Finite-range corrections and Universality in Efimov physic

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Outline

Efimov Physics

- Efimov Effect
- Discrete Scale Invariance

Finite-range Effect

- 3-Body Bound States
- Scattering Length
- Recombination

N-body Universality

- N-Body States
- Universality

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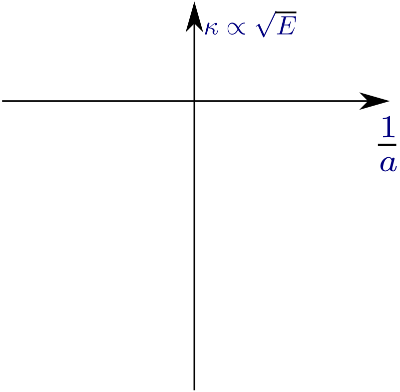
Efimov Effect



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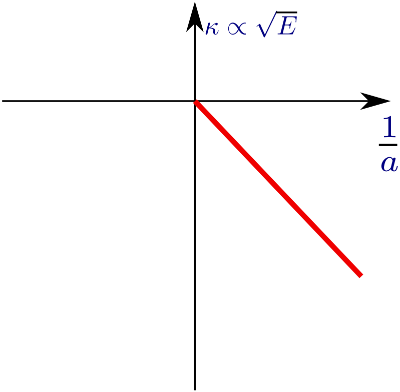

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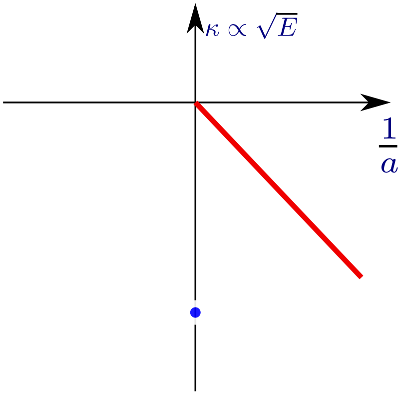

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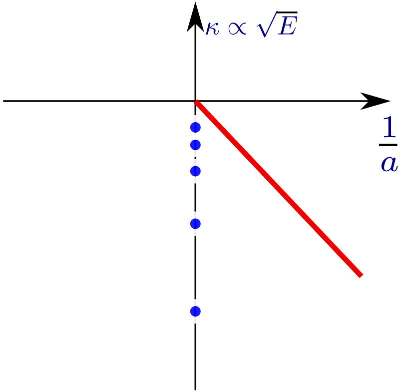

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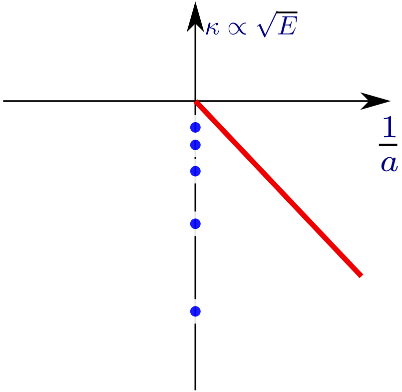

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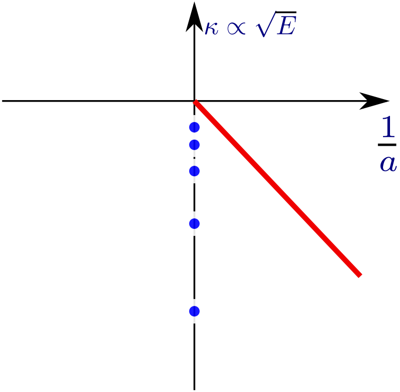

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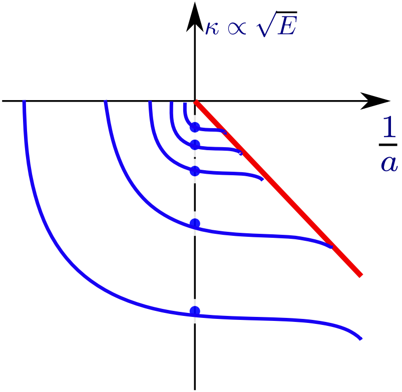

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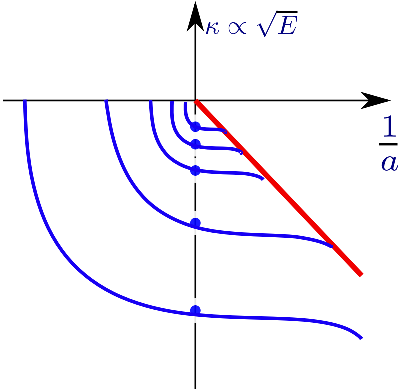

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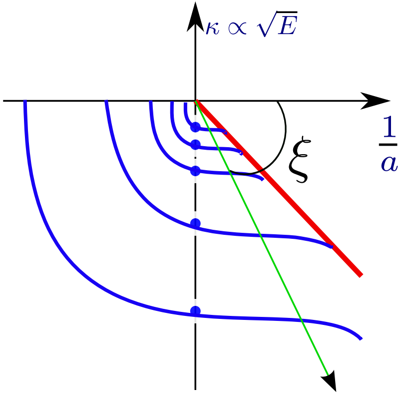

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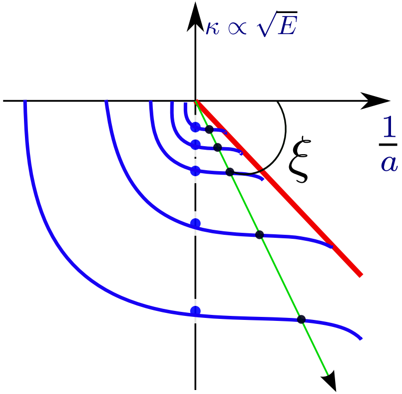

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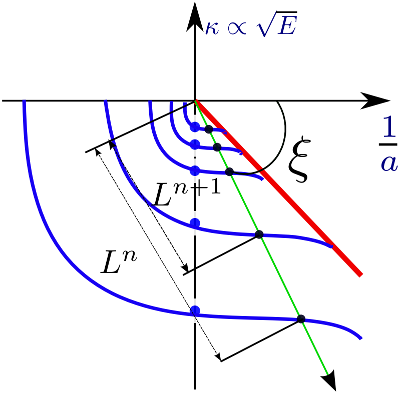
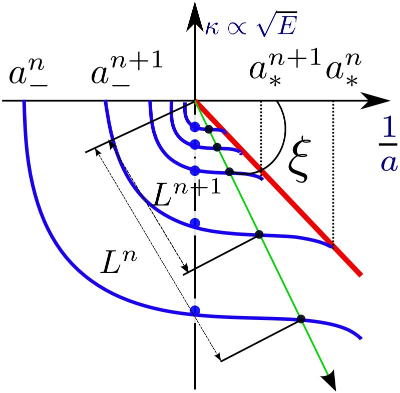


For each ξ

$L^{n+1}/L^n \to 1/22.7$

Discrete Scale Invariance



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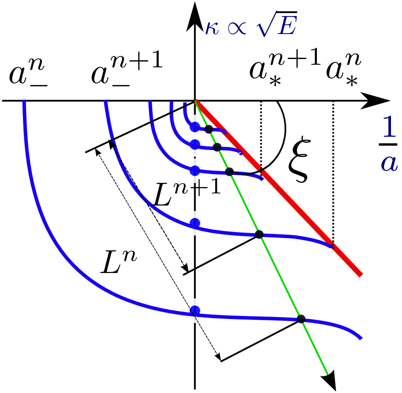

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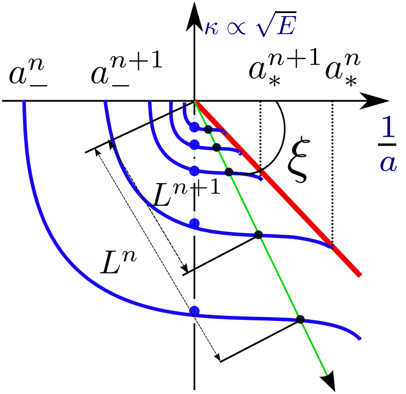

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Recombination Rate at the threshold

$$K_3 = \frac{128\pi^2(4\pi - 3\sqrt{3})}{\sinh^2(\pi s_0) + \cosh^2(\pi s_0)\cot^2[s_0 \ln(\kappa_* a) + \gamma]} \frac{\hbar a^4}{m},$$

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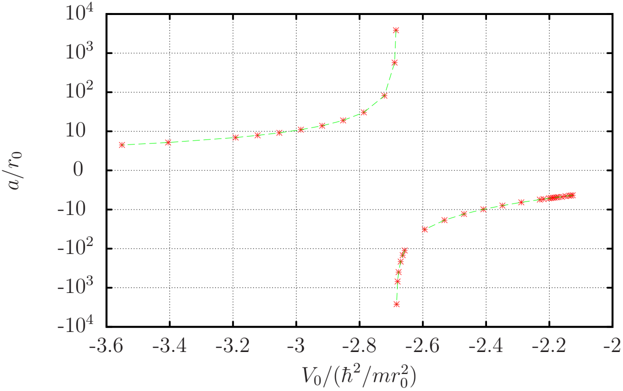
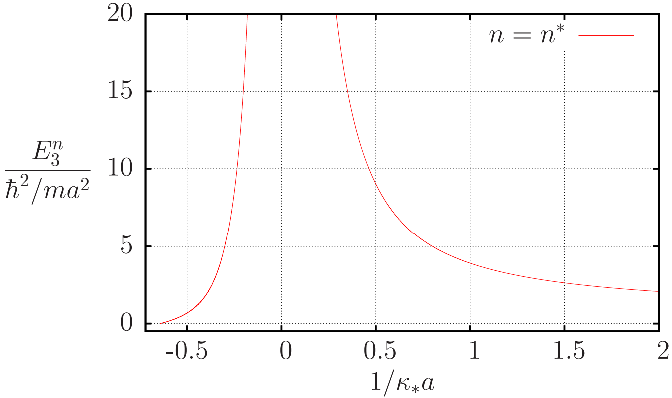


3-Body Bound States



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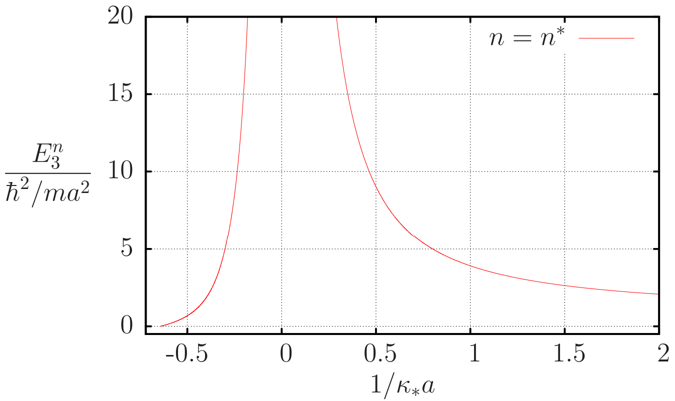

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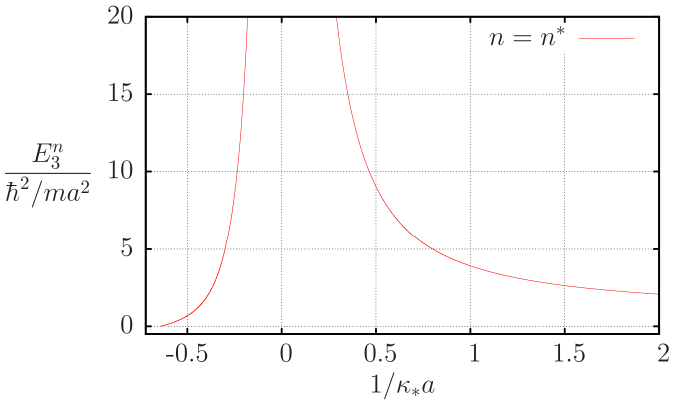

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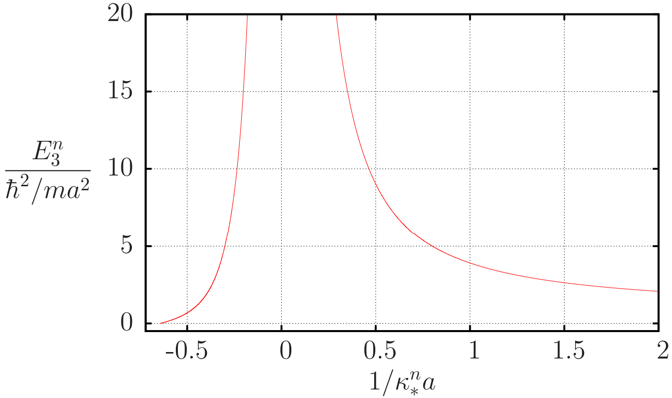

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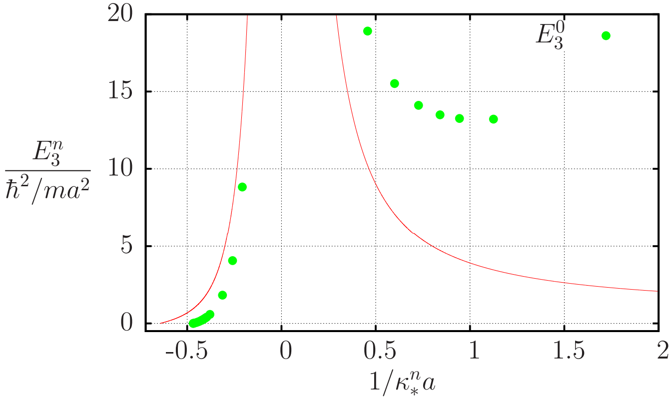

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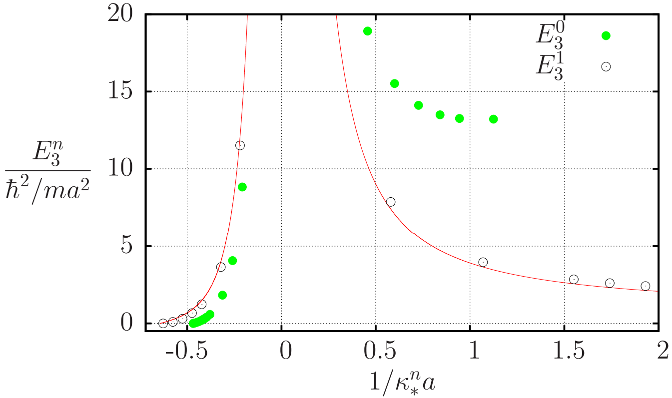

$$
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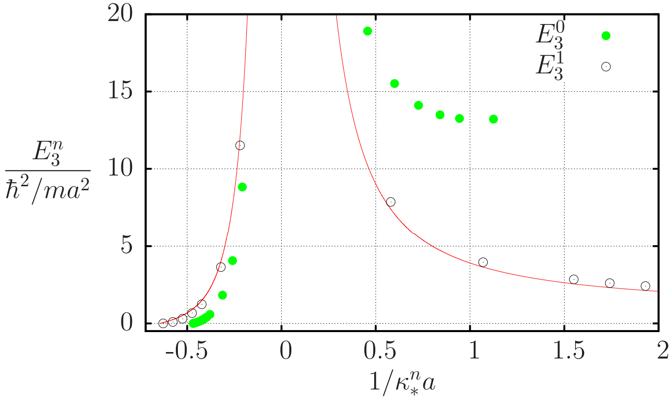

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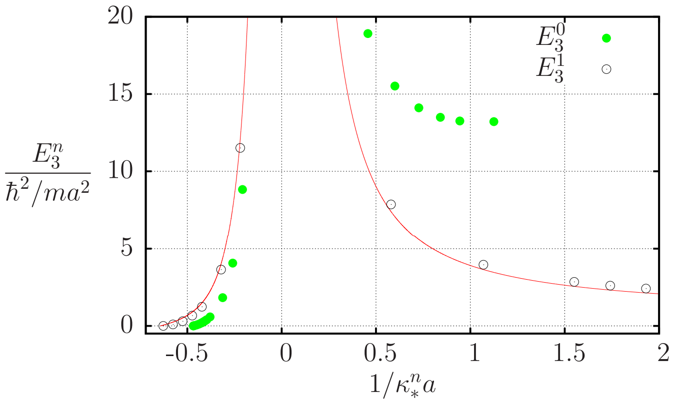

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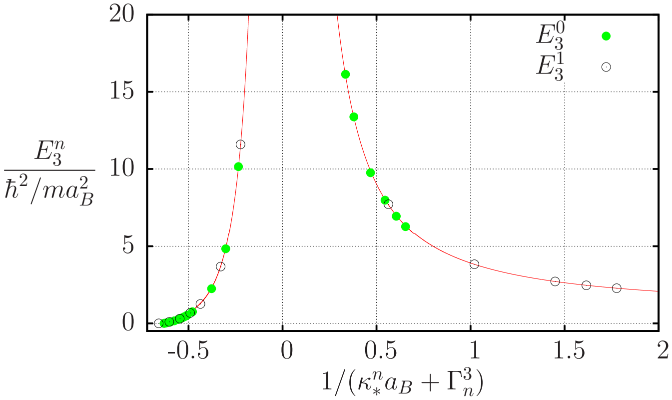

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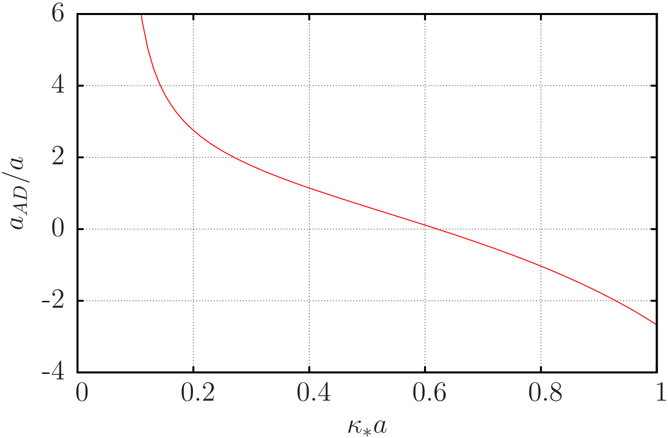
Particle-Dimer Scattering Length



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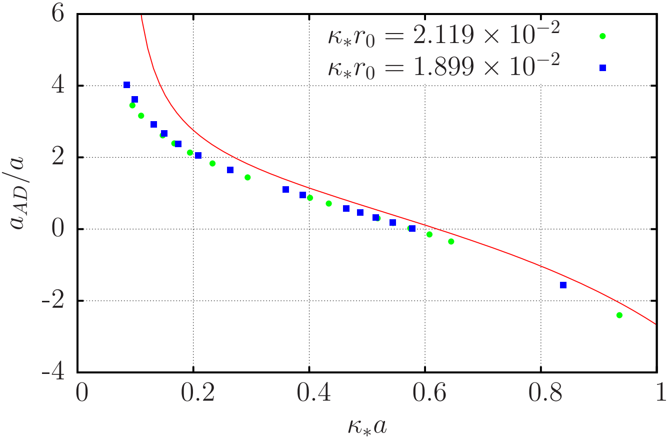

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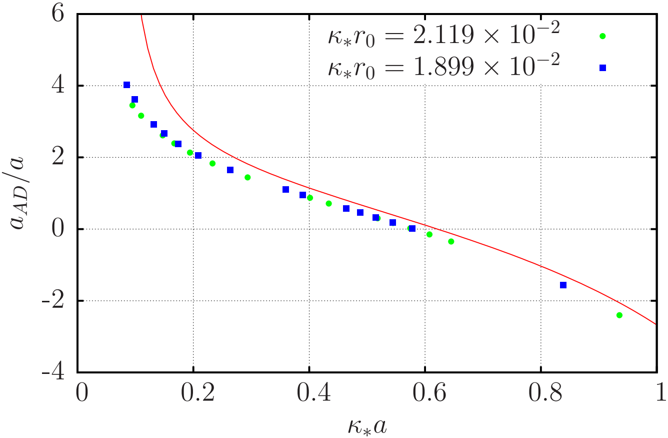

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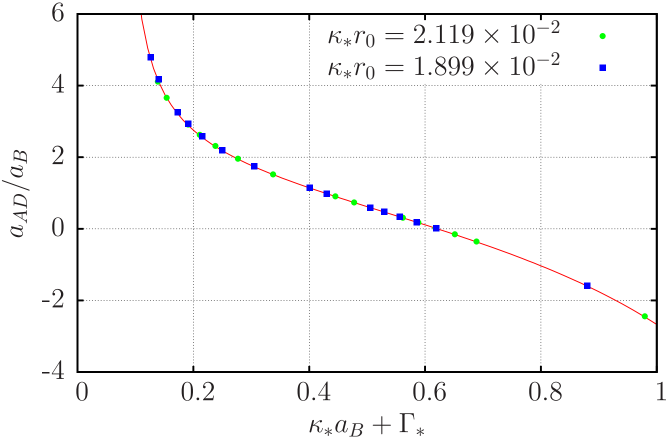

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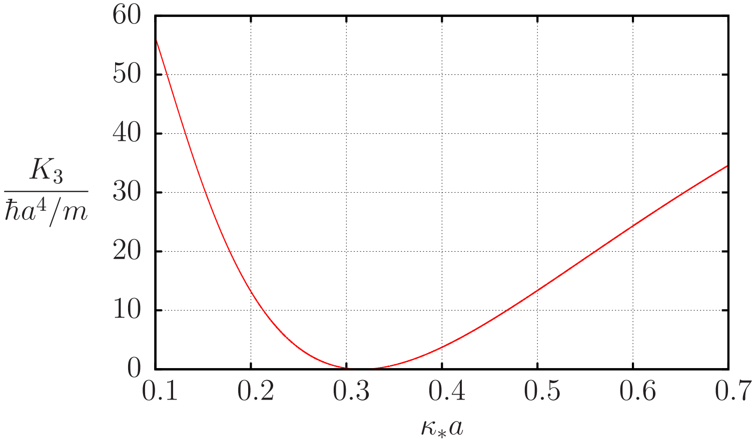
Recombination at the threshold



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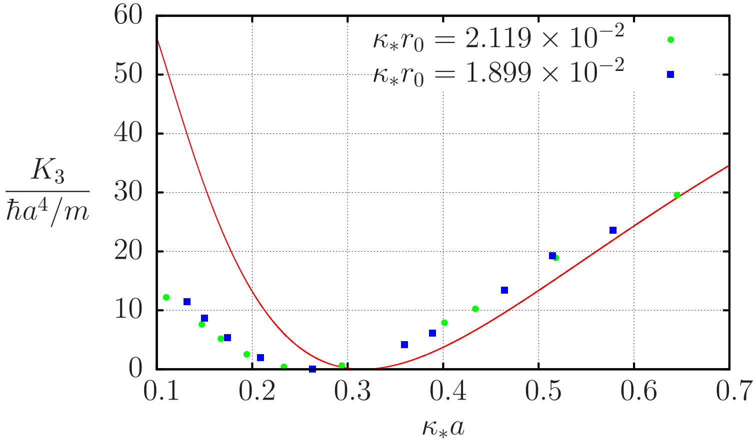

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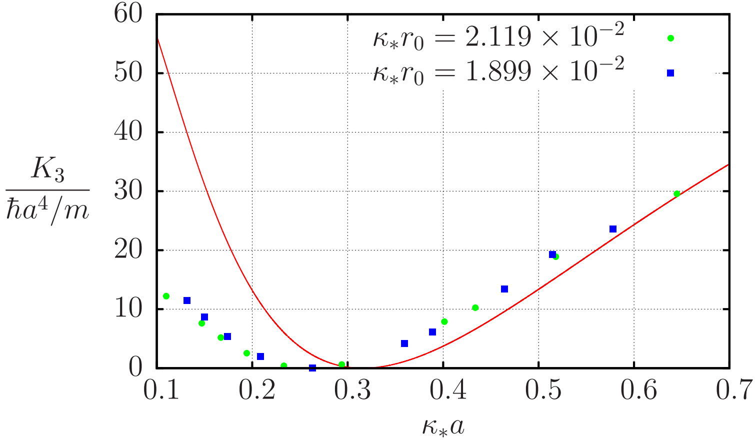

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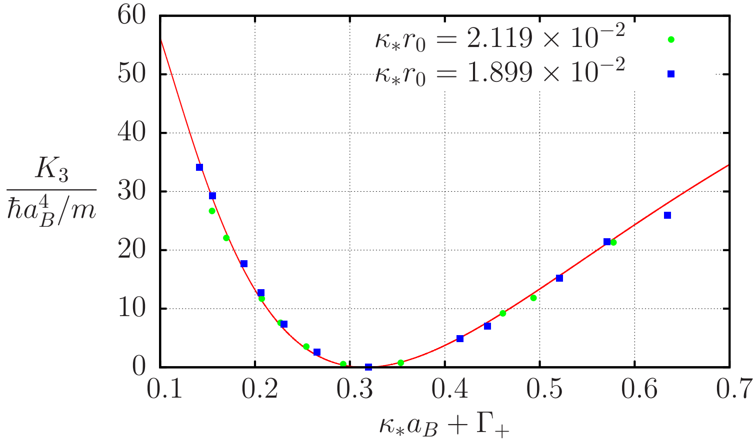

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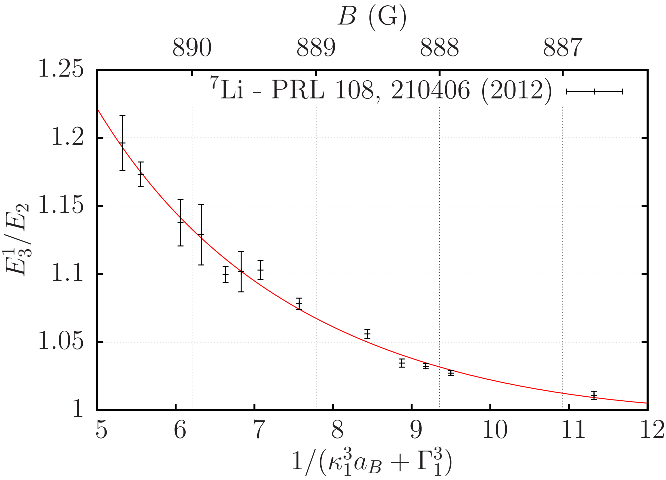
Experimental data



Olga Machtey, Zav Shotan, Noam Gross, and Lev Khaykovich
Phys. Rev. Lett. 108, 210406 (2012)

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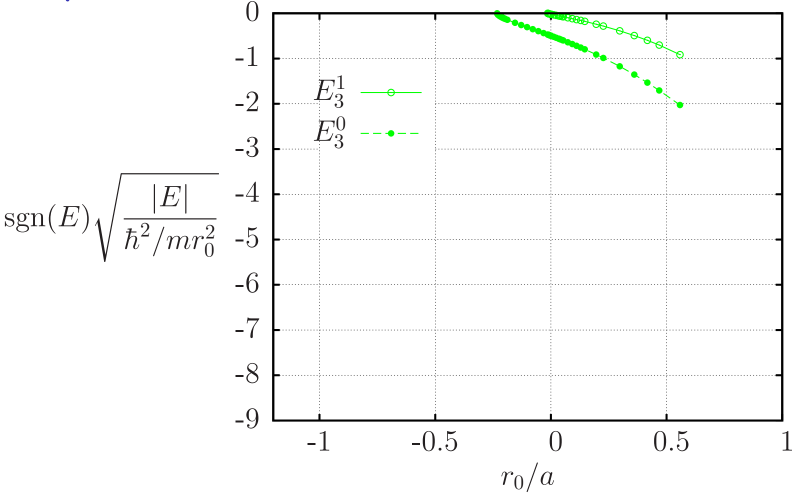
N-body Efimov Plot



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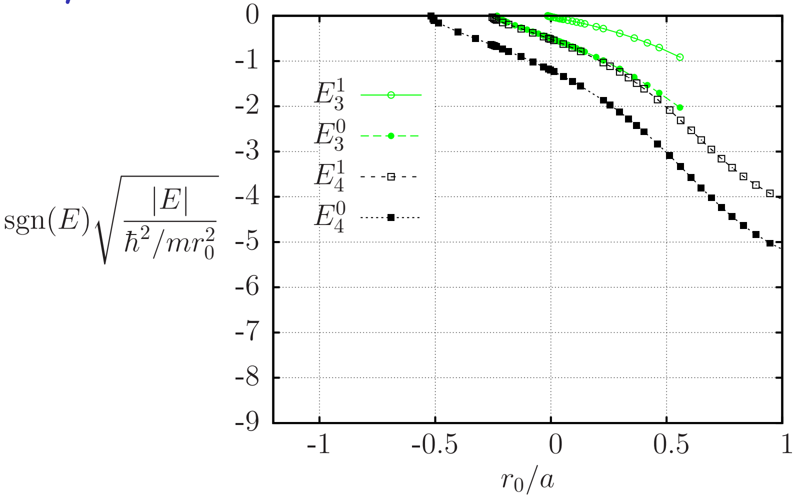

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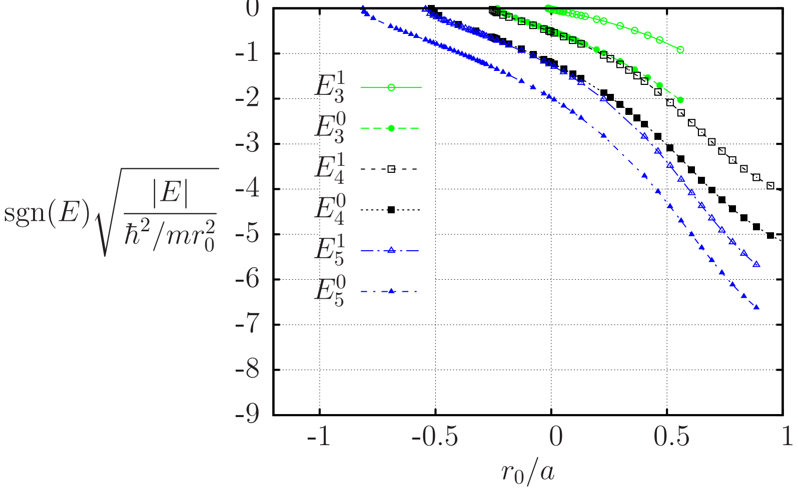

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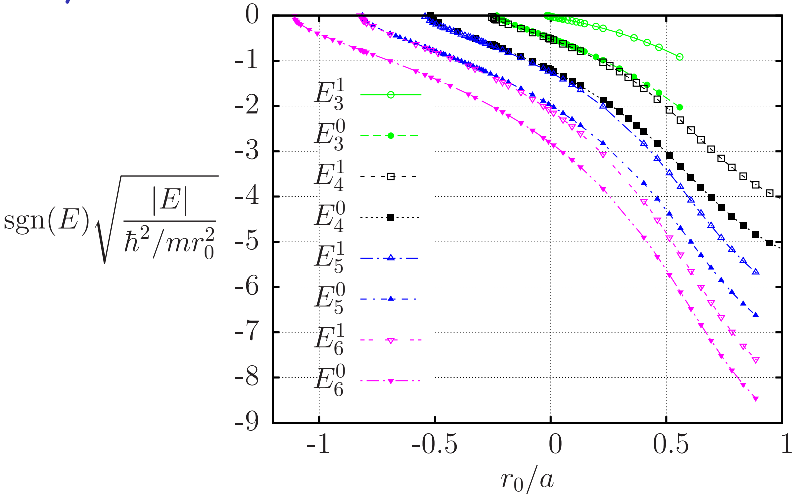

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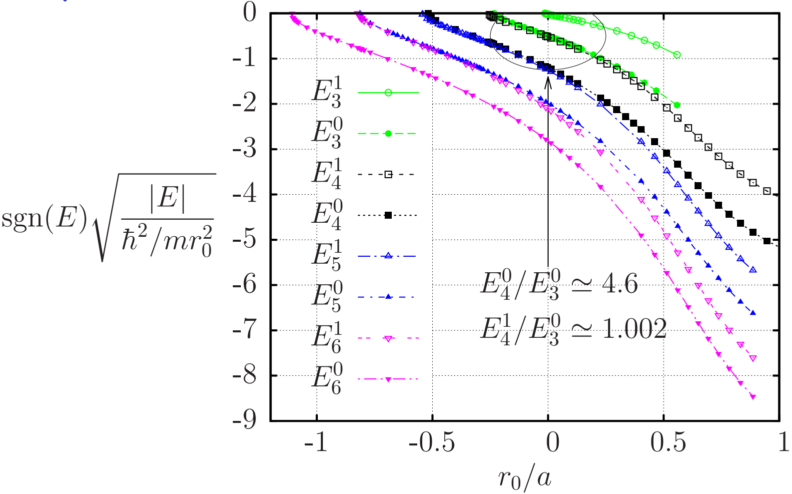

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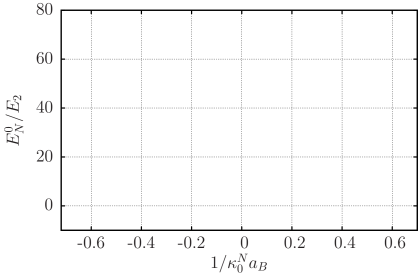
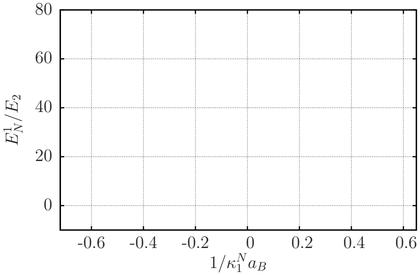
Universality





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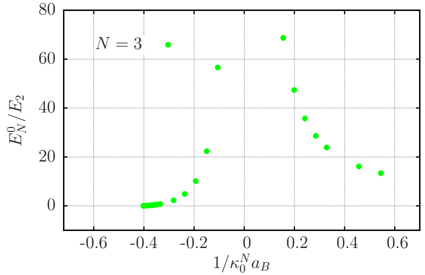

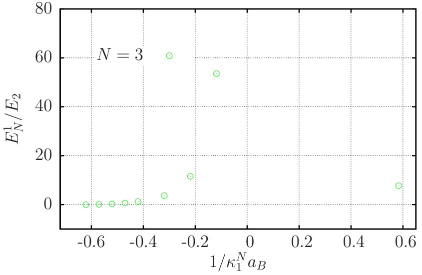

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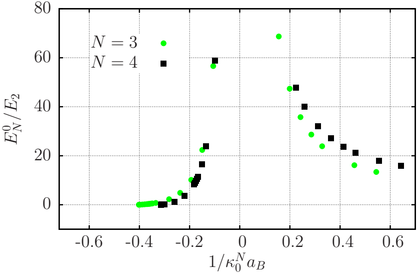

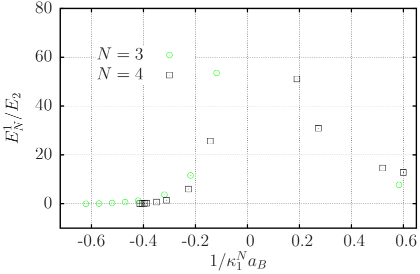

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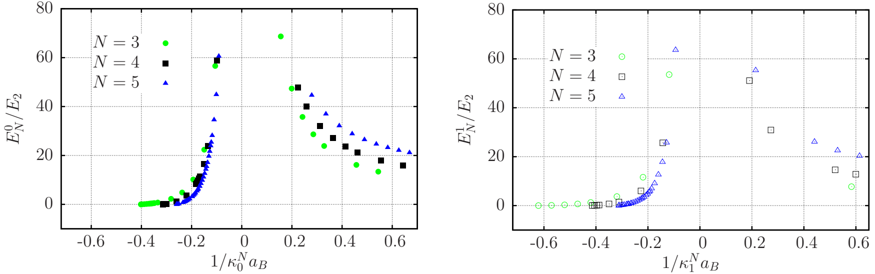
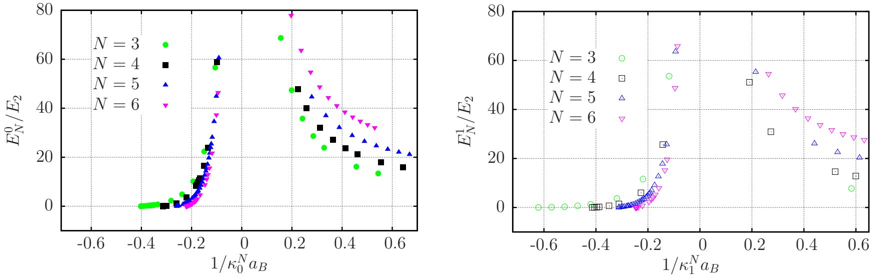


Universality



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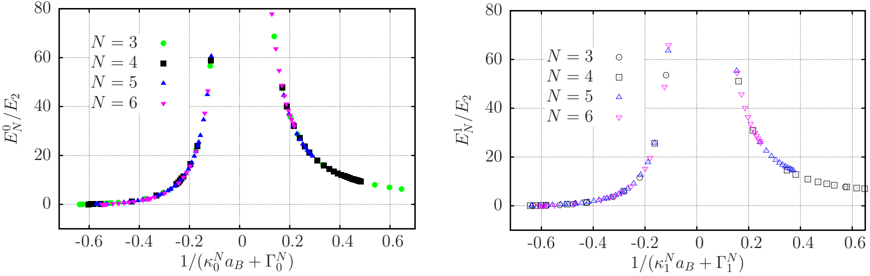

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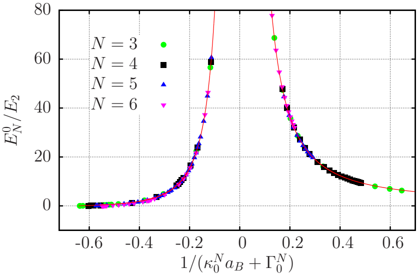
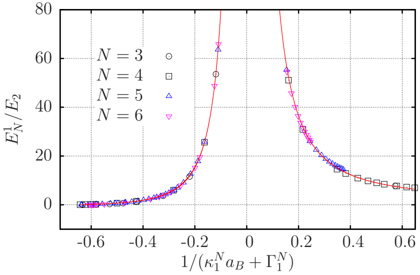


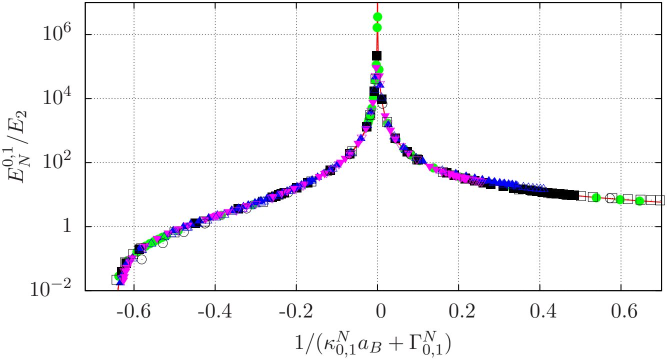


Universal Formula

$$E_N^n/E_2 = \tan^2 \xi$$

$$\kappa_n^N a_B + \Gamma_n^N = \frac{e^{-\Delta(\xi)/2s_0}}{\cos \xi}$$

Universality



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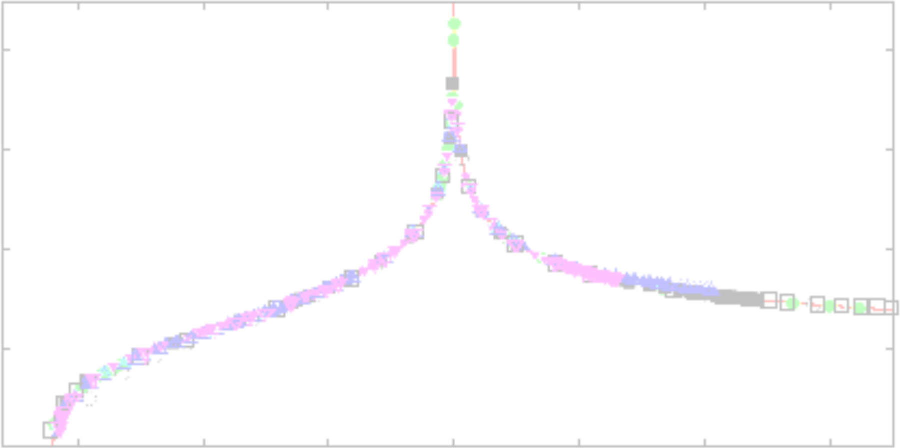
References and Collaborators



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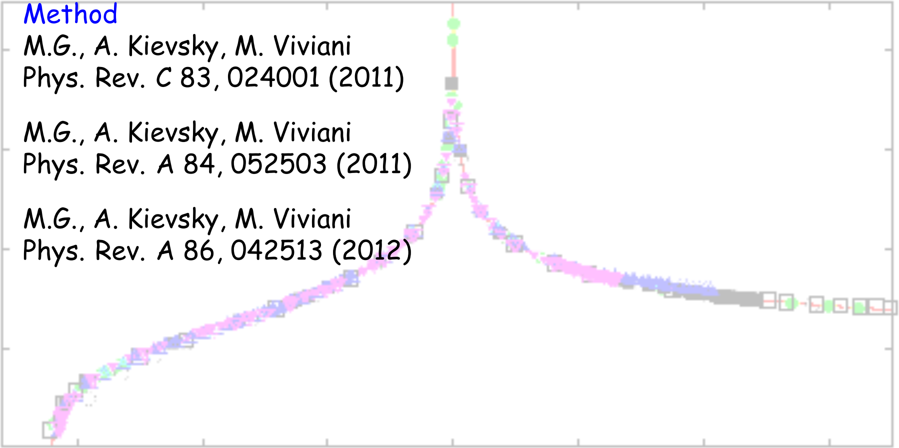

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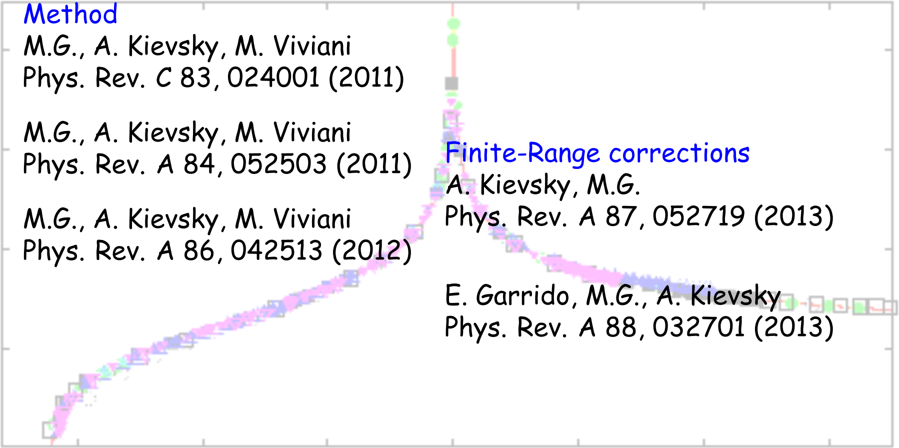

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Phys. Rev. A 87, 052719 (2013)

E. Garrido, M.G., A. Kievsky
Phys. Rev. A 88, 032701 (2013)

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Phys. Rev. A 88, 032701 (2013)

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Thanks!