Phases of QCD

lattice thermodynamics, quasiparticles and Polyakov loop

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Can results of

Lattice QCD Thermodynamics

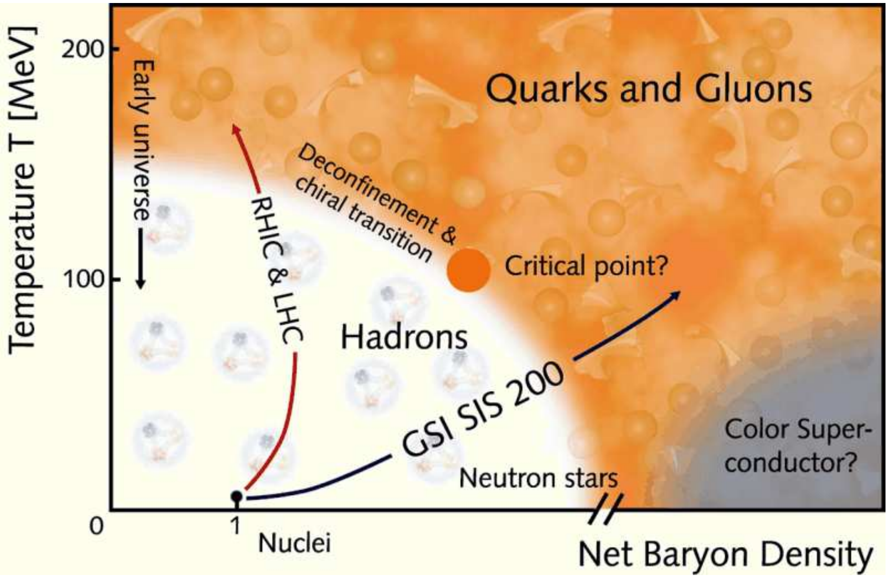
be understood in terms of

QUASIPARTICLE

degrees of freedom?

In collaboration with Simon Rößner, Michael A. Thaler and Wolfram Weise

Introduction



- QCD has a rich phase structure
- Many challenging items:
  - order of the phase transition
  - critical point
  - deconfinement and chiral symmetry
  - colour superconductivity at high μ

- Status of lattice QCD thermodynamics:
  - data available in pure gauge sector
  - quarks easily introduced at $\mu = 0$
  - first lattice data at small μ.

- Taylor expansion method
  - power series of operators in μ/T
  - expansion coefficients at μ/T=0
  - data up to sixth order in μ/T.

Purpose of our work

- MODEL FORMULATION
  - improve NJL model by including Polyakov loop dynamics
  - Polyakov loop dynamics fixed by comparison with pure gauge QCD.
  - parameter fixing in hadronic and pure gauge sectors

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- Taylor expansion coefficients at $\mu/T = 0$.
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MODEL PREDICTIONS AT HIGH μ

- test the validity of the Taylor expansion by comparing the full result and the truncated one for different observables;
- phase diagram and quark mass dependence
- position of critical point and quark mass dependence.

P. Meisinger and M. Ogilvie (1996)- K. Fukushima (2004)

PNJL (Polyakov loop extended NJL) model

Starting point: NJL model in temporal background gauge field

$$\mathcal{L}_{PNJL} = \bar{\psi}\left(i\gamma_\mu D^\mu - \hat{m}_0\right)\psi + \frac{G}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right] - V\left(\Phi, T\right),$$

where:

$$D_\mu = \partial_\mu - igA_\mu \quad \text{and} \quad A_\mu = \delta_{\mu 0}A_0 \ .$$

Coupling between Polyakov loop and quarks uniquely determined by covariant derivative D_μ.
We recall that:

$$\Phi\left(x\right) = \frac{1}{N_c}\mathrm{Tr}\left[\mathcal{P}\exp\left(i\int_0^\beta A_4\left(x,\tau\right)d\tau\right)\right] = \frac{1}{3}\mathrm{Tr}\exp\left[\frac{iA_4^a\lambda_a}{T}\right]$$

Parameters

Λ [GeV]	0.651
G [GeV^{-2}]	10.078
m_0 [MeV]	5.5

Physical quantities

f_π [MeV]	92.4
$\lvert\langle\bar{\psi}\psi\rangle\rvert^{1/3}$ [MeV]	247
m_π [MeV]	139.3

Polyakov loop potential

- The Polyakov loop is the order parameter related to the $Z(N_c)$ symmetry

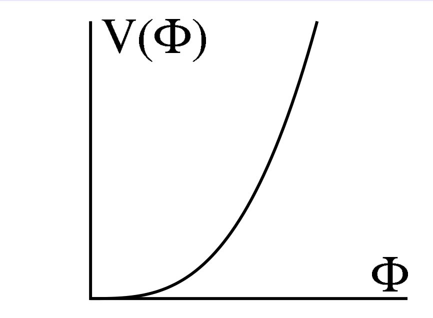
$$\frac{V(\Phi,T)}{T^4} = -\frac{b_2(T)}{2}\Phi^*\Phi - b_4\left(\frac{T_0}{T}\right)^3 \ln[1-6\Phi^*\Phi+4\left(\Phi^{*3}+\Phi^3\right)-3\left(\Phi^*\Phi\right)^2]$$

with

$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2$$

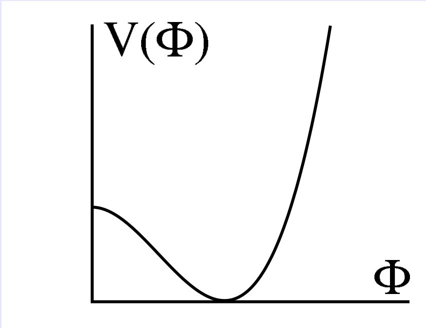
$T < T_0$

- color confinement
- $\langle\Phi\rangle = 0 \longrightarrow Z(3)$ unbroken



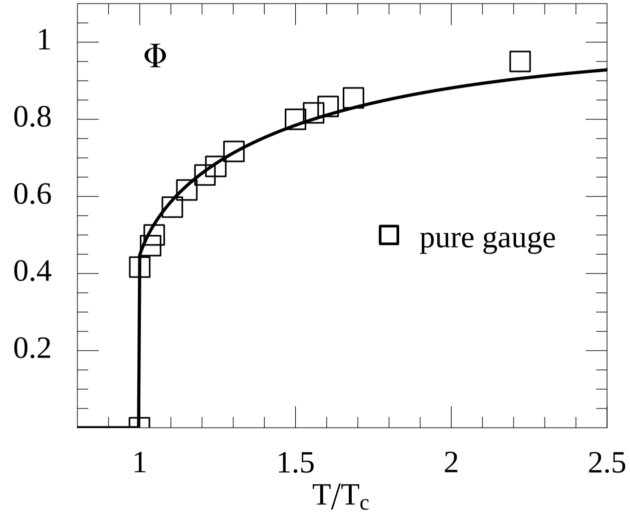
$T > T_0$

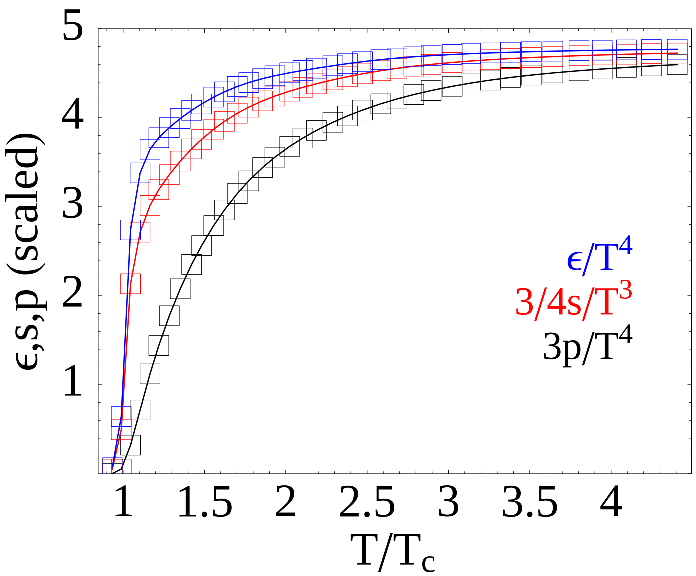
- color deconfinement
- $\langle\Phi\rangle \neq 0 \longrightarrow Z(3)$ broken



Fit to Pure Gauge QCD lattice data

- Minimization of $V(\Phi, T)$: Polyakov loop behaviour as a function of T
- Comparison with lattice data from Kaczmarek *et al.* PLB 543 (2002)





- $p(T) = -V(\Phi(T), T)$
- $s(T) = \frac{dp}{dT} = -\frac{dV(\Phi(T),T)}{dT}$
- $\epsilon(T) = T\frac{dp}{dT} - p = Ts(T) - p(T)$
- Comparison with lattice data from Boyd *et al.* NPB 469 (1996)

PNJL model at finite temperature and quark chemical potential

1-quark (antiquark) states, suppressed below T_c

2-quark (antiquark) states, suppressed below T_c

$$\Omega\left(T,\mu,\sigma,\Phi\right) = V\left(\Phi,T\right) + \frac{\sigma^2}{2G}$$
$$-2N_f \int \frac{\mathrm{d}^3p}{(2\pi)^3} \left\{ 3E_p + T \left[\ln\left[1 + 3\Phi \mathrm{e}^{-(E_p-\mu)/T} + 3\Phi^* \mathrm{e}^{-2(E_p-\mu)/T} + \mathrm{e}^{-3(E_p-\mu)/T}\right] \right.\right.$$
$$\left.\left. + \ln\left[1 + 3\Phi^* \mathrm{e}^{-(E_p+\mu)/T} + 3\Phi \mathrm{e}^{-2(E_p+\mu)/T} + \mathrm{e}^{-3(E_p+\mu)/T}\right]\right]\right\}$$

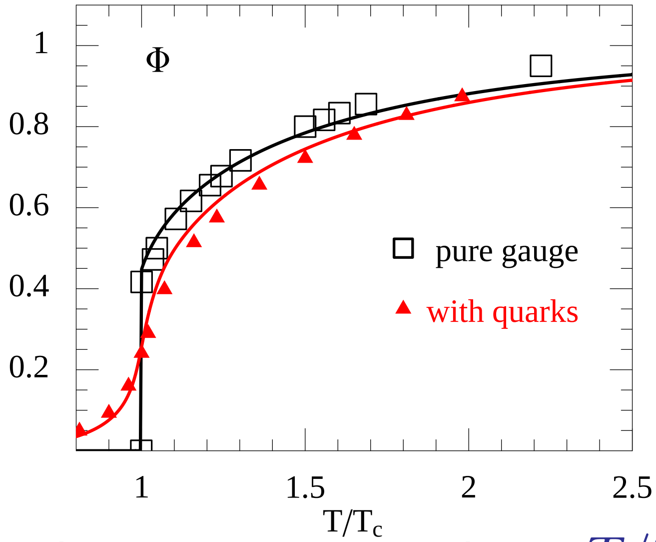
3-quark (antiquark) states, not suppressed even below T_c

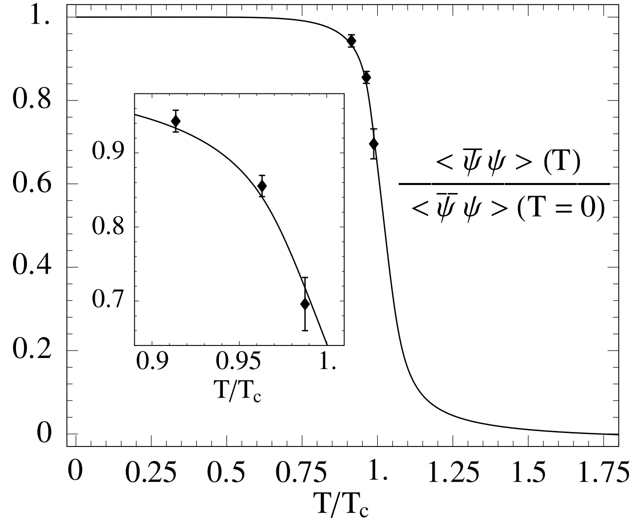
with $E_p = \sqrt{\vec{p}^{\,2} + m^2}$ and $m = m_0 - \langle\sigma\rangle = m_0 - 2G\langle\bar{\psi}\psi\rangle$ is the constituent quark mass.

High temperature limit: $\Phi \longrightarrow 1$, $\Phi^* \longrightarrow 1$: we re-obtain the standard NJL formula:

$$\ln\left[1 + 3\Phi \mathrm{e}^{-(E_p-\mu)/T} + 3\Phi^* \mathrm{e}^{-2(E_p-\mu)/T} + \mathrm{e}^{-3(E_p-\mu)/T}\right]$$
$$\downarrow \quad T \rightarrow \infty$$
$$\ln\left[1 + \mathrm{e}^{-(E_p-\mu)/T}\right]^3 = 3\ln\left[1 + \mathrm{e}^{-(E_p-\mu)/T}\right]$$

$\mu = 0$ PREDICTIONS





- Scaled pressure as a function of T/T_c

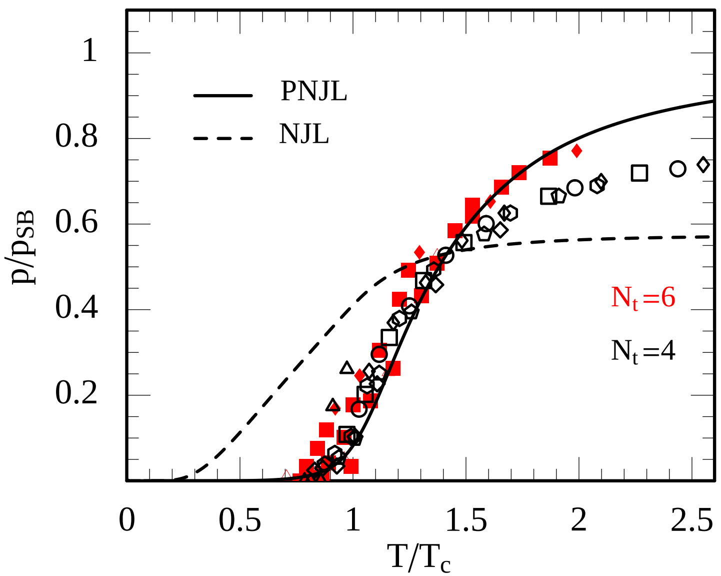
$$\frac{p(T,\mu=0)}{T^4} = -\frac{\Omega(T,\mu=0)}{T^4}$$

- Comparison with lattice data from

  Kaczmarek and Zantow (2005)

  Boyd *et al. (1995)*

  CP-PACS collaboration (2001)



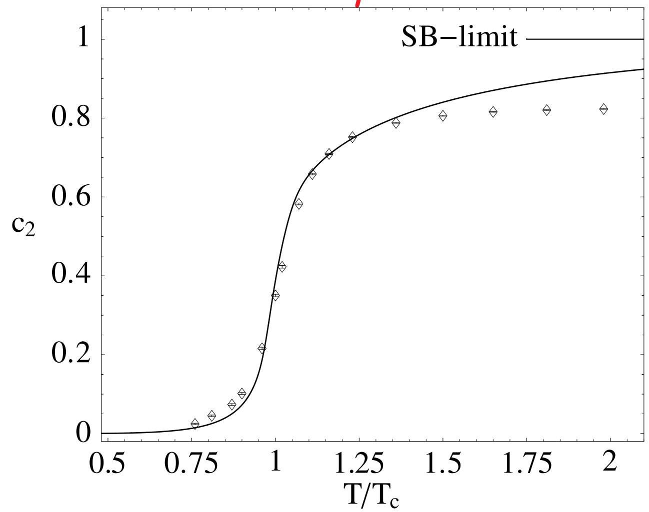
C.R., M. A. Thaler and W. Weise, PRD 73 (2006).

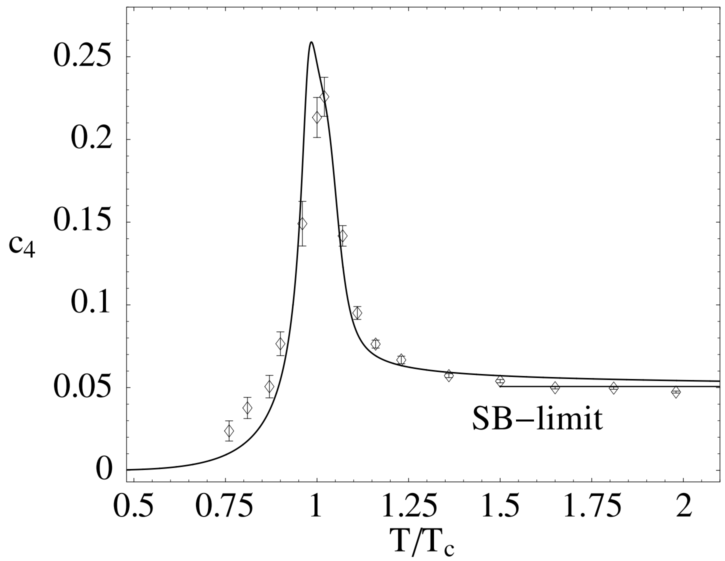
C.R., S. Rößner and W. Weise, hep-ph/0609281.

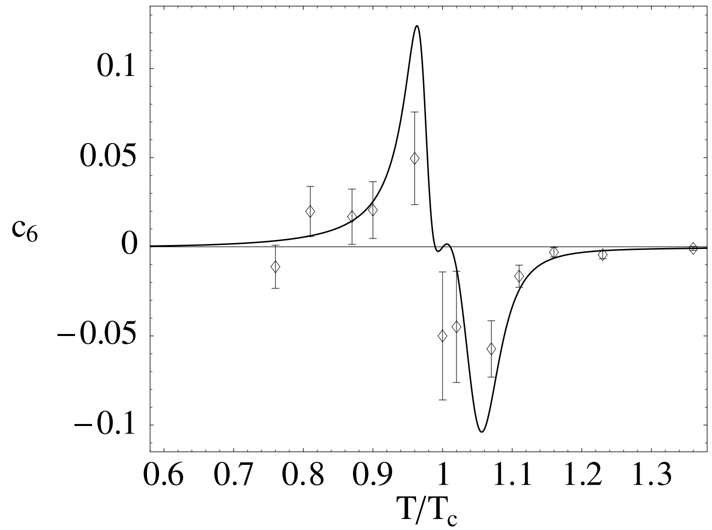
Taylor expansion of the pressure around $\mu = 0$

$$\frac{p(T,\mu)}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu}{T}\right)^n ;$$

$$c_n(T) = \frac{1}{n!} \frac{\partial^n \left(p(T,\mu)/T^4\right)}{\partial(\mu/T)^n}\Bigg|_{\mu=0}$$

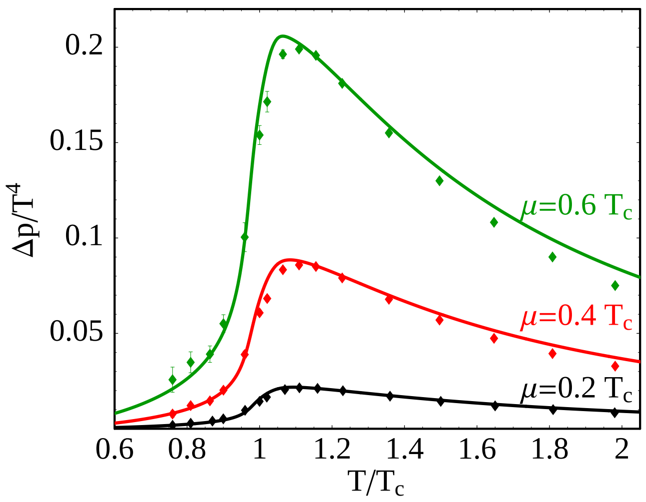


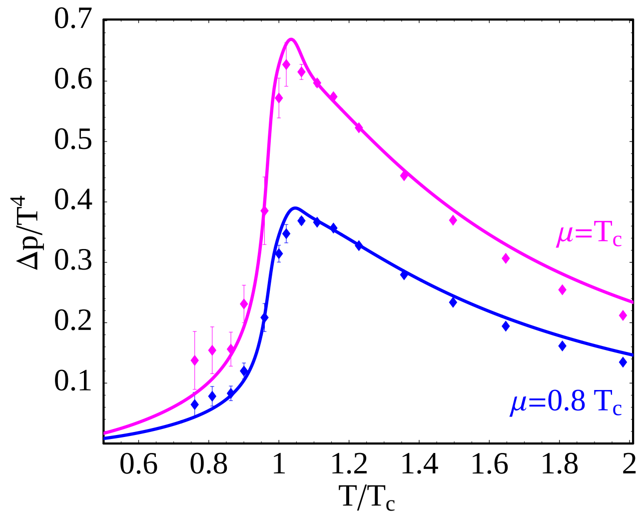




C.R., S. Rößner, M. A. Thaler and W. Weise, hep-ph/0609218, to appear in EPJC.
C.R., S. Rößner and W. Weise, hep-ph/0609281. Lattice data from Allton *et al. (2005)*.

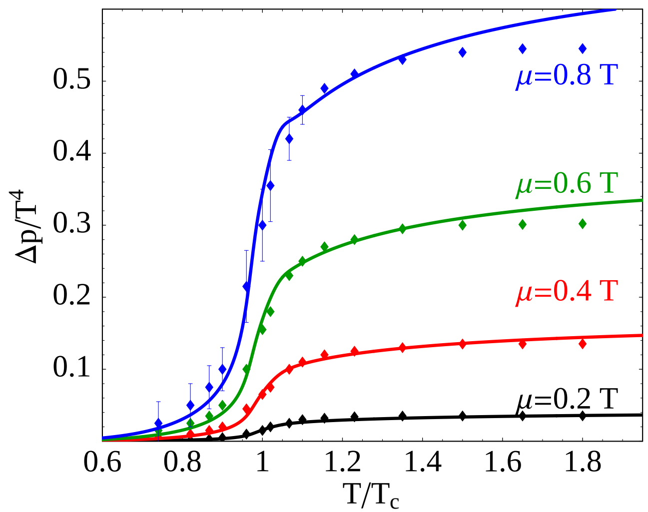
Finite μ PREDICTIONS: pressure





$$\frac{p(T,\mu)}{T^4} = \sum_{n=0}^{\infty} c_n(T)\left(\frac{\mu}{T}\right)^n ;$$

$$c_n(T) = \frac{1}{n!}\frac{\partial^n\left(p(T,\mu)/T^4\right)}{\partial(\mu/T)^n}\Bigg|_{\mu=0}$$



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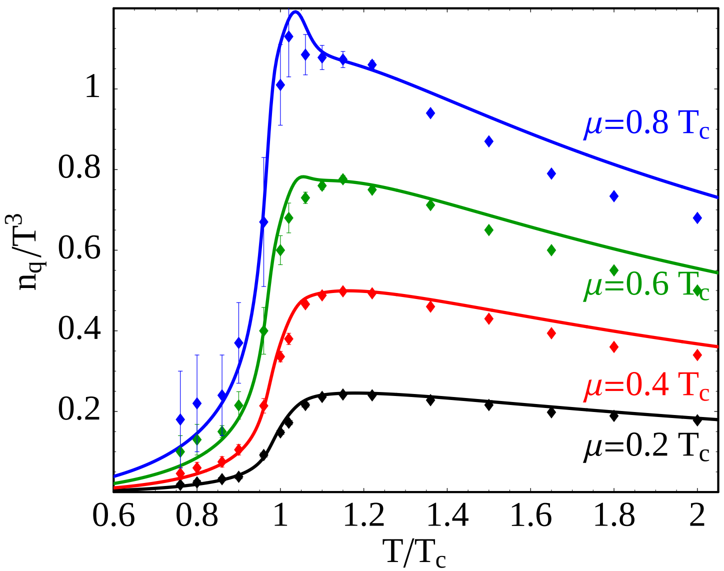
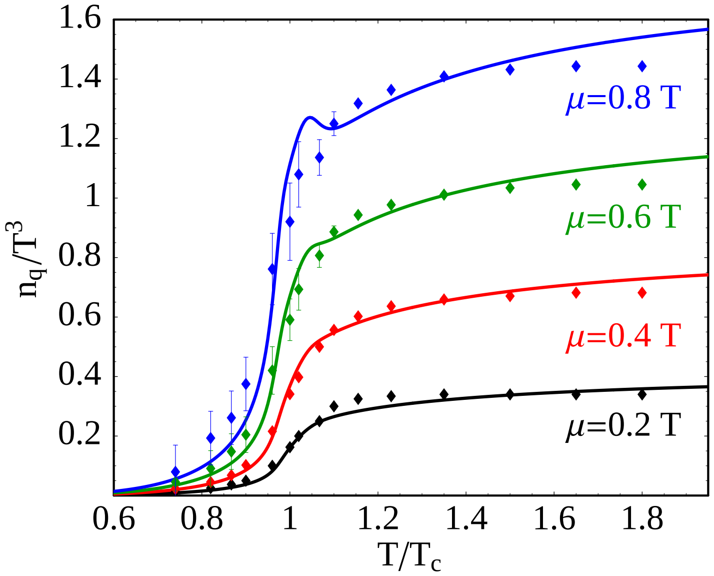
Finite μ PREDICTIONS: quark number density

$$\frac{n_q(T,\mu)}{T^3} = \frac{\partial\left(p/T^4\right)}{\partial\left(\mu_q/T\right)} = 2\,c_2\,\frac{\mu_q}{T} + 4\,c_4\left(\frac{\mu_q}{T}\right)^3 + 6\,c_6\left(\frac{\mu_q}{T}\right)^5$$





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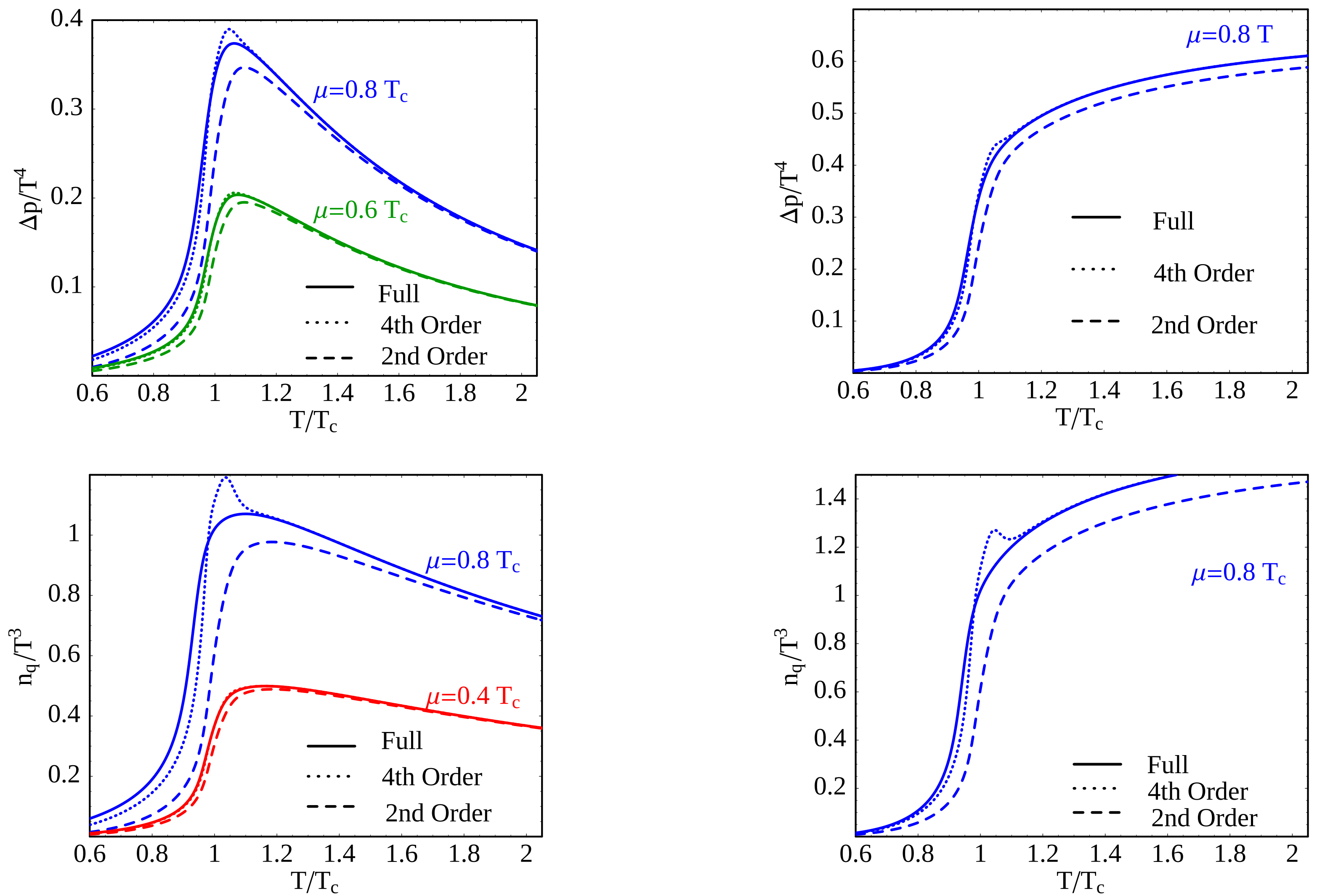
Lattice data from Allton *et al. (2005)*.

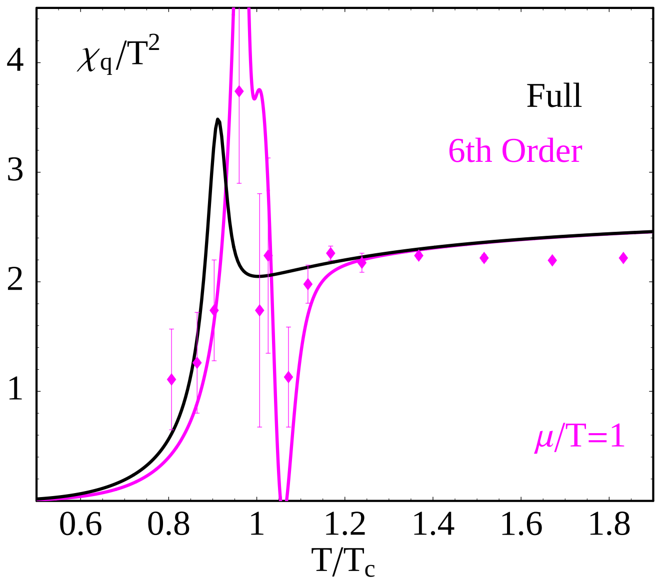
Comparison between Taylor-expanded and full results

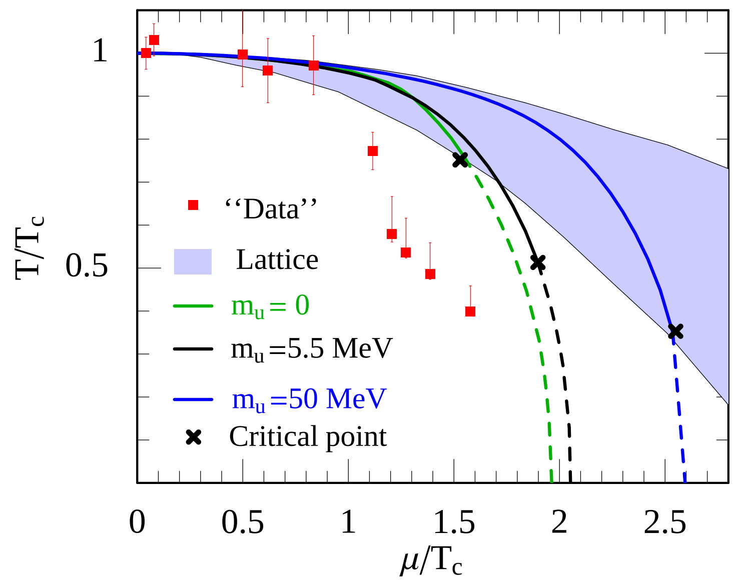


C.R., S. Rößner, M. A. Thaler and W. Weise, hep-ph/0609218, to appear in EPJC.

Quark number susceptibility at finite μ: First order phase transition?

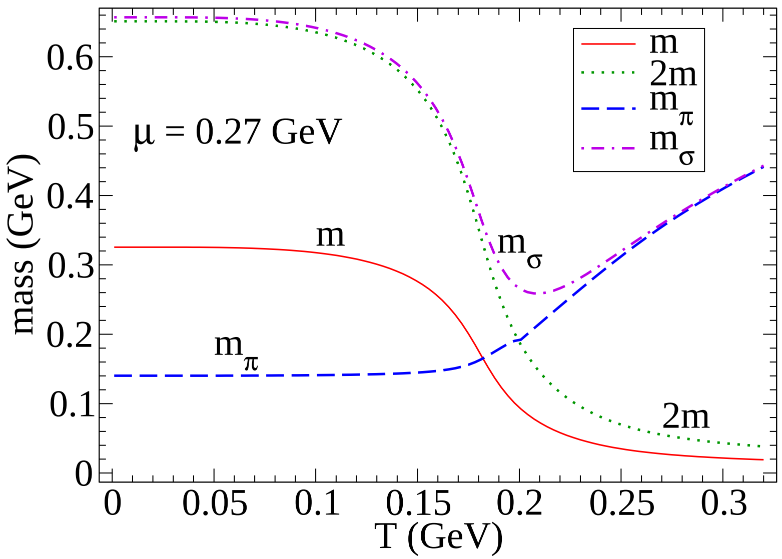
$$\frac{\chi_q(T,\mu)}{T^2} = \frac{\partial\left(n_q/T^3\right)}{\partial\left(\mu_q/T\right)} = 2\,c_2 + 12\,c_4\left(\frac{\mu_q}{T}\right)^2 + 30\,c_6\left(\frac{\mu_q}{T}\right)^4$$

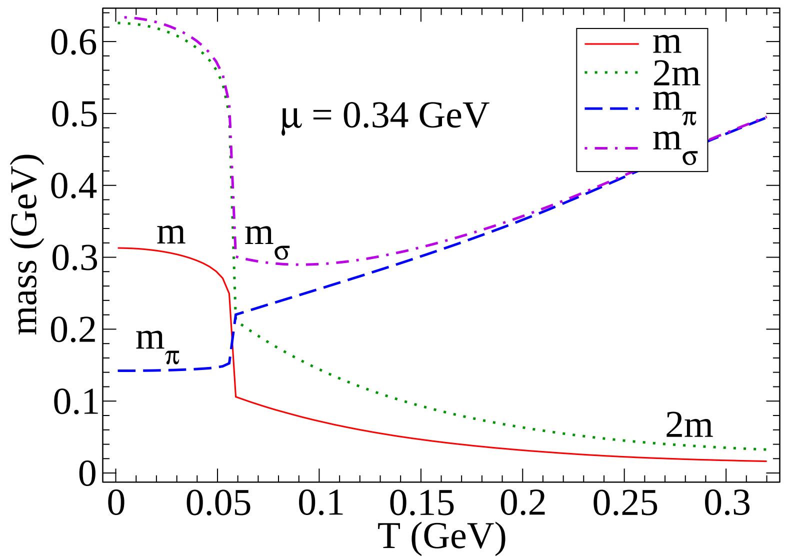




C.R., S. Rößner, W. Weise, hep-ph/0609281.
Lattice data from Allton *et al. (2005)*.
Therm. fit data from Andronic *et al.* (2005).

Mesonic properties in the PNJL model



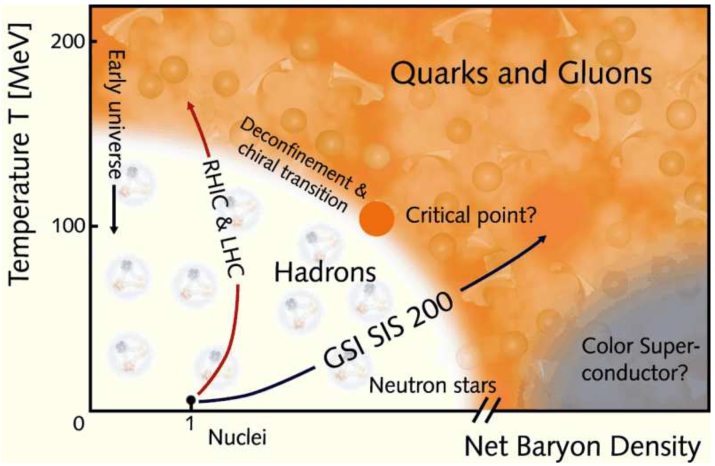


H. Hansen, W. M. Alberico, A. Beraudo, A. Molinari, M. Nardi, C. R., hep-ph/0609116.

Conclusions

- The standard NJL model fails in reproducing QCD thermodynamics
- PNJL model as a minimal synthesis of confinement and chiral symmetry breaking
- A description of QCD thermodynamics with our simple model works very well
- Taylor series converging very quickly at relatively small chemical potentials
- Discrepancy between truncated and full results observed at larger chemical potentials
  - → Quark number susceptibilities

Outlook



- Exploration of the thermodynamics and phase diagram for $N_f = 2 + 1$ and $N_f = 3$
- Improvement of approximation: going beyond mean field approximation