VNIVERSITÀ DEGLI·STVDI DI·FERRARA

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the Freeze-Out in ECHO-QGP

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Table of Contents

ECHO-QGP Collaboration

The ECHO-QGP collaboration involves the Universities of Ferrara, Firenze and Torino. My role inside this project is to develop all the post-hydro tools, which shall produce the observables to be compared with experimental results.

ECHO-QGP

L. Del Zanna, V. Chandra, G. Inghirami, V. Rolando, A. Beraudo, A. De Pace, G. Pagliara, A. Drago, and F. Becattini, *Relativistic viscous hydrodynamics for heavy-ion collisions with ECHO-QGP*, arXiv(nucl-th):1305.7052

Two words on ECHO-QGP

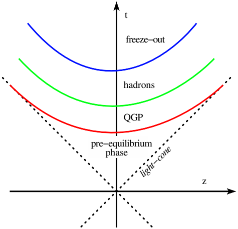
The code has been built on top of the Eulerian Conservative High-Order astrophysical code for general relativistic magnetohydrodynamics:

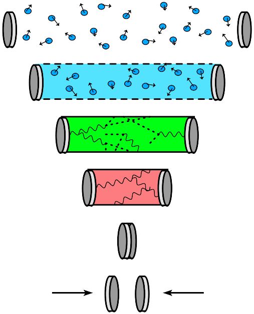
ECHO

L. Del Zanna, O. Zanotti, N. Bucciantini, and P. Londrillo, *ECHO: an Eulerian Conservative High Order scheme for general relativistic magnetohydrodynamics and magnetodynamics*
arXiv(astro-ph):0704.3206

The original ECHO code can handle non-vanishing conserved-number currents as well as electromagnetic fields, which are essential for the astrophysical computations, in any (3+1)-D metric of General Relativity.

Heavy ion collisions





freeze-out

unknown

equation of state

unknown

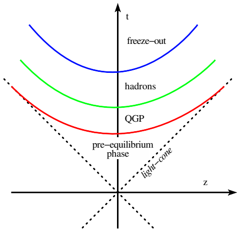
hydrodynamics

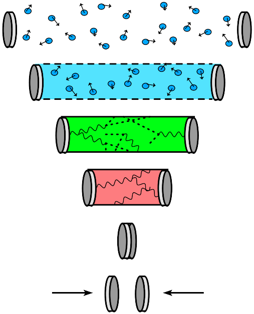
assumption

initial conditions

unknown

Heavy ion collisions





freeze-out

complementary theories

equation of state

want to study

hydrodynamics

assumption

initial conditions

complementary theories

Relativistic Ideal Hydrodynamics

Orthogonal projector

$$\Delta^{\mu\nu} \equiv g^{\mu\nu} + u^\mu u^\nu$$

Covariant derivative

$$d_\mu = \underbrace{-u_\mu D}_{D \equiv u^\alpha d_\alpha} + \underbrace{\nabla_\mu}_{\nabla_\mu \equiv \Delta_\mu^\alpha d_\alpha}$$

Set of equations

$$\begin{cases} d_\mu N^\mu = 0, \\ d_\mu T^{\mu\nu} = 0 \\ EoS \end{cases}$$

Conservative form

$$\partial_0 U + \partial_k F^k = S$$

$$N^\mu = n u^\mu + V^\mu$$
$$T^{\mu\nu} = e u^\mu u^\nu + P \Delta^{\mu\nu}$$

Relativistic Ideal Hydrodynamics

- Landau frame
- vanishing baryon density

Set of equations

$$\begin{cases} d_\mu N^\mu = 0, \\ d_\mu T^{\mu\nu} = 0 \\ EoS \end{cases}$$

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Relativistic Viscous Hydrodynamics

Orthogonal projector

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Set of equations

$$\begin{cases} d_\mu N^\mu = 0, \\ d_\mu T^{\mu\nu} = 0 \\ EoS \end{cases}$$

Conservative form

$$\partial_0 U + \partial_k F^k = S$$

$$N^\mu = n u^\mu + V^\mu$$

$$T^{\mu\nu} = e u^\mu u^\nu + (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu} + w^\mu u^\nu + w^\nu u^\mu$$

Conservative form of equations

$$\partial_0 \mathbf{U} + \partial_k \mathbf{F}^k = \mathbf{S},$$

where

$$\mathbf{U} = |g|^{\frac{1}{2}} \begin{pmatrix} N \equiv N^0 \\ S_i \equiv T^0_i \\ E \equiv -T^0_0 \\ N\Pi \\ N\pi^{ij} \end{pmatrix}, \quad \mathbf{F}^k = |g|^{\frac{1}{2}} \begin{pmatrix} N^k \\ T^k_i \\ -T^k_0 \\ N^k\Pi \\ N^k\pi^{ij} \end{pmatrix}$$

$$\mathbf{S} = |g|^{\frac{1}{2}} \begin{pmatrix} 0 \\ \frac{1}{2}T^{\mu\nu}\partial_i g_{\mu\nu} \\ -\frac{1}{2}T^{\mu\nu}\partial_0 g_{\mu\nu} \\ n[-\frac{1}{\tau_\pi}(\Pi + \zeta\theta) - \frac{4}{3}\Pi\theta] \\ n[-\frac{1}{\tau_\pi}(\pi^{ij} + 2\eta\sigma^{ij}) - \frac{4}{3}\pi^{ij}\theta + \mathcal{I}^{ij}_0 + \mathcal{I}^{ij}_1 + \mathcal{I}^{ij}_2] \end{pmatrix}.$$

freeze-out

The Cooper-Frye prescription [1]

$$\frac{1}{\partial u} = \tau_{exp} \backsimeq \tau_{scatt} = \frac{1}{\langle v\sigma\rangle n}$$

$$E\frac{\mathrm{d}^3 N_i}{\mathrm{d}p^3} = \frac{\mathrm{d}^3 N_i}{dy p_T dp_T d\phi} = \frac{g_i}{(2\pi)^3}\int_\Sigma \frac{-p^\mu \mathrm{d}^3\Sigma_\mu}{\exp\left[-\frac{u^\mu p_\mu + \mu_i}{T_{\mathrm{freeze}}}\right] \pm 1}$$

$$f_i(x,p) = \frac{1}{\exp -\frac{u^\mu p_\mu + \mu_i}{T_{\mathrm{FO}}} \pm 1}$$

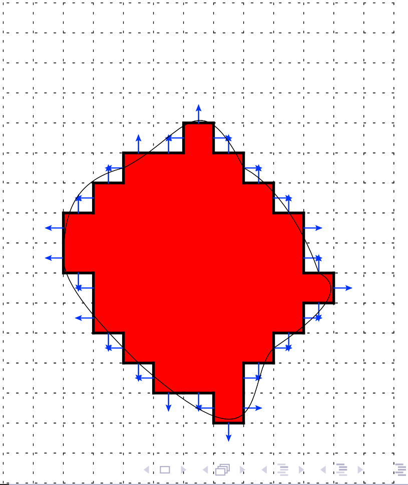
[1] Fred Cooper and Graham Frye, *Single-particle distribution in the hydrodynamic and statistical thermodynamic models of multiparticle production*, Physical Review D 10 (1974), no. 1, 186–189

freeze-out

Isothermal hypersuface: our implementation

$$d^3\Sigma_\mu = \begin{pmatrix} dV^{\perp\tau} \\ dV^{\perp x} \\ dV^{\perp y} \\ dV^{\perp\eta} \end{pmatrix} = \begin{pmatrix} \tau\ \Delta x \Delta y \Delta\eta_s\ s^\tau \\ \tau\ \Delta y \Delta\eta_s \Delta\tau\ s^x \\ \tau\ \Delta\eta_s \Delta\tau \Delta x\ s^y \\ \frac{1}{\tau}\ \Delta\tau \Delta x \Delta y\ s^\eta \end{pmatrix}$$

$$s^\mu = -\mathrm{sign}\left(\frac{\partial T}{\partial x^\mu}\right)$$



freeze-out

Isothermal- discrete freeze-out

$$\frac{(2\pi)^3}{g_i}\frac{d^3N_i}{p_\mathrm{T}dp_\mathrm{T}d\mathrm{y}d\phi} =$$

$$\sum_{T_\mathrm{fr}} \frac{m_\mathrm{T}\cosh(\mathrm{y}-\eta_s)dV^{\perp\tau} - p_\mathrm{T}(\cos\phi dV^{\perp x} + \sin\phi dV^{\perp y}) - m_\mathrm{T}\sinh(\mathrm{y}-\eta_s)dV^{\perp\eta}}{\exp\left[\left(u^\tau m_\mathrm{T}\cosh(\mathrm{y}-\eta_s) - p_\mathrm{T}(u^x\cos\phi + u^y\sin\phi) - \tau u^\eta m_\mathrm{T}\sinh(\mathrm{y}-\eta_s) - \mu_i\right)/T\right] \pm 1}$$

freeze-out

Isothermal- discrete freeze-out

$$\frac{(2\pi)^3}{g_i}\frac{d^3N_i}{p_\mathrm{T}dp_\mathrm{T}dy\,d\phi} =$$

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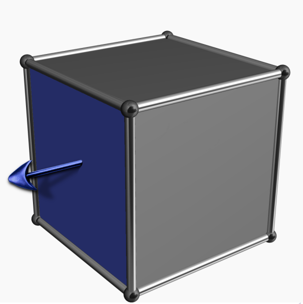
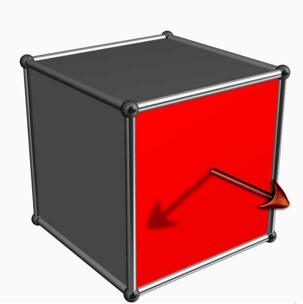


freeze-out

Isothermal- discrete freeze-out

$$\frac{(2\pi)^3}{g_i}\frac{d^3N_i}{p_T dp_T dy d\phi} =$$

$$\sum_{T_{fr}} \frac{m_T \cosh(y-\eta_s)dV^{\perp\tau} - p_T(\cos\phi dV^{\perp x} + \sin\phi dV^{\perp y}) - m_T \sinh(y-\eta_s)dV^{\perp\eta}}{\exp\left[\left(u^\tau m_T \cosh(y-\eta_s) - p_T(u^x\cos\phi + u^y\sin\phi) - \tau u^\eta m_T \sinh(y-\eta_s) - \mu_i\right)/T\right] \pm 1}$$



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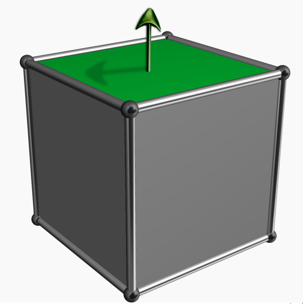

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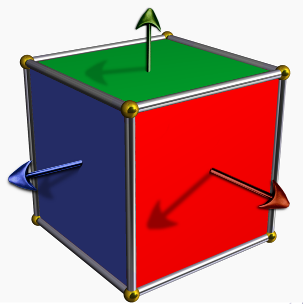


Smooth Hypersurface [2] [3]

$$\Sigma^\mu = (\tau^*(x,y,\eta), x, y, \eta) \quad \mathrm{d}^3\Sigma_\mu = -\epsilon_{\mu\nu\lambda\rho}\frac{\partial\Sigma^\nu}{\partial x}\frac{\partial\Sigma^\lambda}{\partial y}\frac{\partial\Sigma^\rho}{\partial\eta}\sqrt{-\det g}\mathrm{d}x\mathrm{d}y\mathrm{d}\eta$$

The only non vanishing derivatives:

$$\mathrm{d}^3\Sigma_\mu = \left(1, -\frac{\partial\tau^*}{\partial x}, -\frac{\partial\tau^*}{\partial y}, -\frac{\partial\tau^*}{\partial\eta}\right)\tau^*\,\mathrm{d}x\mathrm{d}y\mathrm{d}\eta$$

And the numerator of the Cooper-Frye formula is:

$$p^\mu\mathrm{d}^3\Sigma_\mu = \left\{\frac{\partial}{\partial\eta}\left[\tau^*\sinh(\mathrm{y}-\eta)\right] - \tau^*\frac{\partial\tau^*}{\partial x}p^x - \tau^*\frac{\partial\tau^*}{\partial y}p^y\right\}\mathrm{d}x\mathrm{d}y\mathrm{d}\eta$$

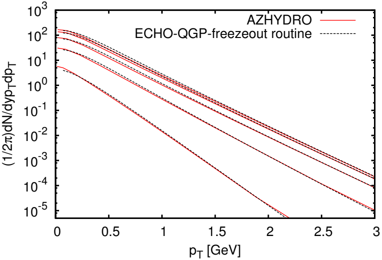
[2] Pasi Huovinen and Hannah Petersen, *Particlization in hybrid models* arXiv(nucl-th):1206.3371

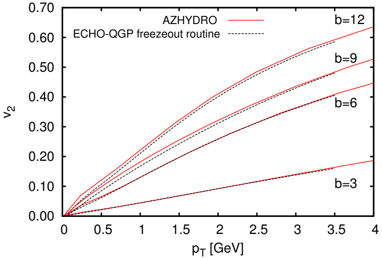
[3] Bjoern Schenke, Sangyong Jeon, and Charles Gale, *(3+1)D hydrodynamic simulation of relativistic heavy-ion collisions*, Phys.Rev. **C82** (2010), 014903

AZHYDRO features

- computes the hypersuface with triangularizations
- works in 2D+1 (assuming boost invariance)
- it approximates the distribution function to a Maxwell-Boltzmann distribution (analytically integrates through modified Bessel functions)

Tests with AZHYDRO code

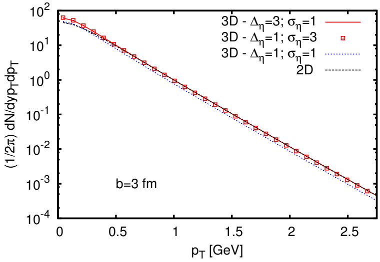


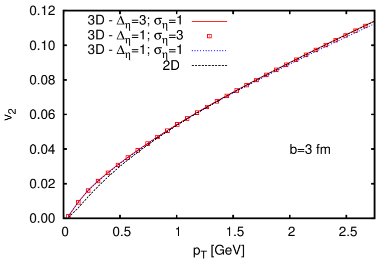


σ_{NN} mb	τ_0 fm	e_0 Gev fm^{-3}	α	b fm	μ_π GeV	T_{freeze} GeV
40	0.6	24.5	1	0,3,6,9,12	0.0622	0.120

Table : The grid spacing here used is: $\Delta x = \Delta y = 0.4$ fm $\Delta\tau = 0.16$ fm.

Consistency between 2+1D and 3+1D





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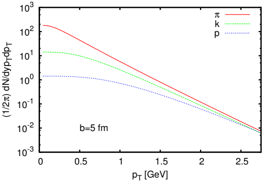
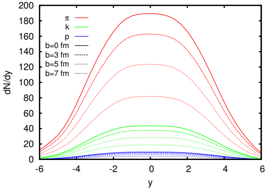
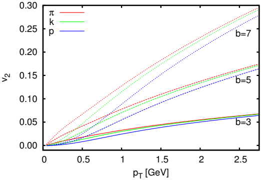


resonance feed down still not implemented: at low p_{T} the spectra is enhanced by ~ 4





Josef Sollfrank, Peter Koch, and Ulrich W. Heinz, *Is there a low p(T) 'anomaly' in the pion momentum spectra from relativistic nuclear collisions?*, Z.Phys. C52 (1991), 593–610

The effect of viscosity[4]

$$E\frac{d^3N_i}{dp^3} = \frac{d^3N_i}{dy p_T dp_T d\phi} = \frac{g_i}{(2\pi)^3}\int_\Sigma -p^\mu d^3\Sigma_\mu f_i(x,p)$$

$$f(x,p) = f_0(x,p) + \delta f(x,p) \quad (1)$$

$$\delta f(x,p) = \delta f_\eta(x,p) + \delta f_\Pi(x,p) \quad (2)$$

[4] Paul Romatschke, *New Developments in Relativistic Viscous Hydrodynamics*, Int.J.Mod.Phys. **E19** (2010), 1–53

The effect of viscosity[4]

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The effect of viscosity[4]

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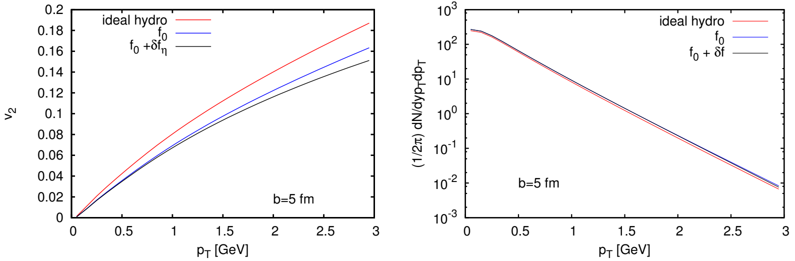
$$f(x,p) = f_0(x,p) + \delta f(x,p) \tag{1}$$

$$\delta f(x,p) = \delta f_\eta(x,p) = f_0(1 \pm f_0)\frac{p^\alpha p^\beta \pi_{\alpha\beta}}{2T^2(e+p)} \tag{2}$$

[4] Paul Romatschke, *New Developments in Relativistic Viscous Hydrodynamics*, Int.J.Mod.Phys. **E19** (2010), 1–53

The effect of viscosity

$$\eta/s = 0.08$$



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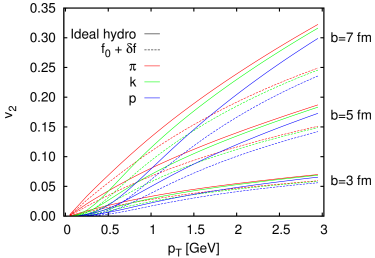

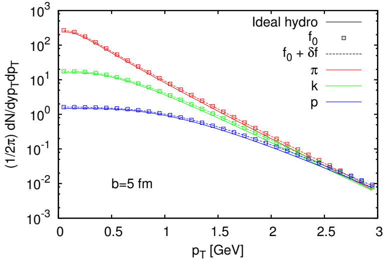

Summary and Outlook

tested
- consistency between our results and AZHYDRO
- extension of the benchmark to the 3D
- viscosity behaviour

developing
- particlization
- δf_{Π}

to do
- Smooth 3-D hypersuface interpolation

next future
- Include finite baryon density and test EoS at FAIR/NICA energy
- Include magnetic effects
- Include polarization effects through vorticity
- Include strangeness conservation

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