# Constituent Quark Models in Point Form Relativistic Quantum Mechanics

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Project P19035, Structure of Baryon Resonances

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## Collaborators

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- *INFN, Sezione di Padova*
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INFN *Sezione di Padova* and the *University of Padova*

# Outline

## Why Relativistic Quantum Mechanics?

- Requirements of special relativity are satisfied
- Finite number of degrees of freedom
- Description of composite particles
- Large class of admissible interactions
- Few-body calculations are tractable

## Challenges

- Connection to local quantum field theory?
- No microscopic locality
  (can be replaced by macroscopic locality)
- Definition of suitable spectator-models

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# Procedure to formulate RQM

## Poincaré Invariance

- Symmetries of special relativity $\longrightarrow$ Poincaré group
  - Translations with four-momentum $P^\mu$
  - Lorentz transformations with rotations $\mathbf{J}$ and boosts $\mathbf{K}$
- Generators adhere to a set of commutation relations
- Properties must be guarantued by systems in question

## Free system

- Representation for one-body system straightforward
  - Define mass operator
  - Define spin operator
- Multi-particle representation given by combining single-particle representations
- Suitable coupling gives total momentum, mass and spin

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*. . . (constraints on interactions) are not easily fulfilled and provide the real difficulty in the problem of constructing a theory of a relativistic dynamical system. . .*

*Dirac, Forms of relativistic dynamics, Rev. Mod. Phys.* **21** *(1949), 392*

# Interacting Systems

## What is the problem?

- Generally, interaction in *all ten generators*
- (Interacting) generators $\longrightarrow$ commutation relations
- Commutators $\longrightarrow$ non-linear constraints for interaction
- Example: $\left[P^j, K^k\right] = H\delta^{jk}$

## Bakamjian-Thomas construction

- Generators for free system
- Define set of auxiliary operators (includes $M$)
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# Poincaré Invariant Transition Amplitude (Point Form)

$$\left\langle V', M', J', \Sigma' \right| \hat{O} \left| V, M, J, \Sigma \right\rangle = \frac{2}{MM'} \sum_{\sigma_i \sigma_i'} \sum_{\mu_i \mu_i'} \int d^3\vec{k}_2 d^3\vec{k}_3 d^3\vec{k}_2' d^3\vec{k}_3'$$

$$\sqrt{\frac{(\omega_1+\omega_2+\omega_3)^3}{2\omega_1 2\omega_2 2\omega_3}} \sqrt{\frac{(\omega_1'+\omega_2'+\omega_3')^3}{2\omega_1' 2\omega_2' 2\omega_3'}}$$

$$\Psi^{\star}_{M'J'\Sigma'}\left(\vec{k}_i'; \mu_i'\right) \prod_{\sigma_i'} D^{\star \frac{1}{2}}_{\sigma_i' \mu_i'} \left\{ R_W \left[ k_i'; B\left(V'\right) \right] \right\}$$

$$\left\langle p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3' \right| \hat{O}_{\mathrm{rd}} \left| p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \right\rangle$$

$$\prod_{\sigma_i} D^{\frac{1}{2}}_{\sigma_i \mu_i} \left\{ R_W \left[ k_i; B\left(V\right) \right] \right\} \Psi_{MJ\Sigma}\left(\vec{k}_i; \mu_i\right)$$

$$2MV_0 \delta^3\left(M\vec{V} - M'\vec{V}' - \vec{Q}\right)$$

## Spectator Electromagnetic Current

$$\left\langle p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3' \right| \hat{J}^\mu_{\mathrm{rd}} \left| p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \right\rangle$$
$$= 3\mathcal{N} \left\langle p_1', \sigma_1' \right| \hat{J}^\mu_{\mathrm{spec}} \left| p_1, \sigma_1 \right\rangle$$
$$2p_{20}\delta\left(\vec{p}_2 - \vec{p}_2'\right) 2p_{30}\delta\left(\vec{p}_3 - \vec{p}_3'\right) \delta_{\sigma_2\sigma_2'}\delta_{\sigma_3\sigma_3'}$$

## Formal Single Particle Current

$$\left\langle p_1', \sigma_1' \right| \hat{J}^\mu_{\mathrm{spec}} \left| p_1, \sigma_1 \right\rangle$$
$$= e_1 \bar{u}\left(p_1', \sigma_1'\right) \left[ f_1(\tilde{Q}^2)\gamma^\mu + \frac{i}{2m_1} f_2(\tilde{Q}^2)\sigma^{\mu\nu}\tilde{q}_\nu \right] u\left(p_1, \sigma_1\right)$$

structureless quarks: $f_1(\tilde{Q}^2) = 1$ and $f_2(\tilde{Q}^2) = 0$

## Problems in PFSM Construction

- No direct microscopic derivation
- Global symmetry constrain microscopic structure

- PFSM construction not unique with Poincaré invariance

$\longrightarrow$ Find additional constraints

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# Nucleon Sachs Form factors

## Definition in Breit Frame

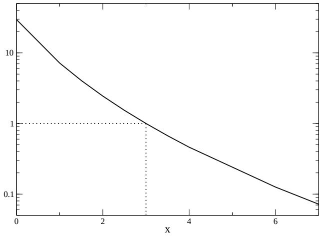
$$F^{\nu}_{\Sigma',\Sigma}\left(Q^2\right) = \left\langle V', m_{\mathrm{N}}, \frac{1}{2}, \Sigma' \right| \hat{J}^{\nu}_{\mathrm{rd}} \left| V, m_{\mathrm{N}}, \frac{1}{2}, \Sigma \right\rangle$$

$$\begin{aligned} F^{0}_{\Sigma',\Sigma}\left(Q^2\right) &= 2MG_{\mathrm{E}}\left(Q^2\right)\delta_{\Sigma',\Sigma} \\ \vec{F}_{\Sigma',\Sigma}\left(Q^2\right) &= iQG_{\mathrm{M}}\left(Q^2\right)\chi^{\dagger}_{\Sigma'}\left(\vec{\sigma}\times\vec{e}_z\right)\chi_{\Sigma} \end{aligned}$$

## Parameterization of $\mathcal{N}$

$$\mathcal{N}(x, y) = \left(\frac{M}{\sum_i \omega_i}\right)^{xy} \left(\frac{M'}{\sum_i \omega'_i}\right)^{x(1-y)}$$

## Charge normalization, $G_E^p(0)$




$$\longrightarrow x = 3$$

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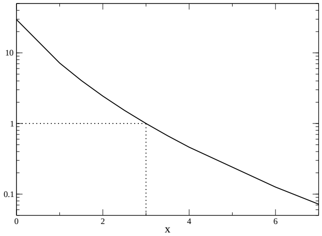


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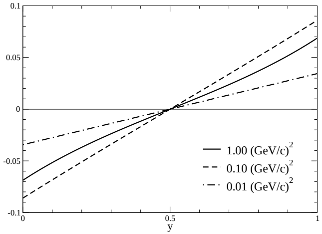
## Parameterization of $\mathcal{N}$

$$\mathcal{N}(3,y) = \left(\frac{M}{\sum_i \omega_i}\right)^{3y} \left(\frac{M'}{\sum_i \omega'_i}\right)^{3(1-y)}$$

## Time reversal invariance $\hat{J}^{\mu=3}$ (in Breit frame)




$$\mathcal{N}(z) = \frac{1}{2}\left[\left(\frac{M}{\sum_i \omega_i}\right)^{3z} \left(\frac{M'}{\sum_i \omega'_i}\right)^{3(1-z)} + \left(\frac{M'}{\sum_i \omega'_i}\right)^{3z} \left(\frac{M}{\sum_i \omega_i}\right)^{3(1-z)}\right]$$

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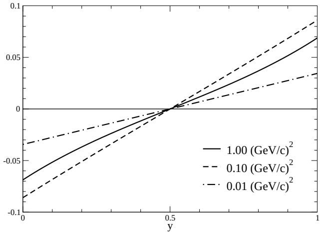


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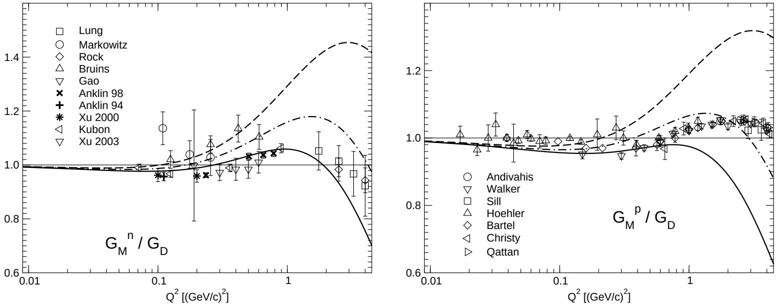
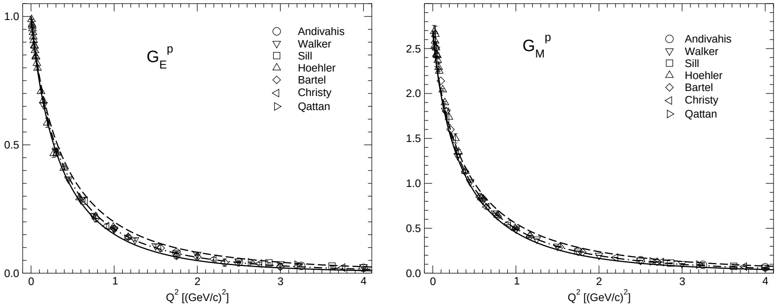
# Magnetic Form Factor to Dipole Ratios

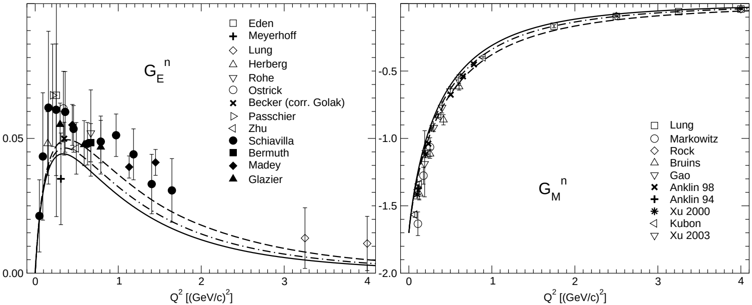



Top: $\mathcal{N}_{ari}$ $(z = 0)$ Middle: $\mathcal{N}_{fix}$ $(z = \frac{1}{6})$ Bottom: $\mathcal{N}_{ari}$ $(z = 0.5)$

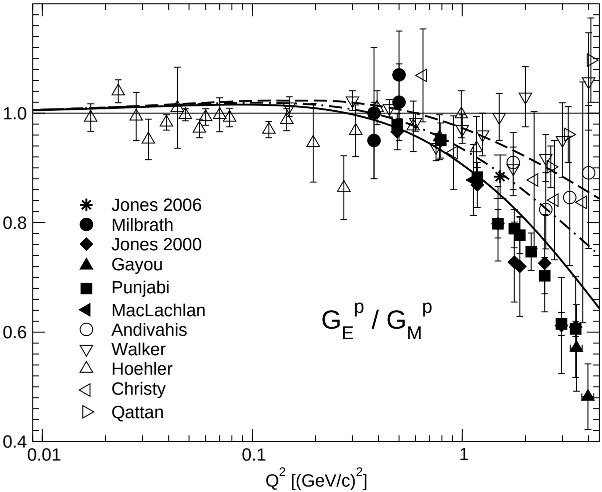
# Form Factors of the Proton



# Form Factors of the Neutron



# Electric/Magnetic Form Factor Ratio of the Proton



## Summary

- Global Constraints detail admissible PFSM constructions:
  EM form factor uncertainty in point form congruent with experiment

## Additional Results

- Relativistic mesonic decay results are generally too small
- Systematics in the theoretical predictions leads to new multiplet classification:
  Inclusion of (non-established) baryons in CQMs

## Outlook

- Future challenge is the treatment of explicit mesonic degrees of freedom