THE AMAZING PROPERTIES OF CRYSTALLINE COLOR SUPERCONDUCTORS

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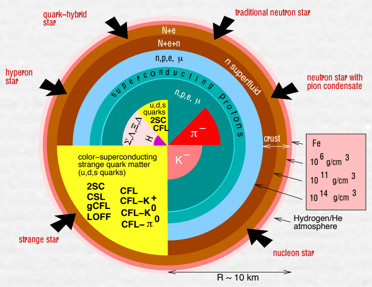
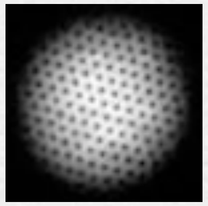
Cortona Oct. 2013

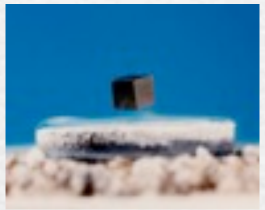
Outline

- Motivations

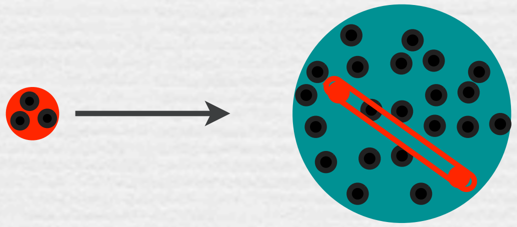


- Superfluids and Superconductors

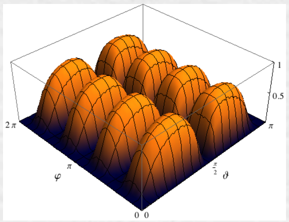




- Color Superconductors



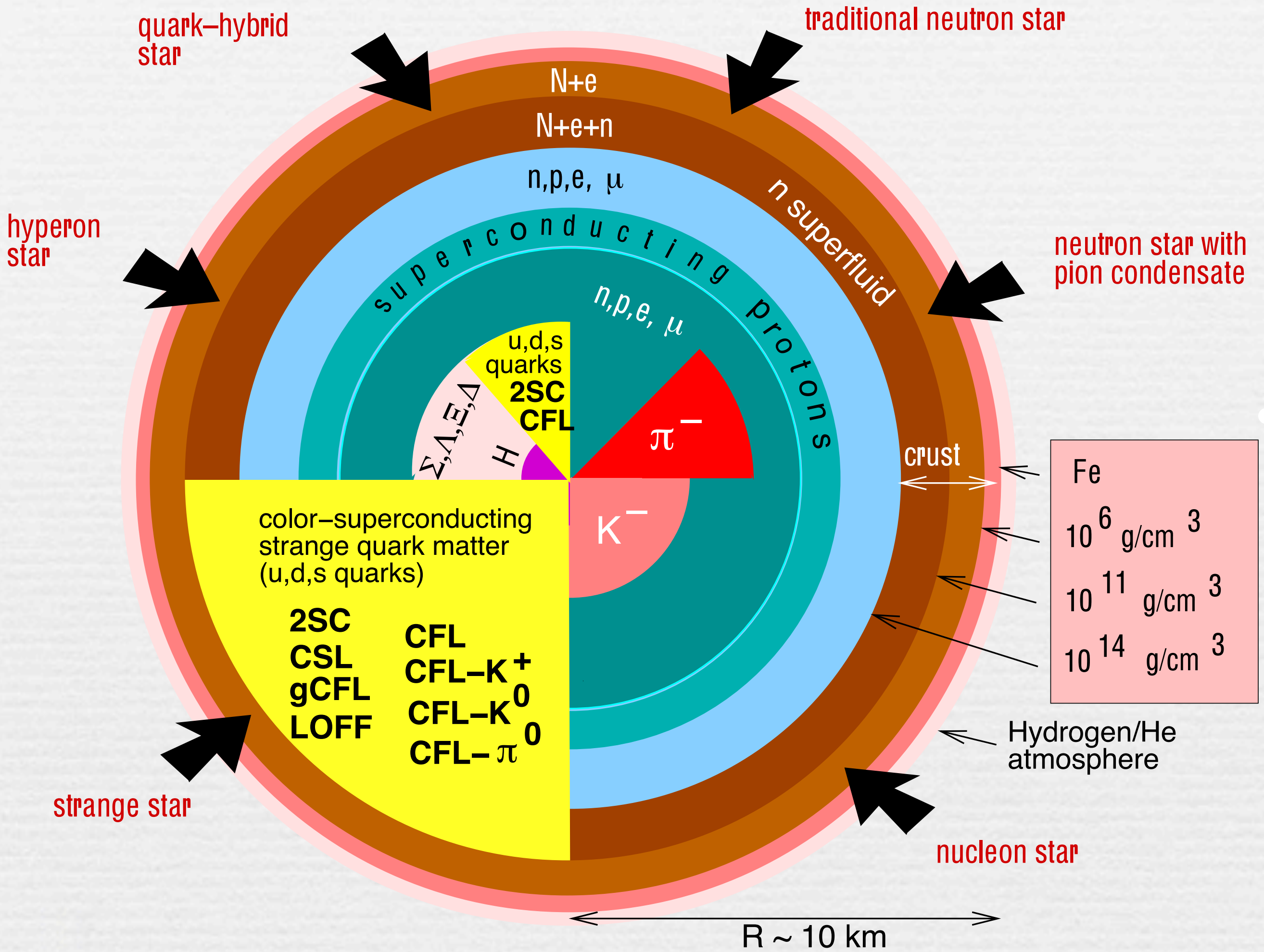
- Crystalline Color Superconductors



Review [arXiv:1302.4264]

MOTIVATIONS

Compact stars



F. Weber, Prog.Part.Nucl.Phys. 54 (2005) 193

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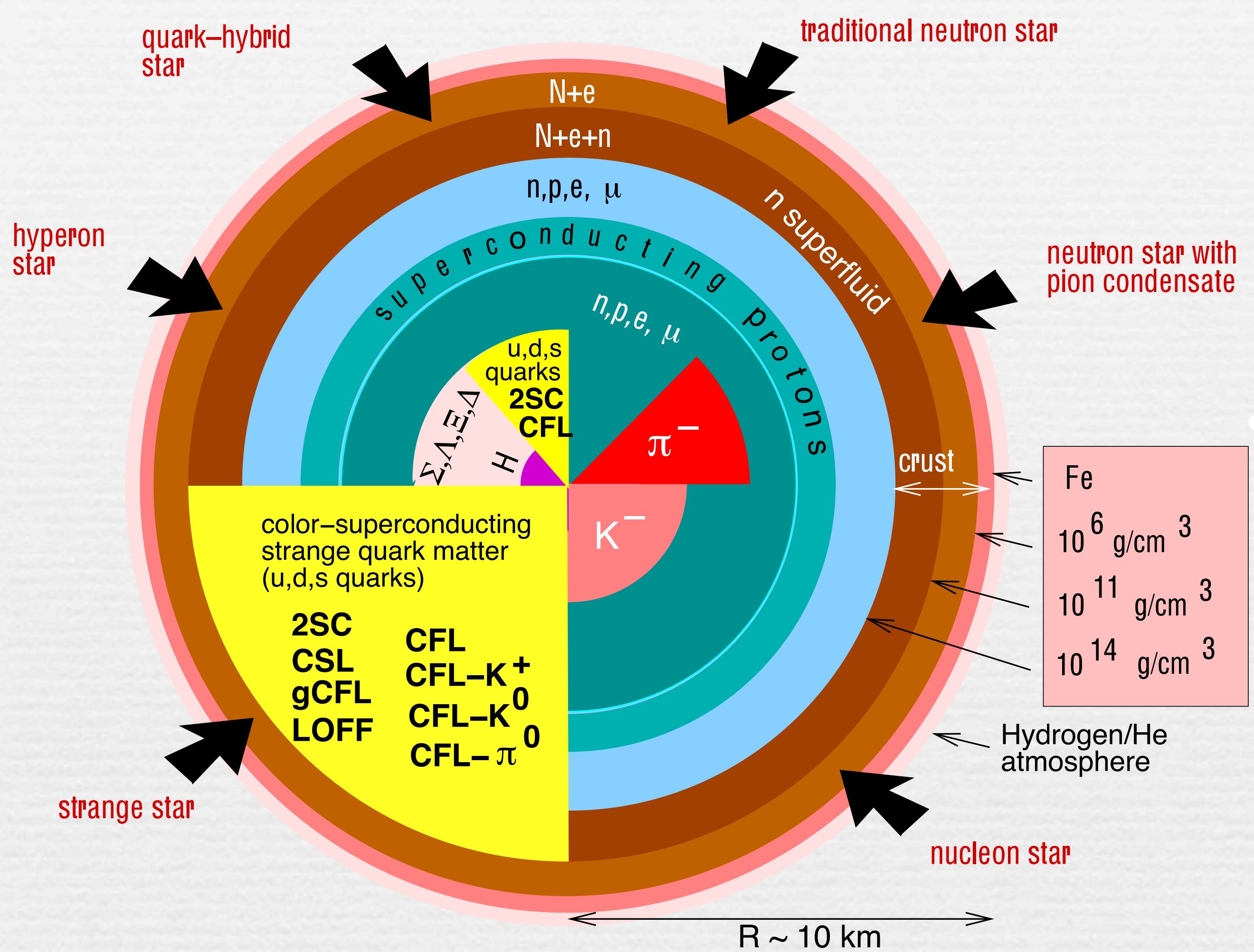

"Probes"

cooling

glitches

instabilities

mass-radius

magnetic field

GW

……

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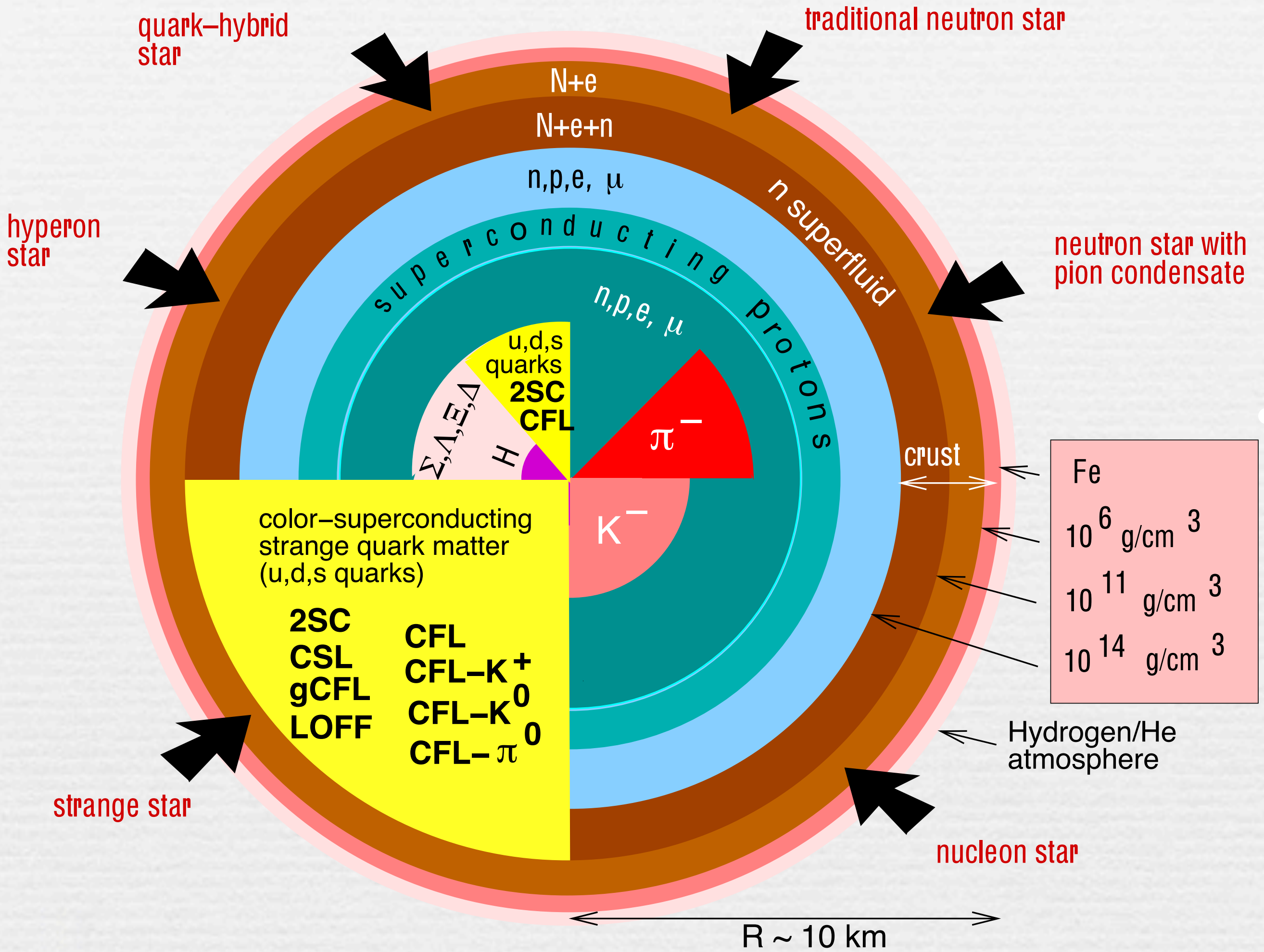

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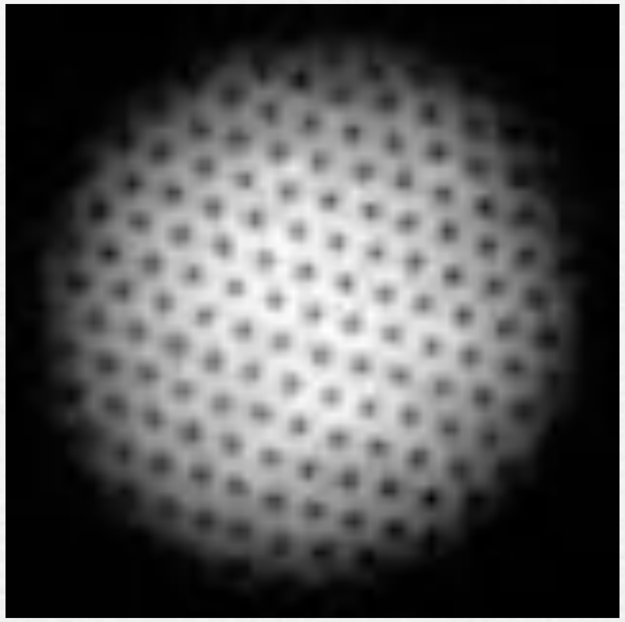
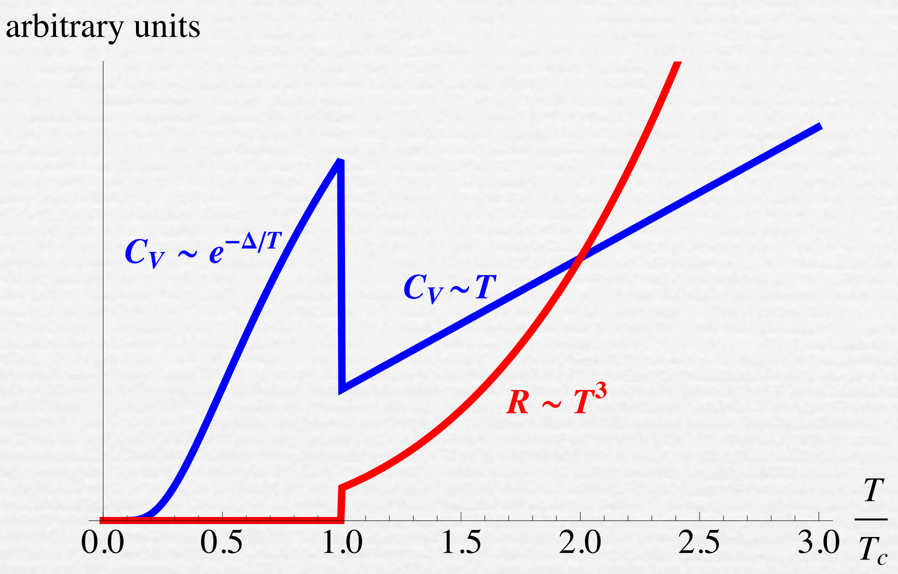
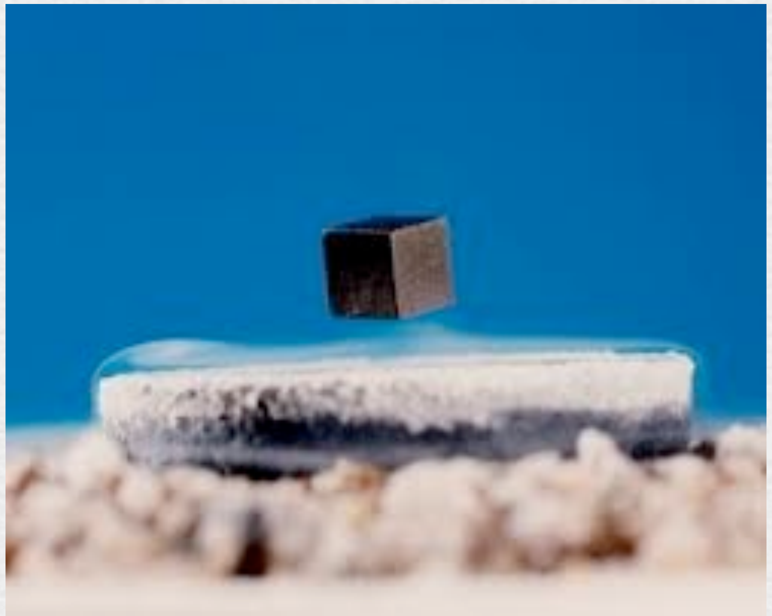
......

Very massive compact stars?
PSR J1614-2230 mass M ~ 2 M⊙ Demorest et al Nature 467, (2010) 1081
Tension with quark matter models Bombaci et al. Phys. Rev. C 85, (2012) 55807
Unlikely that one single model can explain everything, see for example Drago et al. arXiv:1309.7263





SUPERFLUIDS AND SUPERCONDUCTORS



Superfluid vs Superconductors

Definitions

Superfluid: frictionless fluid with $\boldsymbol{v} = \boldsymbol{\nabla}\varphi \quad \Rightarrow \quad \boldsymbol{\nabla} \times \boldsymbol{v} = 0$ (irrotational or quantized vorticity)

Superconductor: "screening" of the magnetic field: Meissner effect (almost perfect diamagnet)

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Superfluid

Broken global symmetry

Goldstone theorem → Transport of the quantum numbers of the broken group with almost no dissipation

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Superconductor

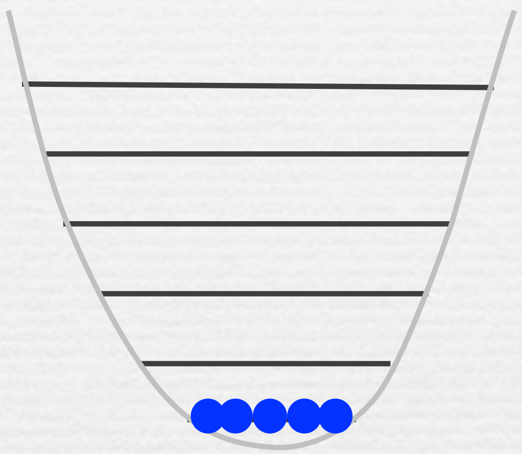
Broken gauge symmetry

Higgs mechanism → Broken gauge fields with mass, M, penetrate for a length $\lambda \propto 1/M$

Fermionic and bosonic superfluids at T=0

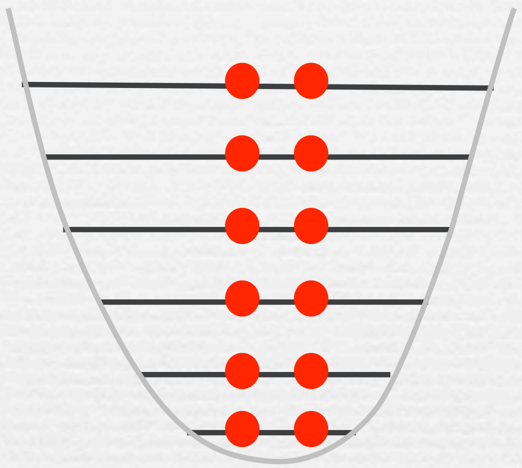
^4He

BOSONS



^3He

FERMIONS



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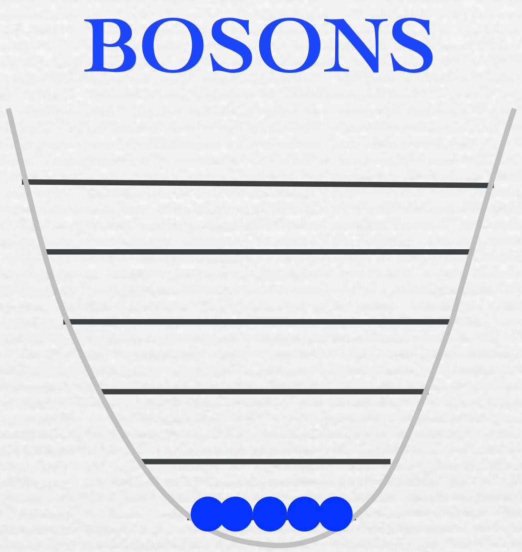

Bosons "like" to move together, no dissipation

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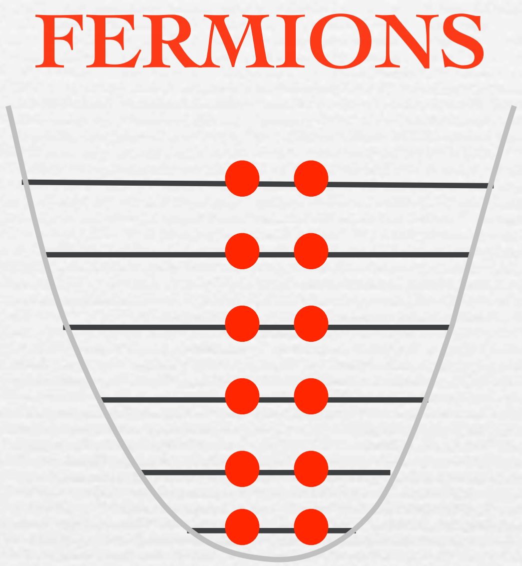
$T_c \simeq 2.17$ K, Kapitsa et al (1938)

^{3}He



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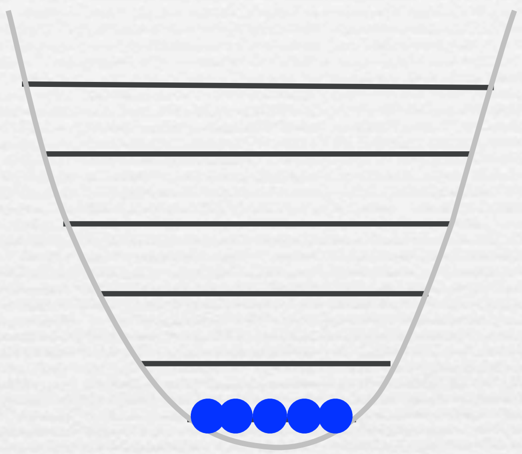

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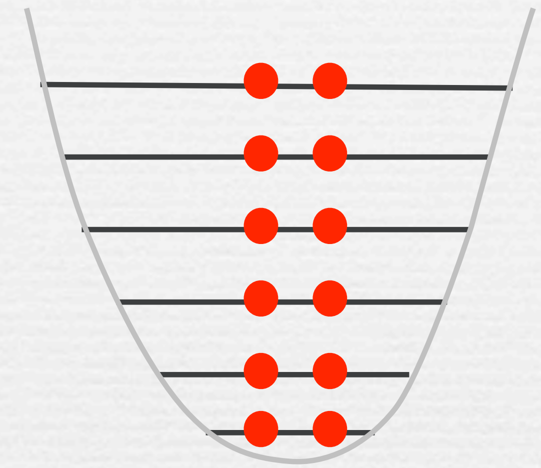

An arbitrary weak interaction leads to the formation of Cooper pairs

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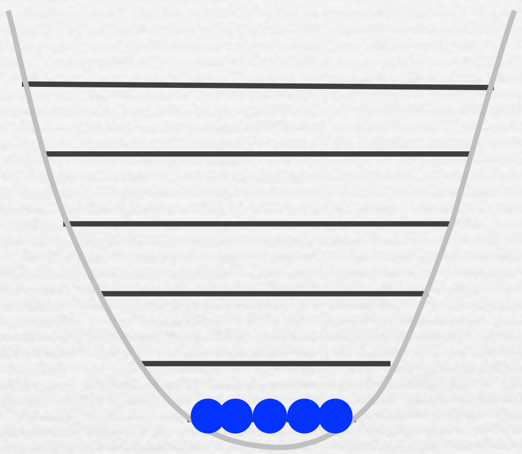

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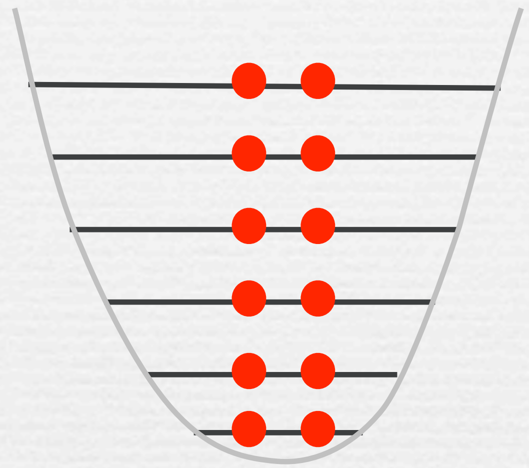

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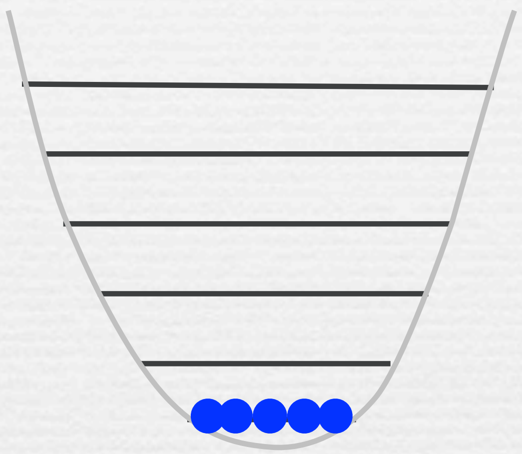

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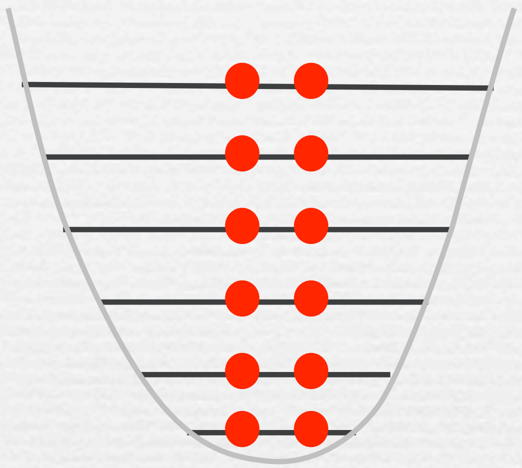

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BEC

BCS

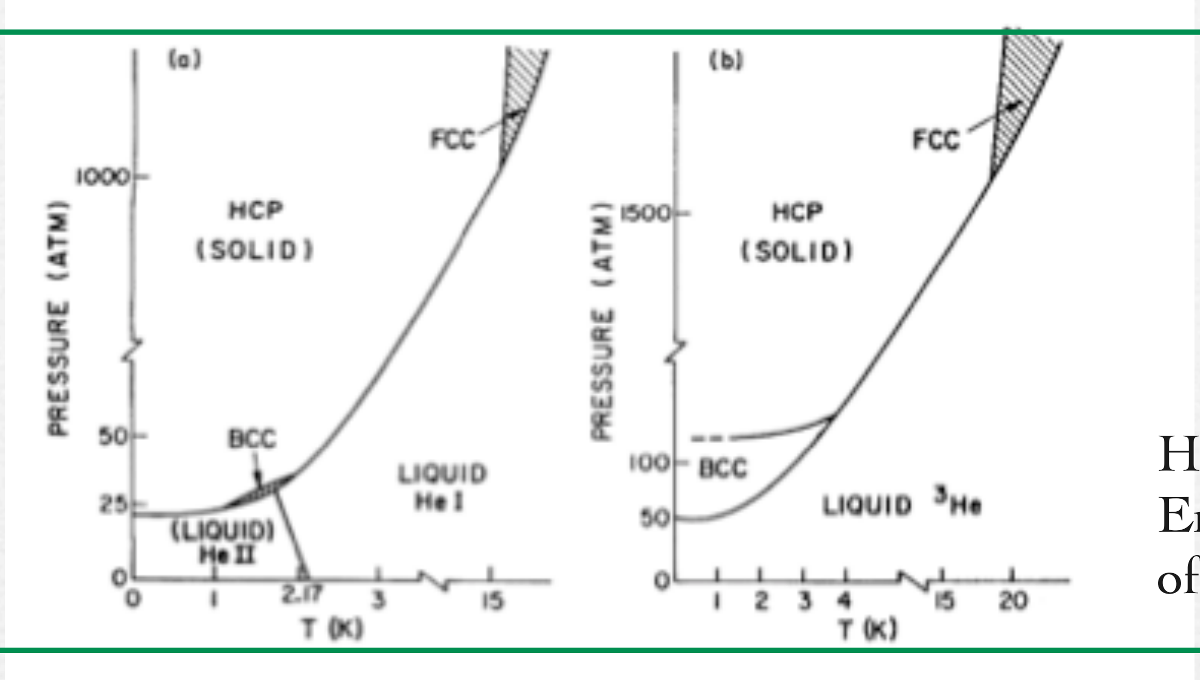
increasing the attractive interaction between fermions

THAT'S IT?

BCS AND BEC

THESE ARE THE ONLY TWO SUPERFLUID PHASE?

Solid helium !!



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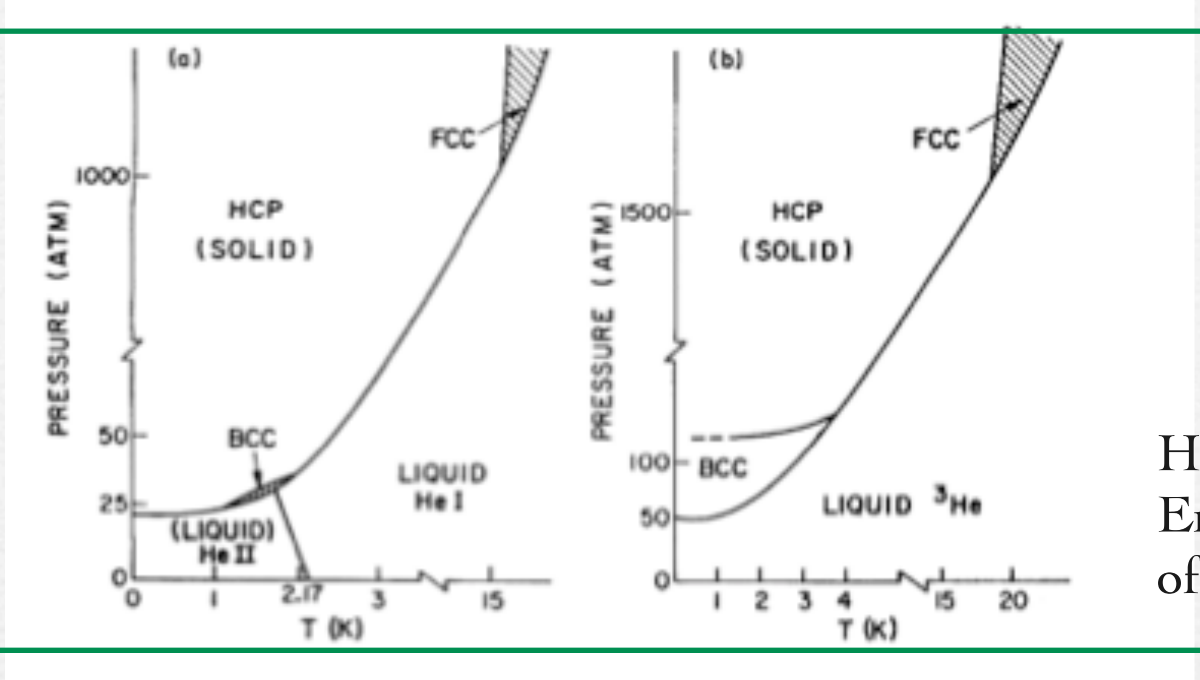

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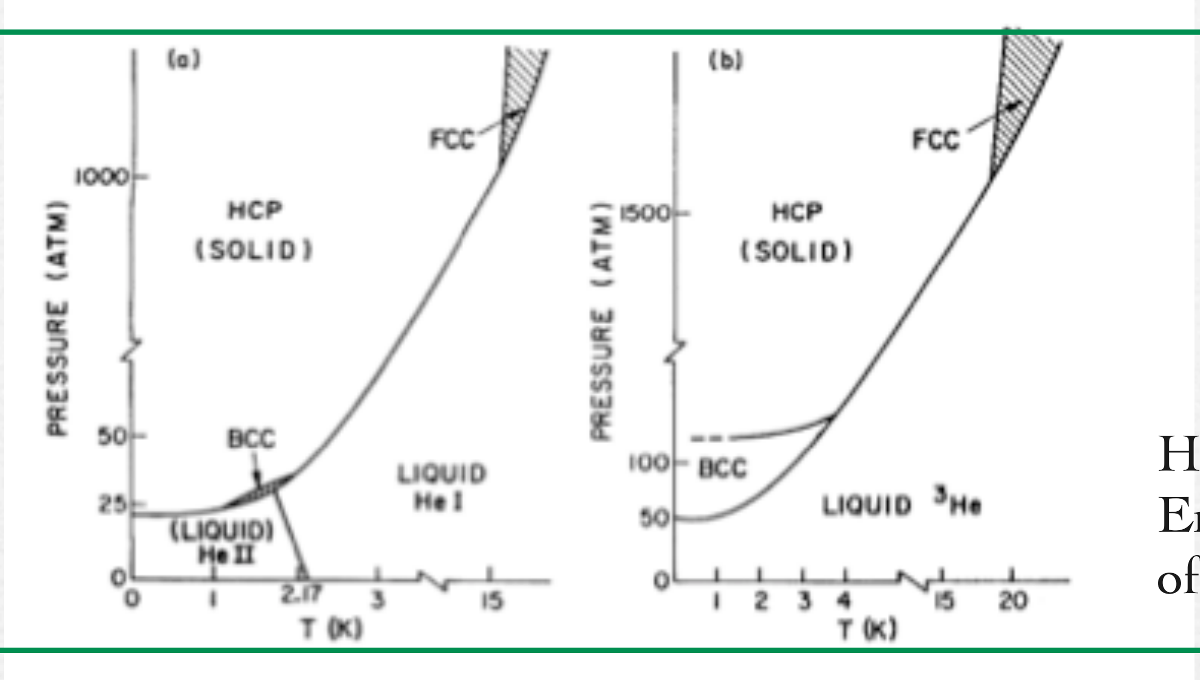

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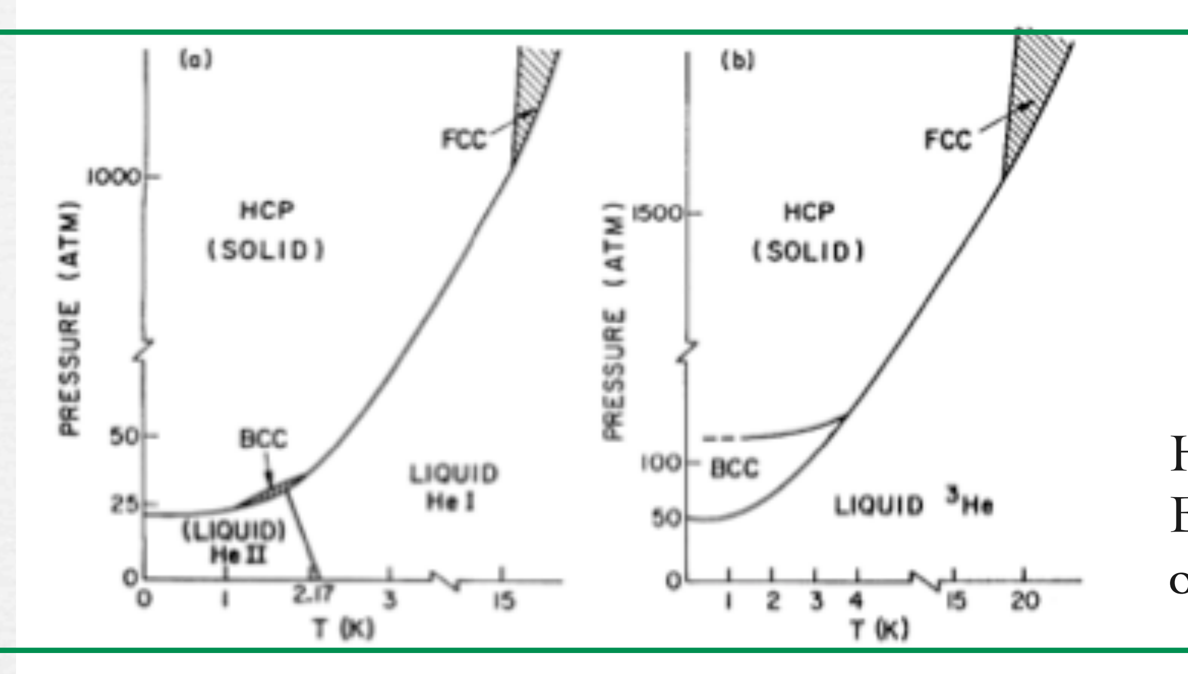

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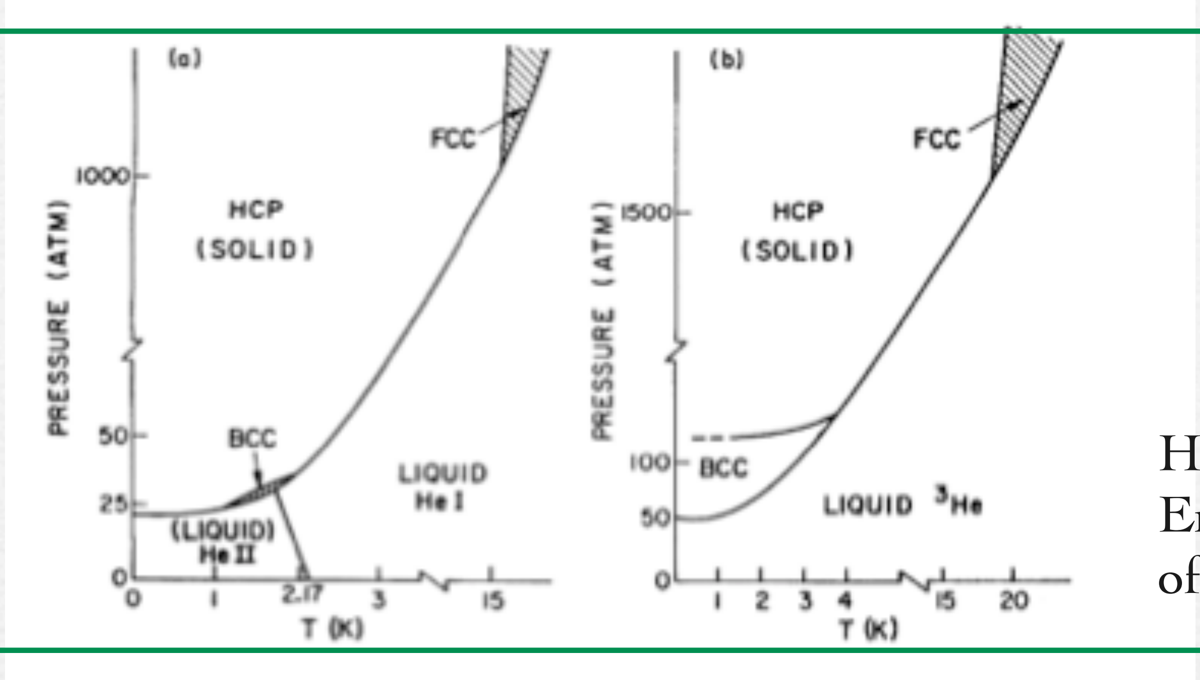
Can a superfluid be solid? Can solids become superfluid?

The naive answer seems NO WAY! (Penrose and Onsager, 1956)

Supersolid

- Solid
  - Long range order spontaneously produced
  - Difficult to deform (shear modulus)
- Superfluid
  - Highly delocalized system
  - Frictionless motion

Solid helium !!



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It seems helium cannot become a supersolid.
Supersolid with ultracold trapped atoms?

Rev. Mod. Phys. 84, 759 (2012) and arXiv:1110.1323v2 [cond-mat.quant-gas]

Main concepts so far:

- We don’t know what is inside compact stars. It might be that some superfluid and/or superconducting phase is realized

- Superfluids are weird systems: vanishing viscosity, quantized vorticity...

- Both fermions (like ^{3}He) and bosons (like ^{4}He) can become superfluid

- There are some physical situations in which both ^{3}He and ^{4}He become solids, but apparently not supersolids

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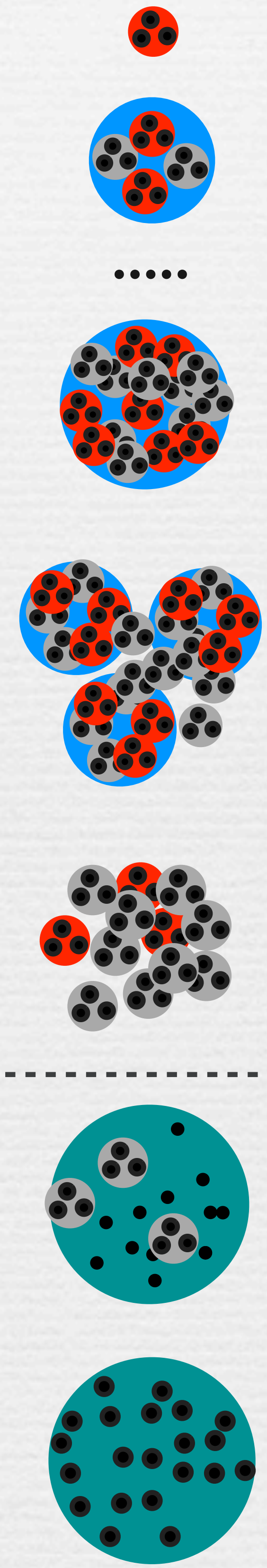
Next:

What about quark matter? Can it look like a supersolid?
Does it happen? Which are the consequences? ....

COLOR SUPERCONDUCTIVITY

Increasing baryonic density



	Density	$\alpha_s \equiv \alpha_s(\mu)$	Degrees of freedom
H	low		light nuclei
He			
......	......		.....
Ni		confining	heavy nuclei
neutron drip			
	high		neutrons and protons
neutron proton soup			
quark drip	very large	strong coupling	quarks and gluons Cooper pairs of quarks? quarkyonic phase?....
quark soup	extreme	weak coupling	Cooper pairs of quarks NGBs

ρ

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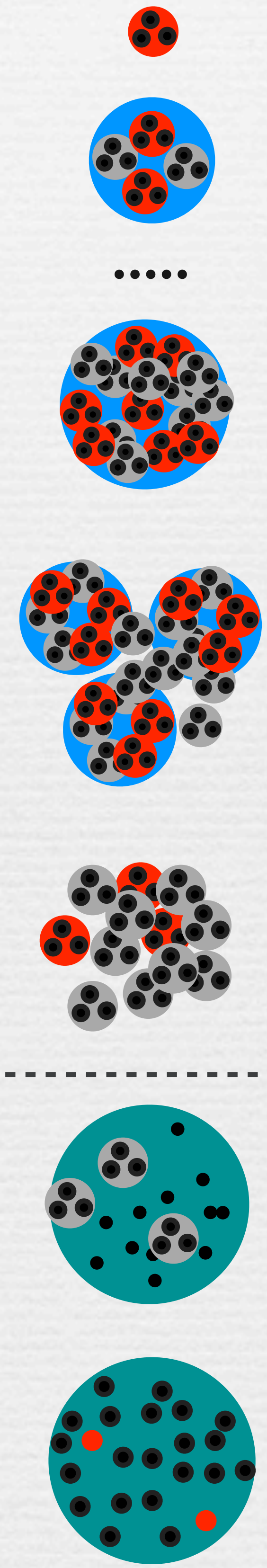

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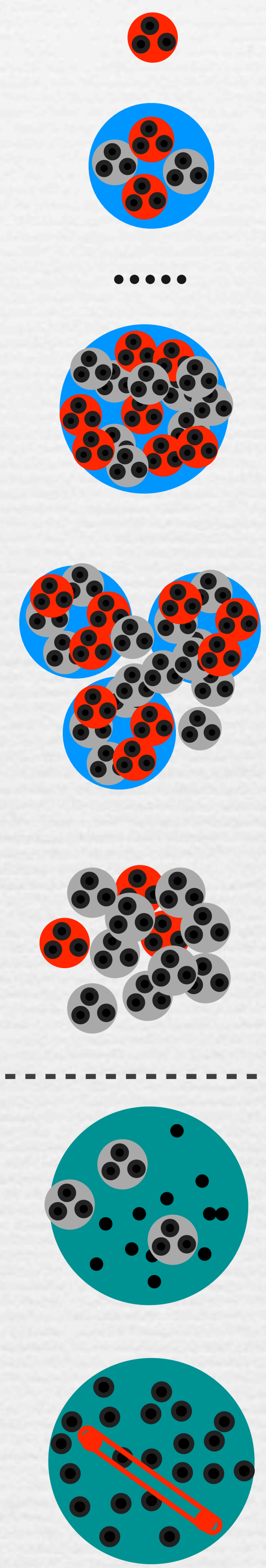

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General recipe for superconductivity

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- Attractive interaction (in some channel)
- $T < T_c$

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N.b. Quarks have color, flavor and spin degrees of freedom: a long menu of colored dishes

The main course: the color flavor locked phase

Massless three flavor quark matter $(m_s \ll \mu)$

CFL condensate
(Alford, Rajagopal, Wilczek hep-ph/9804403)

$$\langle \psi_{\alpha i} C \gamma_5 \psi_{\beta j} \rangle \sim \Delta_{\rm CFL}\, \epsilon_{I\alpha\beta}\epsilon_{Iij}$$

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$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_{\rm Q}} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{\rm Q}}} \times Z_2$$

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- χSB: 8 (pseudo) Nambu-Goldstone bosons (NGBs)
- $U(1)_B$ breaking: 1 NGB
- "Rotated" electromagnetism mixing angle $\cos\theta = \frac{g}{\sqrt{g^2 + 4e^2/3}}$ (analog of the Weinberg angle)

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CRYSTALLINE COLOR SUPERCONDUCTORS

Realistic conditions in compact stars

sizable strange quark mass
+
weak equilibrium
+
electric neutrality

→

mismatch of the Fermi momenta around

$$\mu = \frac{\mu_u + \mu_d + \mu_s}{3}$$

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$$u \to d + \bar{e} + \nu_e$$
$$u \to s + \bar{e} + \nu_e$$
$$u + d \leftrightarrow u + s$$

→

$$\mu_u = \mu_d - \mu_e$$
$$\mu_d = \mu_s$$

electric neutrality →

$$\frac{2}{3}N_u - \frac{1}{3}N_d - \frac{1}{3}N_s - N_e = 0$$

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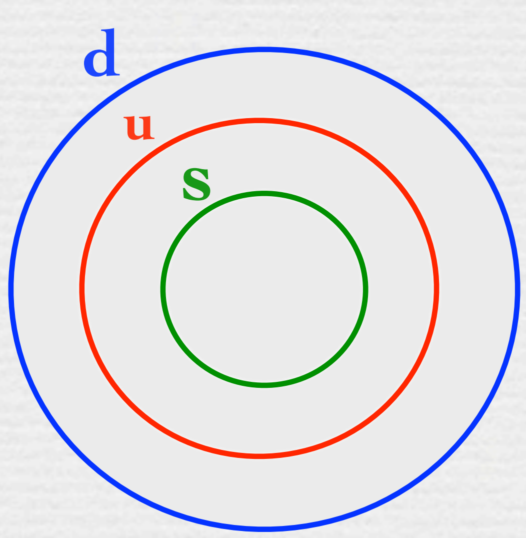
→ $\mu_u = \mu_d - \mu_e$

$\mu_d = \mu_s$

electric neutrality → $\frac{2}{3}N_u - \frac{1}{3}N_d - \frac{1}{3}N_s - N_e = 0$

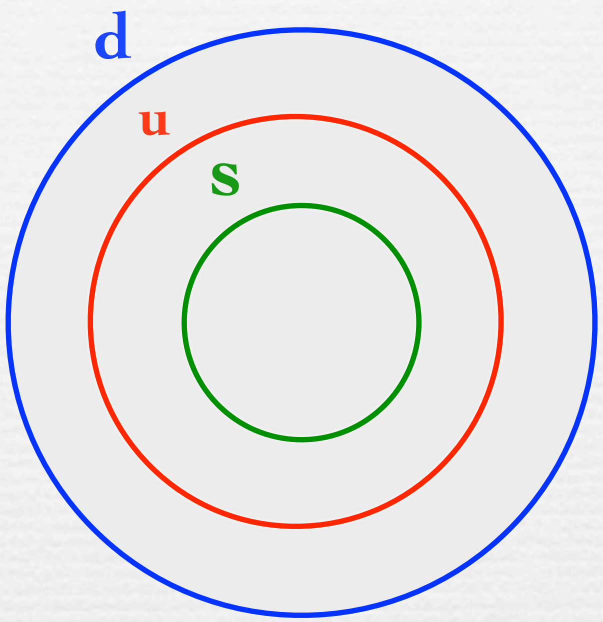
$$\mu_e \simeq \frac{m_s^2}{4\mu} \qquad p_d^F = \mu + \frac{1}{3}\mu_e \quad p_u^F = \mu - \frac{2}{3}\mu_e \quad p_s^F \simeq \mu - \frac{5}{3}\mu_e$$



Fermi spheres of u,d, s quarks

Alford, Rajagopal, JHEP 0206 (2002) 031

Mismatch vs Pairing



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The CFL phase is favored for $\frac{m_s^2}{\mu} \lesssim 2\Delta_{CFL}$

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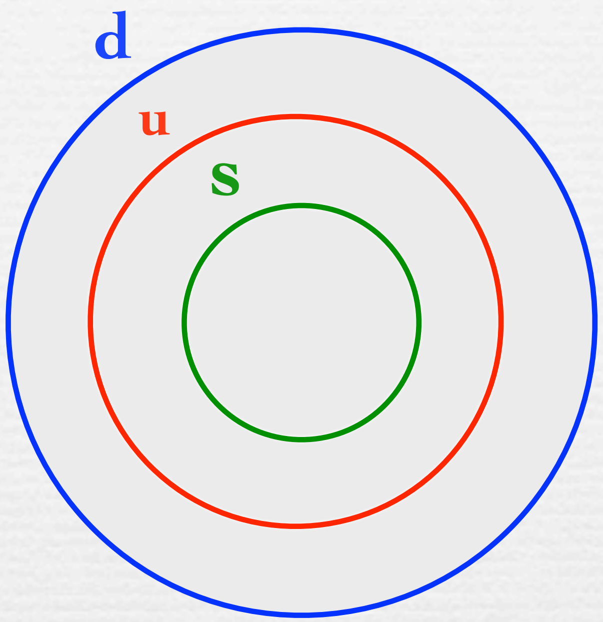

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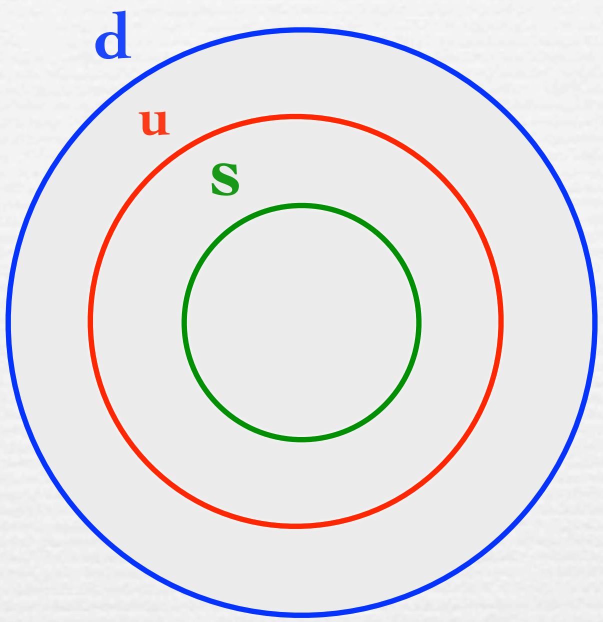
Casalbuoni, MM et al. Phys.Lett. B605 (2005) 362

Forcing the superconductor to a homogenous gapless phase $E(p) = -\delta\mu + \sqrt{(p-\mu)^2 + \Delta_{CFL}^2}$

Leads to the "chromomagnetic instability" $M_{\text{gluon}}^2 < 0$

Mismatch vs Pairing



- **Energy gained in pairing** $\sim 2\Delta_{CFL}$
- **Energy cost of pairing** $\sim \delta\mu \sim \dfrac{m_s^2}{\mu}$

The CFL phase is favored for $\dfrac{m_s^2}{\mu} \lesssim 2\Delta_{CFL}$

Casalbuoni, MM et al. Phys.Lett. B605 (2005) 362

Forcing the superconductor to a homogenous gapless phase $E(p) = -\delta\mu + \sqrt{(p-\mu)^2 + \Delta_{CFL}^2}$

Leads to the "chromomagnetic instability" $M^2_{\text{gluon}} < 0$

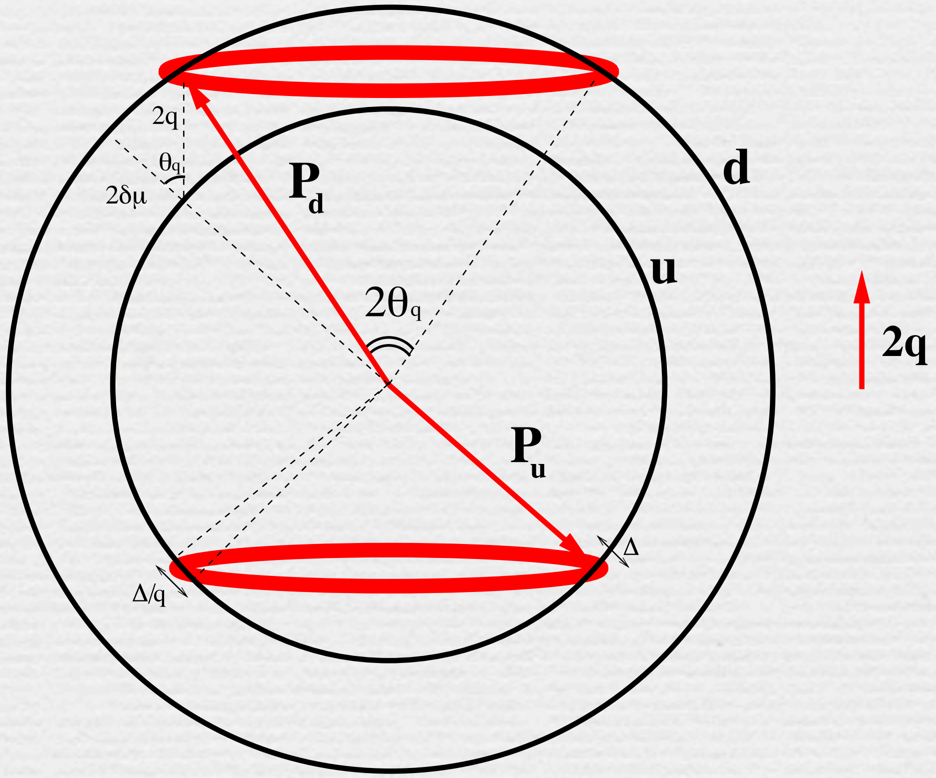
For $\dfrac{m_s^2}{\mu} \gtrsim 2\Delta_{CFL}$ **some less symmetric CSC phase should be realized**

LOFF-phase

LOFF (or FFLO) Larkin, Ovchinnikov (1964) and Fulde, Ferrel (1964)

For $\delta\mu_1 < \delta\mu < \delta\mu_2$ the superconducting LOFF phase is favored

Cooper pairs with nonzero total momentum



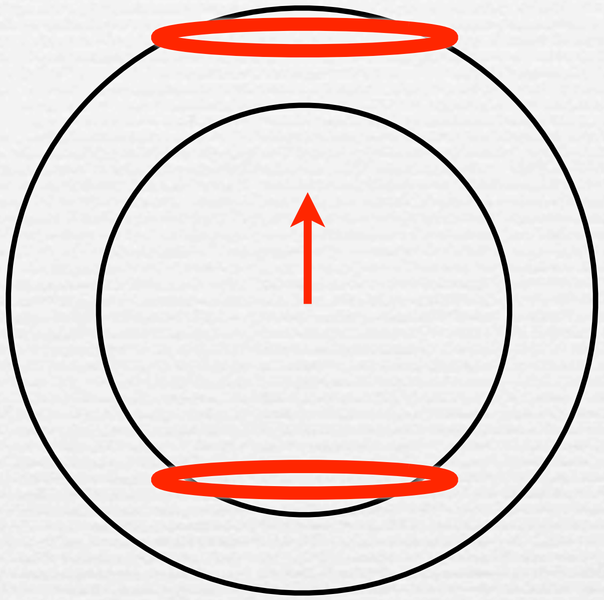
- **In momentum space**

$$< \psi(\mathbf{p_1})\psi(\mathbf{p_2}) > \sim \Delta\, \delta(\mathbf{p_1} + \mathbf{p_2} - \mathbf{2q})$$

- **In coordinate space**

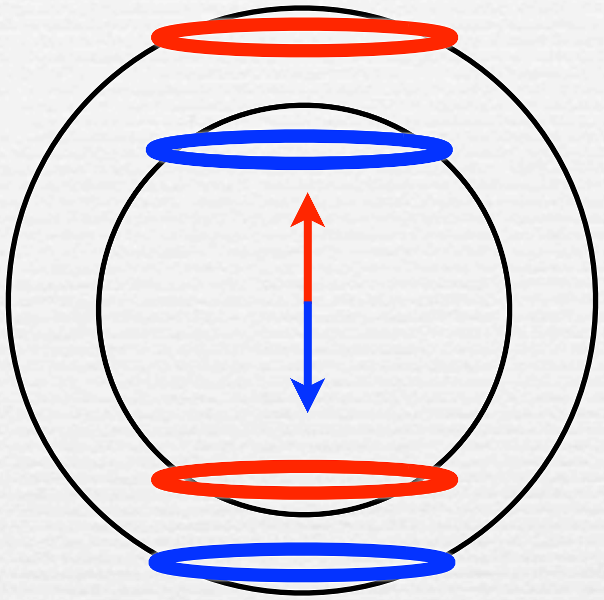
$$< \psi(\mathbf{x})\psi(\mathbf{x}) > \sim \Delta\, e^{i\mathbf{2q}\cdot\mathbf{x}}$$

Crystalline structures



- **Structures combining more plane waves**

Crystalline structures



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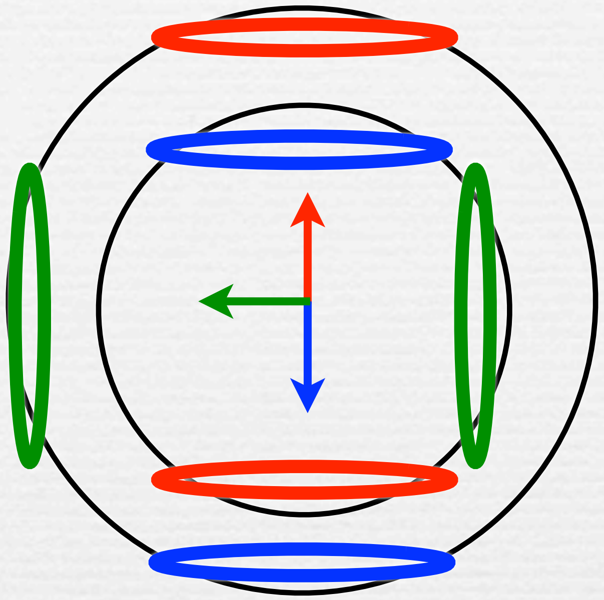

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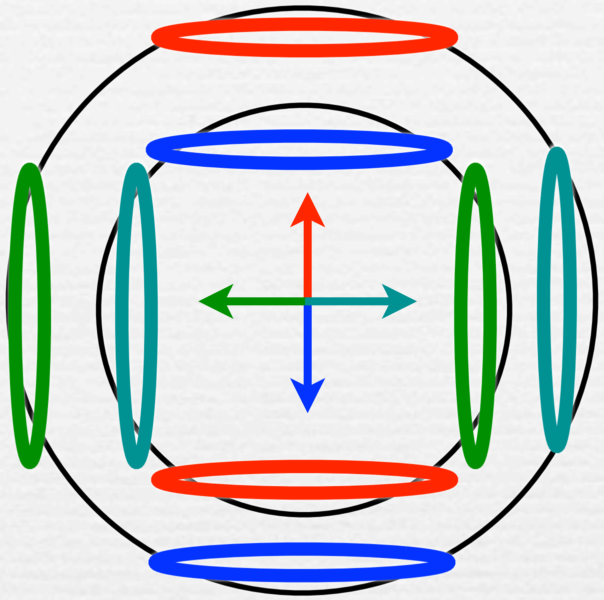

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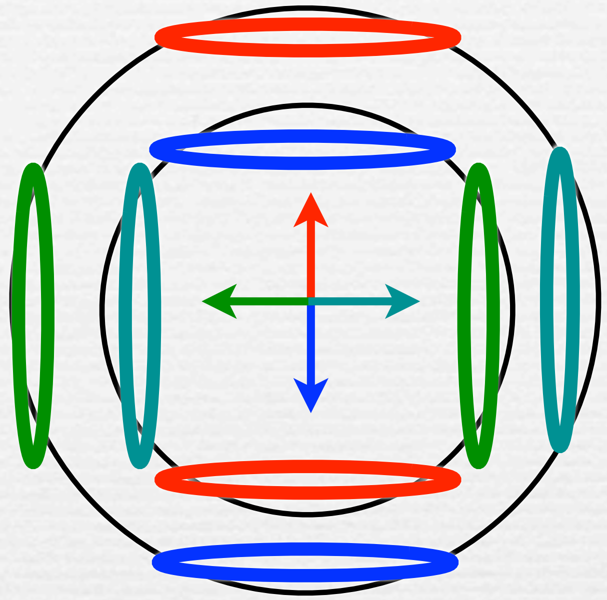

- Structures combining more plane waves
- “No-overlap” condition between ribbons

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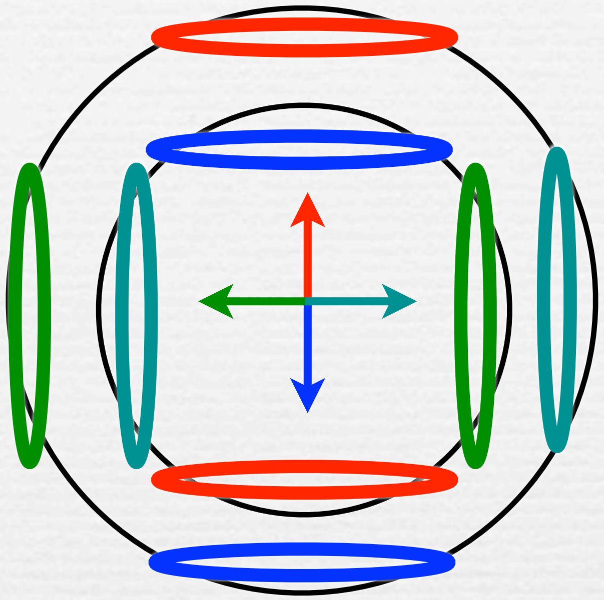

- Structures combining more plane waves
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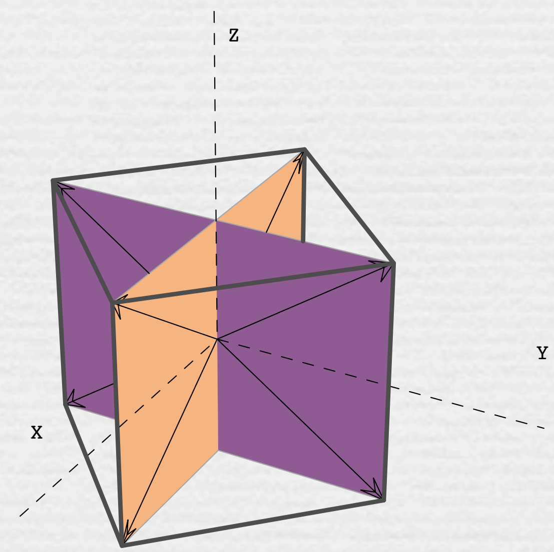
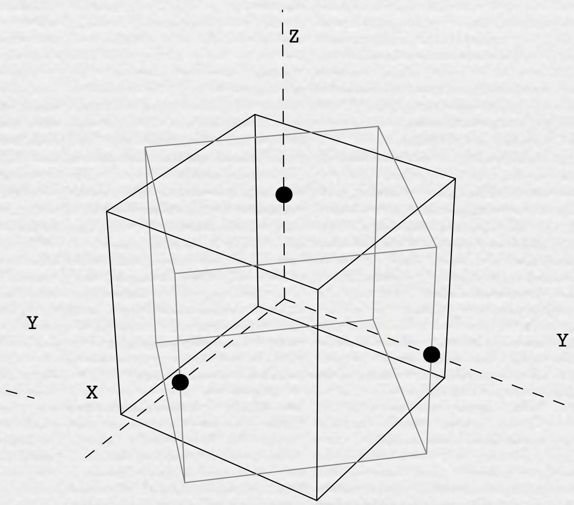
Three flavors

$$< \psi_{\alpha i} C\gamma_5 \psi_{\beta j} > \sim \sum_{I=2,3} \Delta_I \sum_{\mathbf{q}_I^a \in \{\mathbf{q}_I^a\}} e^{2i\mathbf{q}_I^a\cdot\mathbf{r}} \epsilon_{I\alpha\beta}\epsilon_{Iij}$$

CX

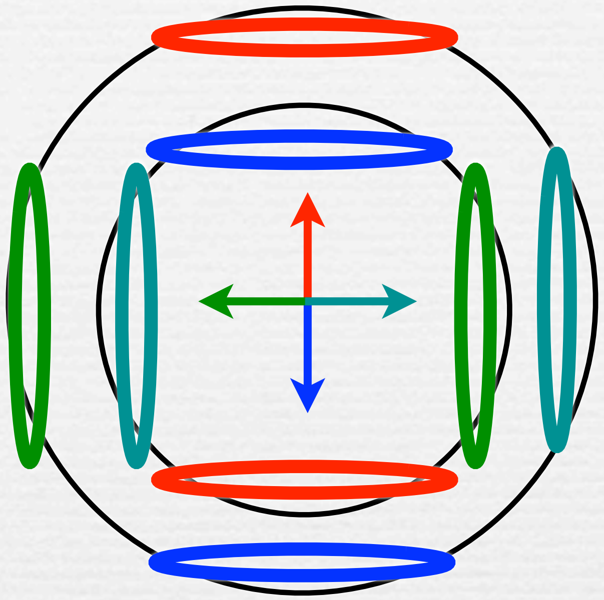


2cube45z



Rajagopal and Sharma Phys.Rev. D74 (2006) 094019

Crystalline structures



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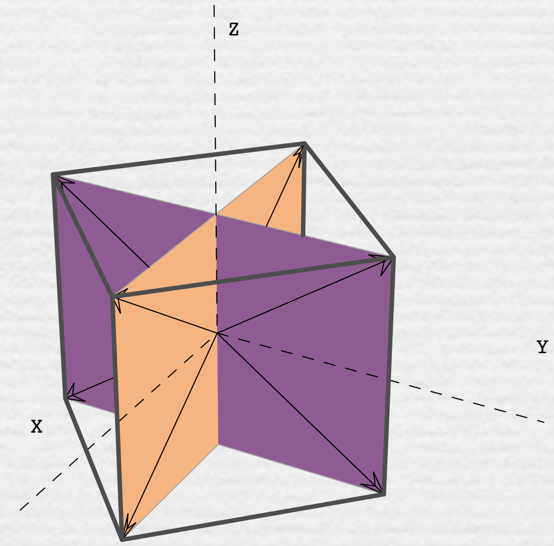

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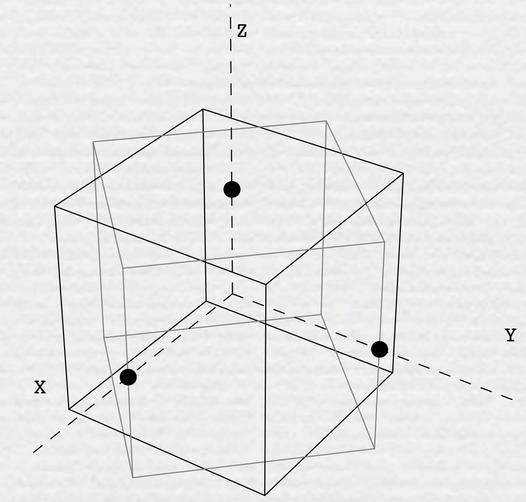


Rajagopal and Sharma Phys.Rev. D74 (2006) 094019

- Crystal oscillations

Casalbuoni, MM et al. Phys.Rev. D66 (2002) 094006

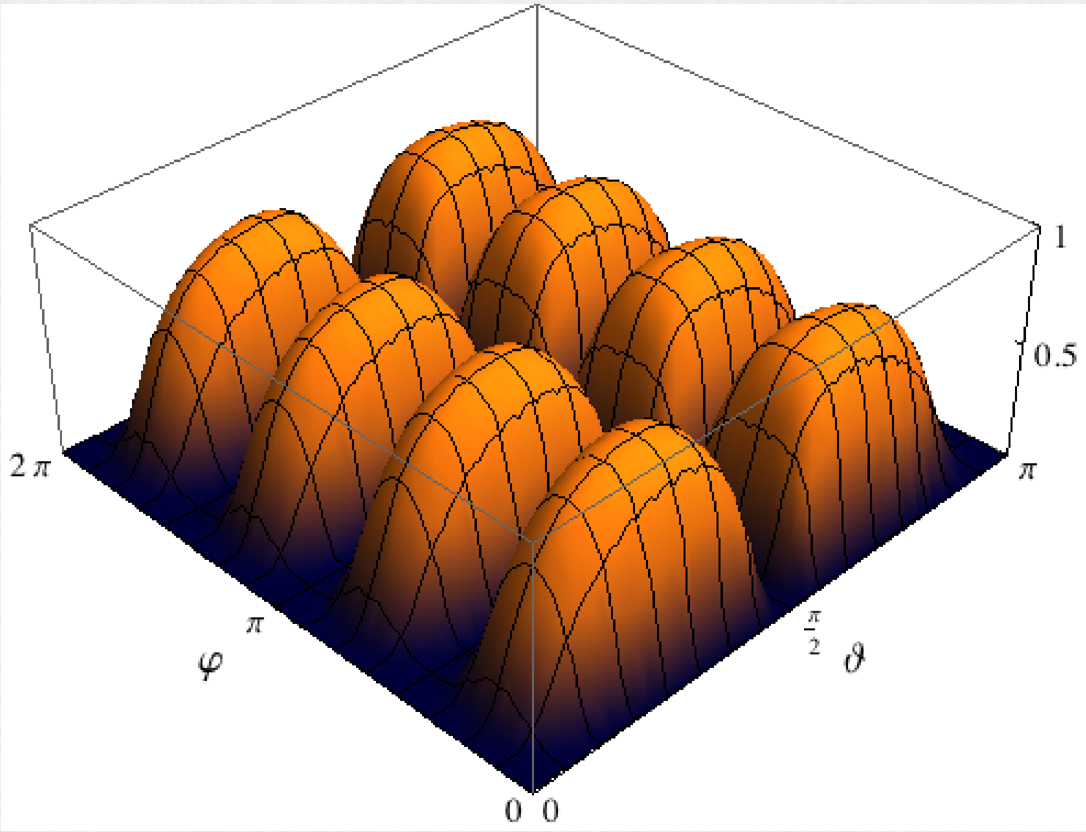
MM, Rajagopal and Sharma Phys.Rev. D76 (2007) 074026

Fermionic dispersion laws

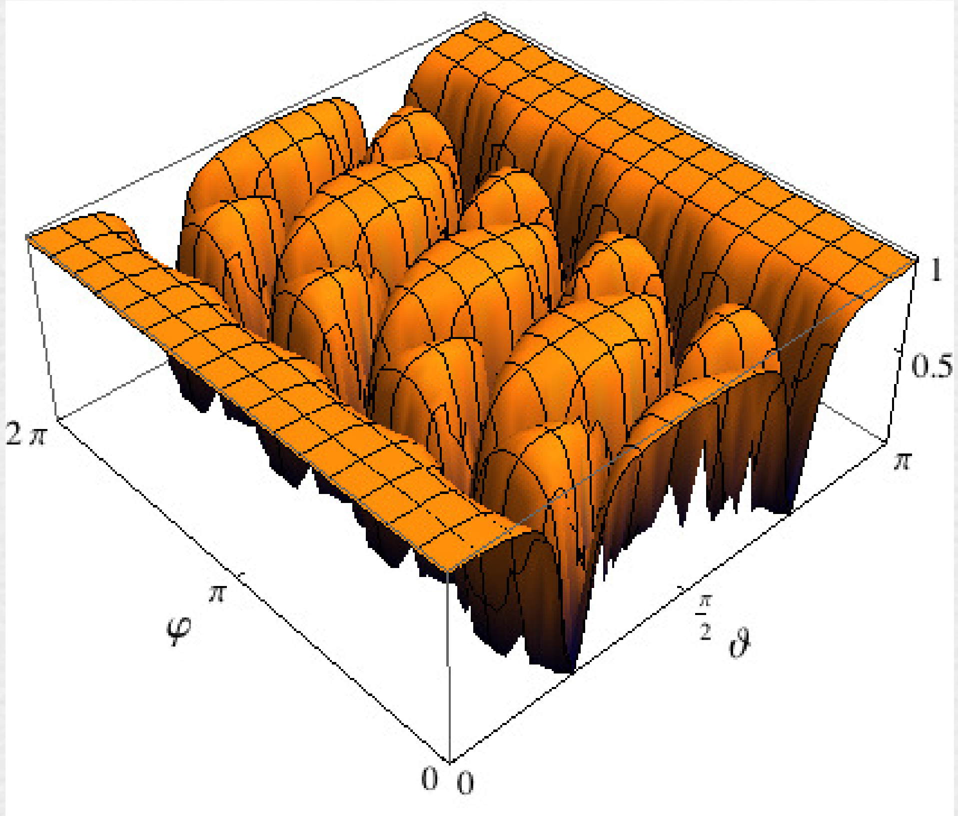
Fermions have an unisotropic gapless dispersion law. Defining $\xi = |p - \mu|$ one has

$$E = c(\theta, \phi)\, \xi$$

Velocity of fermions in two different structures

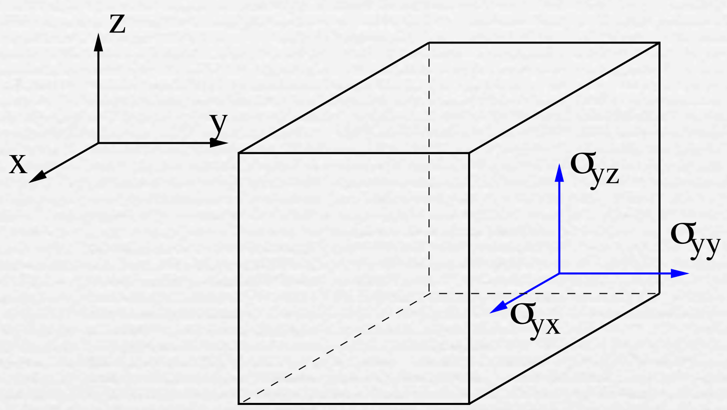


BCC



FCC

Is this phase rigid?



The shear modulus describes the response of a crystal to a shear stress

$$\nu^{ij} = \frac{\sigma^{ij}}{2s^{ij}} \qquad \text{for} \quad i \neq j$$

σ^{ij} **stress tensor acting on the crystal**

s^{ij} **strain (deformation) matrix of the crystal**

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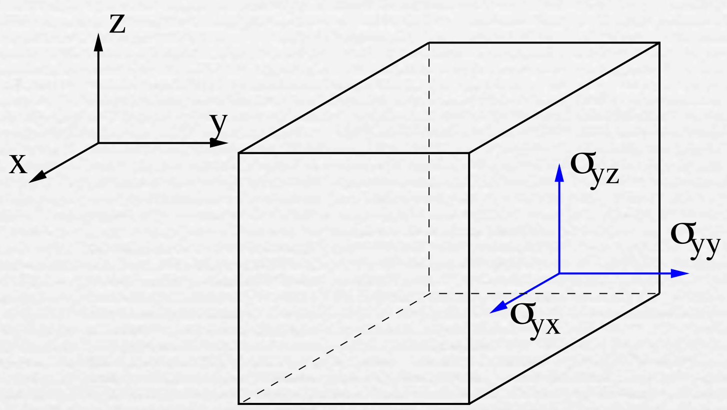

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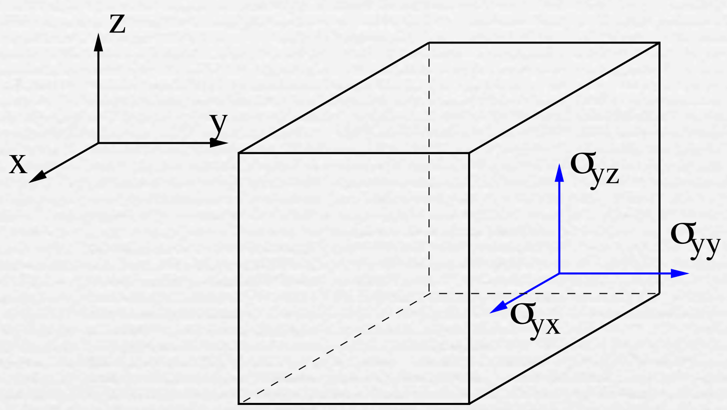

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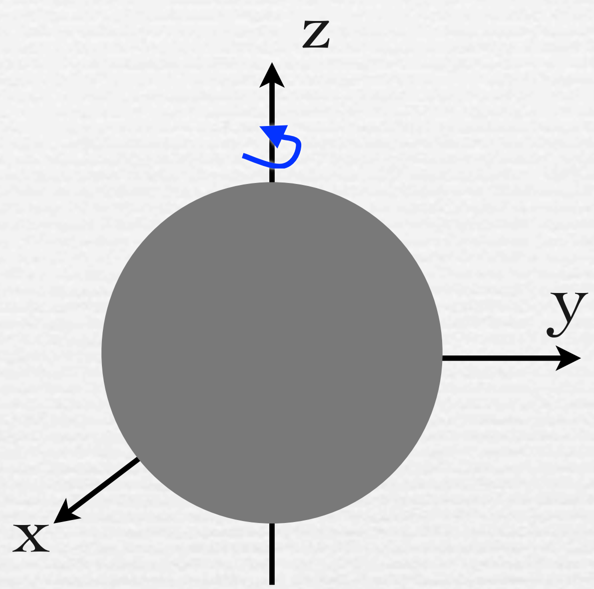
$$\nu = 2.47 \frac{\text{MeV}}{\text{fm}^3} \left(\frac{\Delta}{10\text{MeV}}\right)^2 \left(\frac{\mu}{400\text{MeV}}\right)^2$$

More rigid than diamond!!
20 to 1000 times more rigid than the crust of neutron star

MM, Rajagopal and Sharma Phys.Rev. D76 (2007) 074026

Gravitational waves from "mountains"



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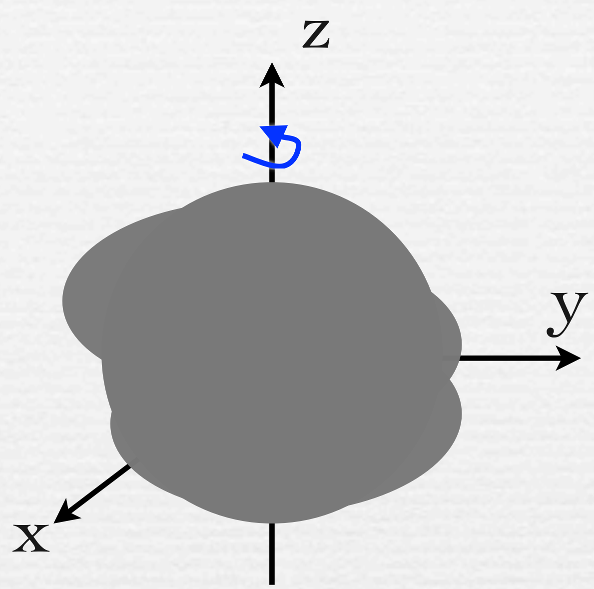

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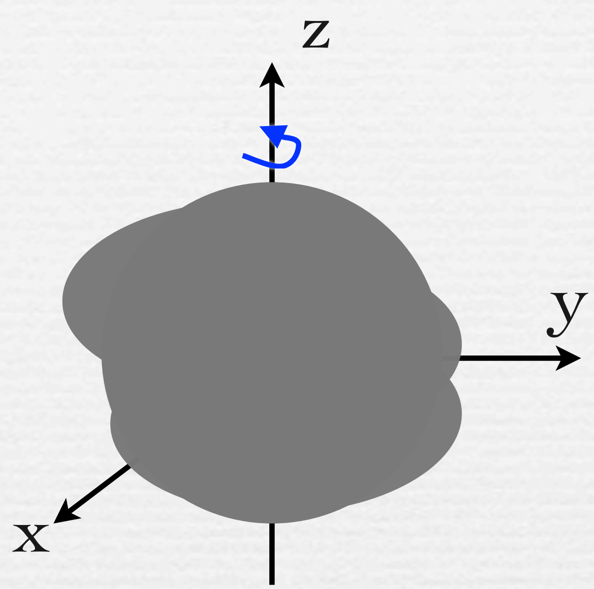

If the star has a non-axial symmetric deformation (mountain) it can emit gravitational waves

ellipticity

$$\epsilon = \frac{I_{xx} - I_{yy}}{I_{zz}}$$

GW amplitude

$$h = \frac{16\pi^2 G}{c^4} \frac{\epsilon I_{zz} \nu^2}{r}$$

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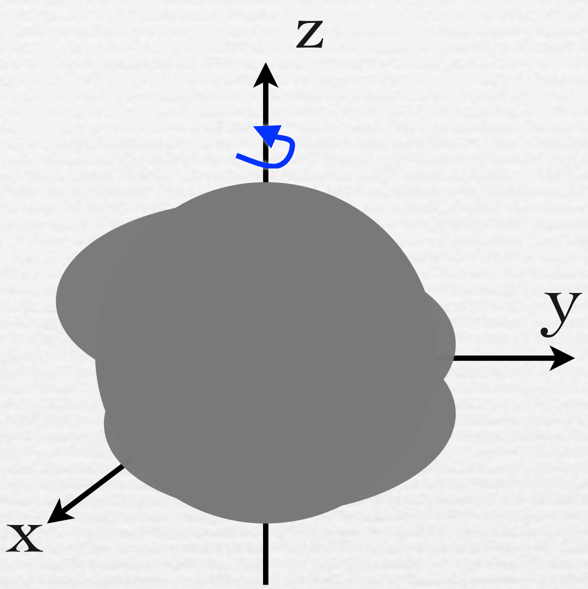

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- **The maximum deformation depends on the breaking strain**

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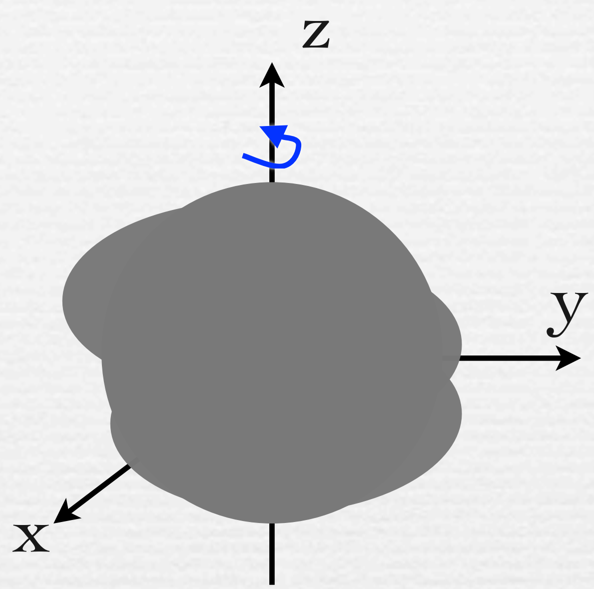

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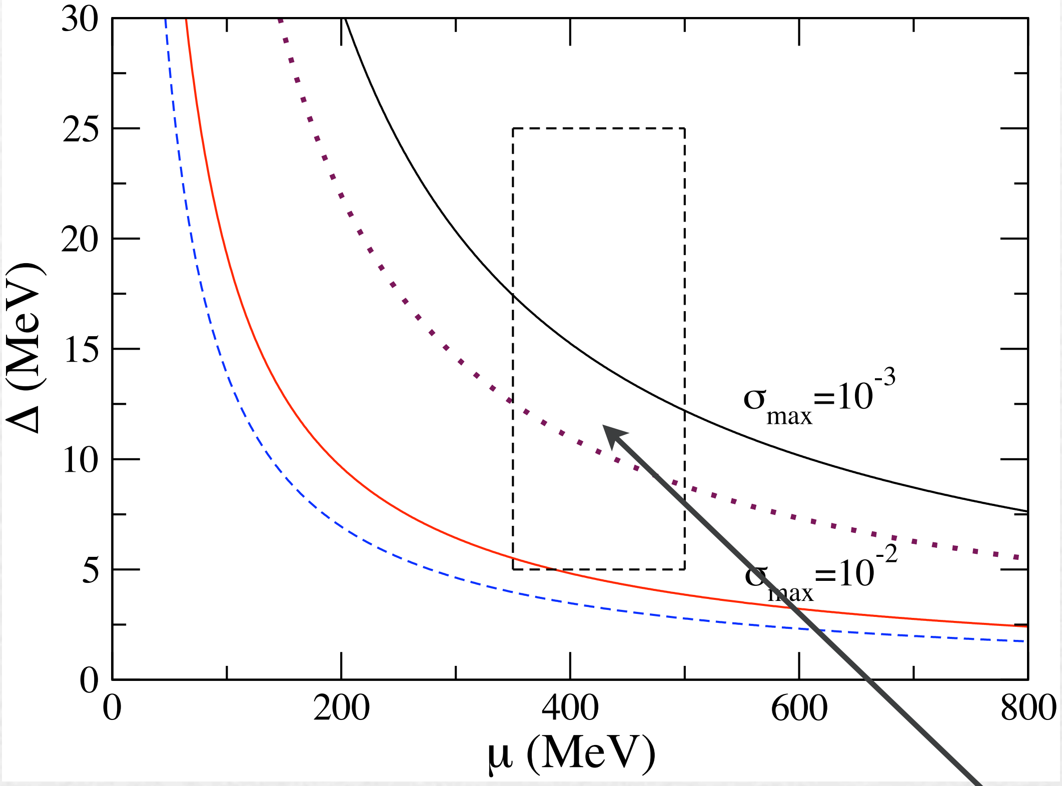
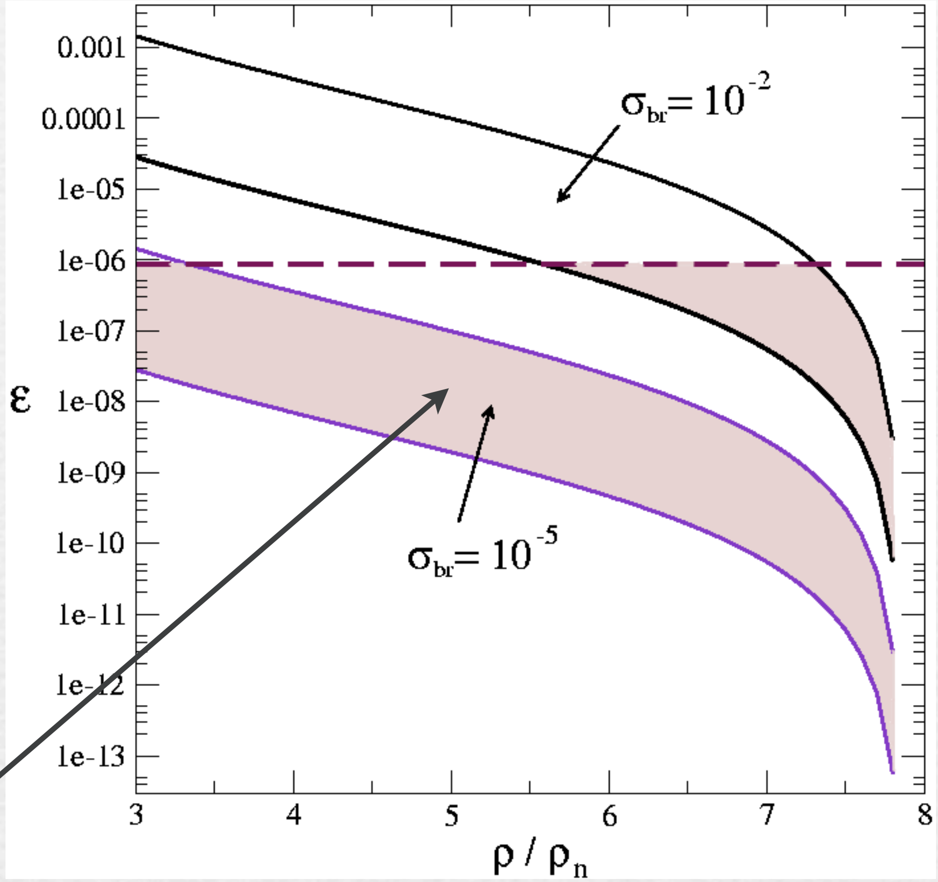
To have a "large" GW amplitude

- Large shear modulus
- Large breaking strain

Using the non-observation of GW from the Crab by the LIGO experiment



Lin, Phys.Rev. D76 (2007) 081502



Andersson et al. Phys.Rev. Lett.99. 231101 (2007)

allowed regions

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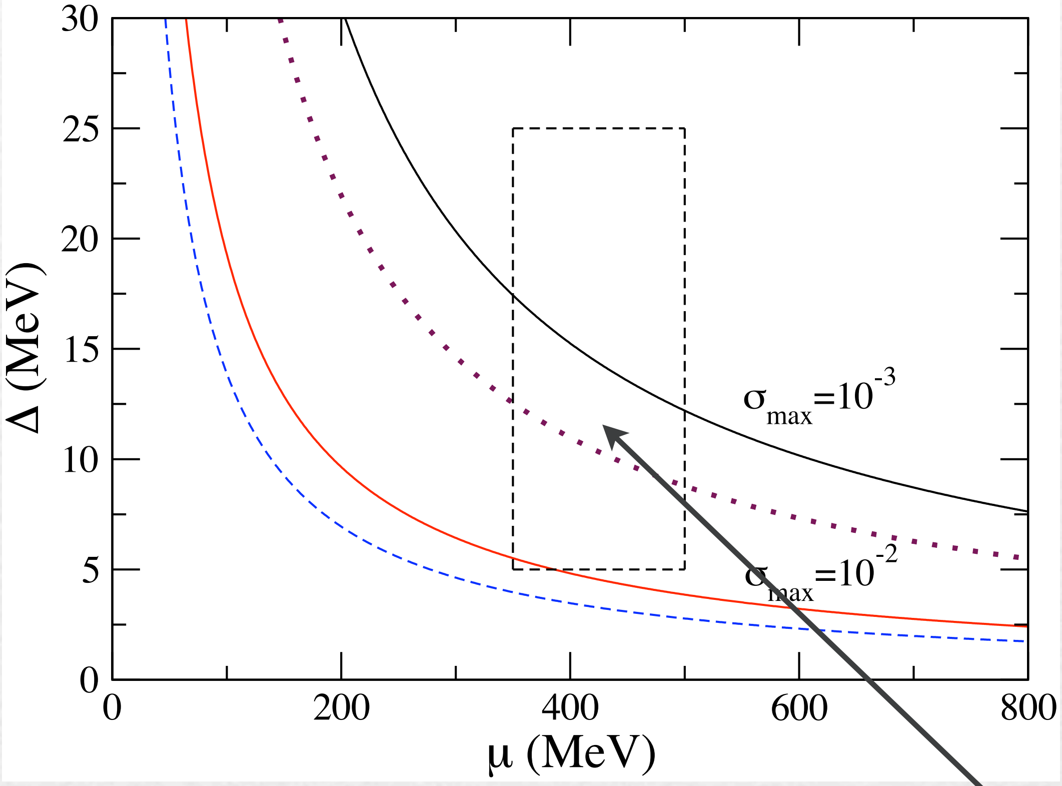
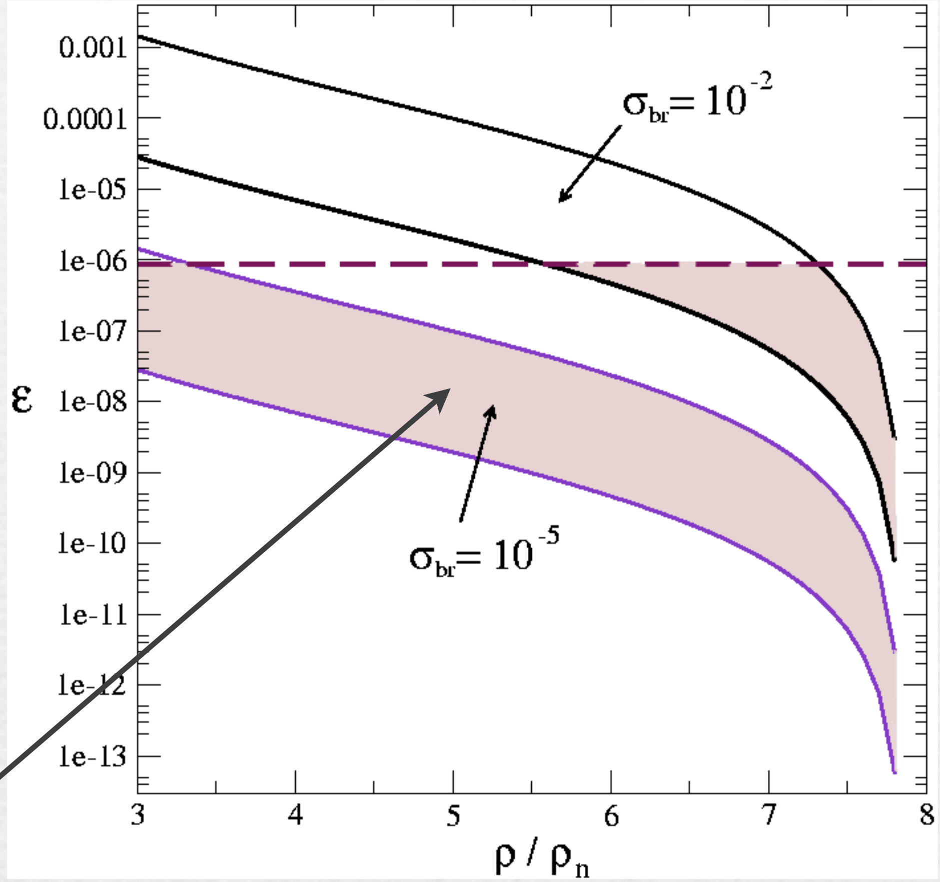


Lin, Phys.Rev. D76 (2007) 081502



Andersson et al. Phys.Rev. Lett.99. 231101 (2007)

allowed regions

...we can restrict the parameter space!

Summary

- The study of matter in extreme conditions allows to shed light on the basic properties of QCD

- Color superconductivity is a phase of matter predicted by QCD

- In realistic conditions a crystalline rigid color superconducting phase should be favored

- We are looking for signatures of this phase

Back-up slides

A bit of history

- Quark matter inside compact stars, Ivanenko and Kurdgelaidze (1965), Paccini (1966) ...
- Quark Cooper pairing was proposed by Ivanenko and Kurdgelaidze (1969)
- With asymptotic freedom (1973) more robust results by Collins and Perry (1975), Baym and Chin (1976)
- Classification of some color superconducting phases: Bailin and Love (1984)

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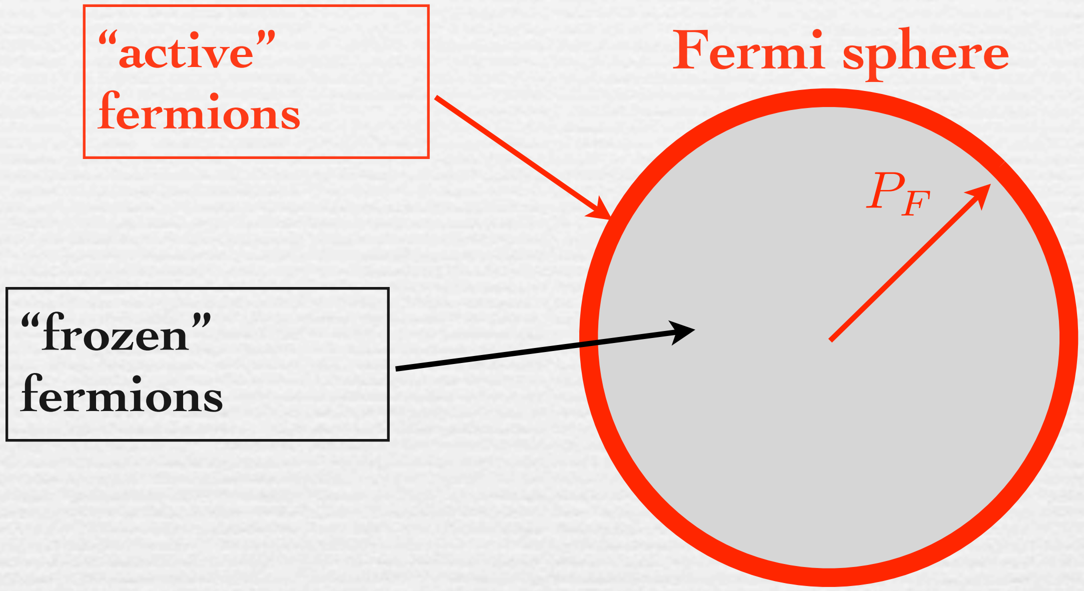
- A large gap with instanton models by Alford et al. (1998) and by Rapp et al. (1998)
- The color flavor locked (CFL) phase was proposed by Alford et al. (1999)

BCS Theory

Bardeen-Cooper-Schrieffer (BCS) in 1957 proposed a microscopic theory of fermionic superfluidity

weak interaction and T=0



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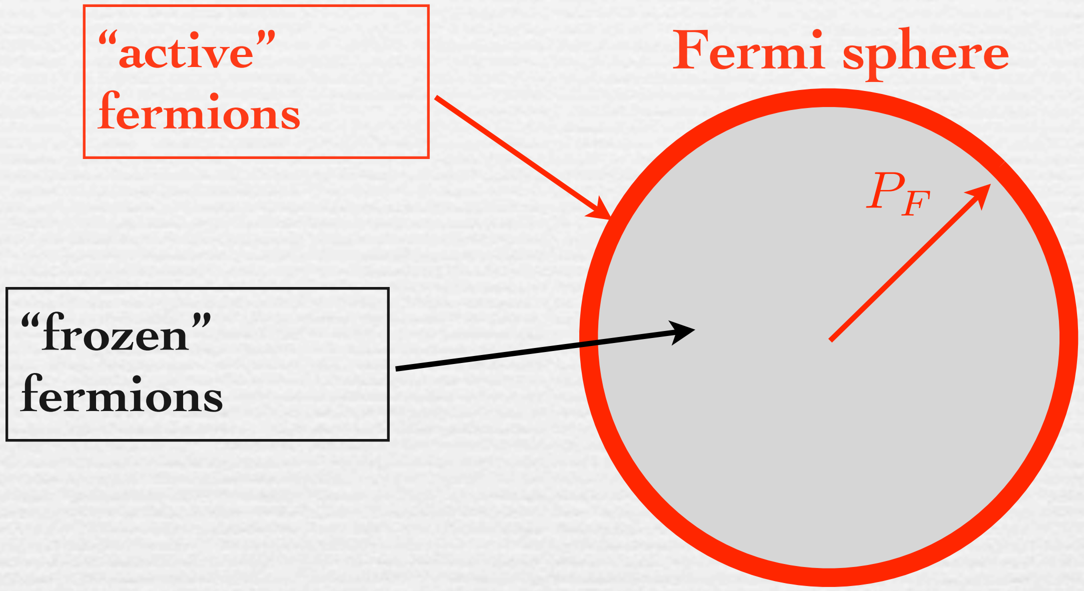

Cooper pairing : Any attractive interaction produces correlated pairs of **"active" fermions**

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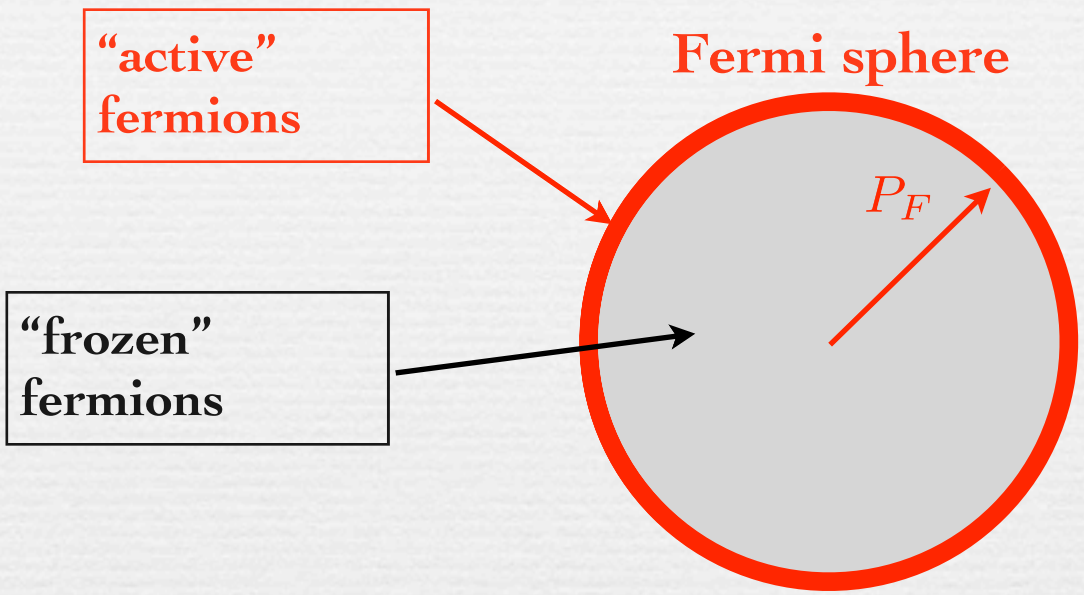

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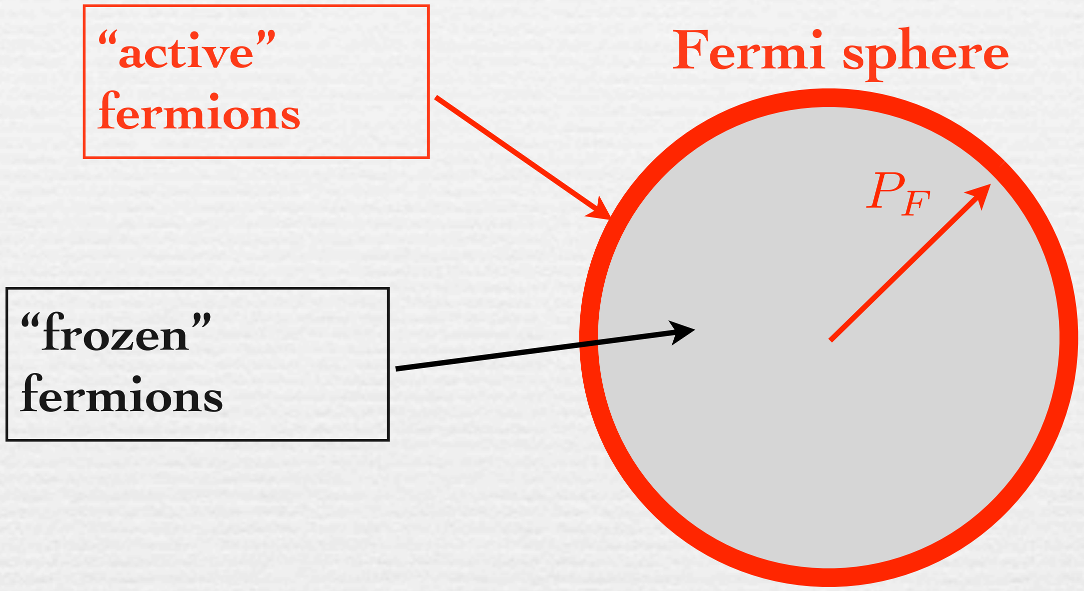

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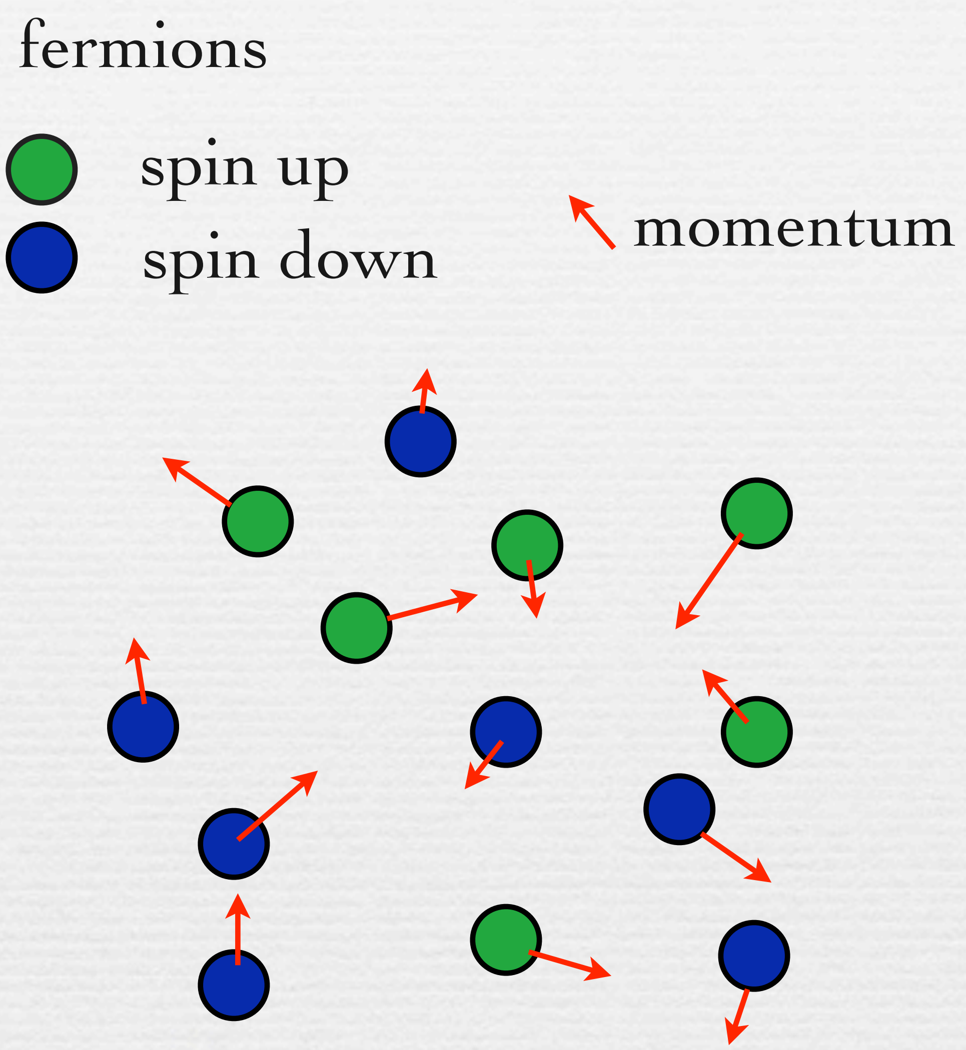
Quasiparticle dispersion law $$E(p) = \sqrt{(\epsilon(p) - \mu)^2 + \Delta(p,T)^2}$$

Increasing the temperature the coherence is lost at $$T_c \simeq 0.3\,\Delta_0$$

BCS-BEC



- **Cooper pairs**: di-fermions with total spin 0 and total momentum 0

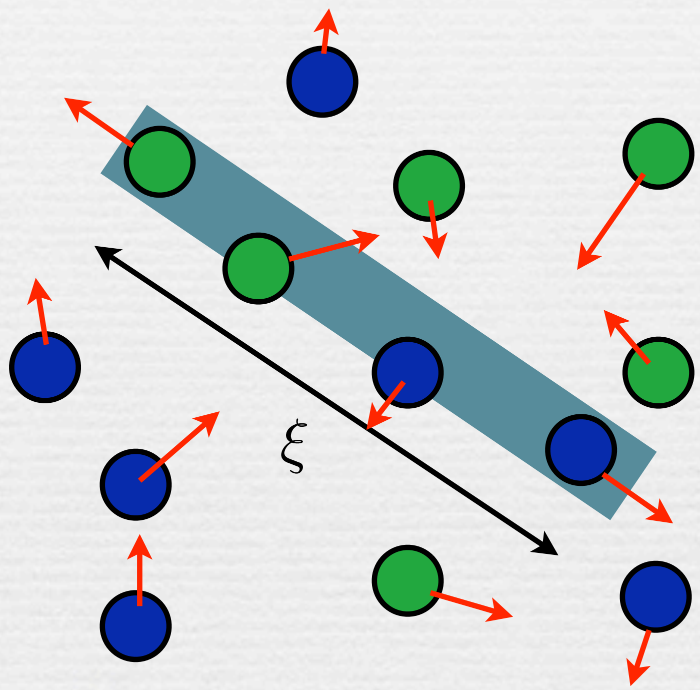
BCS-BEC

fermions

spin up

spin down

momentum



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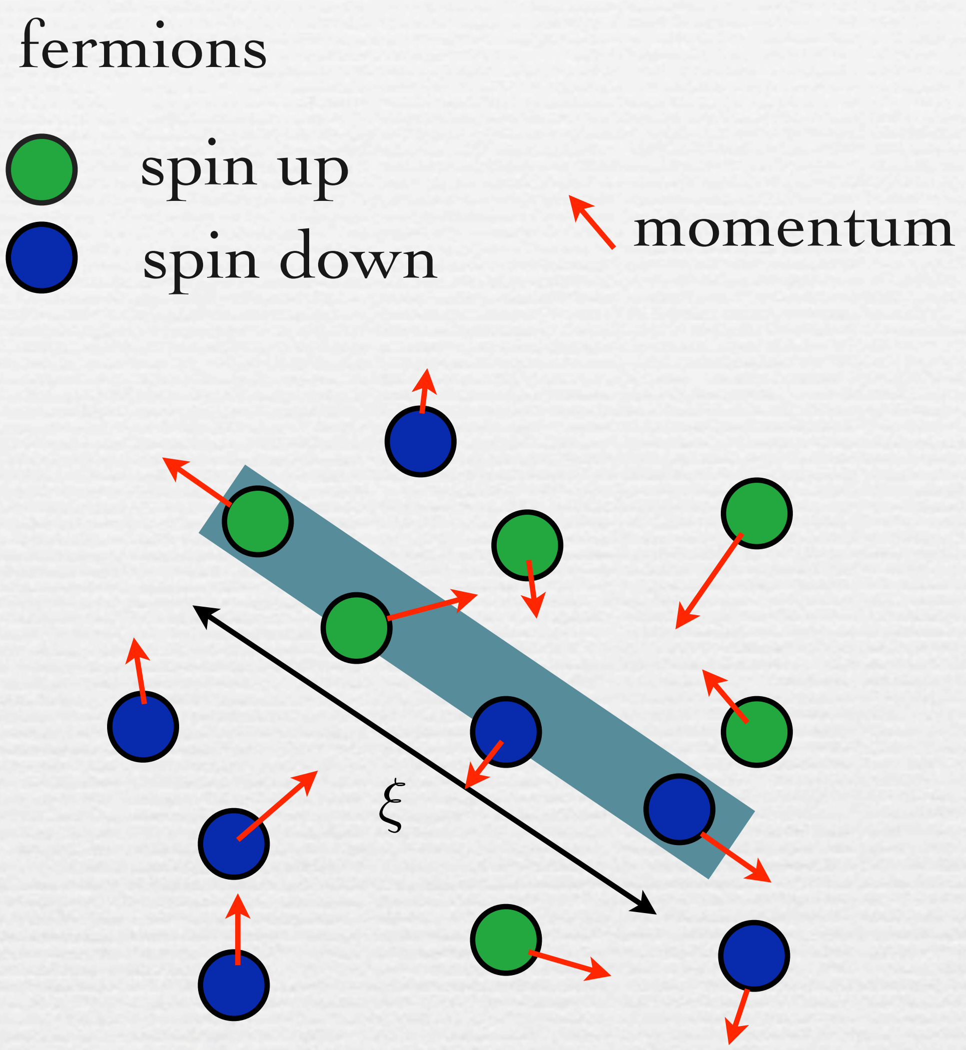

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BCS-BEC

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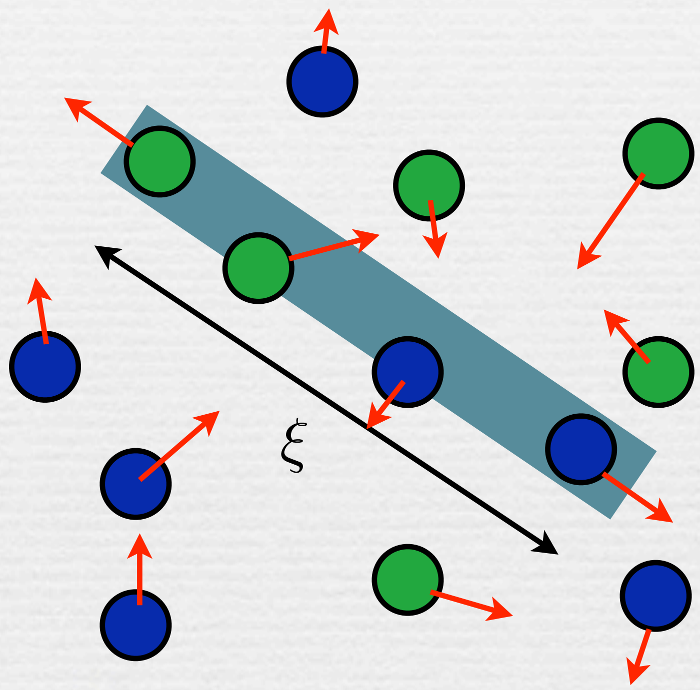

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Type I (Pippard): $\lambda \ll \xi$ first order phase transition to the normal phase

Type II (London): $\lambda \gg \xi$ second order phase transition to the normal phase

Chiral symmetry breaking

At low density the χSB is due to the condensate that locks left-handed and right-handed fields $\langle \bar{\psi}\, \psi \rangle$

In the CFL phase we can write the condensate as

$$\langle \psi^L_{\alpha i} \psi^L_{\beta j} \rangle = -\langle \psi^R_{\alpha i} \psi^R_{\beta j} \rangle = \kappa_1 \delta_{\alpha i} \delta_{\beta j} - \kappa_2 \delta_{\alpha j} \delta_{\beta i}$$

Color is locked to both left-handed and right-handed rotations.