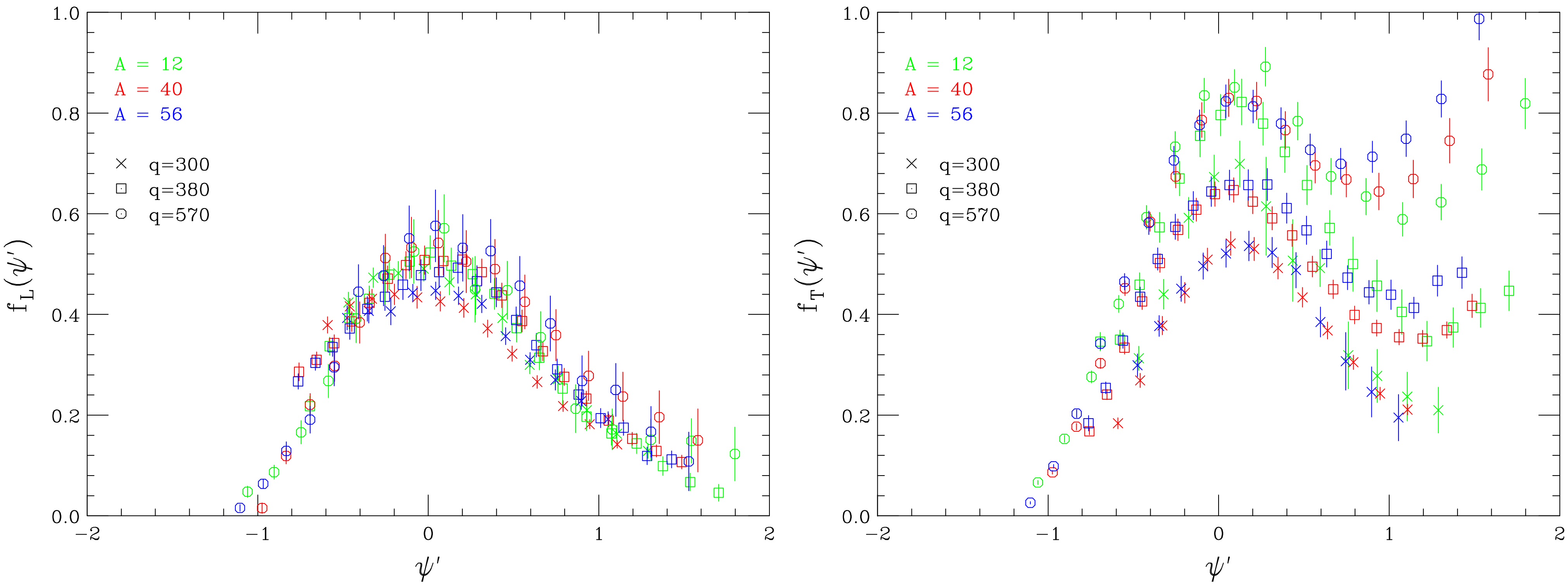
SRC





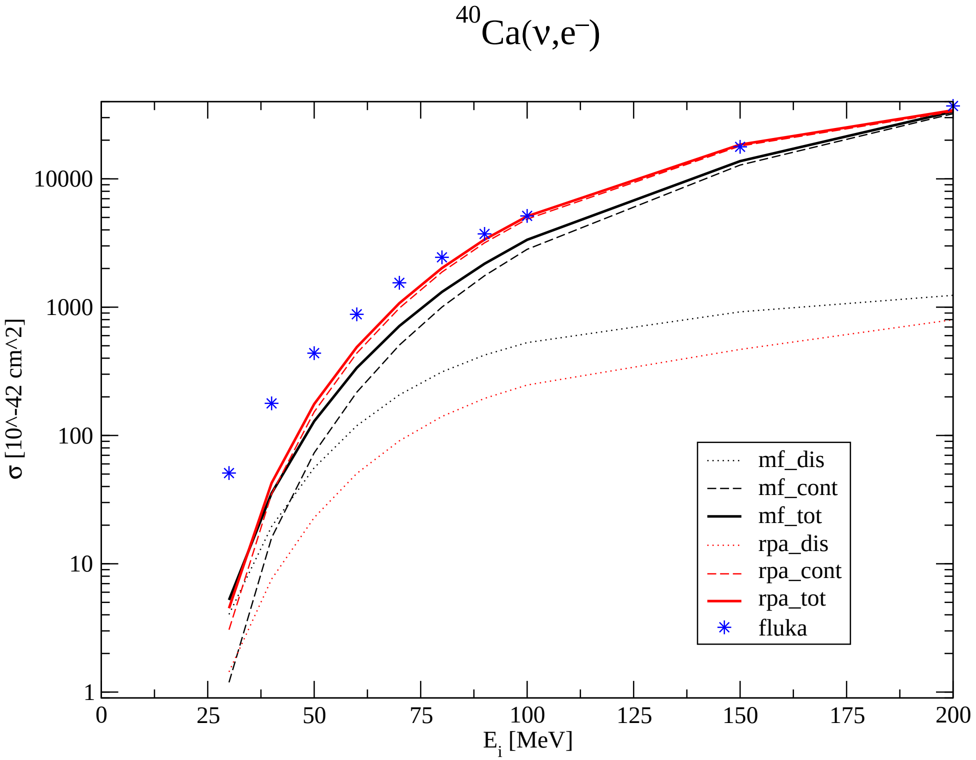
$^{12}C(\bar{\nu},e^+)$
1000
100
10
1
0.1
50
100
150
200
250
E_i [MeV]
σ [10^-42 cm^2]
MF_disc
MF_cont

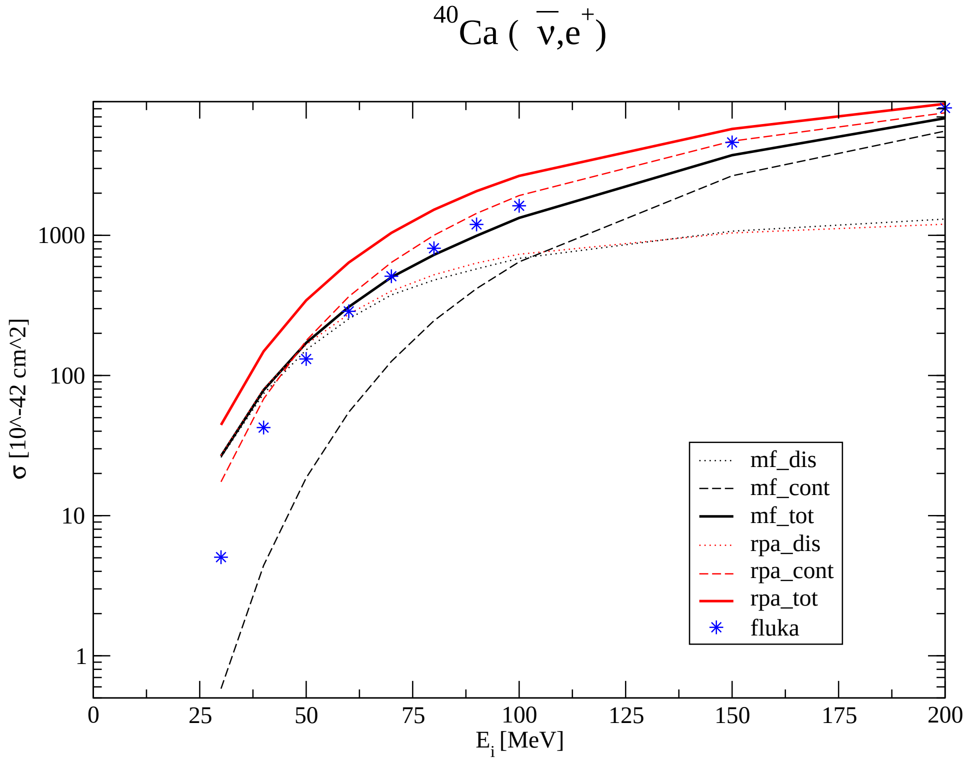
Electron scattering



The longitudinal part superscales

^{40}Ca: MF-RPA vs FLUKA





(ν, e^-): * good agreement above 80 MeV between RPA and FLUKA

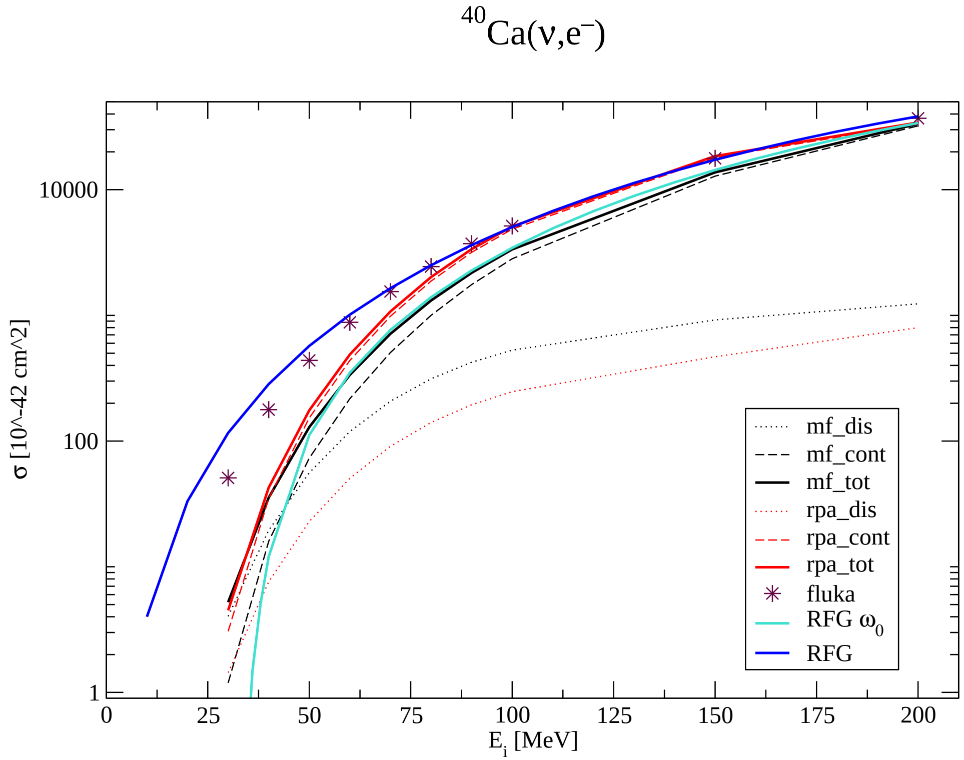
* RPA=MF=FLUKA @ E_i= 200 MeV

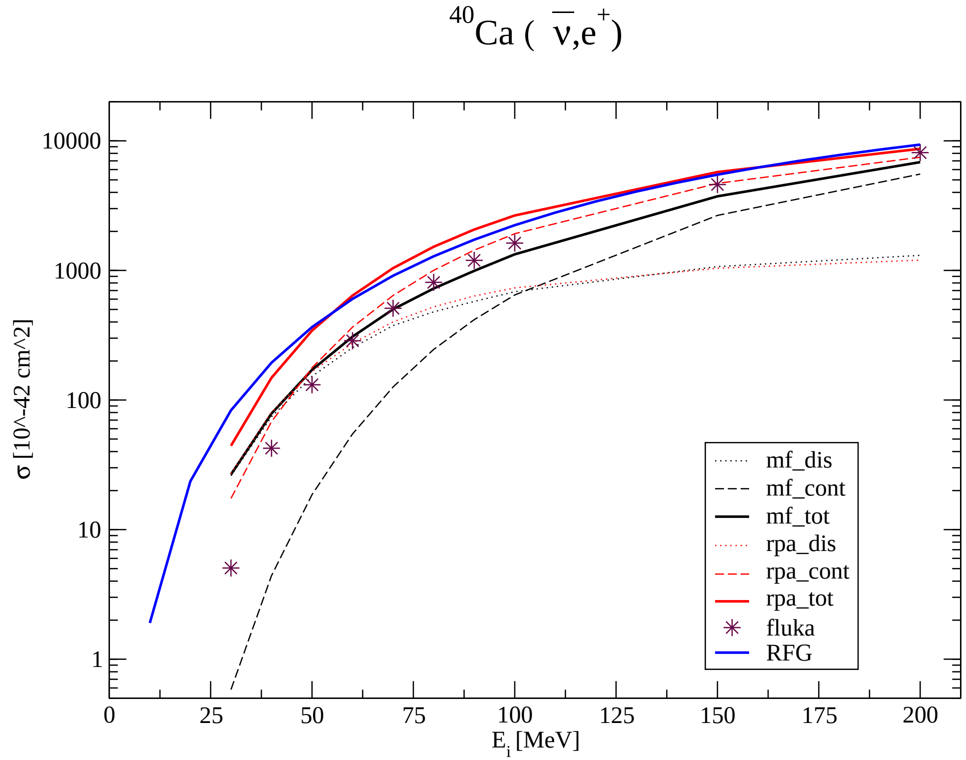
$(\bar{\nu}, e^+)$: * FLUKA $\simeq$ continuum RPA

* relevant discrete contribution up to E_i= 100 MeV

* RPA $\simeq$ MF in discrete case

^{40} Ca: MF-RPA vs RFG



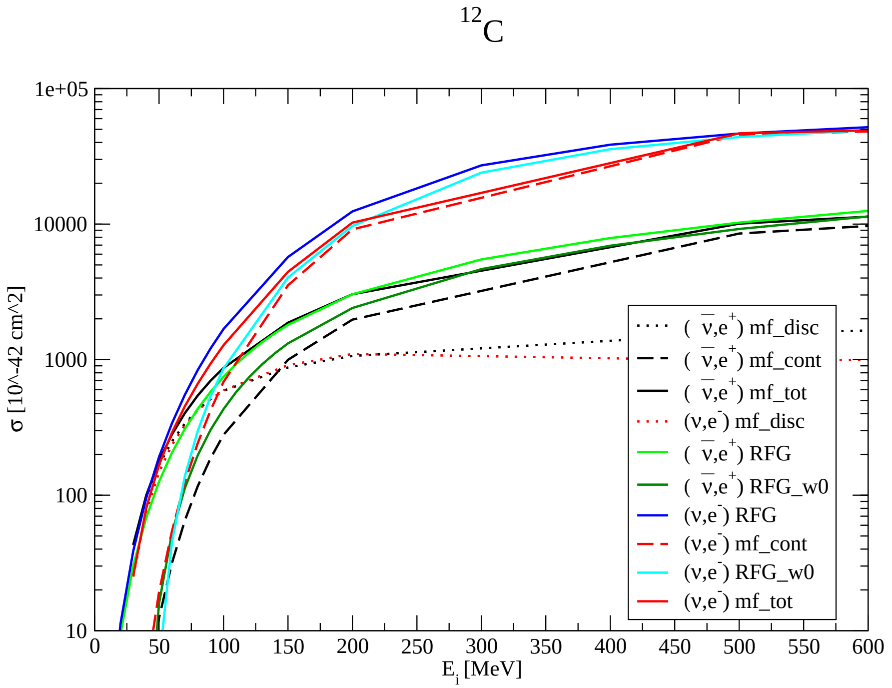


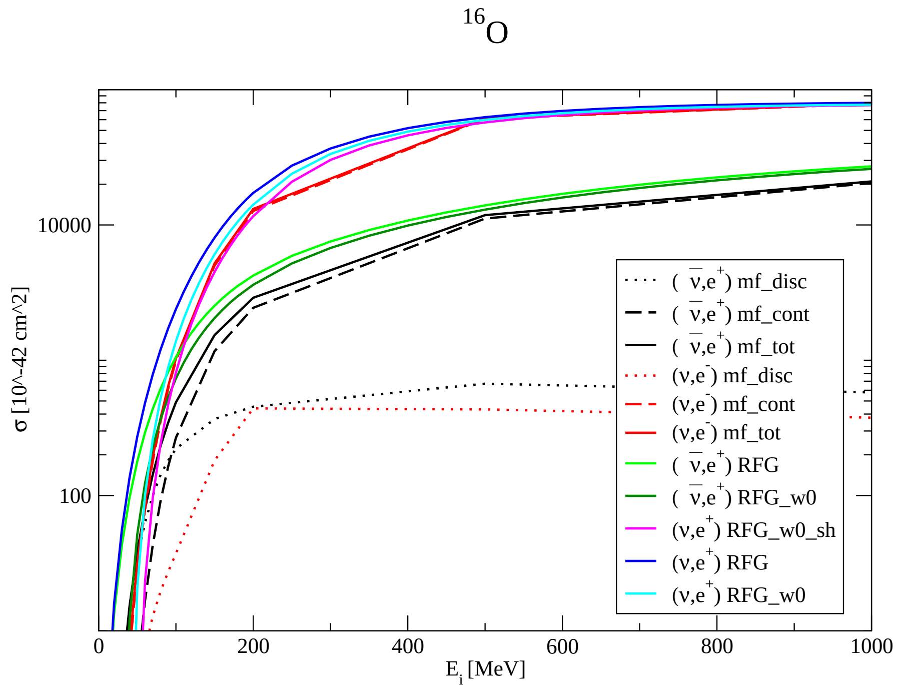
(ν, e^-): * good agreement above 60 MeV between RFG and FLUKA

* MF$\simeq$ RFGω_0 above 50 MeV

PRC 71, 015501 (2005), Amaro et al.: $\omega_0 \equiv \frac{1}{2M_i}(M_f^2 - M_i^2) \geq 0$; $\omega = \omega_0 + \frac{|Q^2|}{2M_i}$

^{12}C and ^{16}O: disc. vs cont., $(\bar{\nu}, e^+)$-(ν, e^-), MF vs RFG





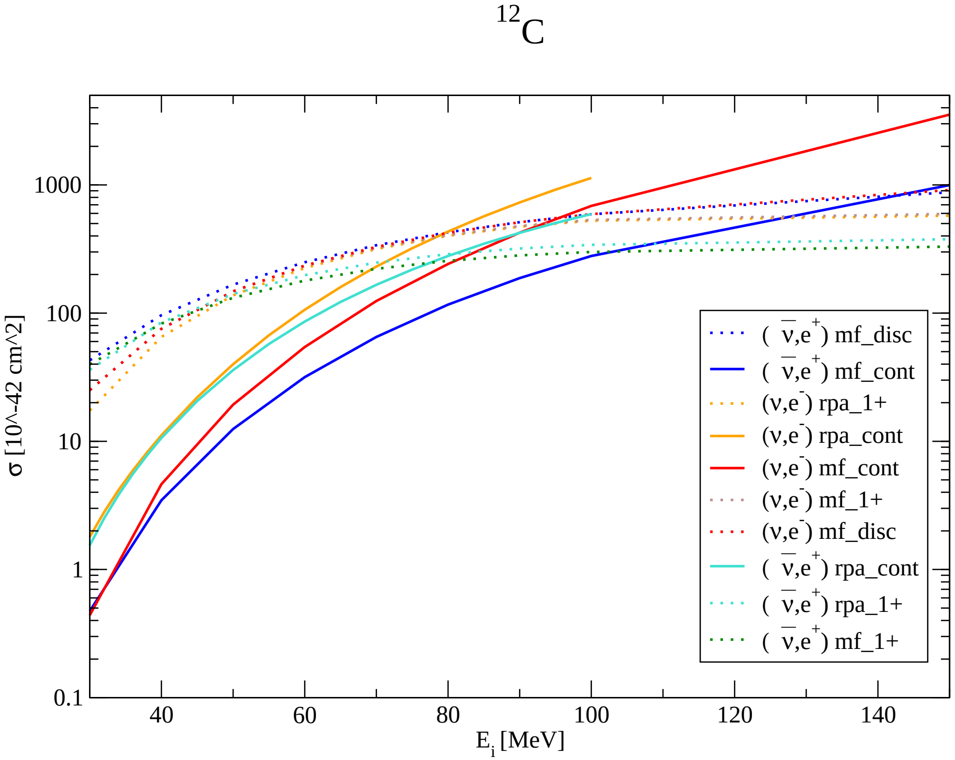
^{12}C:

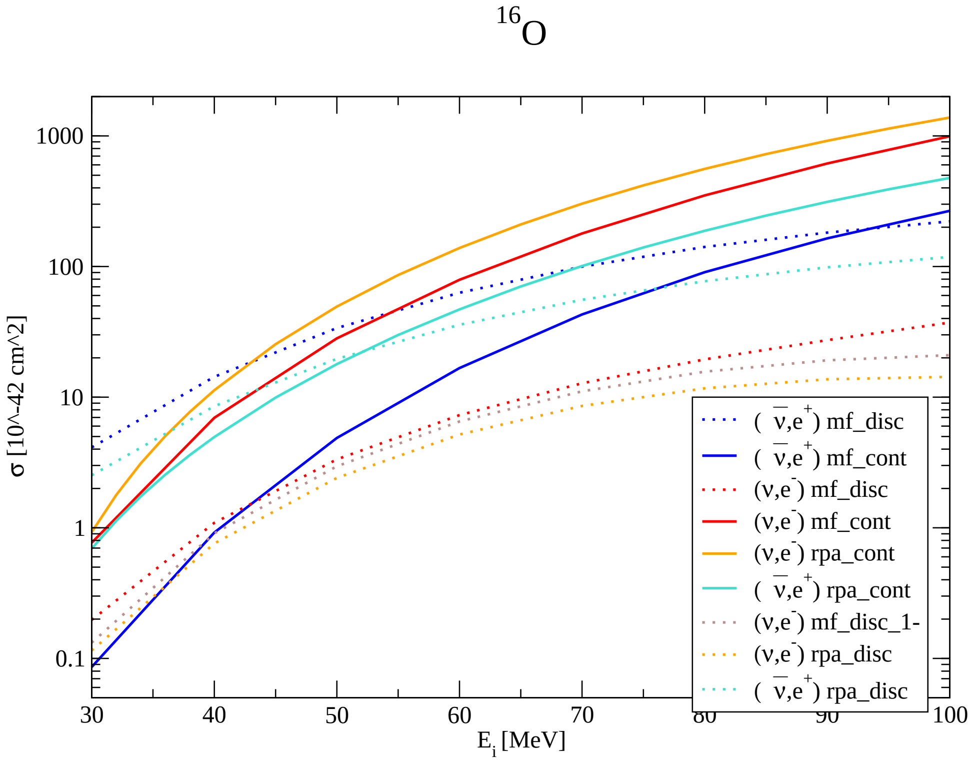
* relevant discrete contribution up to E_i= 100 or 150 MeV

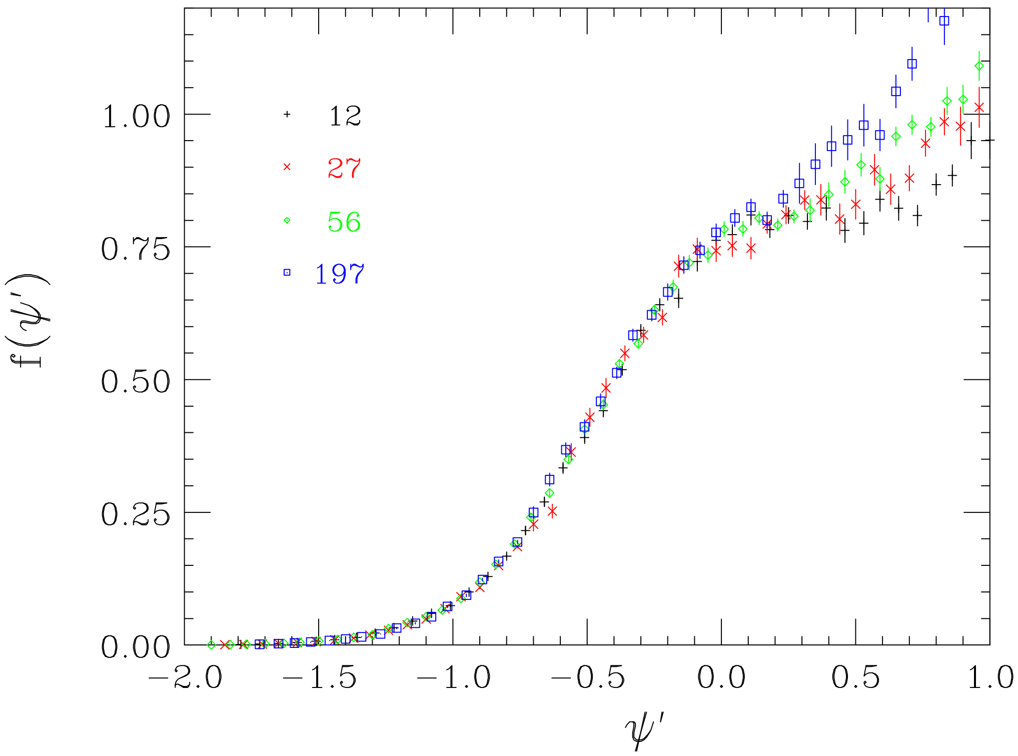
* $\sigma(\nu, e^-)_{DISC} \simeq \sigma(\bar{\nu}, e^+)_{DISC}$

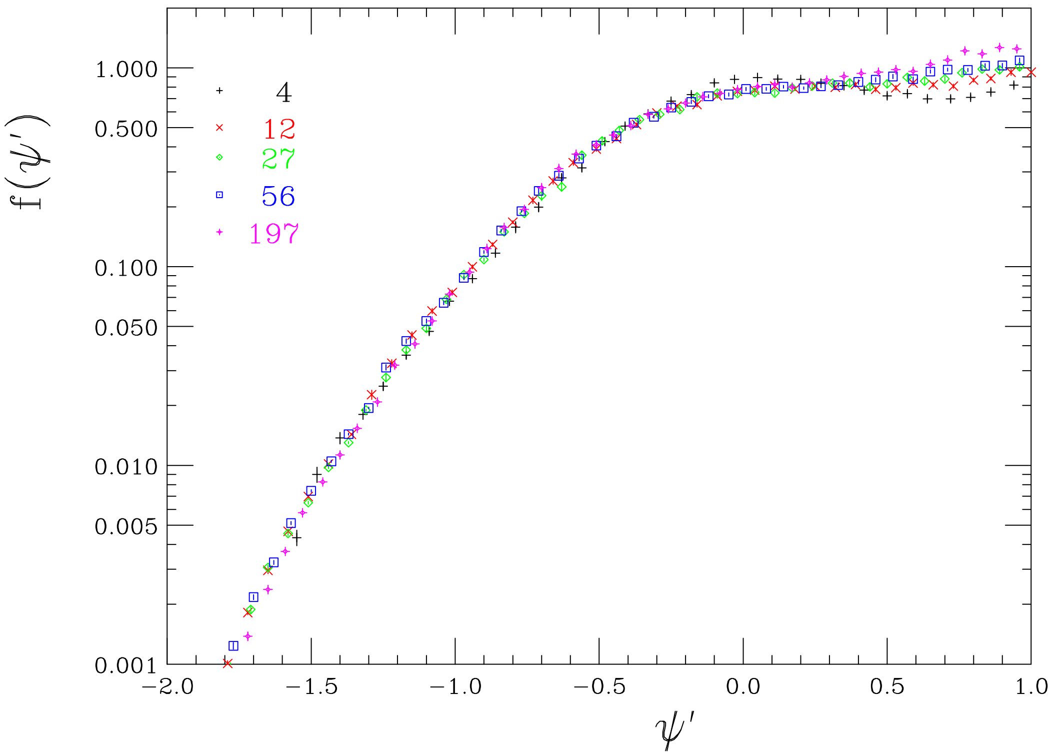
* (ν, e^-):MF$_{CONT}$ $\simeq$ RFGω_0

Nuclear Structure





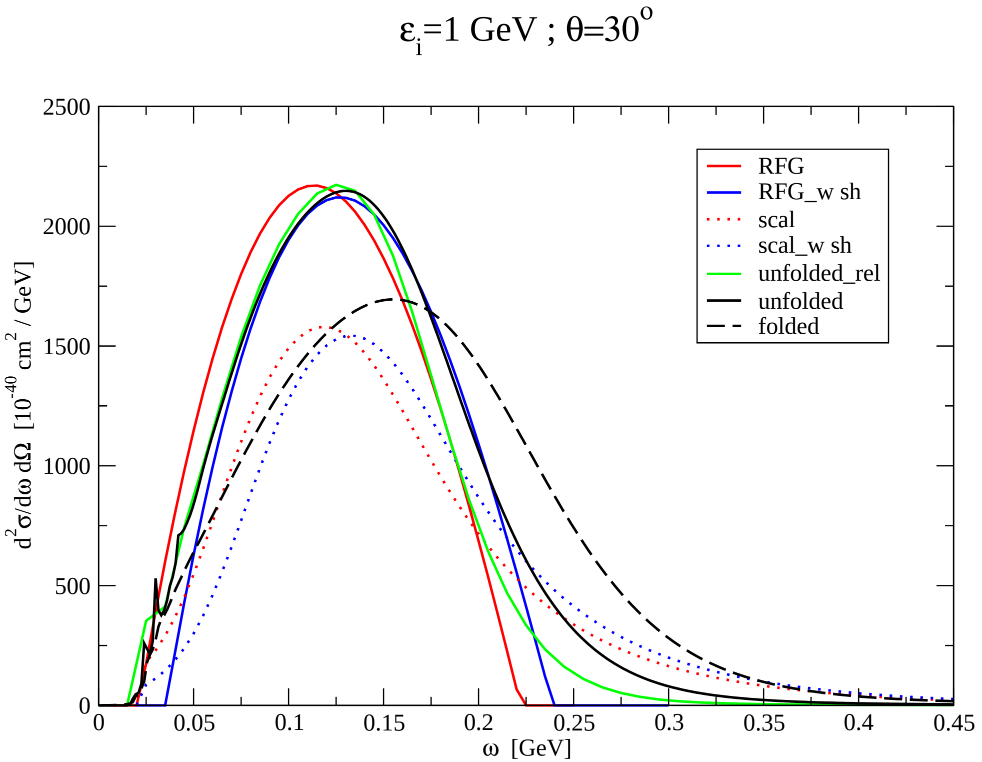


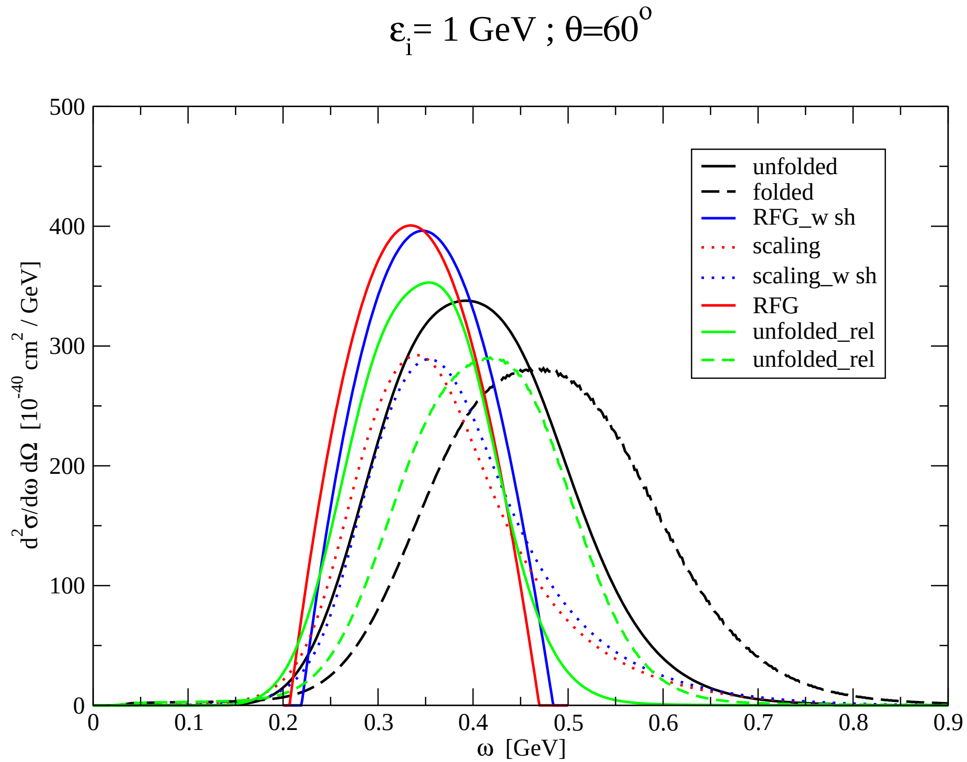


Scaling of the second kind

^{16}O: MF vs RFG, Folding vs Scaling

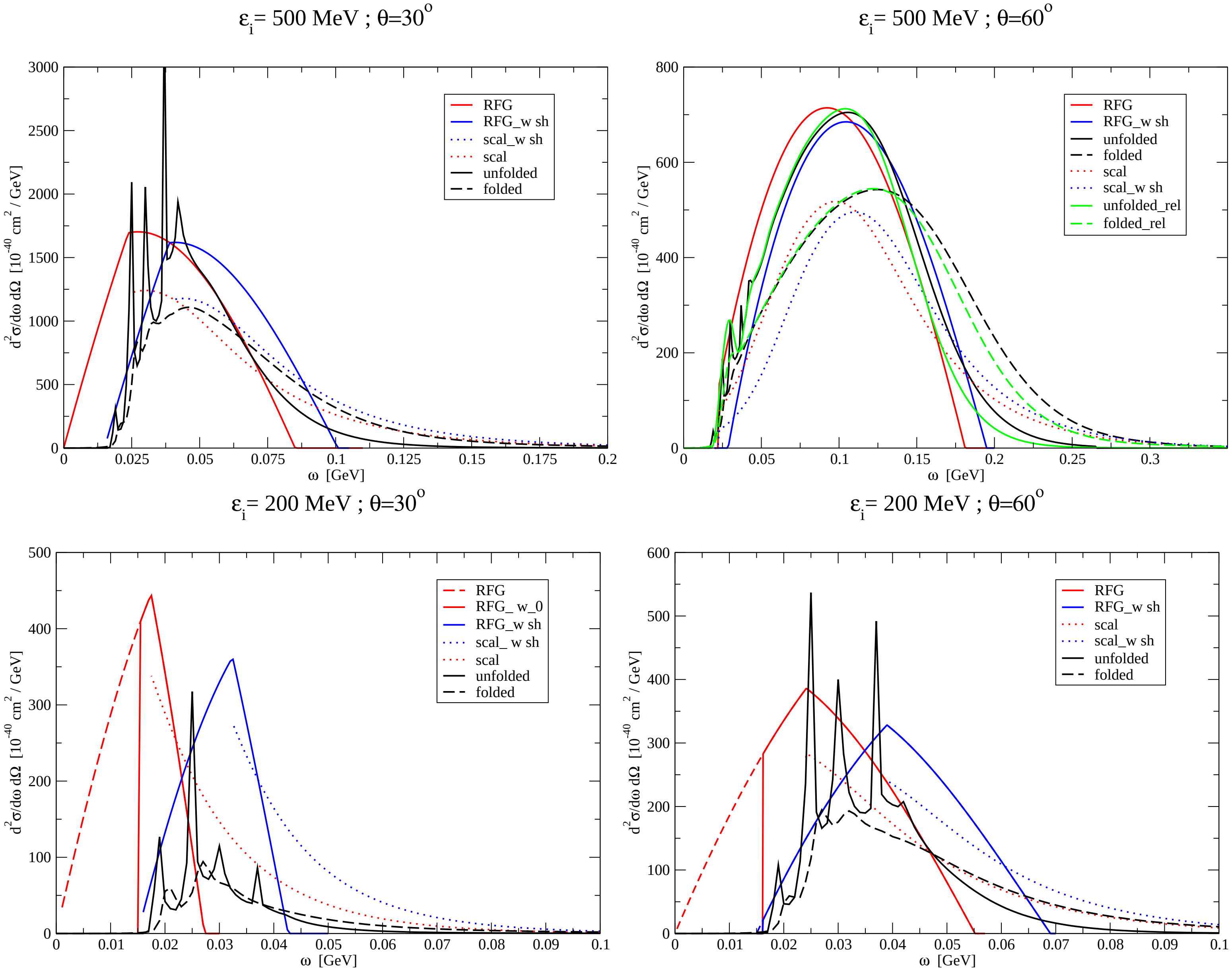
Double differential cross section



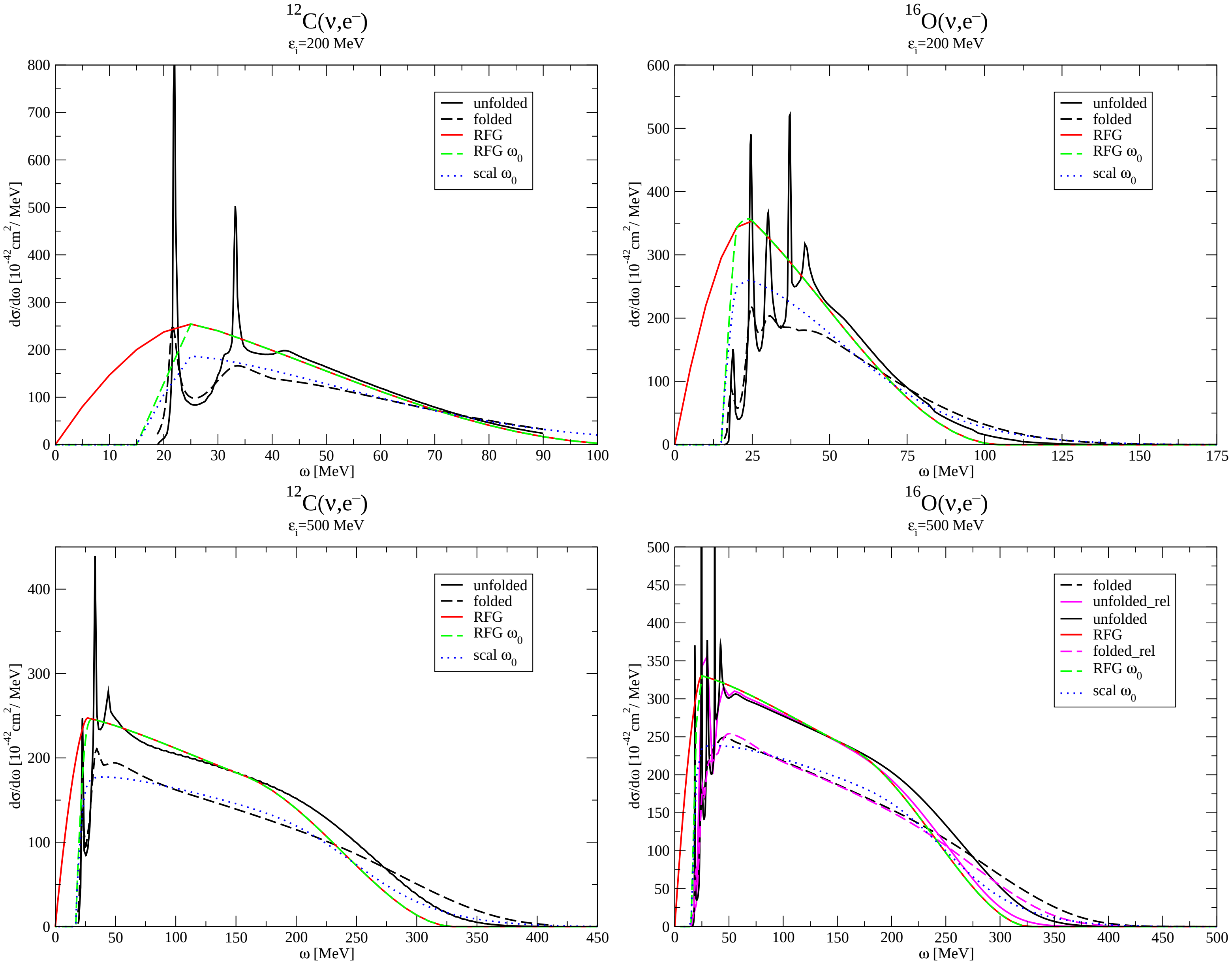


Relativistic kinematics:

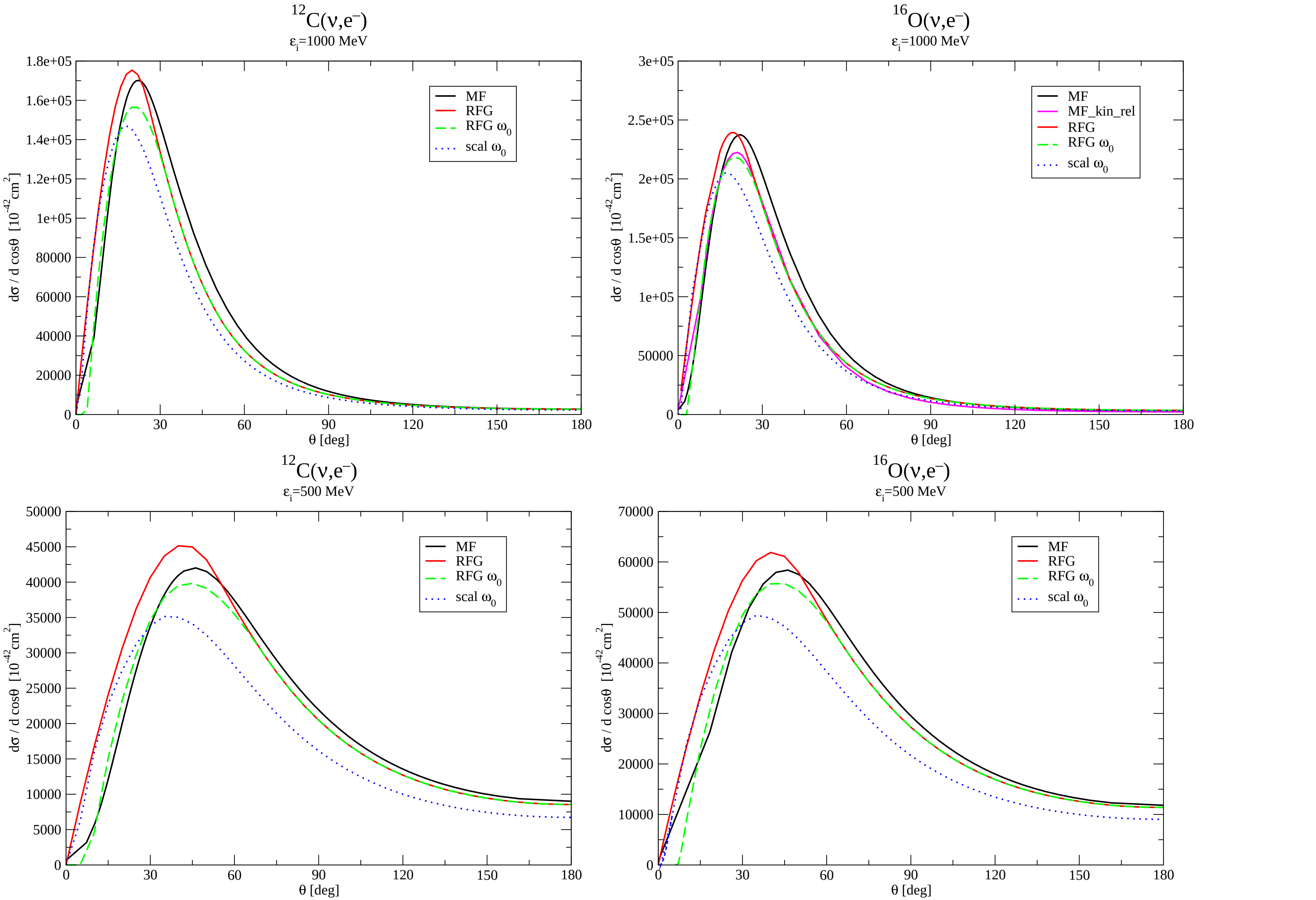
$$\epsilon_p \to \epsilon_p(1 + \epsilon_p/2m_N)$$



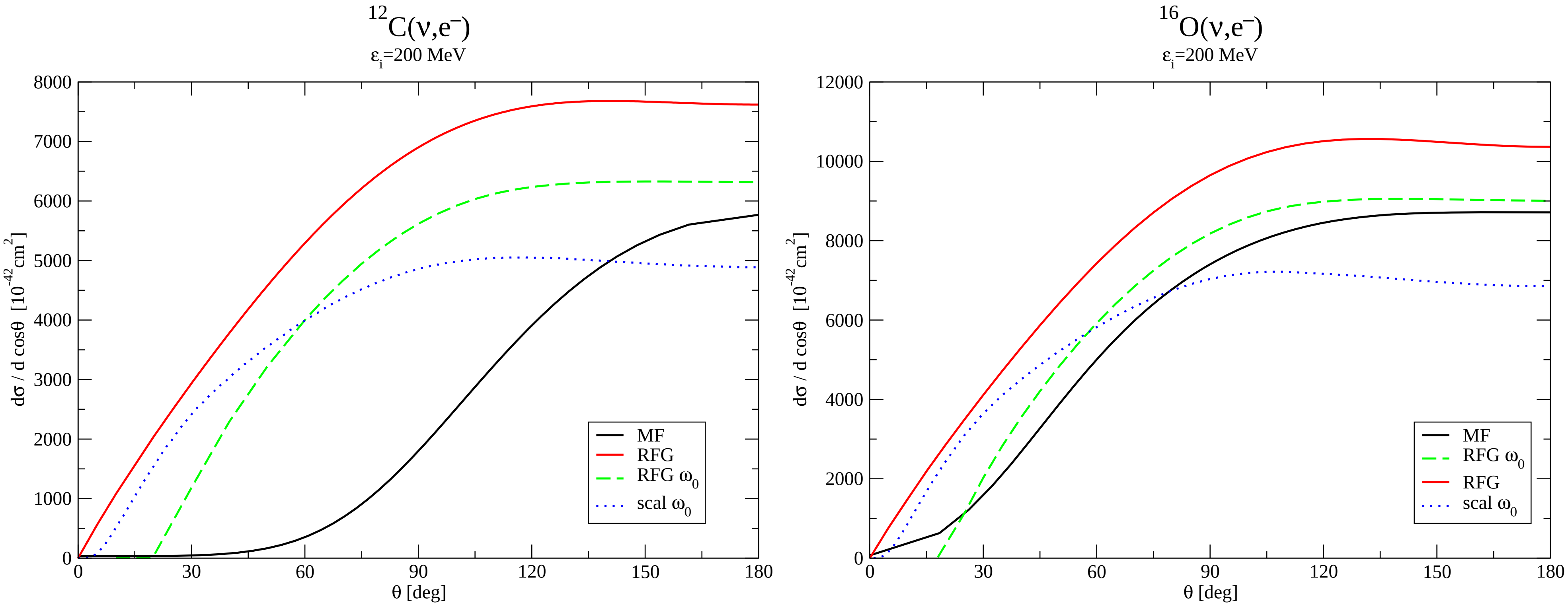
ε_i= 500 MeV ; θ=30^o
ε_i= 500 MeV ; θ=60^o
ε_i= 200 MeV ; θ=30^o
ε_i= 200 MeV ; θ=60^o
d^2σ/dω dΩ [10^-40 cm^2 / GeV]
ω [GeV]
RFG
RFG_w sh
scal_w sh
scal
unfolded
folded
unfolded_rel
folded_rel
RFG_ w_0
scal_ w sh



$^{12}C(\nu,e^-)$
ε_i=200 MeV
unfolded
folded
RFG
RFG ω_0
scal ω_0
$d\sigma/d\omega$ [10^{-42}cm^2/ MeV]
ω [MeV]
$^{16}O(\nu,e^-)$
ε_i=200 MeV
unfolded
folded
RFG
RFG ω_0
scal ω_0
$d\sigma/d\omega$ [10^{-42}cm^2/ MeV]
ω [MeV]
$^{12}C(\nu,e^-)$
ε_i=500 MeV
unfolded
folded
RFG
RFG ω_0
scal ω_0
$d\sigma/d\omega$ [10^{-42}cm^2/ MeV]
ω [MeV]
$^{16}O(\nu,e^-)$
ε_i=500 MeV
folded
unfolded_rel
unfolded
RFG
folded_rel
RFG ω_0
scal ω_0
$d\sigma/d\omega$ [10^{-42}cm^2/ MeV]
ω [MeV]



12C(ν,e−)
εi=1000 MeV
MF
RFG
RFG ω0
scal ω0
dσ / d cosθ [10^-42 cm^2]
θ [deg]
16O(ν,e−)
εi=1000 MeV
MF
MF_kin_rel
RFG
RFG ω0
scal ω0
dσ / d cosθ [10^-42 cm^2]
θ [deg]
12C(ν,e−)
εi=500 MeV
MF
RFG
RFG ω0
scal ω0
dσ / d cosθ [10^-42 cm^2]
θ [deg]
16O(ν,e−)
εi=500 MeV
MF
RFG
RFG ω0
scal ω0
dσ / d cosθ [10^-42 cm^2]
θ [deg]



$^{12}C(\nu,e^-)$
ε_i=200 MeV
dσ / d cosθ [$10^{-42}cm^2$]
θ [deg]
MF
RFG
RFG ω_0
scal ω_0
$^{16}O(\nu,e^-)$
ε_i=200 MeV
dσ / d cosθ [$10^{-42}cm^2$]
θ [deg]
MF
RFG ω_0
RFG
scal ω_0