

# Nuclear electromagnetic processes in ChEFT

Luca Girlanda

Università del Salento & INFN Lecce

work with: L.E. Marcucci, S. Pastore, M. Piarulli, R. Schiavilla, M. Viviani

S. Pastore, et al. Phys. Rev. C80 (2009) 034004

L. Girlanda et al. Phys. Rev. Lett. 105 (2010) 232502

S. Pastore et al. Phys. Rev. C84 (2011) 024001

M. Piarulli et al. Phys. Rev. C87 (2013) 014006

## Outline

- ▶ Noether currents in ChEFT
- ▶ Calculational framework: recoil-corrected TOPT
- ▶ Anatomy of charge and current operators up to 1-loop
- ▶ Strategies to fix the LECs
- ▶ Applications: elastic electron scattering on  $A = 2, 3$  systems
- ▶ Summary and conclusions

# External currents in ChEFT

$\Gamma_c$  generates connected Green functions of quark bilinears

$$e^{\Gamma_c[\mathcal{V}_\mu, \mathcal{A}_\mu, \mathcal{S}, \mathcal{P}]} = \int [\mathcal{D}\mu]_{\text{QCDE}} i \int dx \{ \mathcal{L}_{YM} + \bar{q} [i \not{D} + \not{\mathcal{V}} + \gamma_5 \not{\mathcal{A}} - \mathcal{S} + i \gamma_5 \mathcal{P}] q \}$$

If  $\mathcal{S} = 0 \implies$  (global) chiral symmetry:  $q_L \rightarrow g_L q_L, \quad q_R \rightarrow g_R q_R$

# External currents in ChEFT

$\Gamma_c$  generates connected Green functions of quark bilinears

$$e^{\Gamma_c[\mathcal{V}_\mu, \mathcal{A}_\mu, \mathcal{S}, \mathcal{P}]} = \int [\mathcal{D}\mu]_{\text{QCDE}} e^{i \int dx \{ \mathcal{L}_{YM} + \bar{q} [i \not{D} + \not{\mathcal{V}} + \gamma_5 \not{\mathcal{A}} - \mathcal{S} + i \gamma_5 \mathcal{P}] q \}}$$

If  $\mathcal{S} = 0 \implies$  (global) chiral symmetry:  $q_L \rightarrow g_L q_L$ ,  $q_R \rightarrow g_R q_R$

Chiral Ward Identities can be obtained in the following way:

1. Promote  $g_L$  and  $g_R$  to local transformations

# External currents in ChEFT

$\Gamma_c$  generates connected Green functions of quark bilinears

$$e^{\Gamma_c[\mathcal{V}_\mu, \mathcal{A}_\mu, \mathcal{S}, \mathcal{P}]} = \int [\mathcal{D}\mu]_{\text{QCDE}} i \int dx \{ \mathcal{L}_{YM} + \bar{q} [i \not{D} + \not{\mathcal{V}} + \gamma_5 \not{\mathcal{A}} - \mathcal{S} + i \gamma_5 \mathcal{P}] q \}$$

If  $\mathcal{S} = 0 \implies$  (global) chiral symmetry:  $q_L \rightarrow g_L q_L$ ,  $q_R \rightarrow g_R q_R$   
Chiral Ward Identities can be obtained in the following way:

1. Promote  $g_L$  and  $g_R$  to local transformations
2. Take the sources to transform as gauge fields

$$\mathcal{V}_\mu + \mathcal{A}_\mu \rightarrow \mathcal{V}'_\mu + \mathcal{A}'_\mu = g_R(x) (\mathcal{V}_\mu + \mathcal{A}_\mu) g_R^\dagger(x) + i g_R(x) \partial_\mu g_R^\dagger(x)$$

$$\mathcal{V}_\mu - \mathcal{A}_\mu \rightarrow \mathcal{V}'_\mu - \mathcal{A}'_\mu = g_L(x) (\mathcal{V}_\mu - \mathcal{A}_\mu) g_L^\dagger(x) + i g_L(x) \partial_\mu g_L^\dagger(x)$$

$$\mathcal{S} + i\mathcal{P} \rightarrow \mathcal{S}' + i\mathcal{P}' = g_R(x) (\mathcal{S} + i\mathcal{P}) g_L^\dagger(x)$$

$$\mathcal{S} - i\mathcal{P} \rightarrow \mathcal{S}' - i\mathcal{P}' = g_L(x) (\mathcal{S} + i\mathcal{P}) g_R^\dagger(x)$$

# External currents in ChEFT

$\Gamma_c$  generates connected Green functions of quark bilinears

$$e^{\Gamma_c[\mathcal{V}_\mu, \mathcal{A}_\mu, \mathcal{S}, \mathcal{P}]} = \int [\mathcal{D}\mu]_{\text{QCDE}} e^{i \int dx \{ \mathcal{L}_{YM} + \bar{q} [i \not{D} + \not{\mathcal{V}} + \gamma_5 \not{\mathcal{A}} - \mathcal{S} + i \gamma_5 \mathcal{P}] q \}}$$

If  $\mathcal{S} = 0 \implies$  (global) chiral symmetry:  $q_L \rightarrow g_L q_L$ ,  $q_R \rightarrow g_R q_R$

Chiral Ward Identities can be obtained in the following way:

1. Promote  $g_L$  and  $g_R$  to local transformations
2. Take the sources to transform as gauge fields

$$\mathcal{V}_\mu + \mathcal{A}_\mu \rightarrow \mathcal{V}'_\mu + \mathcal{A}'_\mu = g_R(x) (\mathcal{V}_\mu + \mathcal{A}_\mu) g_R^\dagger(x) + i g_R(x) \partial_\mu g_R^\dagger(x)$$

$$\mathcal{V}_\mu - \mathcal{A}_\mu \rightarrow \mathcal{V}'_\mu - \mathcal{A}'_\mu = g_L(x) (\mathcal{V}_\mu - \mathcal{A}_\mu) g_L^\dagger(x) + i g_L(x) \partial_\mu g_L^\dagger(x)$$

$$\mathcal{S} + i\mathcal{P} \rightarrow \mathcal{S}' + i\mathcal{P}' = g_R(x) (\mathcal{S} + i\mathcal{P}) g_L^\dagger(x)$$

$$\mathcal{S} - i\mathcal{P} \rightarrow \mathcal{S}' - i\mathcal{P}' = g_L(x) (\mathcal{S} + i\mathcal{P}) g_R^\dagger(x)$$

$\implies$  it amounts to local transformations of the quark fields,  
 $q_L \rightarrow g_L(x) q_L$ ,  $q_R \rightarrow g_R(x) q_R$ , that leave  $\Gamma_c$  (almost) invariant

The effective theory is a low-energy representation of  $\Gamma_c$  given in terms of the active degrees of freedom

$$e^{\Gamma_c[\mathcal{V}_\mu, \mathcal{A}_\mu, \mathcal{S}, \mathcal{P}|\bar{\eta}]} = \int [\mathcal{D}U \mathcal{D}\psi] e^{i \int dx \{ \mathcal{L}_{\text{eff}}[U, \psi; \mathcal{V}_\mu, \mathcal{A}_\mu, \mathcal{S}, \mathcal{P}] + \bar{\eta} \psi \}}$$

with  $\mathcal{L}_{\text{eff}}$  including all possible chiral invariant terms involving pions ( $U$ ) and nucleons ( $\psi$ )

The effective theory is a low-energy representation of  $\Gamma_c$  given in terms of the active degrees of freedom

$$e^{\Gamma_c[\mathcal{V}_\mu, \mathcal{A}_\mu, \mathcal{S}, \mathcal{P} | \bar{\eta}]} = \int [\mathcal{D}U \mathcal{D}\psi] e^{i \int d^4x \{ \mathcal{L}_{\text{eff}}[U, \psi; \mathcal{V}_\mu, \mathcal{A}_\mu, \mathcal{S}, \mathcal{P}] + \bar{\eta} \psi \}}$$

with  $\mathcal{L}_{\text{eff}}$  including all possible chiral invariant terms involving pions ( $U$ ) and nucleons ( $\psi$ )

At the end the functional is to be evaluated at

$$\mathcal{S} = \begin{pmatrix} m_u & \\ & m_d \end{pmatrix}, \quad \mathcal{V}_\mu = eA_\mu \begin{pmatrix} m_u & \\ & m_d \end{pmatrix}, \dots$$



The effective theory is a low-energy representation of  $\Gamma_c$  given in terms of the active degrees of freedom

$$e^{\Gamma_c[\mathcal{V}_\mu, \mathcal{A}_\mu, \mathcal{S}, \mathcal{P}|\bar{\eta}]} = \int [\mathcal{D}U \mathcal{D}\psi] e^{i \int dx \{ \mathcal{L}_{\text{eff}}[U, \psi; \mathcal{V}_\mu, \mathcal{A}_\mu, \mathcal{S}, \mathcal{P}] + \bar{\eta} \psi \}}$$

with  $\mathcal{L}_{\text{eff}}$  including all possible chiral invariant terms involving pions ( $U$ ) and nucleons ( $\psi$ )

At the end the functional is to be evaluated at

$$\mathcal{S} = \begin{pmatrix} m_u & \\ & m_d \end{pmatrix}, \quad \mathcal{V}_\mu = eA_\mu \begin{pmatrix} m_u & \\ & m_d \end{pmatrix}, \dots$$

Power counting is the organizing principle

-it works because of chiral symmetry: Goldstone bosons have derivative interactions

The effective theory is a low-energy representation of  $\Gamma_c$  given in terms of the active degrees of freedom

$$e^{\Gamma_c[\mathcal{V}_\mu, \mathcal{A}_\mu, \mathcal{S}, \mathcal{P} | \bar{\eta}]} = \int [\mathcal{D}U \mathcal{D}\psi] e^{i \int dx \{ \mathcal{L}_{\text{eff}}[U, \psi; \mathcal{V}_\mu, \mathcal{A}_\mu, \mathcal{S}, \mathcal{P}] + \bar{\eta} \psi \}}$$

with  $\mathcal{L}_{\text{eff}}$  including all possible chiral invariant terms involving pions ( $U$ ) and nucleons ( $\psi$ )

At the end the functional is to be evaluated at

$$\mathcal{S} = \begin{pmatrix} m_u & \\ & m_d \end{pmatrix}, \quad \mathcal{V}_\mu = eA_\mu \begin{pmatrix} m_u & \\ & m_d \end{pmatrix}, \dots$$

Power counting is the organizing principle

-it works because of chiral symmetry: Goldstone bosons have derivative interactions

-it explains the hierarchy of nuclear forces and gives rise to realistic potentials (Entem-Machleidt, Epelbaum-Gloeckle-Meissner)

# History

nuclear electroweak currents were addressed immediately after Weinberg's proposal, and used in the hybrid approach

# History

nuclear electroweak currents were addressed immediately after Weinberg's proposal, and used in the hybrid approach

(some) related work on electromagnetic processes:

- ▶ Park, Min, Rho, 1996  
application to hybrid calculations in  $A=2-4$  systems (Song, Lazauskas, Park, 2009-2011)
- ▶ Meissner, Walzl, 2001; D. Phillips 2003  
isoscalar component, applied to deuteron static properties and form factors
- ▶ Koelling, Epelbaum, Krebs, Meissner, 2009-2011  
within the unitary transformation formalisms;  
hybrid application to  $d$  and  $^3\text{He}$  photodisintegration (Rozpedzik et al, 2011)

## Power counting

Consider the  $NN$  amplitude

$$\langle f|T|i\rangle = \langle f|H_I \sum_n \left( \frac{1}{E_i - H_0 + i\epsilon} H_I \right)^{N-1} |i\rangle$$

a generic (reducible or irreducible) contribution with  $N$  vertices will scale like

$$\left[ \prod_{i=1}^N p^{\nu_i} \right] p^{-(N-N_K-1)} p^{-2N_K}$$

out of the  $N - 1$  energy denominators,  $N_K$  are purely nucleonic (small) the remaining (large) energy denominators can be further expanded in  $E/\omega_\pi$

$$\frac{1}{E_i - E_I - \omega_\pi} \sim -\frac{1}{\omega_\pi} \left[ 1 + \frac{E_i - E_I}{\omega_\pi} + \dots \right]$$

## Power counting

Consider the  $NN$  amplitude

$$\langle f|T|i\rangle = \langle f|H_I \sum_n \left( \frac{1}{E_i - H_0 + i\epsilon} H_I \right)^{N-1} |i\rangle$$

a generic (reducible or irreducible) contribution with  $N$  vertices will scale like

$$\left[ \prod_{i=1}^N p^{\nu_i} \right] p^{-(N-N_K-1)} p^{-2N_K}$$

out of the  $N - 1$  energy denominators,  $N_K$  are purely nucleonic (small) the remaining (large) energy denominators can be further expanded in  $E/\omega_\pi$

$$\frac{1}{E_i - E_I - \omega_\pi} \sim -\frac{1}{\omega_\pi} \left[ 1 + \frac{E_i - E_I}{\omega_\pi} + \dots \right]$$

at the end

$$T = T^{(0)} + T^{(1)} + T^{(2)} + \dots, \quad T^{(n)} \sim O(p^n)$$

## Power counting

Consider the  $NN$  amplitude

$$\langle f|T|i\rangle = \langle f|H_I \sum_n \left( \frac{1}{E_i - H_0 + i\epsilon} H_I \right)^{N-1} |i\rangle$$

a generic (reducible or irreducible) contribution with  $N$  vertices will scale like

$$\left[ \prod_{i=1}^N p^{\nu_i} \right] p^{-(N-N_K-1)} p^{-2N_K}$$

out of the  $N - 1$  energy denominators,  $N_K$  are purely nucleonic (small) the remaining (large) energy denominators can be further expanded in  $E/\omega_\pi$

$$\frac{1}{E_i - E_f - \omega_\pi} \sim -\frac{1}{\omega_\pi} \left[ 1 + \frac{E_i - E_f}{\omega_\pi} + \dots \right]$$

at the end

$$T = T^{(0)} + T^{(1)} + T^{(2)} + \dots, \quad T^{(n)} \sim O(p^n)$$

We can define  $v = v^{(0)} + v^{(1)} + \dots$  such that

$$T = v + vG_0v + vG_0vG_0v + \dots$$

order by order in the chiral expansion

# From amplitudes to potentials

Solving for  $v^{(n)}$  we have

$$\begin{aligned}v^{(0)} &= T^{(0)} , \\v^{(1)} &= T^{(1)} - \left[ v^{(0)} G_0 v^{(0)} \right] , \\v^{(2)} &= T^{(2)} - \left[ v^{(0)} G_0 v^{(0)} G_0 v^{(0)} \right] \\&\quad - \left[ v^{(1)} G_0 v^{(0)} + v^{(0)} G_0 v^{(1)} \right] , \\v^{(3)} &= T^{(3)} - \left[ v^{(0)} G_0 v^{(0)} G_0 v^{(0)} G_0 v^{(0)} \right] \\&\quad - \left[ v^{(1)} G_0 v^{(0)} G_0 v^{(0)} + \text{permutations} \right] \\&\quad - \left[ v^{(2)} G_0 v^{(0)} + v^{(0)} G_0 v^{(2)} \right] - \left[ v^{(1)} G_0 v^{(1)} \right] .\end{aligned}$$

where  $G_0 \sim p^{-2} d^3 p \sim O(p)$



# From amplitudes to potentials

Solving for  $v^{(n)}$  we have

$$\begin{aligned}v^{(0)} &= T^{(0)} , \\v^{(1)} &= T^{(1)} - \left[ v^{(0)} G_0 v^{(0)} \right] , \\v^{(2)} &= T^{(2)} - \left[ v^{(0)} G_0 v^{(0)} G_0 v^{(0)} \right] \\&\quad - \left[ v^{(1)} G_0 v^{(0)} + v^{(0)} G_0 v^{(1)} \right] , \\v^{(3)} &= T^{(3)} - \left[ v^{(0)} G_0 v^{(0)} G_0 v^{(0)} G_0 v^{(0)} \right] \\&\quad - \left[ v^{(1)} G_0 v^{(0)} G_0 v^{(0)} + \text{permutations} \right] \\&\quad - \left[ v^{(2)} G_0 v^{(0)} + v^{(0)} G_0 v^{(2)} \right] - \left[ v^{(1)} G_0 v^{(1)} \right] .\end{aligned}$$

where  $G_0 \sim p^{-2} d^3 p \sim O(p)$

- ▶ this procedure allows to systematically subtract the terms due to the iteration of the dynamical equation
- ▶ nevertheless it is ambiguous, because we need the  $v^{(n)}$  *off shell*

## Off-shell ambiguity

There exists a whole class of 2nd order recoil corrections to OPE which are equivalent on shell, parametrized by a parameter  $\nu$  (Friar 1980)

$$v_{RC}^{(2)}(\nu = 0) = v_{\pi}^{(0)}(\mathbf{k}) \frac{(E'_1 - E_1)^2 + (E'_2 - E_2)^2}{2\omega_k^2}$$

$$v_{RC}^{(2)}(\nu = 1) = -v_{\pi}^{(0)}(\mathbf{k}) \frac{(E'_1 - E_1)(E'_2 - E_2)}{\omega_k^2}$$

## Off-shell ambiguity

There exists a whole class of 2nd order recoil corrections to OPE which are equivalent on shell, parametrized by a parameter  $\nu$  (Friar 1980)

$$v_{RC}^{(2)}(\nu = 0) = v_{\pi}^{(0)}(\mathbf{k}) \frac{(E'_1 - E_1)^2 + (E'_2 - E_2)^2}{2\omega_k^2}$$

$$v_{RC}^{(2)}(\nu = 1) = -v_{\pi}^{(0)}(\mathbf{k}) \frac{(E'_1 - E_1)(E'_2 - E_2)}{\omega_k^2}$$

The off-shell ambiguities will affect successive terms  $v^{(n)}$ : for each  $v^{(2)}(\nu)$  there is a corresponding  $v^{(3)}(\nu)$

However, the different choices are related by a unitary transformation,

$$H(\nu) = e^{-iU(\nu)} H(\nu = 0) e^{iU(\nu)}$$

with  $U = U^{(0)} + U^{(1)} + \dots$  explicitly

$$i U^{(0)}(\nu) = -\nu \frac{v_{\pi}^{(0)}(\mathbf{p}' - \mathbf{p})}{(\mathbf{p}' - \mathbf{p})^2 + m_{\pi}^2} \frac{p'^2 - p^2}{2m_N}, \quad U^{(1)}(\nu) = -\frac{\nu}{2} \int_s \frac{v_{\pi}^{(0)}(\mathbf{p}' - \mathbf{s}) v_{\pi}^{(0)}(\mathbf{s} - \mathbf{p})}{(\mathbf{p}' - \mathbf{s})^2 + m_{\pi}^2}$$

thus extending the unitary equivalence to the TPEP

Analogously for the electromagnetic transition operator  $v_\gamma = A^0 \rho - \mathbf{A} \cdot \mathbf{j}$  we start by expanding the amplitude  $T_\gamma = T_\gamma^{(-3)} + T_\gamma^{(-2)} + \dots$  and then match order by order

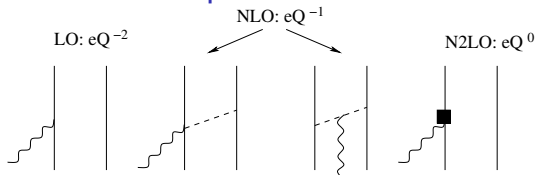
$$\begin{aligned}
 v_\gamma^{(-3)} &= T_\gamma^{(-3)} \\
 v_\gamma^{(-2)} &= T_\gamma^{(-2)} - \left[ v_\gamma^{(-3)} G_0 v^{(0)} + v^{(0)} G_0 v_\gamma^{(-3)} \right] , \\
 v_\gamma^{(-1)} &= T_\gamma^{(-1)} - \left[ v_\gamma^{(-3)} G_0 v^{(0)} G_0 v^{(0)} + \dots \right] - \left[ v_\gamma^{(-2)} G_0 v^{(0)} + v^{(0)} G_0 v_\gamma^{(-2)} \right] , \\
 v_\gamma^{(0)} &= T_\gamma^{(0)} - \left[ v_\gamma^{(-3)} G_0 v^{(0)} G_0 v^{(0)} G_0 v^{(0)} + \dots \right] - \left[ v_\gamma^{(-2)} G_0 v^{(0)} G_0 v^{(0)} + \dots \right] \\
 &\quad - \left[ v_\gamma^{(-1)} G_0 v^{(0)} + v^{(0)} G_0 v_\gamma^{(-1)} \right] - \left[ v_\gamma^{(-3)} G_0 v^{(2)} + v^{(2)} G_0 v_\gamma^{(-3)} \right] \\
 v_\gamma^{(1)} &= T_\gamma^{(1)} - \left[ v_\gamma^{(-3)} G_0 v^{(0)} G_0 v^{(0)} G_0 v^{(0)} G_0 v^{(0)} + \dots \right] \\
 &\quad - \left[ v_\gamma^{(-2)} G_0 v^{(0)} G_0 v^{(0)} G_0 v^{(0)} + \dots \right] - \left[ v_\gamma^{(-1)} G_0 v^{(0)} G_0 v^{(0)} + \dots \right] \\
 &\quad - \left[ v_\gamma^{(0)} G_0 v^{(0)} + v^{(0)} G_0 v_\gamma^{(0)} \right] - \left[ v_\gamma^{(-3)} G_0 v^{(2)} G_0 v^{(0)} + \dots \right] \\
 &\quad - \left[ v_\gamma^{(-3)} G_0 v^{(3)} + v^{(3)} G_0 v_\gamma^{(-3)} \right] ,
 \end{aligned}$$

Analogously for the electromagnetic transition operator  $v_\gamma = A^0 \rho - \mathbf{A} \cdot \mathbf{j}$  we start by expanding the amplitude  $T_\gamma = T_\gamma^{(-3)} + T_\gamma^{(-2)} + \dots$  and then match order by order

$$\begin{aligned}
 v_\gamma^{(-3)} &= T_\gamma^{(-3)} \\
 v_\gamma^{(-2)} &= T_\gamma^{(-2)} - \left[ v_\gamma^{(-3)} G_0 v^{(0)} + v^{(0)} G_0 v_\gamma^{(-3)} \right] , \\
 v_\gamma^{(-1)} &= T_\gamma^{(-1)} - \left[ v_\gamma^{(-3)} G_0 v^{(0)} G_0 v^{(0)} + \dots \right] - \left[ v_\gamma^{(-2)} G_0 v^{(0)} + v^{(0)} G_0 v_\gamma^{(-2)} \right] , \\
 v_\gamma^{(0)} &= T_\gamma^{(0)} - \left[ v_\gamma^{(-3)} G_0 v^{(0)} G_0 v^{(0)} G_0 v^{(0)} + \dots \right] - \left[ v_\gamma^{(-2)} G_0 v^{(0)} G_0 v^{(0)} + \dots \right] \\
 &\quad - \left[ v_\gamma^{(-1)} G_0 v^{(0)} + v^{(0)} G_0 v_\gamma^{(-1)} \right] - \left[ v_\gamma^{(-3)} G_0 v^{(2)} + v^{(2)} G_0 v_\gamma^{(-3)} \right] \\
 v_\gamma^{(1)} &= T_\gamma^{(1)} - \left[ v_\gamma^{(-3)} G_0 v^{(0)} G_0 v^{(0)} G_0 v^{(0)} G_0 v^{(0)} + \dots \right] \\
 &\quad - \left[ v_\gamma^{(-2)} G_0 v^{(0)} G_0 v^{(0)} G_0 v^{(0)} + \dots \right] - \left[ v_\gamma^{(-1)} G_0 v^{(0)} G_0 v^{(0)} + \dots \right] \\
 &\quad - \left[ v_\gamma^{(0)} G_0 v^{(0)} + v^{(0)} G_0 v_\gamma^{(0)} \right] - \left[ v_\gamma^{(-3)} G_0 v^{(2)} G_0 v^{(0)} + \dots \right] \\
 &\quad - \left[ v_\gamma^{(-3)} G_0 v^{(3)} + v^{(3)} G_0 v_\gamma^{(-3)} \right] ,
 \end{aligned}$$

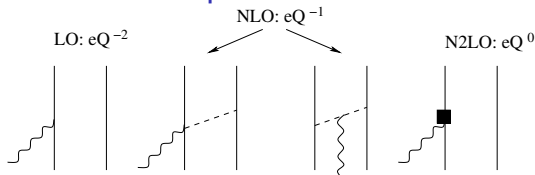
At this order, the offshell ambiguity in  $v$  affects only the charge operator. However,  $\rho(\nu) = e^{-iU(\nu)} \rho(\nu=0) e^{iU(\nu)}$  with the same  $U(\nu)$  as before

# Current operator to 1 loop



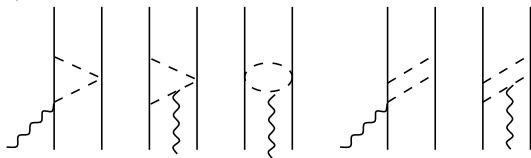
- ▶ 1-body operator (convection current and spin-magnetization)
- ▶ Two-body currents (seagull and pion-in-flight) - only isovector
- ▶ Relativistic corrections to the 1-body operator

# Current operator to 1 loop



- ▶ 1-body operator (convection current and spin-magnetization)
- ▶ Two-body currents (seagull and pion-in-flight) - only isovector
- ▶ Relativistic corrections to the 1-body operator

At N3LO,  $O(eQ)$



- ▶ Two-pion exchange diagrams - only isovector



## Contact terms from the subleading Lagrangian

There are two classes of contributions:

- ▶ terms from the gauging of the subleading two nucleon contact Lagrangian (minimal substitution)  
these can be expressed in terms of the same LECs entering the  $NN$  potential
- ▶ terms involving the electromagnetic field strength tensor - 1 isoscalar and 1 isovector

$$\mathbf{j}^{(1)} = -i e \left[ C'_{15} \boldsymbol{\sigma}_1 + C'_{16} (\tau_{1,z} - \tau_{2,z}) \boldsymbol{\sigma}_1 \right] \times \mathbf{q} + 1 \rightleftharpoons 2 ,$$





## Contact terms from the subleading Lagrangian

There are two classes of contributions:

- ▶ terms from the gauging of the subleading two nucleon contact Lagrangian (minimal substitution)  
these can be expressed in terms of the same LECs entering the  $NN$  potential
- ▶ terms involving the electromagnetic field strength tensor - 1 isoscalar and 1 isovector

$$\mathbf{j}^{(1)} = -i e \left[ C'_{15} \boldsymbol{\sigma}_1 + C'_{16} (\tau_{1,z} - \tau_{2,z}) \boldsymbol{\sigma}_1 \right] \times \mathbf{q} + 1 \rightleftharpoons 2 ,$$



## Subleading corrections to one pion exchange

$$\mathbf{j}^{(1)} = i e \frac{g_A}{F_\pi^2} \frac{\boldsymbol{\sigma}_2 \cdot \mathbf{k}_2}{\omega_{k_2}^2} \left[ \left( d'_8 \tau_{2,z} + d'_9 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \right) \mathbf{k}_2 - d'_{21} (\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)_z \boldsymbol{\sigma}_1 \times \mathbf{k}_2 \right] \times \mathbf{q} + 1 \rightleftharpoons 2 ,$$

two isovector and one isoscalar. One additional contribution vanishes for real photons

# Charge operator to 1 loop

$$e Q^{(-3)}$$


(a)

$$e Q^{(-1)}$$


(b)

leading seagull and pion-in-flight vanish

$$e Q^{(0)}$$


(c)



(d)



(e)

$$e Q^{(1)}$$


(f)



(g)



(h)



(i)



(j)



(k)



(l)



(m)

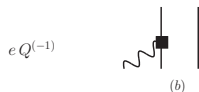
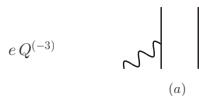


(n)

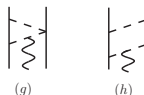
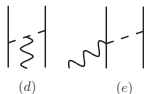
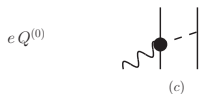


(o)

# Charge operator to 1 loop



leading seagull and pion-in-flight vanish



All divergences cancel, since there are no LECs contributing.

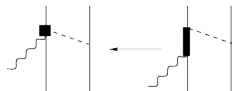
Renormalization of OPE and relativity corrections not included yet.

# Fixing the LECs

- ▶ in the minimal coupling current we take the  $C_i$  from N3LO potentials
- ▶ in the subleading OPE current

$$\mathbf{j}^{(1)} = i e \frac{g_A}{F_\pi^2} \frac{\boldsymbol{\sigma}_2 \cdot \mathbf{k}_2}{\omega_{k_2}^2} \left[ \left( d'_8 \tau_{2,z} + d'_9 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \right) \mathbf{k}_2 - d'_{21} (\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)_z \boldsymbol{\sigma}_1 \times \mathbf{k}_2 \right] \times \mathbf{q} + 1 \Rightarrow 2 ,$$

the LECs could in principle be taken from  $\pi N$  observables. Instead, we fix them from nuclear data. However, isovector contributions can be saturated by  $\Delta$ -excitation diagrams



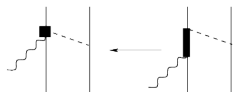
$$\text{with } d'_8 = 4d'_{21} = 4\mu^* h_A / (9m_N \Delta)$$

# Fixing the LECs

- ▶ in the minimal coupling current we take the  $C_i$  from N3LO potentials
- ▶ in the subleading OPE current

$$\mathbf{j}^{(1)} = i e \frac{g_A}{F_\pi^2} \frac{\boldsymbol{\sigma}_2 \cdot \mathbf{k}_2}{\omega_{k_2}^2} \left[ \left( d'_8 \tau_{2,z} + d'_9 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \right) \mathbf{k}_2 - d'_{21} (\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)_z \boldsymbol{\sigma}_1 \times \mathbf{k}_2 \right] \times \mathbf{q} + 1 \Rightarrow 2 ,$$

the LECs could in principle be taken from  $\pi N$  observables. Instead, we fix them from nuclear data. However, isovector contributions can be saturated by  $\Delta$ -excitation diagrams



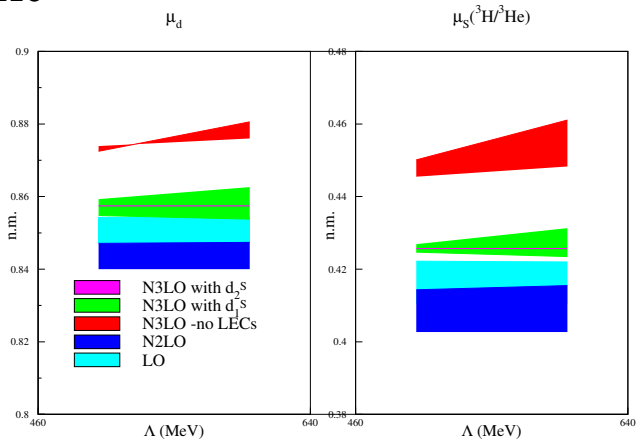
$$\text{with } d'_8 = 4d'_{21} = 4\mu^* h_A / (9m_N \Delta)$$

We fix  $d'_{21} = d'_8/4 \Rightarrow 4$  adjustable LECs

$$d_1^S = \Lambda^4 C'_{15}, \quad d_1^V = \Lambda^4 C'_{16}, \quad d_2^S = \Lambda^2 d'_9, \quad d_2^V = \Lambda^2 d'_8$$

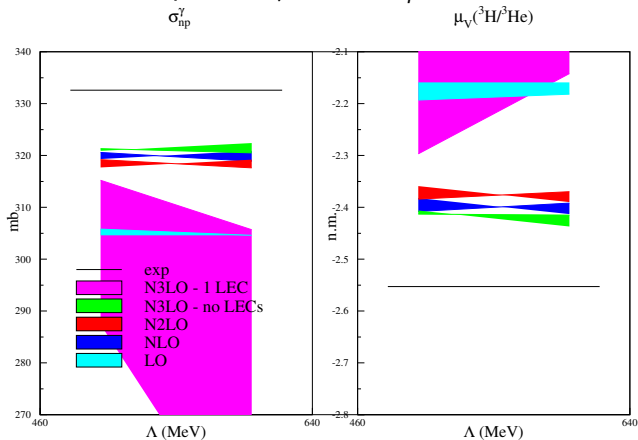
# Isoscalar LECs

For each  $\Lambda$ , we fixed isoscalar LECs to reproduce  $\mu_d$  and  $\mu_S$   
Accurate nuclear wave functions from HH method with AV18+UIX and N3LO+N2LO



# Isovector LECs

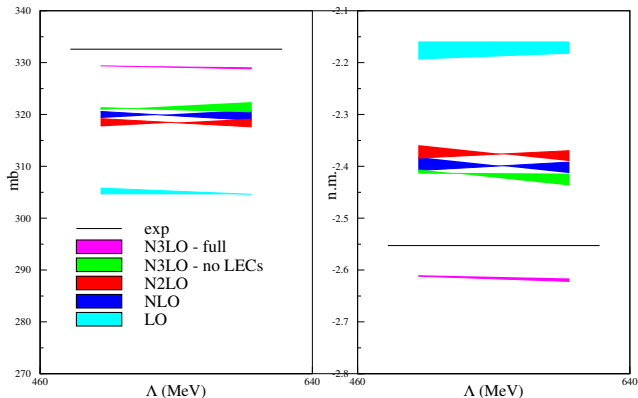
Set I: fix the LECs to reproduce  $\mu_V$  and  $\sigma_{np}^\gamma$



unnatural convergence pattern, huge model and cutoff dependence

# Isovector LECs

Set II-III: fix  $d_2^V$  from  $\Delta$  saturation and  $d_1^V$  from either  $\mu_V$  or  $\sigma_{np}^\gamma$

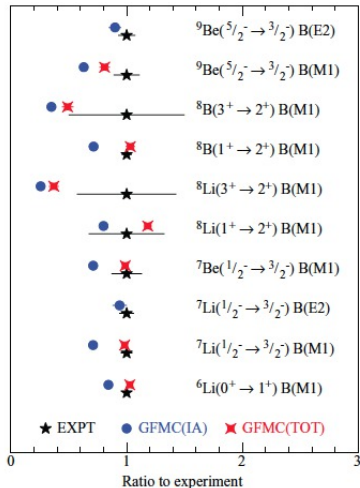
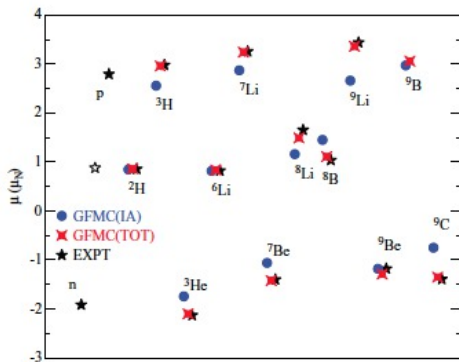


stable, model-independent prediction to 1% and 2% respectively



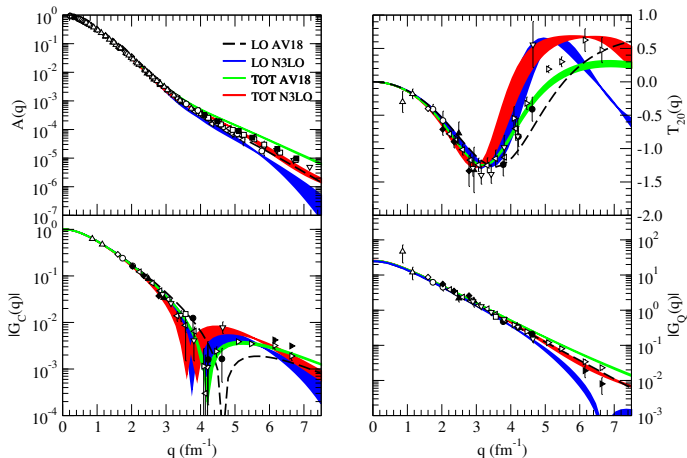
# Predictions for $A \leq 9$

using Set III



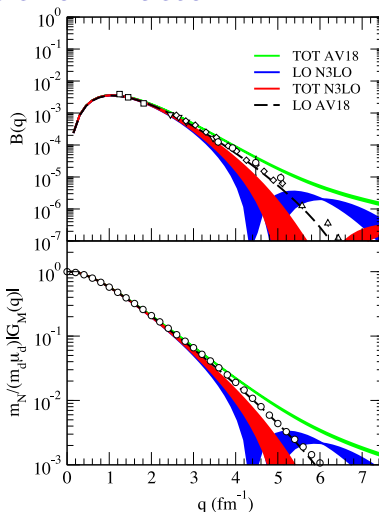
S. Pastore, S. Pieper, R. Schiavilla, R. Wiringa, PRC87 (2013) 035503

# Deuteron charge and quadrupole form factors



agreement with data extends to  $q \sim 3 - 4 \text{ fm}^{-1}$  or beyond

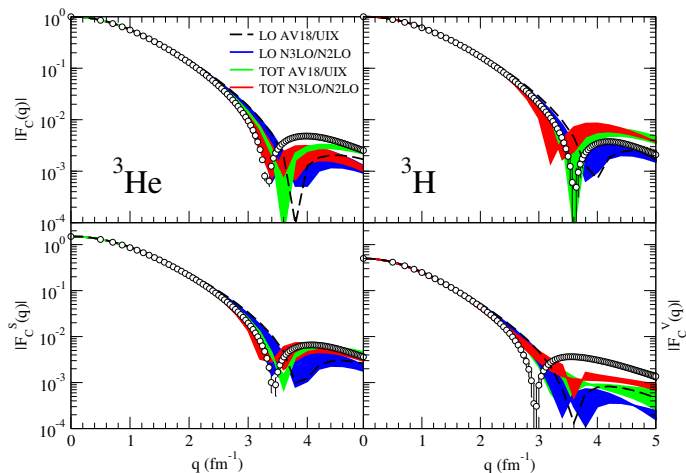
# Deuteron magnetic form factor



good agreement up to  $q < 3 \text{ fm}^{-1}$

large cutoff dependence with the chiral potentials, presumably due to scheme dependence

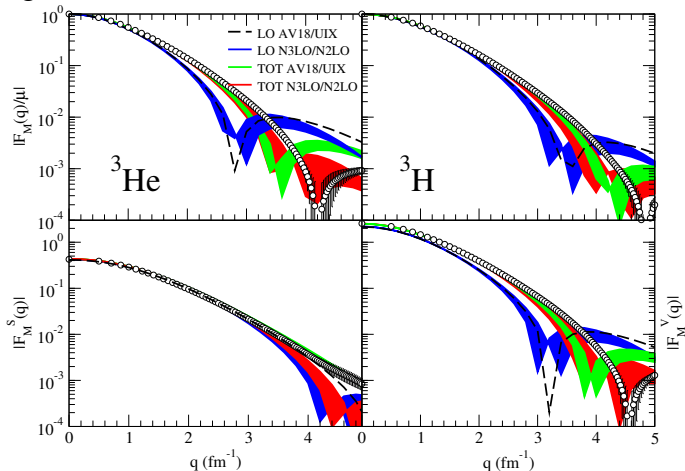
# Trinucleon charge form factors



chiral loops bring theory closer to experiment in the diffraction region  
good agreement below  $2.5 \text{ fm}^{-1}$

# Trinucleon magnetic form factors

Set III: magnetic moments are fitted



large effect of two-body currents

# Summary and outlook

- ▶ electroweak currents are naturally embedded in the ChEFT machinery
- ▶ special care is required to properly isolate the truly irreducible contribution, accounting systematically for the nucleon recoil
- ▶ we encountered ambiguities due to the off-shell behaviour of the potential. However, they don't affect observables, since different choices are unitarily equivalent, both at the level of OPE (Friar, '77) and TPE
- ▶ we have applied our charge and current operators to compute static properties and elastic form factors of the deuteron and trinucleons, finding good agreement up to  $q \sim 2 - 3 \text{ fm}^{-1}$
- ▶ large cutoff dependence and convergence pattern not yet satisfactory, pointing to the necessity including the  $\Delta$  in the effective theory.