

# Fission at Intermediate Nucleon Energies

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Fission induced by intermediate energy nucleons (200 MeV – 2.5 GeV) is usually considered as a two-stage process :

**Fast intranuclear cascade**, described as a succession of binary collisions, leading to the emission of fast nucleons, light fragments, pions ... and leaving an excited residual nucleus, which decays by **Evaporation – fission**, described by evaporation, fission or other mechanism

# Theoretical features for Fast Cascade (1/3)

Liège intranuclear cascade model:

**INCL++** (C++ version)

(A. Boudard, J. Cugnon, J.-C. David, S. Leray and D. Mancusi, Phys. Rev. C 87 , 2013)

Main features of INCL++:

- 1) Target nucleus at the centre of a calculation volume (**CV**) of radius  $R_{max} = R_0 + 8a$ , with  $R_0$  the radius and  $a$  the diffuseness of target nucleus density.
- 2) Incident nucleon located on the **CV** surface with its own impact parameter.

# Theoretical features for Fast Cascade (2/3)



- 3) Particles move along straight-line trajectories between collisions inside the **CV**.
- 4) Binary collision model valid at  $E_{\text{proj}} > 100 \text{ MeV}$ .
- 5) Potential well inside the **CV** is isospin dependent, so that neutron and proton Fermi levels have the same energy.
- 6) Potential well decreases with increasing nucleon energy and goes to zero at  $E \approx 200 \text{ MeV}$ .
- 7) Pauli blocking strictly applied in the first collision, stochastically applied in subsequent collisions on the basis of final-state blocking factors  $\rightarrow$  depletion of the Fermi sphere during the cascade stage.

# Theoretical features for Fast Cascade (3/3)



- 8) Coalescence model based on phase space considerations permits formation and **emission of light clusters ( $A \leq 8$ )** in the cascade stage.
- 9) Possible excitation of nucleons to  **$\Delta$  resonances** and subsequent production of **pions**, which can be reabsorbed or emitted through the **CV  $\rightarrow E_{\text{proj}} < 2.5 \text{ GeV}$** .
- 10) Duration of cascade connected with the target mass by the empirical law  **$t_{\text{stop}} = 29.8 A_T^{0.16} \text{ fm/c}$**  ( $1 \text{ fm/c} \approx 3.3 \times 10^{-24} \text{ sec}$ ).

## Model for Evaporation-fission of residual nuclei:

### 1) GEMINI++ (C++ version)

*(D. Mancusi, R.J. Charity and J. Cugnon, Phys. Rev. C 82 (2010) 044610 and references therein)*

### 2) ABLA07

*(A. Kelic, M.V. Ricciardi and K.-H. Schmidt, report INDC(NDS)-530, IAEA, Vienna, 2008, p. 181).*

## Main features of GEMINI++:

Light particle evaporation described with the Hauser Feshbach formalism (conservation of angular momentum).

An important ingredient of the decay width is

1) the **level density formula**:

$\rho(E^*, J) \sim (2J+1) \exp[2\sqrt{a(U, J)U}]$  (Fermi gas-like) ,

where  $U = E^* - E_{rot}(J) + \delta P$  (pairing correction)

and  $a(U) = \tilde{a}(U)[1 + (\delta W/U)(1 - \exp(-\eta U))]$

(damping of shell correction  $\delta W$  according to Ignatyuk, but

$\tilde{a}(U)$  contains energy-dependent long-range correlations)

# Theoretical features for Evaporation (3/6)

$\tilde{a}(U)$  admits the limits

$\tilde{a}(0) = A/k_0$ , with  $k_0 = 7.3$  MeV, from neutron resonance counting,

$\tilde{a}(\infty) = A/k_\infty$ , with  $k_\infty = 12$  MeV, from kinetic energy spectra of evaporated particles.

At intermediate energies it is taken of the form

$$\tilde{a}(U) = \frac{A}{k_\infty - (k_\infty - k_0) \exp\left(-\frac{\gamma}{k_\infty - k_0} \frac{U}{A}\right)}$$

Here,  $\gamma = 0.00493 \exp(0.0332 A)$ .



# Theoretical features for Evaporation (4/6)



2) It is customary to assume that  $\tilde{a}_f(0)$ , the level density parameter at the saddle point, used in fission calculations, is larger than  $\tilde{a}_n(0)$ , the level density parameter used in neutron evaporation. The default value is  $\tilde{a}_f(0)/\tilde{a}_n(0) = 1.036$ .

3) Symmetric fission is described by Bohr-Wheeler decay

$$\text{Width } \Gamma_{BW} = \int d\varepsilon \rho_{sad} [E^* - B_f(J_{cn}) - \varepsilon] / [2\pi \rho_{cn}(E^*, J_{cn})]$$

Binary decay (asymmetric fission) is described by Moretto's conditional decay width

$$\Gamma_{ZA} = \int d\varepsilon \rho_{sad} [E^* - B_{ZA}(J_{cn}) - \varepsilon] / [2\pi \rho_{cn}(E^*, J_{cn})]$$

where  $Z$  and  $A$  are charge and mass of one of the nascent fragments.

## Main features of ABLA07:

- 1) Particle evaporation : Weisskopf-Ewing formalism with semiclassical approximation for degradation of intrinsic angular momentum.
- 2) Nuclear level density with collective enhancement factor :

$$\rho(E) = \rho_{in}(E) K_{vib}(E) K_{rot}(E)$$

$\rho_{in}(E)$  : constant temperature formula at low energy  
combined with Bethe formula at high energy, with shell  
and pairing corrections and Ignatyuk asymptotic level  
density parameter

$$\tilde{a} = 0.073 A + 0.095 B_s A^{2/3}$$

with  $B_s$  the ratio of the surface of the deformed nucleus and  
a spherical one with same  $A$ .

# Theoretical features for Evaporation (6/6)



3) High-energy fission : time-dependent fission width from approximate solution of one-dimensional Fokker-Planck equation in fission coordinate  $\beta$ :

$$\Gamma_f(t) = K(\gamma) \Gamma_{BW} W(\beta = \beta_s; t, \gamma) / W(\beta = \beta_s; t \rightarrow \infty, \gamma) \text{ where :}$$

$K(\gamma)$  : Kramers factor depending on dissipation coefficient  $\gamma$  ;

$\Gamma_{BW}$  : Bohr – Wheeler fission width ;

$W$  : time-dependent distribution of fission coordinate  $\beta$ .

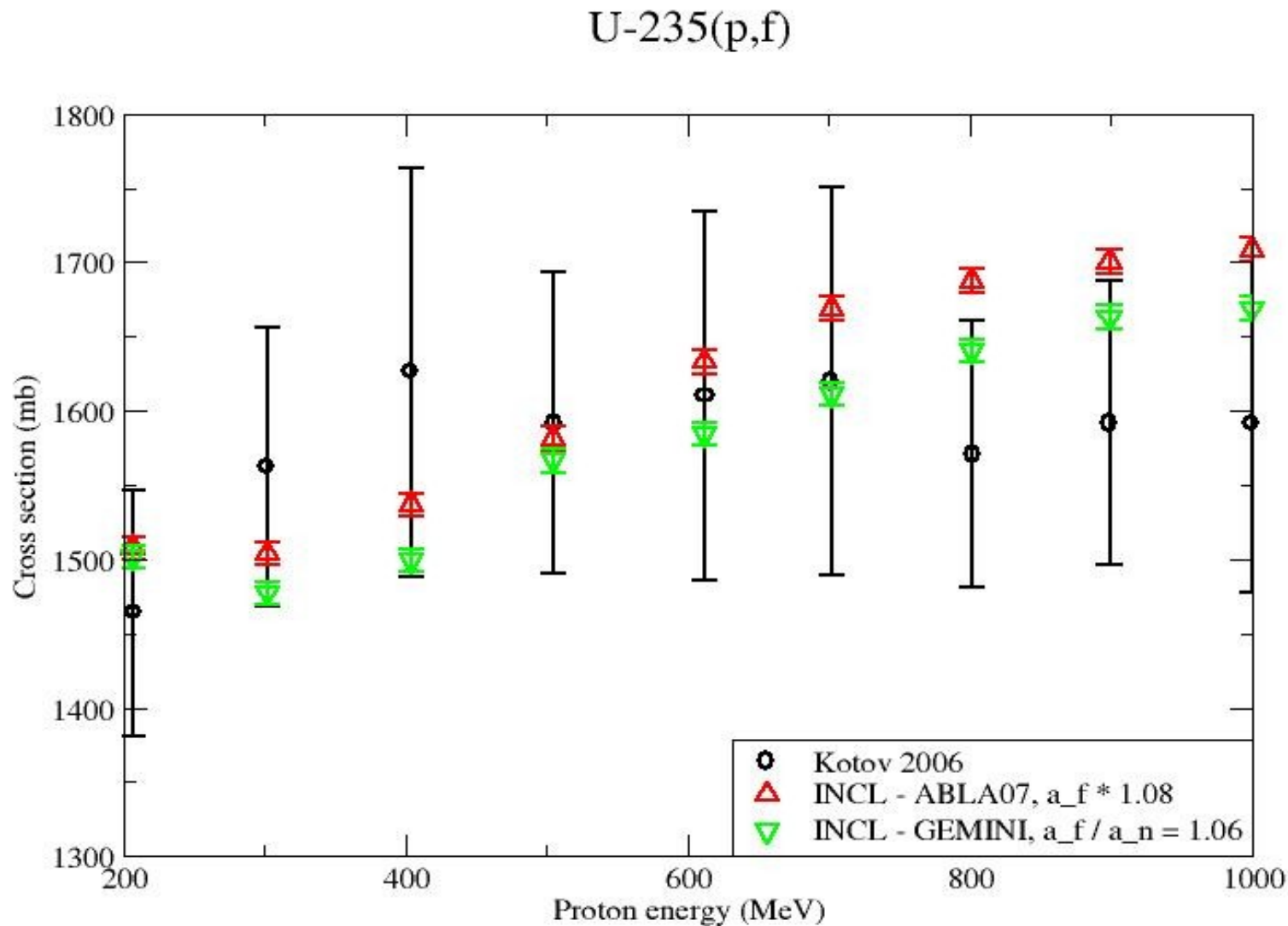
$\beta_s$  : fission coordinate at the saddle point.

4) Low-energy fission : standard two-humped fission barrier in the limit of full damping of resonances in intermediate well :

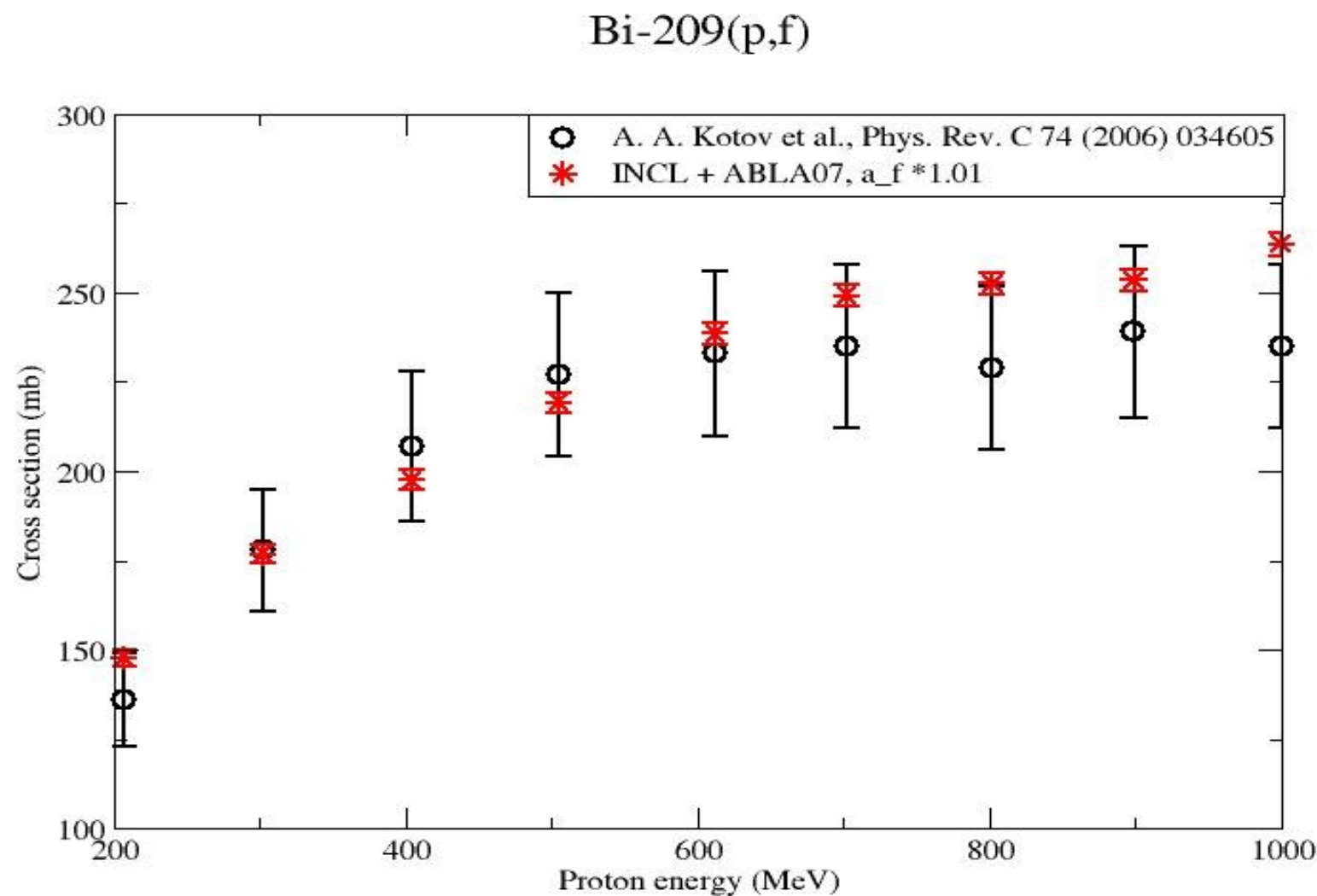
$$\Gamma_f = \Gamma_A \Gamma_B / (\Gamma_A + \Gamma_B) .$$

- 1) Parameters of fission models (basically  $\tilde{a}_f / \tilde{a}_n$  in GEMINI and  $\tilde{a}_f$  in ABLA) are calibrated on experimental **(p,f)** cross sections in the energy range 200 MeV – 1 GeV (Kotov et al., Phys. Rev. C 74 (2006) 034605) and are then applied to the calculation of **(n,f)** cross sections.
- 2) The theoretical ratios to the  $^{235}\text{U}(\text{n,f})$  cross section are compared with the **n\_TOF** (<https://ntof-exp.web.cern.ch/ntof-exp/>) ratios in the actinide region ( C. Paradela et al., Phys. Rev. C 82 (2010) 034601 ) and in the Pb-Bi region ( D. Tarrío et al., Phys. Rev. C 83 (2011) 044620).

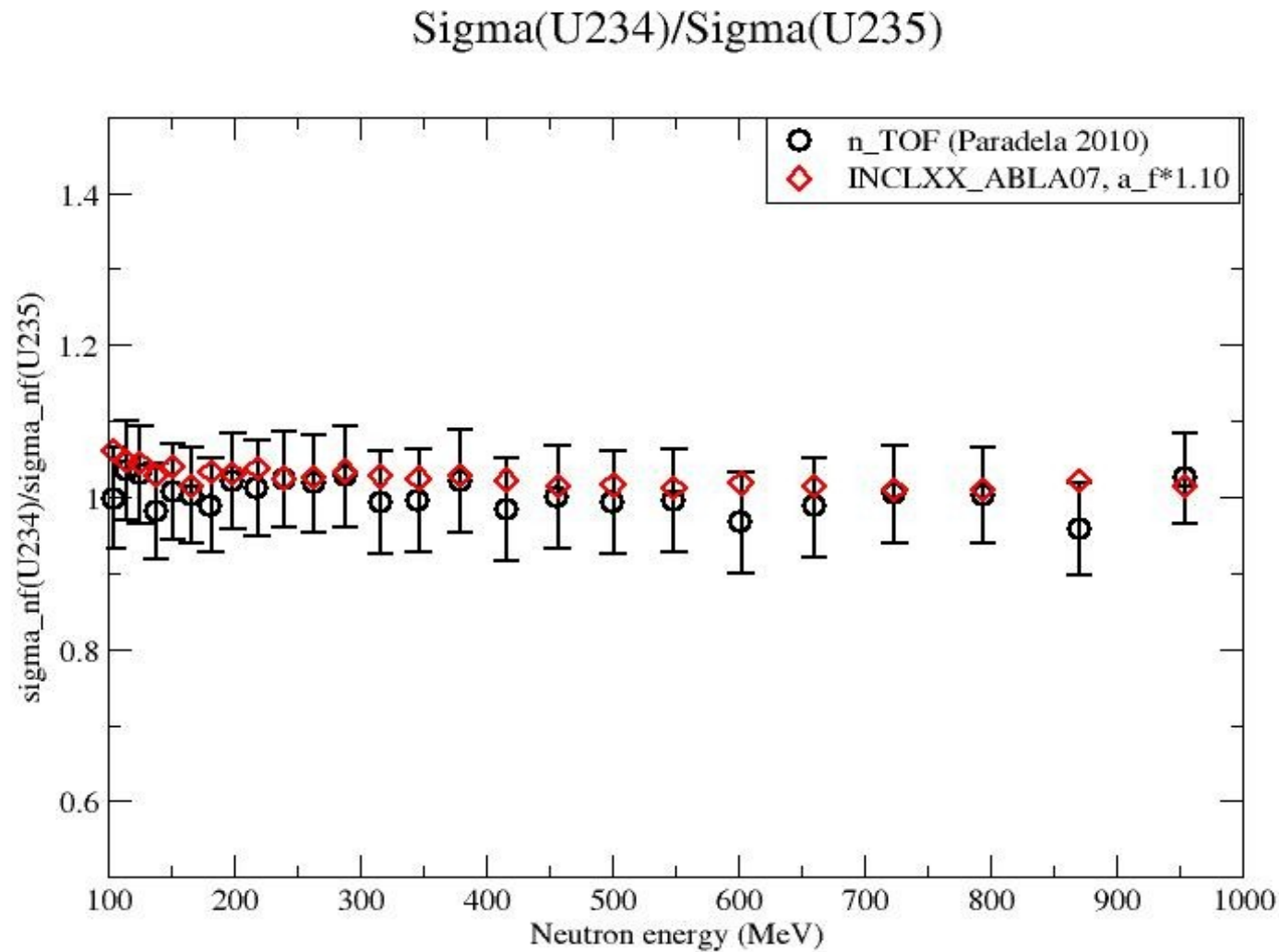
# Computational results: $^{235}\text{U}(p,f)$



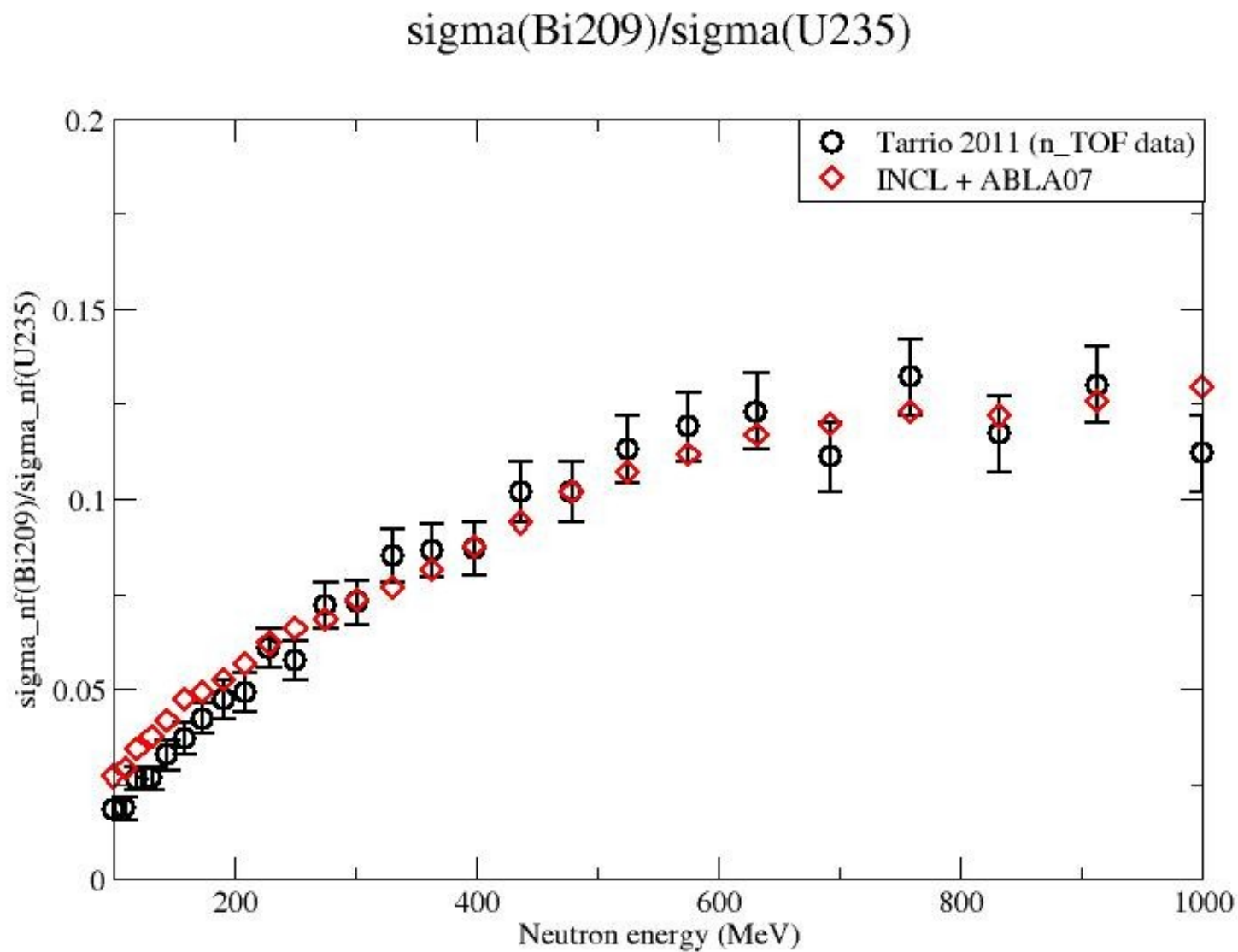
# Computational results: $^{209}\text{Bi}(p,f)$



# Computational results: $\sigma(^{234}\text{U}(n,f))/\sigma(^{235}\text{U}(n,f))$

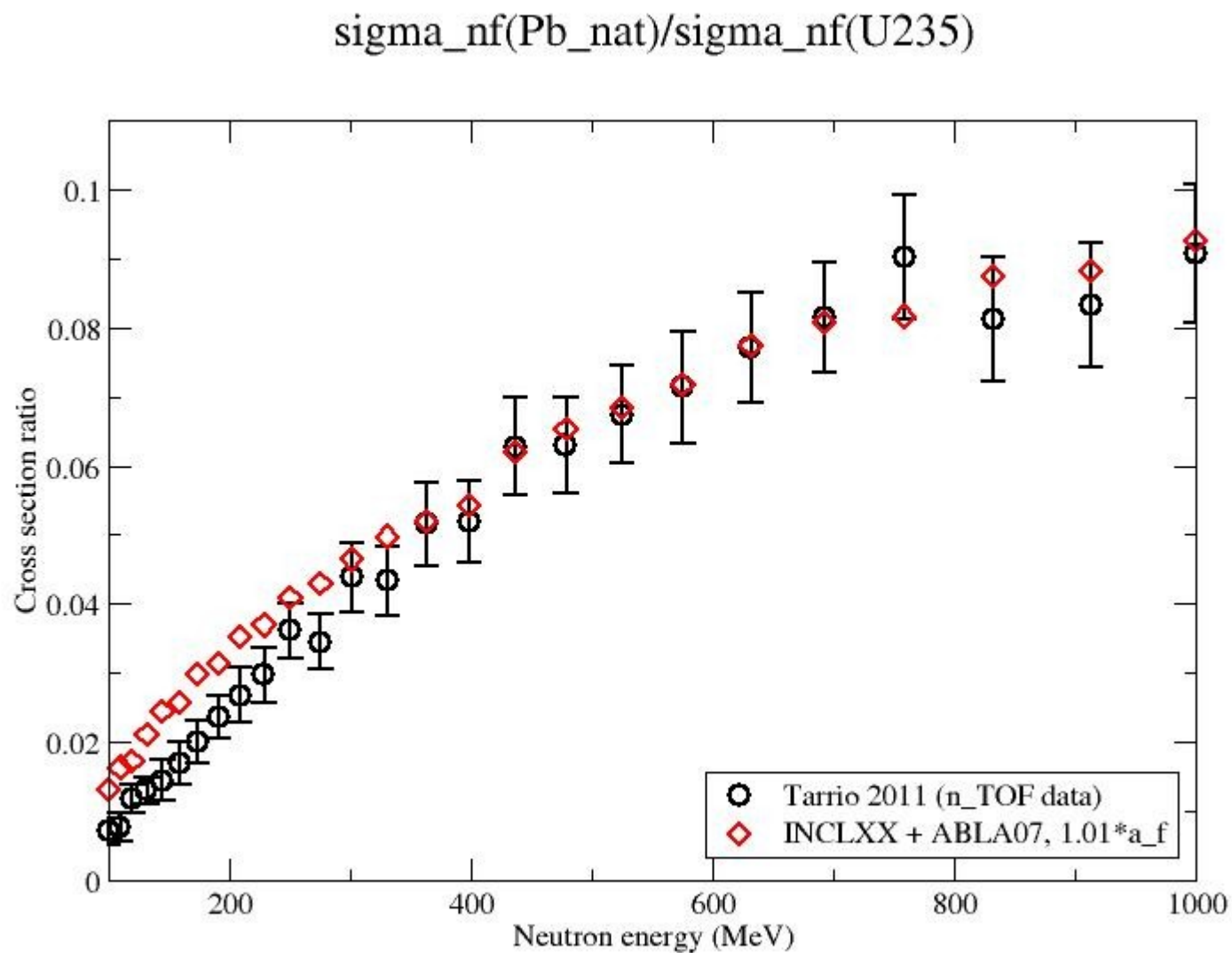


# Computational results: $\sigma(^{209}\text{Bi}(n,f))/\sigma(^{235}\text{U}(n,f))$

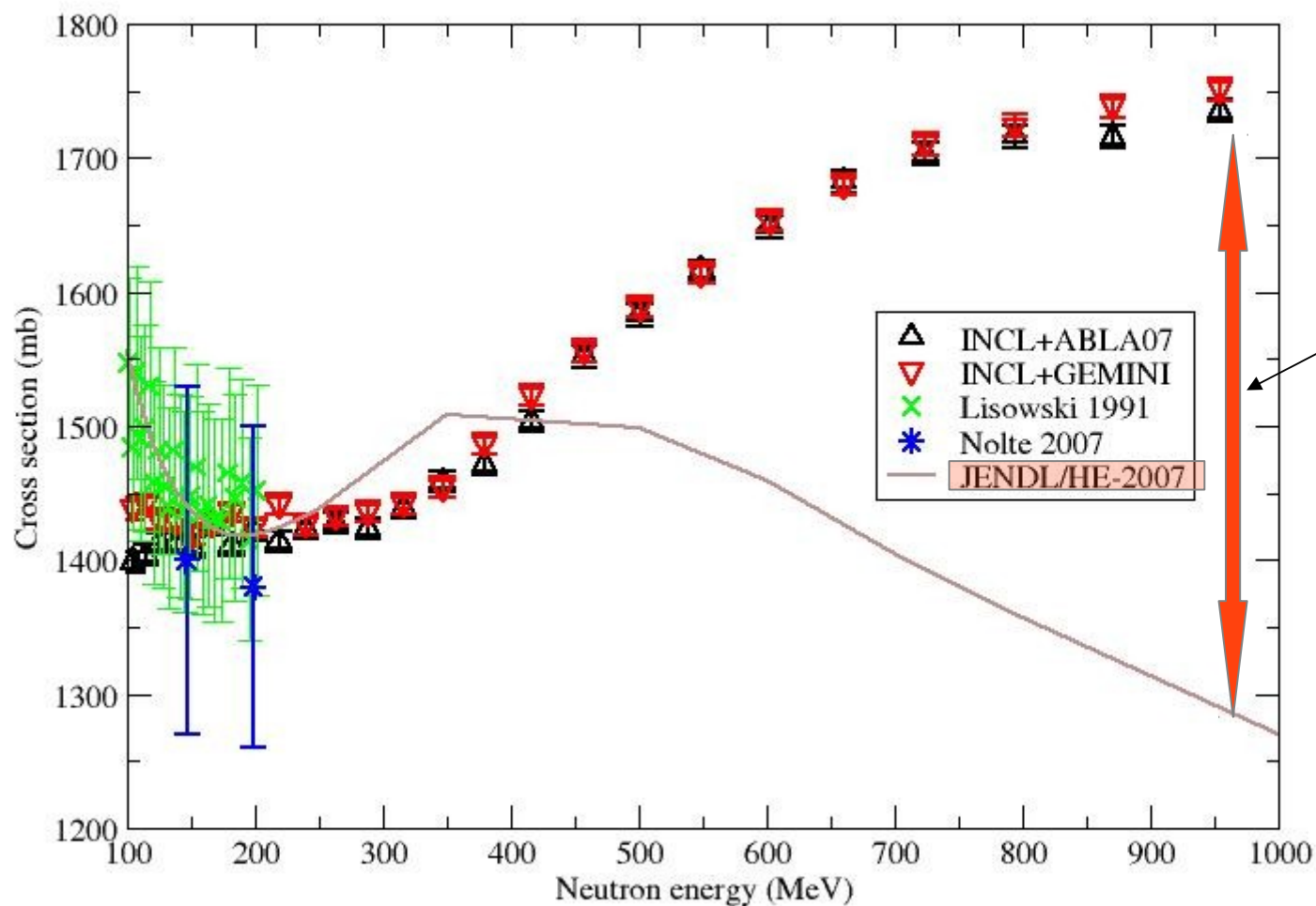




# Computational results: $\sigma(\text{natPb}(n,f))/\sigma(^{235}\text{U}(n,f))$

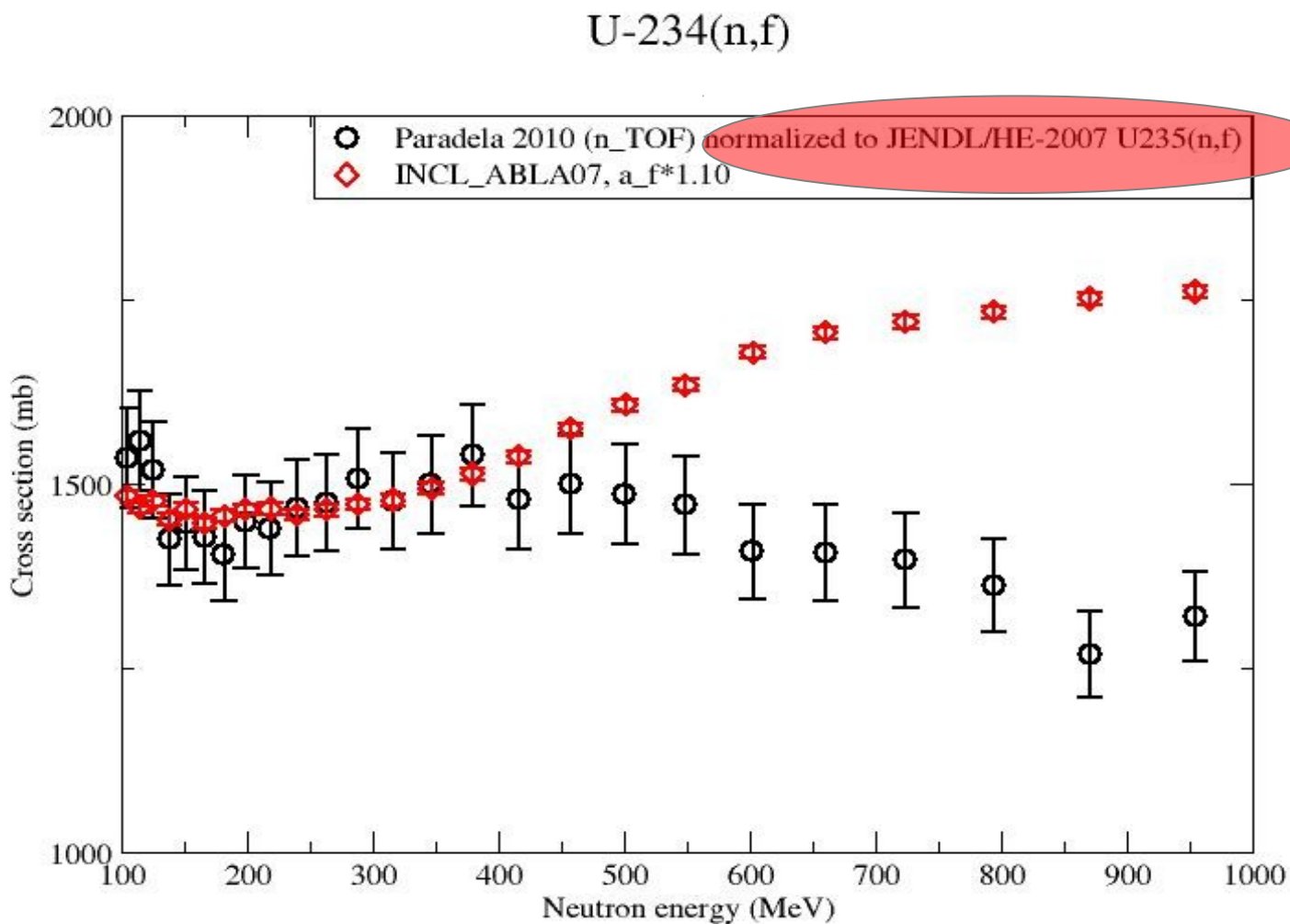


# Computational results: $^{235}\text{U}(n,f)$

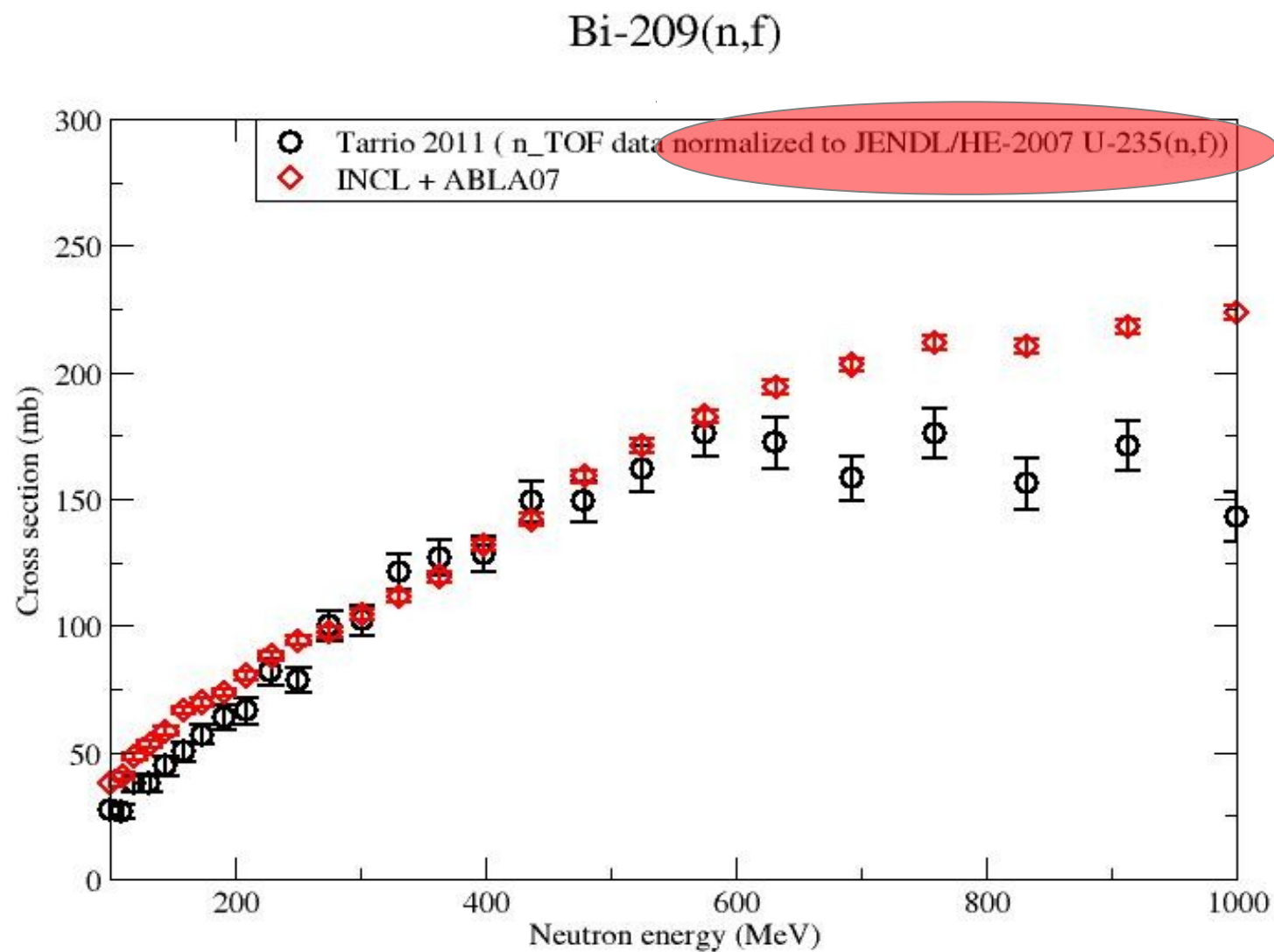


**Big  
Difference!!**

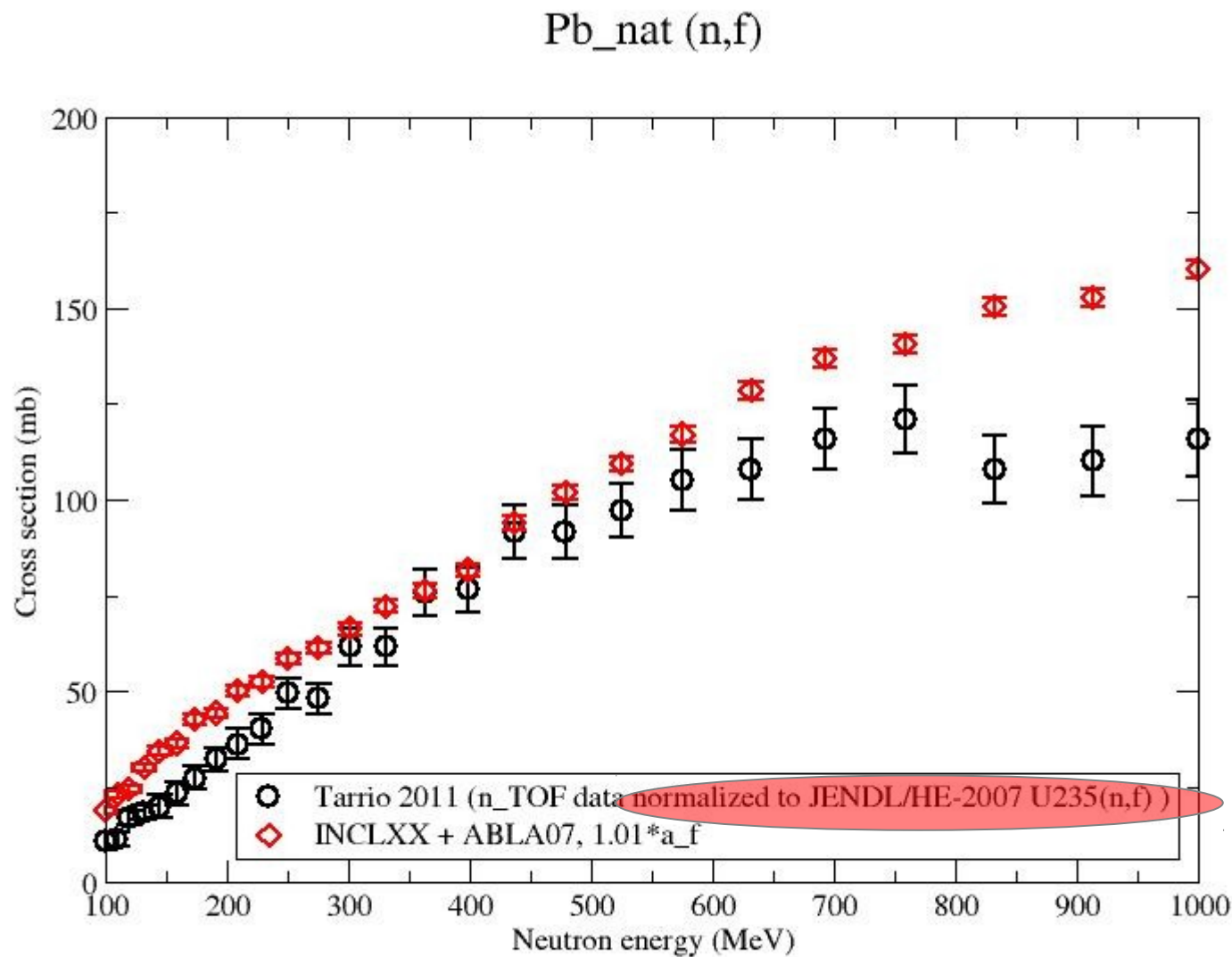
# Computational results: $^{234}\text{U}(n,f)$



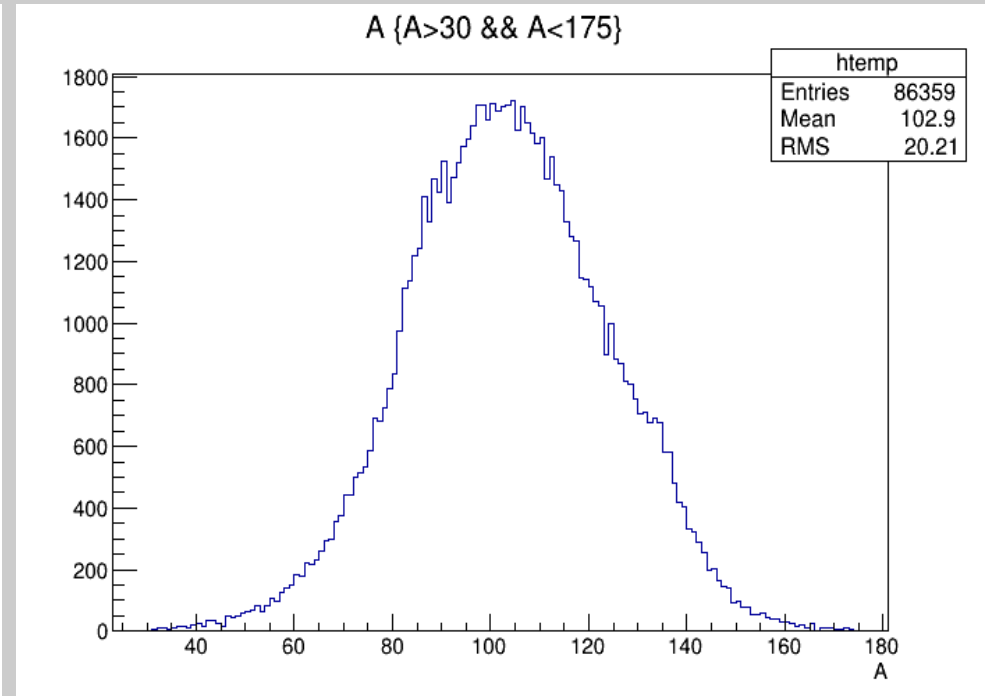
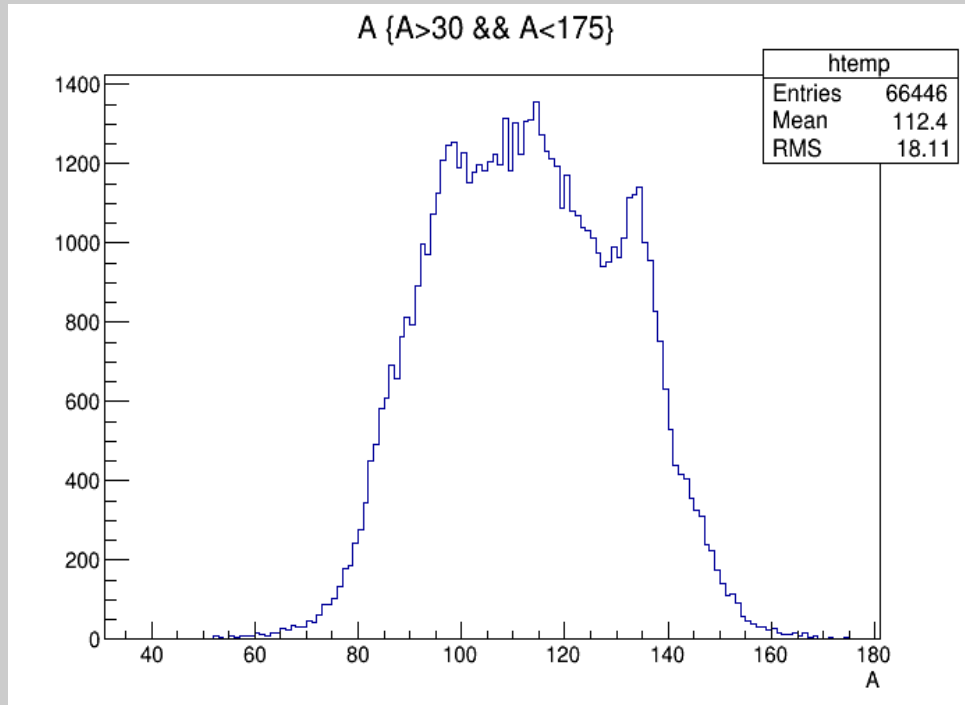
# Computational results: $^{209}\text{Bi}(n,f)$



# Computational results: $^{\text{nat}}\text{Pb}(n,f)$



# Mass distribution of Fission Fragments $^{235}\text{U}(n,f)$



Mass distribution of fission fragments at  
 $E_n = 104.71$  MeV (left panel) and  $E_n = 954.99$  MeV  
(right panel) computed with INCL + ABLA07.

# Conclusion



- 1) **INCL + ABLA07** is able to reproduce experimental (p,f) cross sections (Kotov 2006) and experimental ratios of (n,f) cross sections to  $^{235}\text{U}(n,f)$  (n\_TOF data) in the Pb-Bi and actinide region.
- 2) **Absolute (n,f)** cross sections are larger than those published by n\_TOF in the energy region from 500 MeV to 1 GeV because of different normalization. The JENDL evaluation of the  $^{235}\text{U}$  (n, f) cross section does not seem to be consistent with the experimental (p, f) cross section in the same energy range.
- 3) **INCL + GEMINI** is equivalent to **INCL + ABLA07** in the actinide region, but less satisfactory in the Pb-Bi region.

- 1) Systematic analysis** of (p,f) and (n,f) cross sections above 100 MeV for determination of  $\tilde{a}_f/\tilde{a}_n$  and other crucial parameters as a function of the fissility parameter and other characteristics of target nuclei.
- 2) Prediction** of (n,f) cross sections to be measured by n\_TOF, with particular reference to Pu isotopes.
- 3) Calculation** of angular distributions of fission fragments for the isotopes of interest to n\_TOF.



For further details:

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**THANKS FOR ATTENTION!!**