Electromagnetic Form Factors of the nucleon in spacelike and timelike regions

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Outline

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- A covariant expression for the nucleon electromagnetic current: the MANDELSTAM FORMULA
- Nucleon Bethe-Salpeter amplitude
- Quark-photon vertex function
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- Nucleon electromagnetic form factors in the spacelike and in the timelike regions: preliminary results
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Motivations

- Experimental puzzle: the ratio $\mu_p G_{Ep}/G_{Mp}$ very different from 1 for Q^2(SL)>1

The investigation of nucleon electromagnetic form factor in the spacelike and in the timelike regions, within the light-front dynamics:

- open a unique possibility to study the hadronic state, both in the valence and in the non-valence sector

$$|meson\rangle = |q\bar{q}\rangle + |q\bar{q}q\bar{q}\rangle + |q\bar{q}\ g\rangle .....$$

$$|baryon\rangle = \underbrace{|qqq\rangle}_{\text{valence}} + \underbrace{|qqq\ q\bar{q}\rangle + |qqq\ g\rangle .....}_{\text{nonvalence}}$$

the Fock expansion is meaningful within LF framework!!

- yields the possibility to address the vast phenomenology of hadronic resonances (Vector Meson propagation...) in the timelike region, and then impose strong constraints on dynamical models pointing to a microscopical description of hadrons
- allows one to obtain insights into the two-body currents associated to the quark-antiquark pair production (very important in reference frame where $q^+ > 0$)

Mandelstam Formula for the EM nucleon current:SL

The matrix element in the spacelike region for the EM current is given by

$$\langle N;\sigma',p'|j^{\mu}|p,\sigma;N\rangle = \bar{U}_N(p',\sigma')\left[-F_2(Q^2)\,\frac{p'^{\mu}+p^{\mu}}{2M_N} + \left(F_1(Q^2)+F_2(Q^2)\right)\,\gamma^{\mu}\right]U_N(p,\sigma)$$

final and initial nucleon momenta

nucleon spinors

$F_1(Q^2)$ and $F_2(Q^2)$ are the Dirac and Pauli form factors. This matrix element can be approximated ***microscopically*** by the ***Mandelstam formula***:

$$\langle N;\sigma',p'|j^{\mu}|p,\sigma;N\rangle = \int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}Tr\left\{\bar{\Phi}_N^{\sigma'}(k_1,k_2,k_3',p')\right.$$
$$\left.\times\, S^{-1}(k_1)\,S^{-1}(k_2)\,\mathcal{I}^{\mu}(k_1,k_2,k_3,q)\;\Phi_N^{\sigma}(k_1,k_2,k_3,p)\right\}$$

$S(k_i)$ are the Feynman propagators, I^{μ} is the quark-photon vertex function, and Φ_N is the nucleon ***Bethe-Salpeter amplitude***.

Nucleon ***Bethe-Salpeter*** *amplitude*

The nucleon Bethe-Salpeter amplitude is given by:

$$\Phi_N^\sigma(k_1,k_2,k_3,p) = \Lambda(k_1,k_2,k_3)\ \mathcal{P}(k_1,k_2,k_3)\,\chi_{\tau_N}\,U_N(p,\sigma)$$

where **Λ** is a momentum dependent function and ***P*** accounts for the Dirac structure of the qqq-N vertex.
P is based on the following ***effective Lagrangian***:

$$\mathcal{L}_{eff}(x,\tau_1,\tau_2,\tau_3,\tau_N) = \epsilon_{abc}\int d^4x_1\,d^4x_2\,d^4x_3\,\mathcal{F}(x_1,x_2,x_3,x)\ \times$$

$$\Big[m_N\,\alpha\,\frac{1}{\sqrt{2}}\sum_{\tau_1,\tau_2,\tau_3}\bar{q}^a(x_1)\mathcal{T}^\dagger_{\tau_1}i\tau_y\,\mathcal{T}^*_{\tau_2}\,\gamma^5\,q^b_C(x_2)\,\bar{q}^c(x_3)\mathcal{T}^\dagger_{\tau_3} +$$

$$-\,(1-\alpha)\,\frac{1}{\sqrt{6}}\sum_{\tau_1,\tau_2,\tau_3}\bar{q}^a(x_1)\mathcal{T}^\dagger_{\tau_1}\,i\tau_y\,\mathcal{T}^*_{\tau_2}\,\gamma^5\,\gamma_\mu\,q^b_C(x_2)\cdot\bar{q}^c(x_3)\mathcal{T}^\dagger_{\tau_3}(-i\,\partial^\mu)\Big]\,\psi_N(x)\mathcal{T}_{\tau_N} +$$

$$+\,\mathrm{cyclic} + \mathrm{h.c.}$$

M. Beyer *et al*,
nucl-th/9804021

We take α=1

Quark-photon vertex function

We have an isoscalar and an isovector contribution:

$$\mathcal{I}^{\mu}_{3}(k_3,q) = \left(\frac{1+\tau_z}{2}\right) \mathcal{I}^{\mu}_{u}(k_3,q) + \left(\frac{1-\tau_z}{2}\right) \mathcal{I}^{\mu}_{d}(k_3,q) =$$

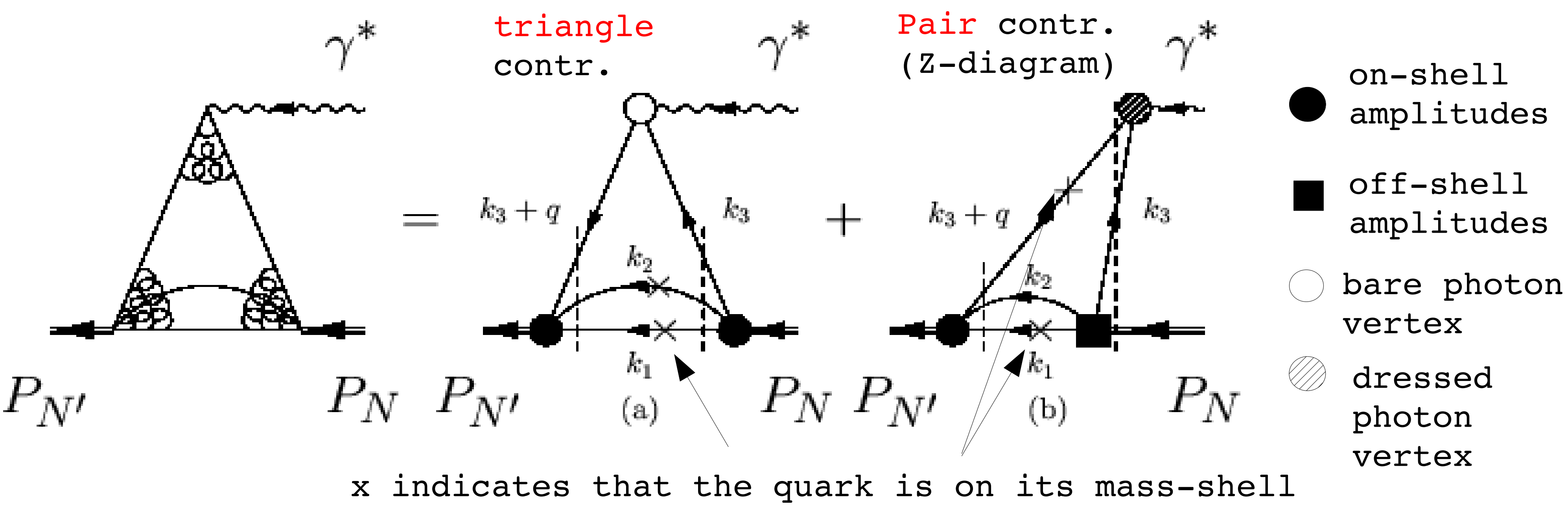
$$= \frac{\mathcal{I}^{\mu}_{u}(k_3,q) + \mathcal{I}^{\mu}_{d}(k_3,q)}{2} + \tau_z \frac{\mathcal{I}^{\mu}_{u}(k_3,q) - \mathcal{I}^{\mu}_{d}}{2} = \mathcal{I}^{\mu}_{IS}(k_3,q) + \tau_z \mathcal{I}^{\mu}_{IV}(k_3,q)$$

Each term contains a purely valence contribution (in the spacelike region only), i. e. an ***elastic term*** (triangle) and a ***pair production term*** (Z-diagram)

Projecting out the Mandelstam formula on the Light-Front: SL

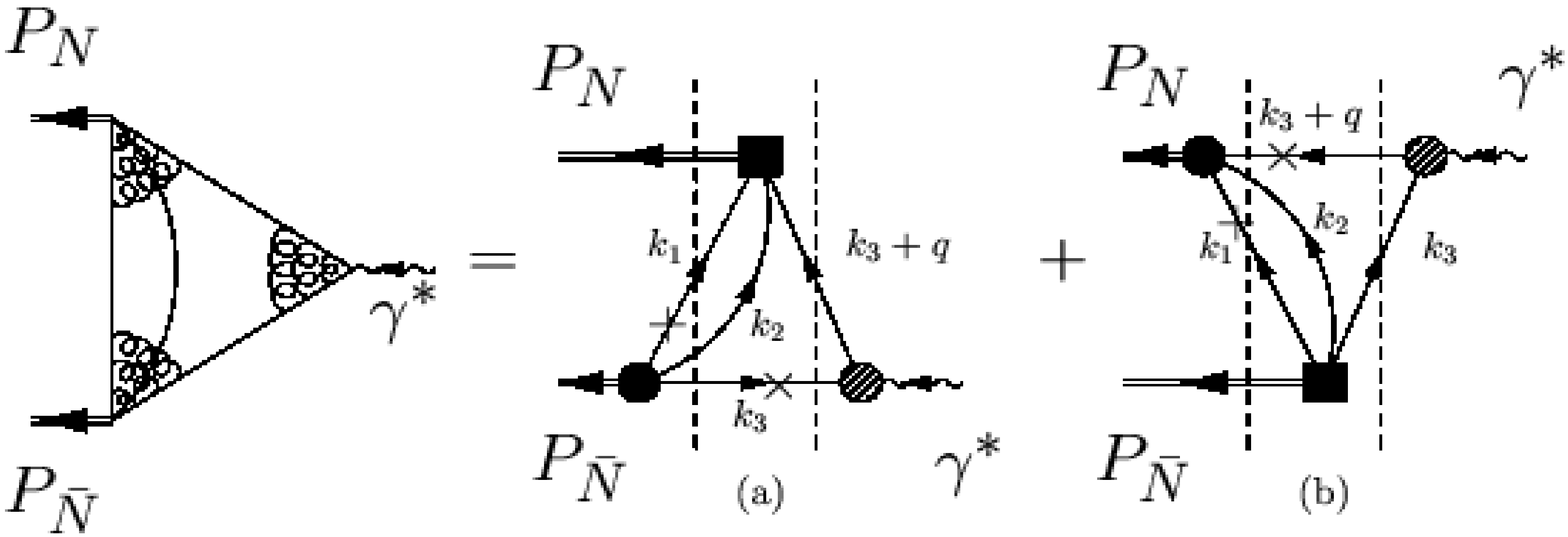
...through a k- integration, by adopting the so-called **PPA** (*Propagator Pole Approximation*), *i.e.* by disregarding the analytical structure form the Bethe-Salpeter amplitudes.
The reference frame we will adopt is $q^+ > 0$ and $q_{perp} = 0$.

Spacelike process:



Projecting out the Mandelstam formula on the Light-Front: TL

...and timelike process:



Phenomenological approximations: 1

We will introduce some phenomenological approximations:
the momentum dependent parts of the nucleon Bethe-Salpeter amplitudes are approximated as follows:

Nucleon:

$$W_N \sim \frac{1}{\left[\beta^2 + M_0^2(1,2,3)\right]^3}$$

In the **valence sector** the spectator quarks are on their own k- -shell, and the momentum dependence, reduced to a 3-momentum dependence through the LF projection, is approximated through a **Nucleon Wave Function** *a la* Brodsky (pQCD ispired)

In the **non-valence** vertex, needed to evaluate the Z-diagram contribution, the momentum dependence is approximated by this expression:

$$G_N \sim \frac{1}{\left[\beta^2 + M_0^2(1,2)\right]^2} \left\{ \frac{1}{\left[\beta^2 + M_0^2(3',2)\right]} + \frac{1}{\left[\beta^2 + M_0^2(3',1)\right]} \right\}$$

Quark-photon vertex function

It is composed by a ***bare*** term and by a ***dressed*** term:

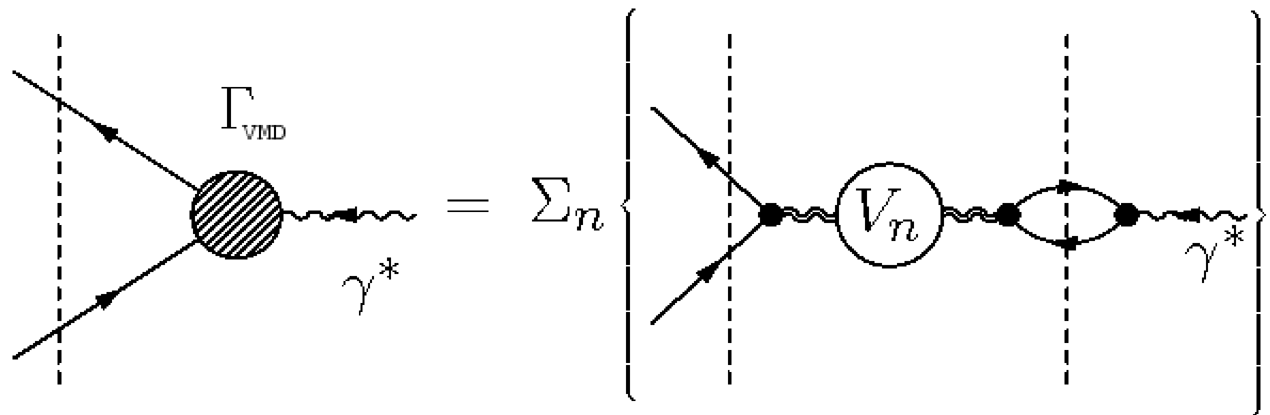
$$\mathcal{I}^{\mu}_{IS(IV)}(k,q) = \frac{1}{2}\,\theta(p^+ - k^+)\,\theta(k^+)\gamma^{\mu} + \theta(q^+ + k^+)\,\theta(-k^+)\left[\frac{Z_B}{2}\gamma^{\mu} + Z_{VMD}\Gamma^{\mu}_{VMD}(k,q,IS(IV))\right]$$

bare terms

proportional to γ^{μ}

dressed term

The dressed term is described through a microscopical ***VMD approximation***:



T. Frederico *et al*, PRD 66 (2002) 116011

where the ***vertex function*** is taken as

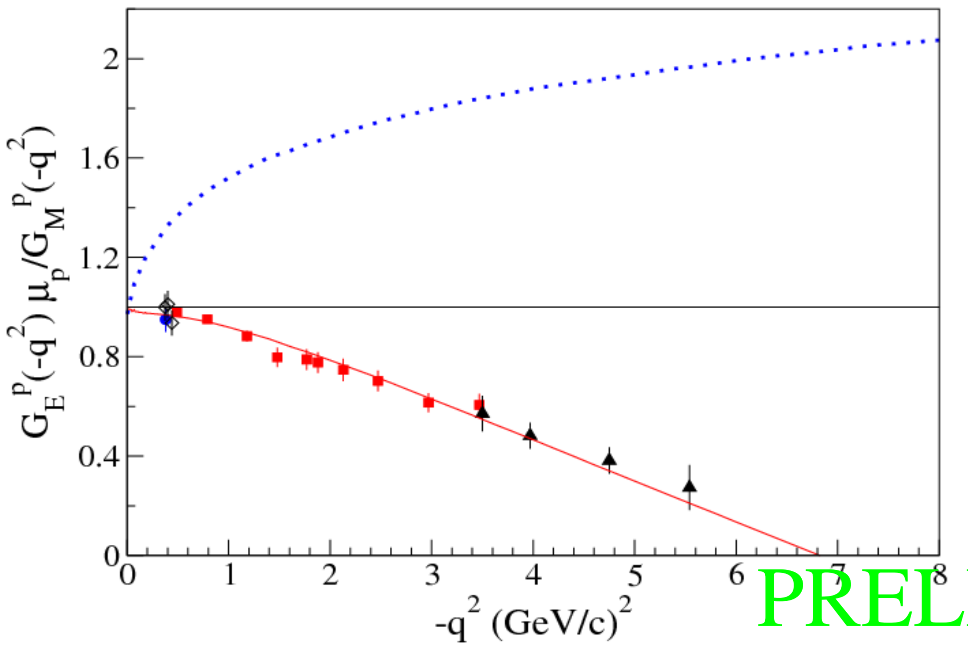
$$\widehat{V}^{\mu}_{n}(k,k-P_n) = \gamma^{\mu} - \frac{k^{\mu}_{on} - (q-k)^{\mu}_{on}}{M_0(k^+,\mathbf{k}_{\perp};q^+,\mathbf{q}_{\perp}) + 2m}$$

Adjusted parameters

In this model, the following parameters appear:

- constituent quark mass $\boldsymbol{m_u = m_d = 220}$ ***MeV***
- the oscillator strength ω = 1.556 GeV^2 appearing in the VMD approximation, chosen in order to reproduce the e.m. decay widths of the first four vector mesons
- β=0.118 GeV, fixed through the anomalous magnetic moments: μ_p= 2.878 (Exp. 2.793), μ_n= -1.859 (Exp. -1,913)
- the widths for the vector meson Γ_n= 0.15 GeV
- the renormalization constants for the pair production term Z_B and Z_{VMD}

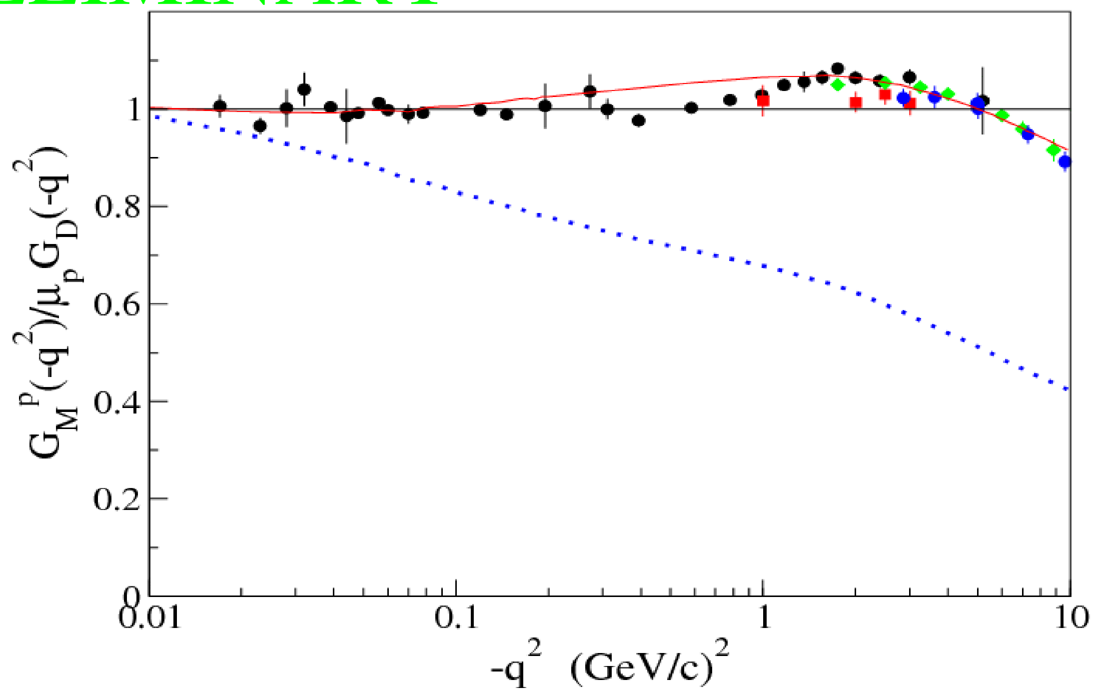
Results in the spacelike region: PROTON



blue dotted line: triangle

red line: triangle + Z-diagram

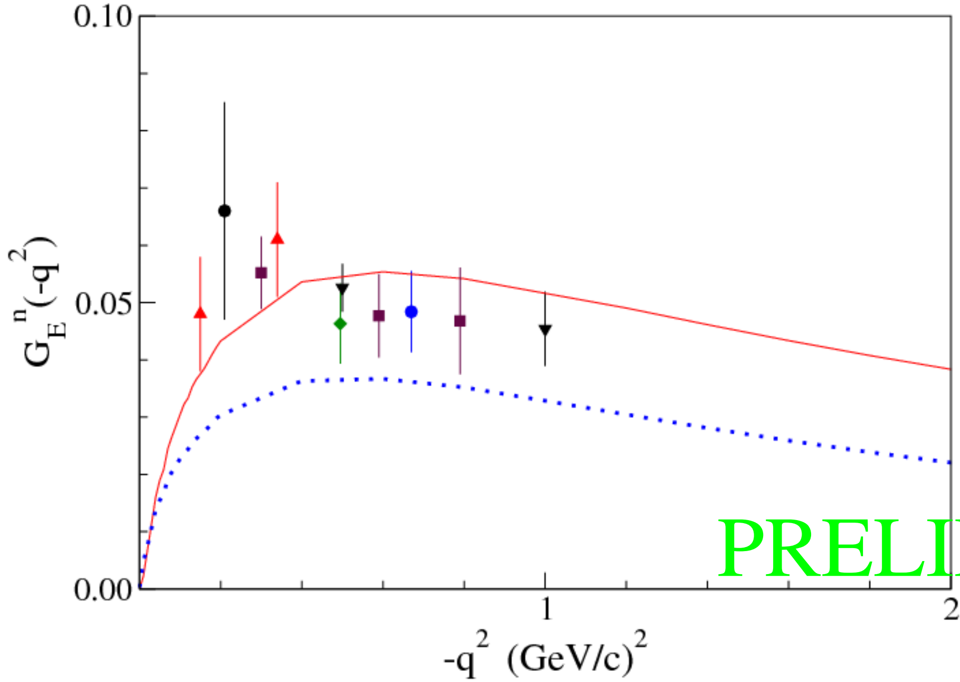
PRELIMINARY



Considerations:

- From these results turns out to be very important the Z-diagram term
- The almost linear decrease in the ratio is well reproduced

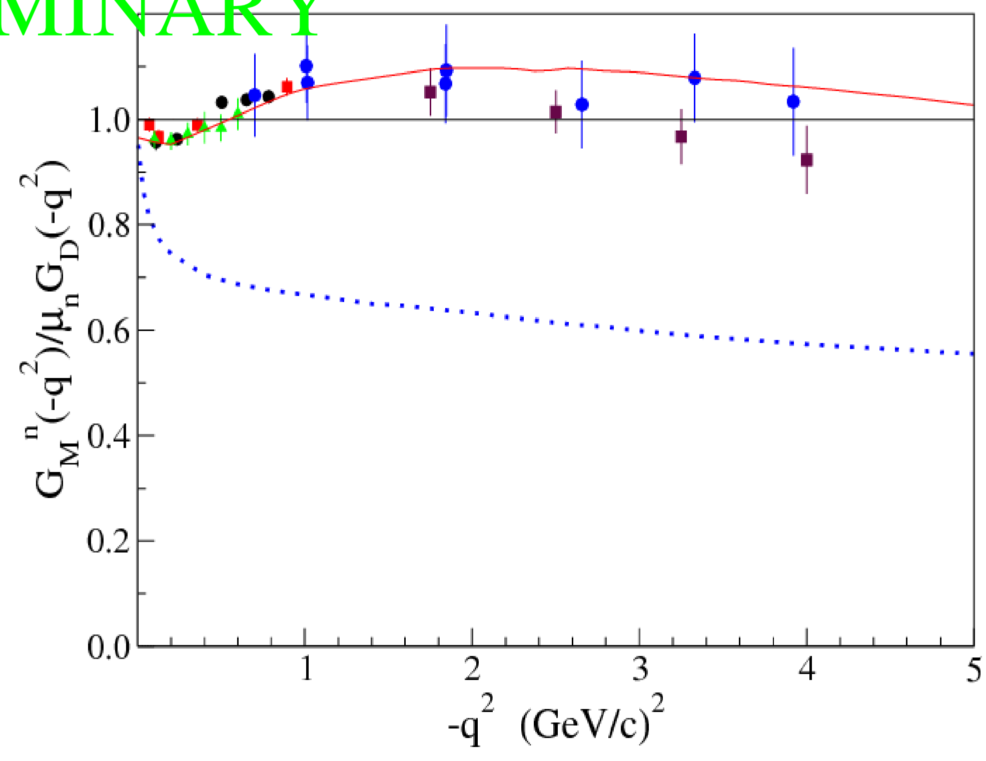
Results in the spacelike region: NEUTRON



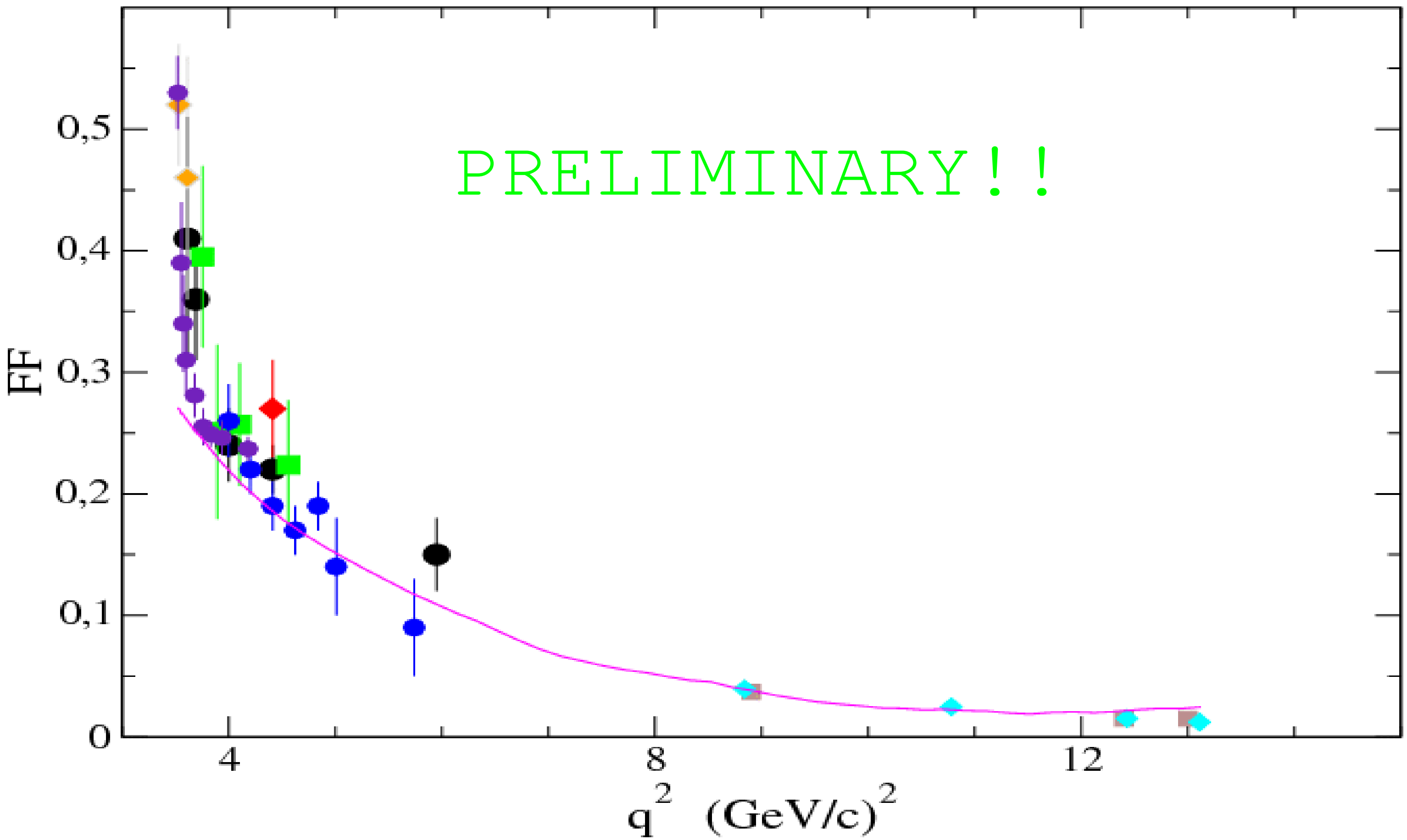
blue dotted line: triangle

red line: triangle + Z-diagram

PRELIMINARY



Proton results in the timelike region



$$FF = \sqrt{\frac{\sigma_{(e^+e^- \to N\bar{N})}}{\sigma_{pl}}}$$

Z_B (TL) ~ 3 Z_B(SL)

Conclusions...

- A microscopical model for nucleon electromagnetic form factors in both spacelike and timelike region has been proposed
- The quark-photon vertex for the process where a virtual photon materializes in a quark-antiquark pair is approximated by a VMD model plus a bare term
- the *Z*-diagram are fundamental in the adopted reference frame ($q^+ > 0$)
- The preliminary results in the spacelike region are quite good, and the possible zero in the ratio μpGEp/GMp seems to be strongly related to the *Z*-diagram contribution

... and Perspectives

- A better description of the quark-photon vertex may improve the results: a fully covariant VMD model is under investigation
- Different approximations for the nucleon wave function may be tested.