Chiral-Odd Generalized Parton Distributions in Light-Front Constituent Quark Models

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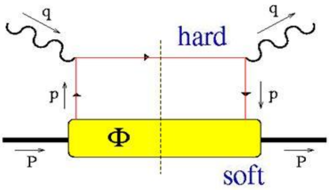
XI Convegno su Problemi di Fisica Nucleare Teorica

Cortona, 11-14 ottobre 2006

From Parton Distributions...

♠ Inclusive **D**eep **I**nelastic **S**cattering process $eP \longrightarrow eX$



In the Bjorken limit, when the photon virtuality $Q^2 = -q^2$ and the squared hadronic c.m. energy $(P+q)^2$ both $\longrightarrow \infty$ with the ratio $x_B = Q^2/(2P \cdot q)$ fixed, we have:

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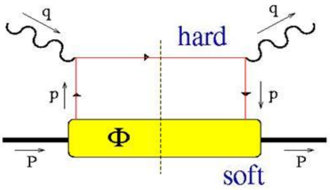

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⋆"Optical Theorem" $\Longrightarrow$ $\sigma(\gamma^* P \to X) \;\propto\; \Im\mathfrak{m}\mathcal{A}(\gamma^* P \to \gamma^* P)$ Forward Compton

⋆Factorization: $\Im\mathfrak{m}\mathcal{A}$ = Hard scattering (pQCD) $\otimes$ Parton Distribution

⋆ The correlation function Φ which describes the soft process:

$$\Phi(p, P, S) = \int \frac{d^4 z}{(2\pi)^4}\, e^{ip\cdot z} \langle PS|\bar{\psi}(0)\Gamma\psi(z)|PS\rangle.$$

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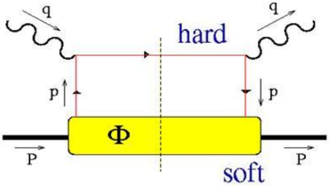

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Parton Distribution $\doteq$ Probability density of finding a parton with specified momentum fraction x in the target, (Initial state $=$ Final state [PS]).

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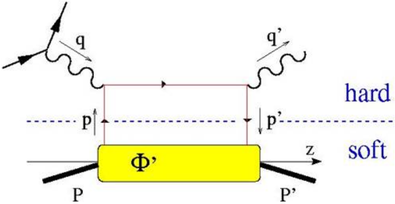

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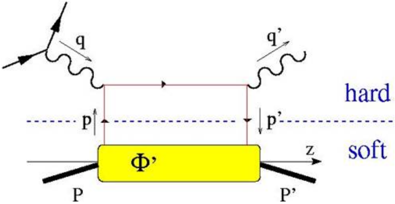

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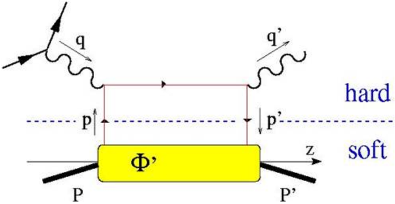

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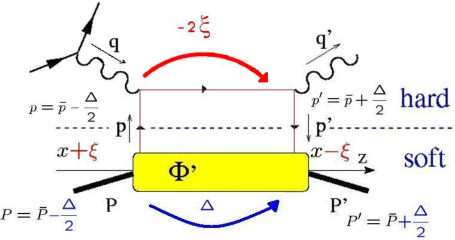
$\Rightarrow$ GPD $\doteq$ Nondiagonal hadronic matrix elements of bilocal products of the light-front quark field operators. Thus, they don't represent probability density (like PDF), but interference between amplitudes describing different quantum fluctuactions of a nucleon.

Kinematics and Correlation Functions



"Average Momentum" FRAME

- $\bar{P} = \frac{1}{2}(P + P') = (\bar{P}^+, \bar{P}^-, \mathbf{0}_\perp)$ "average momentum"
- $\xi = \frac{(P-P')^+}{(P+P')^+}$ "skewedness" (SPDs)
- $\Delta = P - P'$ "momentum transfer"

$$\Phi'^{[\Gamma]} = \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ z^-} \langle P' S' | \bar{\psi}\left(-\frac{z^-}{2}\right) \Gamma \, \psi\left(\frac{z^-}{2}\right) | PS \rangle \Big|_{z^+ = \mathbf{z}_\perp = 0} \quad \left(\Gamma = \gamma^+, \gamma^+\gamma_5, i\sigma^{i+}\gamma_5\right)$$

Depending on which Γ structure we substitute ⇒ 8 different GPDs:

Diehl, EPJ **C 19** '01

- Unpolarized GPDs $\Phi'^{[\gamma^+]} \Rightarrow \mathcal{H}(x,\xi,t)$ & $\mathcal{E}(x,\xi,t)$
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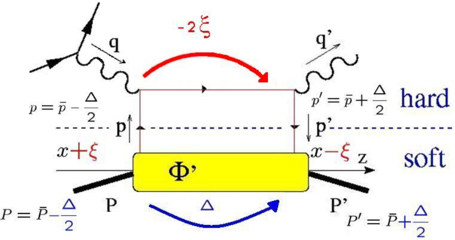

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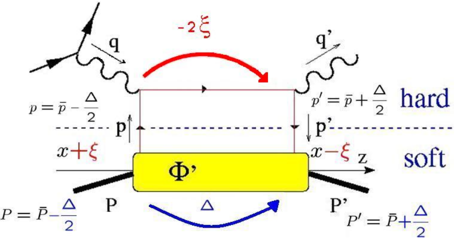

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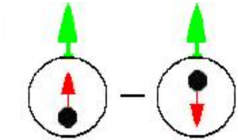
  In particular, from the Chiral-Odd GPDs $\mathcal{H}_T(x, \xi, t)$ we obtain the "TRANSVERSITY" distribution:

$$\mathcal{H}_T(x, 0, 0) = h_1(x) =$$



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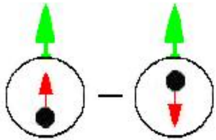

- Test of relativistic effects, i.e. $h_1(x) \neq g_1(x) \Rightarrow$

  Possibility of measure the relativistic nature of quarks inside the nucleon.

GPDs with helicity flip (Chiral-odd)

$$\frac{1}{2}\int \frac{dz^-}{2\pi} e^{i\bar{x}P^+z^-} \langle p',\lambda'|\bar{\psi}(-z/2)\sigma^{+i}\gamma_5\psi(z/2)|p,\lambda\rangle|_{z^+=0,\vec{z}_\perp=0}$$

$$= \frac{1}{2P^+}\bar{u}(p',\lambda')\left[\mathcal{H}_T^q\sigma^{+i}\gamma_5 + \tilde{\mathcal{H}}_T^q\frac{\epsilon^{+i\alpha\beta}\Delta_\alpha P_\beta}{M^2} + \mathcal{E}_T^q\frac{\epsilon^{+i\alpha\beta}\Delta_\alpha\gamma_\beta}{2M} + \tilde{\mathcal{E}}_T^q\frac{\epsilon^{+i\alpha\beta}P_\alpha\gamma_\beta}{M}\right]u(p,\lambda)$$

GPDs can be related to the following helicity amplitudes

M. Diehl, EPJ **C 19**, ('01)

$$\mathbf{A}_{\lambda'+,\lambda-} = \int \frac{dz^-}{2\pi} e^{i\bar{x}P^+z^-} \langle p',\lambda'|\,\mathcal{O}_{+,-}(z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\vec{z}_\perp=0}$$

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with

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Mix RH ↔ LH

⇒ CHIRAL-ODD Nature

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(cont'ed)

$\Rightarrow$

- $\mathbf{A}_{++,+-} = \epsilon \frac{(t_0-t)^{1/2}}{2M}[\tilde{\mathcal{H}}_T^q + (1-\xi)\frac{\mathcal{E}_T^q+\tilde{\mathcal{E}}_T^q}{2}]$
- $\mathbf{A}_{-+,--} = \epsilon \frac{(t_0-t)^{1/2}}{2M}[\tilde{\mathcal{H}}_T^q + (1+\xi)\frac{\mathcal{E}_T^q-\tilde{\mathcal{E}}_T^q}{2}]$
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- $\mathbf{A}_{-+,+-} = -(1-\xi^2)^{1/2}\frac{(t_0-t)}{4M^2}\tilde{\mathcal{H}}_T^q$

Thus inverting:

- $\mathcal{E}_T^q = \frac{2M}{\epsilon\sqrt{t_0-t}}\left(\frac{1}{(1-\xi)}\mathbf{A}_{++,+-} + \frac{1}{(1+\xi)}\mathbf{A}_{-+,--}\right) + \frac{8M^2}{(t_0-t)(1-\xi^2)\sqrt{1-\xi^2}}\mathbf{A}_{-+,+-}$
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Change of base

Helicity Basis

$$\phi_R = P_R\phi = \frac{1}{\sqrt{2}}(1+\gamma_5)\phi$$

$$\phi_L = P_L\phi = \frac{1}{\sqrt{2}}(1-\gamma_5)\phi$$

Transversity Basis

$$\mathcal{Q}_\pm = \frac{1}{\sqrt{2}}(1 \pm \gamma^1\gamma_5)$$

$$\mathcal{Q}_+\phi \equiv \phi_\uparrow$$

$$\mathcal{Q}_-\phi \equiv \phi_\downarrow$$

Introducing the transversity basis also for the nucleon spin states, i.e.

$$|p,\uparrow\rangle = \frac{1}{\sqrt{2}}(|p,+\rangle + |p,-\rangle)$$

$$|p,\downarrow\rangle = \frac{1}{\sqrt{2}}(|p,+\rangle - |p,-\rangle)$$

and defining the following matrix elements

$$T^q_{\lambda'_t\lambda_t} = \langle p',\lambda'_t| \int \frac{dz^-}{2\pi} e^{i\bar{x}P^+z^-} \bar{\psi}(-z/2)\gamma^+\gamma^1\gamma_5\psi(z/2)|p,\lambda_t\rangle$$

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where $\lambda_t(\lambda'_t)$ labels the transverse polarization of initial (final) nucleon in the $\uparrow(\downarrow)$ direction.

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where $\lambda_t(\lambda'_t)$ labels the transverse polarization of initial (final) nucleon in the $\uparrow(\downarrow)$ direction.

Change of base

Helicity Basis

$$\phi_R = P_R\phi = \frac{1}{\sqrt{2}}(1 + \gamma_5)\phi$$

$$\phi_L = P_L\phi = \frac{1}{\sqrt{2}}(1 - \gamma_5)\phi$$

→

Transversity Basis

$$\mathcal{Q}_{\pm} = \frac{1}{\sqrt{2}}(1 \pm \gamma^1\gamma_5)$$

$$\mathcal{Q}_{+}\phi \equiv \phi_{\uparrow}$$

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Introducing the transversity basis also for the nucleon spin states, i.e.

$$|p,\uparrow\rangle = \frac{1}{\sqrt{2}}(|p,+\rangle + |p,-\rangle)$$

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(cont'ed)

we find:

♠ $T^q_{\uparrow\uparrow} = -T^q_{\downarrow\downarrow}$ $\quad T^q_{\uparrow\downarrow} = T^q_{\downarrow\uparrow}$

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"Parity invariance"

and in terms of the matrix elements "A"

⋆ $T^q_{\uparrow\uparrow} = A_{++,--} + A_{-+,+-}$ $\quad T^q_{\uparrow\downarrow} = A_{++,+-} - A_{-+,--}$

⋆ $\tilde{T}^q_{\uparrow\uparrow} = A_{++,+-} + A_{-+,--}$ $\quad \tilde{T}^q_{\downarrow\uparrow} = A_{++,--} - A_{-+,+-}$

Finally, we obtain the chiral-odd GPDs from the transverse matrix elements through the relations

- $\mathcal{H}^q_T = \frac{1}{(1-\xi^2)^{1/2}} T^q_{\uparrow\uparrow} - \frac{2M\xi}{\epsilon\sqrt{t_0-t}(1-\xi^2)} T^q_{\uparrow\downarrow}$
- $\mathcal{E}^q_T = \frac{2M\xi}{\epsilon\sqrt{t_0-t}} \frac{1}{1-\xi^2} T^q_{\uparrow\downarrow} + \frac{2M}{\epsilon\sqrt{t_0-t}(1-\xi^2)} \tilde{T}^q_{\uparrow\uparrow} - \frac{4M^2}{(t_0-t)(1-\xi^2)^{3/2}} (\tilde{T}^q_{\downarrow\uparrow} - T^q_{\uparrow\uparrow})$
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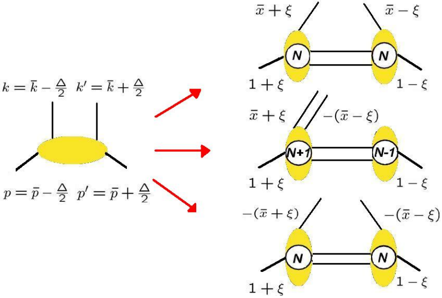
OVERLAP REPRESENTATION

M. Diehl, Th. Feldmann, R. Jakob and P. Kroll,
Eur. Phys. J. **C 8**, '99; Nucl. Phys. **B 596**, '01.



- $\xi < \bar{x} < 1$
- $-\xi < \bar{x} < \xi$
- $-1 < \bar{x} < -\xi$

We restrict our calculation to the region $\xi < \bar{x} < 1$ of plus-momentum fraction, where the GPDs describe the emission of a quark with plus-momentum $(\bar{x} + \xi)\bar{P}^+$ and its reabsorption with plus-momentum $(\bar{x} - \xi)\bar{P}^+$.

At the light-cone time $z^+ = 0$, Fourier expansion of the free quark field (Fock Decomp.):

$$\phi_\uparrow(z^-, \vec{z}_\perp) = \int \frac{dk^+ d^2\vec{k}_\perp}{k^+ 16\pi^3} \Theta(k^+) \Big\{ b_\uparrow(k^+, \vec{k}_\perp) u_+(k,\uparrow) \exp(-i k^+ z^- + i\vec{k}_\perp \cdot \vec{z}_\perp)$$

$$+ d^\dagger_\uparrow(k^+, \vec{k}_\perp)\, v_+(k,\uparrow) \exp(+i k^+ z^- - i\vec{k}_\perp \cdot \vec{z}_\perp) \Big\}$$

with $b_\uparrow$ ($d^\dagger_\uparrow$) annihilation (creation) operator for an on-shell (anti)quark with transverse pol. and u_+ (v_+) "good" projection of the (anti)quark spinor.

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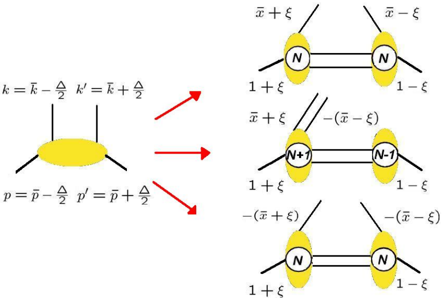

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The representation of the nucleon state reads:

$$|p, \lambda_t\rangle = \sum_{N,\beta} \int [dx]_N [d^2\vec{k}_\perp]_N \, \Psi_{\lambda_t,N,\beta}(r) \, |N, \beta; k_1, \ldots, k_N\rangle,$$

- $\Psi_{\lambda_t,N,\beta}$, Light-Cone Wave Function (LCWF) of the N-parton Fock state;
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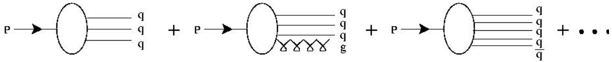
Light-Cone Wave Function in Constituent Quark Models

$$|\Psi\rangle = \Psi_{3q}|qqq\rangle + \Psi_{3q,g}|qqq,g\rangle + \Psi_{3q,q\bar{q}}|3q,q\bar{q}\rangle + \ldots$$



CQM are quantomechanical models with a fixed number of constituents and relativity-consistent, based on two fundamental hypotesis:

1. valence quark dominance (...only Ψ_{3q});
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In such models “relativity” can be incorporated in a straightforward way and the wave functions can be calculated solving the Hamiltonian eigenvalues equation in different forms of relativistic dynamic linked by an unitary transformation.

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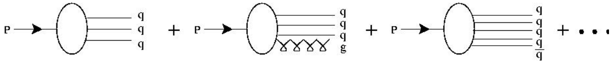

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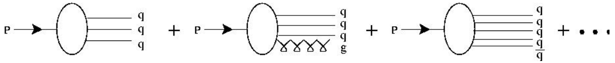

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Connection between Instant Form [IF] and Light-Front Form [LF]

[IF]: x^0, time;

x^1, x^2, x^3, space

[LF]: $x^+ = x^0 + x^3$, time;

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[IF] $M|M, j_c, \mu_c\rangle_c = [M_0 + V]|M, j_c, \mu_c\rangle_c = M|M, j_c, \mu_c\rangle_c$

with $M_0 = \sum_{i=1}^{3} \sqrt{\mathbf{k}_i^2 + m_i^2}$ free mass operator and V interaction operator;

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Thus, $\mathcal{M} = \mathcal{R}^\dagger M \mathcal{R} = M_0 + \mathcal{R}^\dagger V \mathcal{R}$, where $\mathcal{R} = \prod_{i=1}^{3} R_M(\mathbf{k}_{\perp,i}, x_i, m_i)$ is a generalized Melosh rotation

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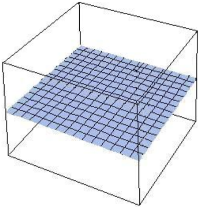

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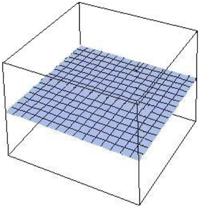

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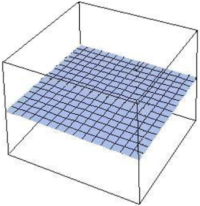

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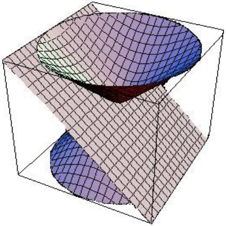

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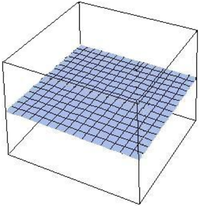

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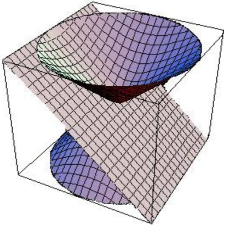

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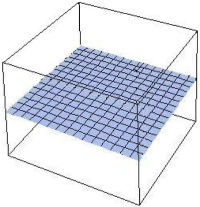

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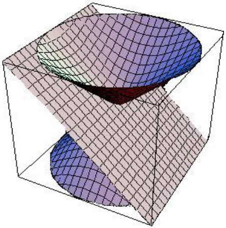

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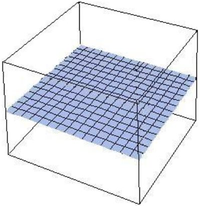

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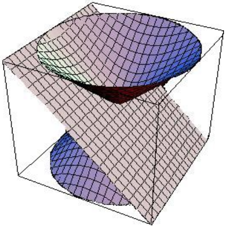

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$$\Rightarrow \Psi_{3q}^{f} = \langle \{x_i, \mathbf{k}_{\perp,i}, \lambda_i\} | M, j_f, \mu_f\rangle_f = \left[\frac{\omega_1 \omega_2 \omega_3}{x_1 x_2 x_3 M_0}\right]^{1/2} \sum_{\{\lambda_i'\}} \langle \{\lambda_i\}|\mathcal{R}^\dagger|\{\lambda_i'\}\rangle \Psi_{3q}^{c}$$

S. Boffi, B. Pasquini and M. Traini, Nucl. Phys. **B 649**,('03).

RESULTS

B. Pasquini, M. P. and S. Boffi,
PR D 72,'05 & hep-ph/0610051

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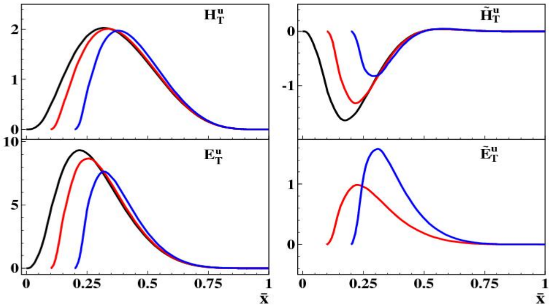

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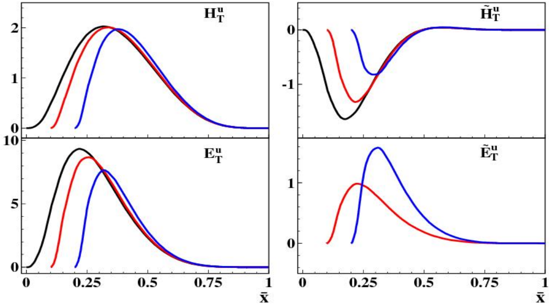

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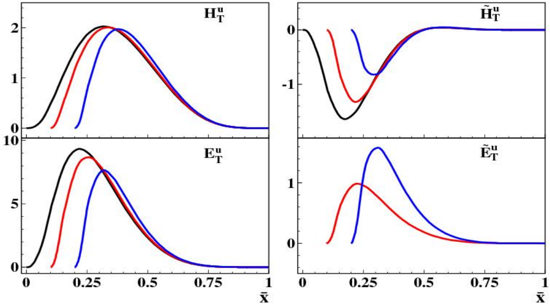

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$$\mathcal{H}_T^q(x,0,0) \quad = \quad h_1^q(x) = \sum_{\lambda_i^t \tau_i} \sum_{j=1}^{3} \delta_{\tau_j \tau_q} \mathrm{sign}(\lambda_j^t) \int [d\bar{x}]_3 [d\mathbf{k}_\perp]_3 \delta(x - x_j) \left| \psi_{\lambda^t}^{[f]} \left(x_i, \mathbf{k}_{\perp i}; \lambda_i^t, \tau_i \right) \right|^2$$

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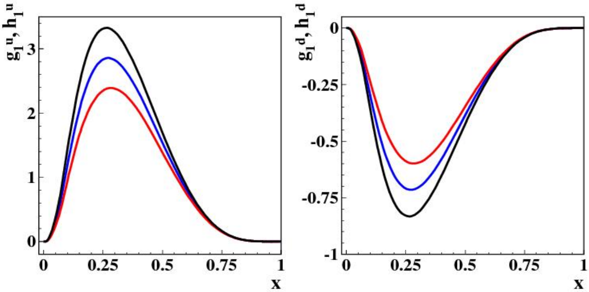

Helicity and Transversity distributions for the u (left panel) and d (right panel) quark. The blue lines correspond to h_1^q, the red lines show g_1^q, and the black lines are the nonrelativistic results when Melosh rotations reduce to the identity ($h_1^q = g_1^q$).

- Axial (Δq) and Tensor (δq) "Charges"

$$\Delta q = \int_{-1}^{1} dx\, g_1^q(x), \quad \delta q = \int_{-1}^{1} dx\, h_1^q(x).$$

The nucleon axial / tensor charge measures the net number of longitudinally / transversely polarized valence quarks in a longitudinally / transversely polarized nucleon.

	NR	HO	SCH
Δu	$4/3$	1.0	1.00
Δd	$-1/3$	-0.25	-0.25
δu	$4/3$	1.17	1.16
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Valence contributions to the axial and tensor charge calculated within different SU(6)-symmetric quark models: the nonrelativistic quark model (NR), the harmonic oscillator model (HO), and the Schlumpf's (SCH) model.

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Angular Momentum Decomposition

Recently Burkardt showed (PR**D 72**,'05) how the angular momentum J^x carried by quarks with transverse polarization in the $\hat{x}$ direction in an unpolarized nucleon at rest is related to the forward limit of chiral-odd GPDs through the following relation:

$$\langle \delta^{\mathbf{x}} \mathbf{J}^{\mathbf{x}}_{\mathbf{q}} \rangle = \langle \mathbf{J}^{\mathbf{x}}_{\mathbf{q},+\hat{\mathbf{x}}} - \mathbf{J}^{\mathbf{x}}_{\mathbf{q},-\hat{\mathbf{x}}} \rangle = \frac{1}{2}\left[A_{T20}(0) + 2\tilde{A}_{T20}(0) + B_{T20}(0) \right],$$

where the invariant form factors A_{T20}, $\tilde{A}_{T20}$ and B_{T20} are the second "Mellin" moments of the chiral-odd GPDs:

$$A_{T20}(t) = \int_{-1}^{1} dx\, x H_T(x,\xi,t),\ \tilde{A}_{T20}(t) = \int_{-1}^{1} dx\, x \tilde{H}_T(x,\xi,t),\ B_{T20}(t) = \int_{-1}^{1} dx\, x E_T(x,\xi,t).$$

Using LCWFs derived from the Schlumpf's CQM we obtain

$$\langle \delta^{\mathbf{x}} \mathbf{J}^{\mathbf{x}}_{\mathbf{u}} \rangle = 0.54, \quad \langle \delta^{\mathbf{x}} \mathbf{J}^{\mathbf{x}}_{\mathbf{d}} \rangle = 0.37, \qquad \text{(SCH)}$$

while using the harmonic oscillator wave function of the nucleon (B.-Q. Ma, I. Schmidt, J. Soffer, PL**B 407**, '97), we obtain:

$$\langle \delta^{\mathbf{x}} \mathbf{J}^{\mathbf{x}}_{\mathbf{u}} \rangle = 0.68, \quad \langle \delta^{\mathbf{x}} \mathbf{J}^{\mathbf{x}}_{\mathbf{d}} \rangle = 0.28. \qquad \text{(HO)}$$

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- "Transverse" anomalous magnetic moment

The forward matrix element of $2\tilde{H}_T + E_T$,

$$\kappa_T^q \equiv \int dx \left[2\tilde{H}_T^q(x,0,0) + E_T^q(x,0,0) \right],$$

describes how far and in which direction the average position of quarks with spin in the $\hat{x}$ direction is shifted in the $\hat{y}$ direction for an unpolarized nucleon (Burkardt '05). Thus κ_T^q governs the transverse spin-flavor dipole moment in an unpolarized nucleon $\sim$ to the anomalous magnetic moment κ^q.
We obtain

$$\begin{array}{lll} \kappa_T^u = 3.98, & \kappa_T^d = 2.60, & \text{(SCH)} \\ \kappa_T^u = 3.60, & \kappa_T^d = 2.36. & \text{(HO)} \end{array}$$

The same sign of κ_T^q is predicted in both models, this may have an impact on the Boer-Mulders function $h_1^{\perp q}$. Since for $\kappa_T > 0$ we expect that quarks polarized in the $\hat{y}$ direction should preferentially be deflected in the $\hat{x}$ direction, $\kappa_T^q > 0 \Rightarrow h_1^{\perp q} < 0$ ("Trento convention", Bacchetta *et al.* PR**D 70**, '04).
Furthermore, keeping in mind that the magnitude of κ^q derived within the same approach are of the order of $1 \Rightarrow$ Boer-Mulders function is predicted here larger than the average Sivers function $f_{1T}^{\perp q} \sim -\kappa^q$.

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Summary

- Derivation of the Overlap representation for the chiral-odd Generalized Parton Distributions in a general (model indipendent) framework using the Fock-state decomposition in the transverse-spin basis;
- Application of the formalism to the case of Light Cone Wave Functions obtained by considering only valence quarks in a Costituent Quark Model;
- Different Helicity and Transversity distributions have been derived in the forward limit in agreement with the relativistic requirements and the Soffer inequality;
- Estimation of the axial and tensor "charges" confirming the different size and sign of the up and down quarks predicted within a relativistic quark models;
- Evaluation of the magnitude of the angular momentum $\langle \delta^{\mathbf{x}} \mathbf{J}^{\mathbf{x}}_{\mathbf{q}} \rangle$ carried by transversely polarized quarks in an unpolarized nucleon;
- Calculation of the "Transverse" anomalous magnetic moment, κ^q_T, whose value gives important indications for the magnitude of the Boer-Mulders and Sivers functions.

...and Outlook

- In the CQM we have taken into account only valence quarks and this limits the average longitudinal momentum fraction $\bar{x}$ between ξ and 1. Nevertheless the inclusion of "sea" contributions (to access ERBL region) is possible following, e.g., the lines of the papers: B. Pasquini and S. Boffi, PR**D 71** ('05) and PR**D 73** ('06).

- With the Transversity distribution, h_1, derived from the forward limit of the GPD $\mathcal{H}_T$, we have the opportunity of give some predictions for the Double Transverse-Spin Asymmetries in Drell-Yan dilepton production, $p^\uparrow \bar{p}^\uparrow \to l^+ l^-$, (see Barone *et al.*, PL **B 639** ,'06). Currently, this process is the favorite one for the extraction of Transversity from experimental data (Proposal of the *PAX* collaboration at **GSI**).

S. Boffi, B. Pasquini, M.P., *in preparation.*

$$A_{TT}^{DY} = |\mathbf{S}_{A\perp}||\mathbf{S}_{B\perp}| \frac{\sin^2\theta\cos(2\phi)}{1+\cos^2\theta} \frac{\sum_a e_a^2 h_1(x_A)\bar{h}_1(x_B)}{\sum_a e_a^2 f_1(x_A)\bar{f}_1(x_B)} + [A \leftrightarrow B].$$

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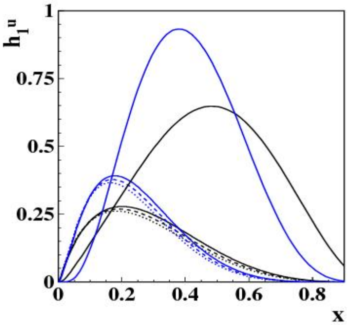
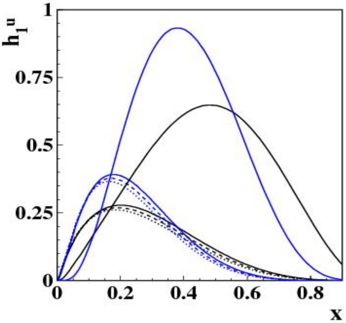


Transversity distribution h_1^u at different Q^2 scales:

- 0.079 GeV^2 solid curve;
- 5 GeV^2 dashed curve;
- 9 GeV^2 dotted curve;
- 16 GeV^2 dashed-dotted curve.

Black curves ⇒ Hypercentral CQM
Blue curves ⇒ Schlumpf's wave functions.



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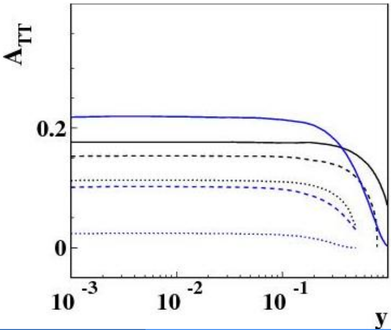

Asymmetry A_{TT} at different Q^2 scales:

- 5 GeV^2 solid curve;
- 9 GeV^2dashed curve;
- 16 GeV^2 dashed-dotted curve.

Black curves ⇒ Hypercentral CQM
Blue curves ⇒ Schlumpf's wave functions.

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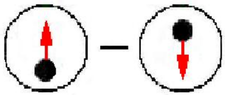
- Boer-Mulders function: $h_1^{\perp q}$ describes the asymmetry of the transverse momentum of quarks perpendicular to the quark spin in an unpolarized nucleon;



- Sivers function: $f_{1T}^{\perp q}$ describes the transverse momentum asymmetry of quarks in a transversely polarized target.