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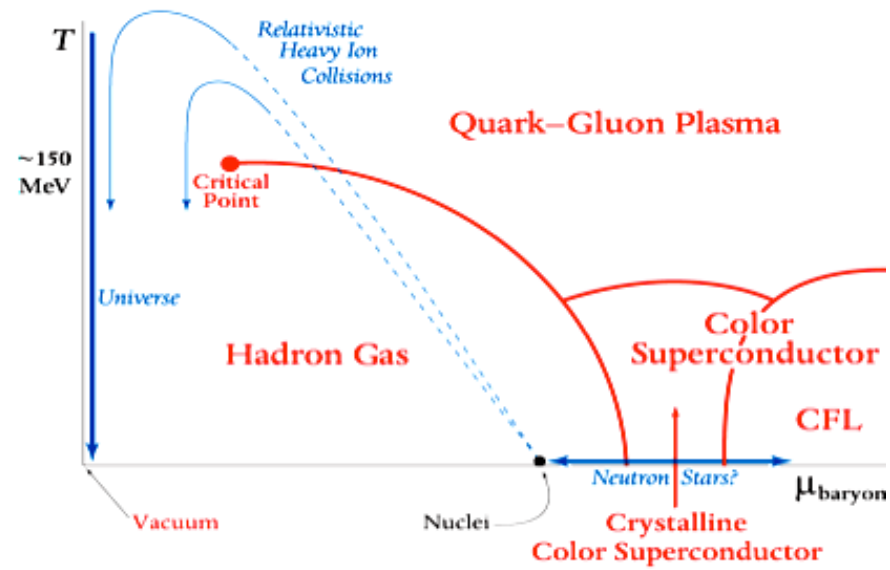
Realistic QCD Equation of State

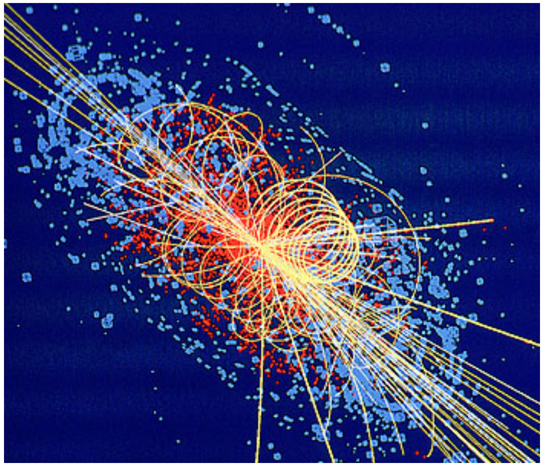
Cortona – 30 October 2013

Based on M. Bluhm, P.A., W. Alberico, A. Beraudo, C. Ratti, arXiv: 1306.6188 [hep-ph]

Introduction

- A lot of theoretical effort is devoted to study the QCD phase diagram.
- A transition from the confined phase to the deconfined phase (Quark-Gluon Plasma (QGP)) is predicted for large temperatures and/or densities.
- Experimentally, by Heavy Ion Collisions (HIC), we are able to create the QGP in the laboratory.





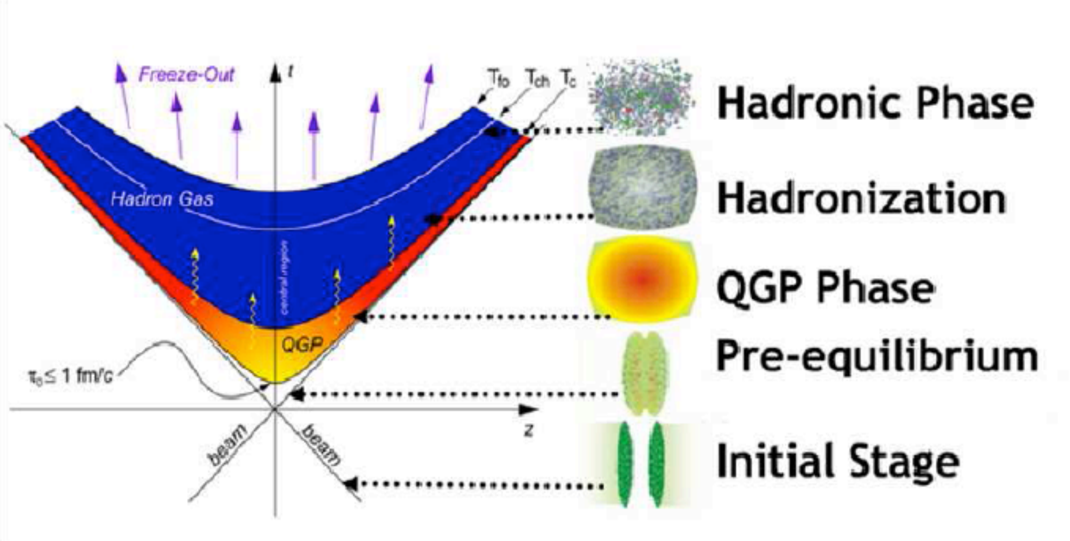
- I concentrate in my talk on the transition at vanishing density. This case is of interest for modelling the HIC at LHC and at RHIC top beam energies.

Hydrodynamics

Simulations of HIC by means of relativistic hydrodynamics show that the QGP behaves like a perfect fluid, i.e. with a small viscosity.

With suitable initial and freeze-out conditions we are able to reproduce the measurements in the HIC:

- Collective behaviour (direct and elliptic flows);
- p_T-spectrum;
- Relative abundance of the various hadron species;



The hydrodynamic codes require an Equation of State (EoS) to determine the evolution of the fluid.

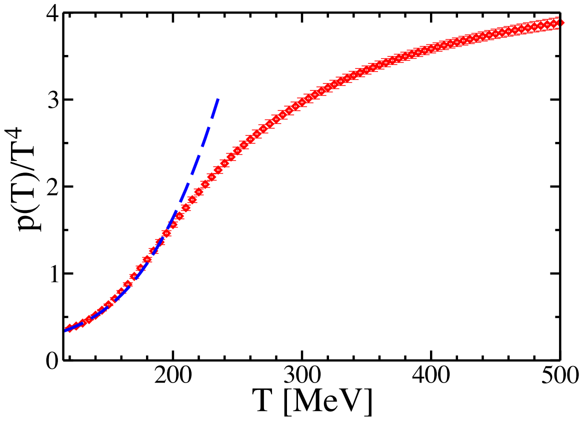
Our approach

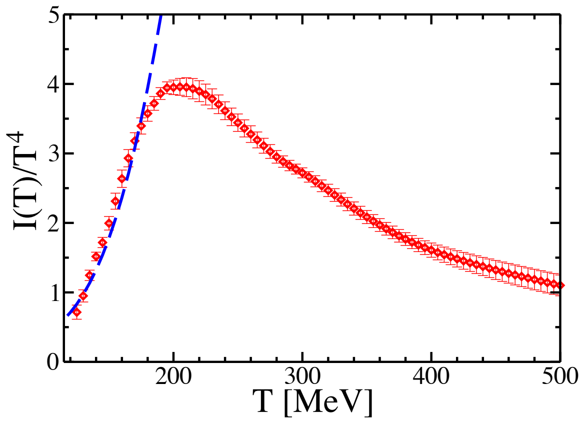
We want to provide a realistic EoS to describe the matter created in HIC.

Main features:

- For $T>T^*$: recent lattice EoS for a system of 2+1 flavour with physical values for the quark masses (WB collaboration, Quark Matter 2012);
- For $T_{ch}<T<T^*$: Hadron-Resonance Gas in chemical equilibrium (HRG in CE);
- For $T<T_{ch}$: HRG in partial chemical equilibrium (PCE).

Similar approach in P. Huovinen, P. Petreczky Nucl. Phys. A, 2010





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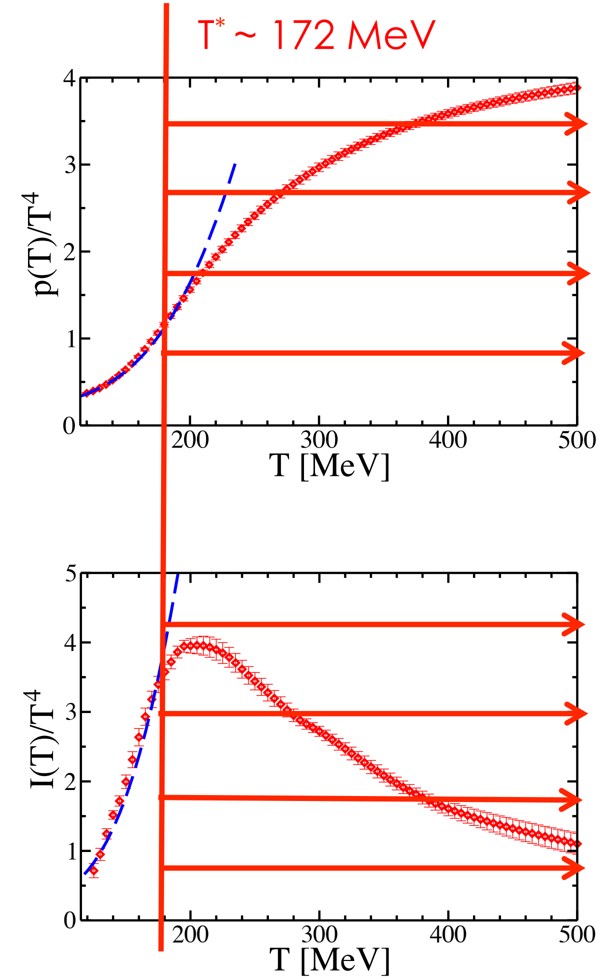

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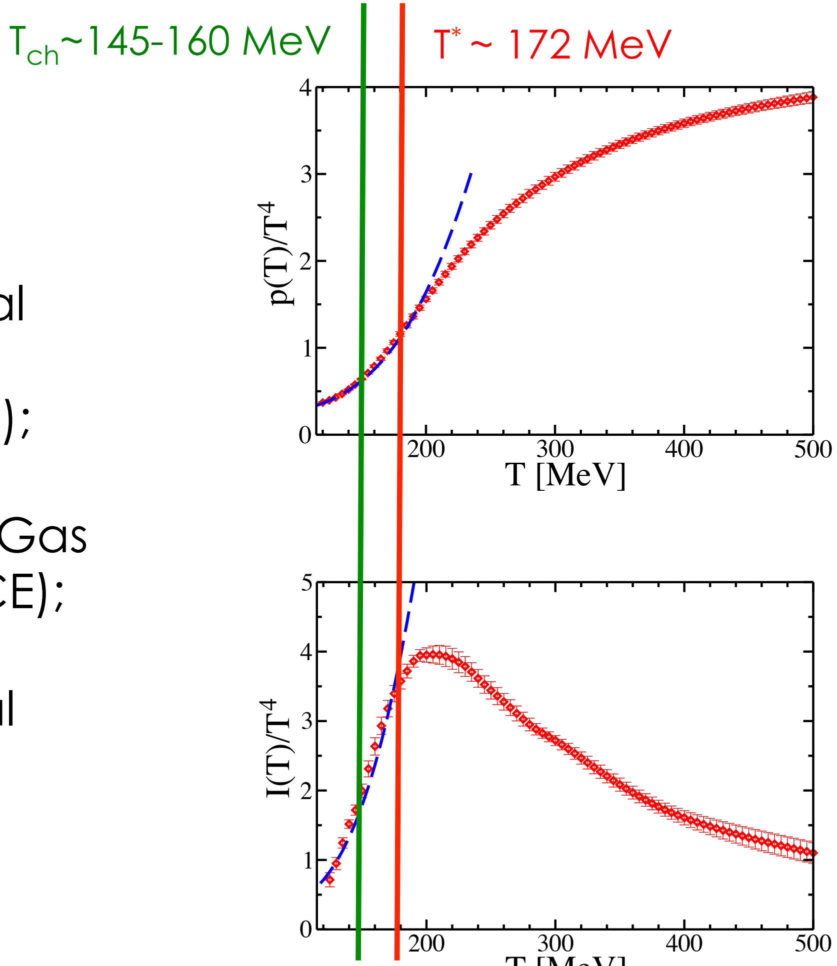

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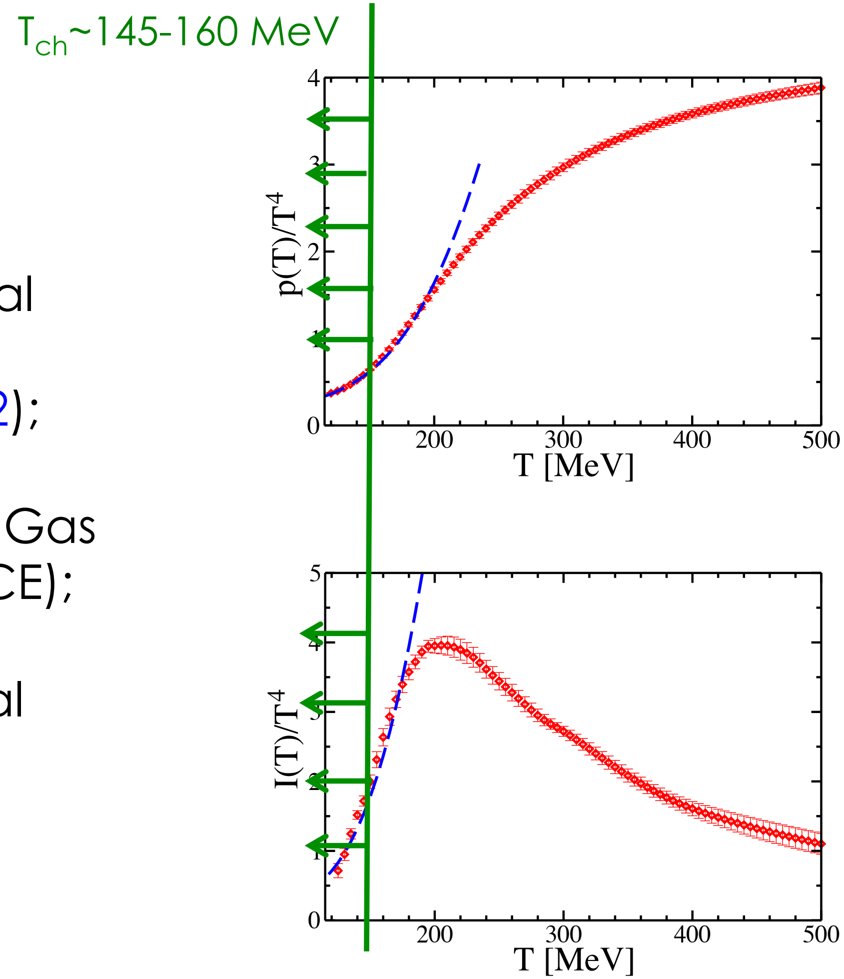

Hadron-Resonance Gas model

This model is based on the idea that a system of interacting hadrons in the ground state can be described by a gas of non-interacting resonances and hadrons.

$$p(T,\{\mu_k\}) = \sum_k (-1)^{B_k+1} \frac{d_k T}{(2\pi)^3} \int d^3\vec{p}\, \ln\left[1 + (-1)^{B_k+1} e^{-(\sqrt{\vec{p}^{\,2}+{m_k}^2}-\mu_k)/T}\right]$$

$$n_k(T,\mu_k) = \left(\frac{\partial p}{\partial \mu_k}\right)_T = \frac{d_k}{(2\pi)^3} \int d^3\vec{p} \frac{1}{(-1)^{B_k+1} + e^{(\sqrt{\vec{p}^{\,2}+{m_k}^2}-\mu_k)/T}}$$

Where B_k, d_k and m_k denote respectively the baryon number, the degeneracy and the mass of the particle k, while its chemical potential in full chemical equilibrium is given by:

$$\mu_k = B_k\mu_B + Q_k\mu_Q + S_k\mu_S$$

The particle properties are taken from the PDG. We took into account particles with mass up to 2 GeV.

Full vs Partial chemical equilibrum

Experimental situation:

- The time scales for inelastic scatterings are much larger than the lifetime of the hadronic stage.

- So it is more reasonable that the hadronic phase in HIC is not in full but in partial chemical equilibrium (PCE).

- At $T<T_{ch}$ (chemical freeze-out temperature), only elastic and quasi-elastic scatterings (mediated by the formation and subsequent strong decay of resonances) are allowed.

$$\pi\pi \to \rho \to \pi\pi \qquad p\pi \to \Delta \to p\pi \qquad K\pi \to K^* \to K\pi$$

- Only a few particles are considered stable:

$$\text{Mesons: } \pi^0, \pi^+, \pi^-, K^+, K^-, K^0, \overline{K}^0, \eta$$

$$\text{Baryons: } p, n, \Lambda^0, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^-, \Omega^-$$

Full vs Partial chemical equilibrum

The main consequence of the PCE is the conservation of the effective number of stable particles N_i.

Resonances contribute to the effective number of stable species through their branching ratios $d_{r->i}$.

$$\bar{N}_i = N_i + \sum_r d_{r\to i} N_r$$

As a consequence, each stable hadron acquires an effective chemical potential.
The chemical potentials of the resonances can be written in term of the stable states in which they decay.

$$\mu_r = \sum_i d_{r\to i} \mu_i$$

The thermodynamic quantities can be calculated by means of this set of 26 effective chemical potentials, which can be determined from conservation laws.

Partial Chemical Equilibrum

- Considering the system as a perfect fluid, we can assume the conservation of its entropy.
- Indeed in PCE we keep the ratio between the effective number and the entropy fixed.

$$\frac{\bar{N}_i}{S} = \frac{\bar{n}_i}{s}$$

We calculate the set of chemical potentials by solving this system of coupled equations, temperature by temperature.

$$\frac{\bar{n}_i(T, \{\mu_{i'}(T)\})}{s(T, \{\mu_{i'}(T)\})} = \frac{\bar{n}_i(T_{ch}, \{0\})}{s(T_{ch}, \{0\})}$$

At T_{ch} the effective chemical potentials are zero because the system is still in full CE.

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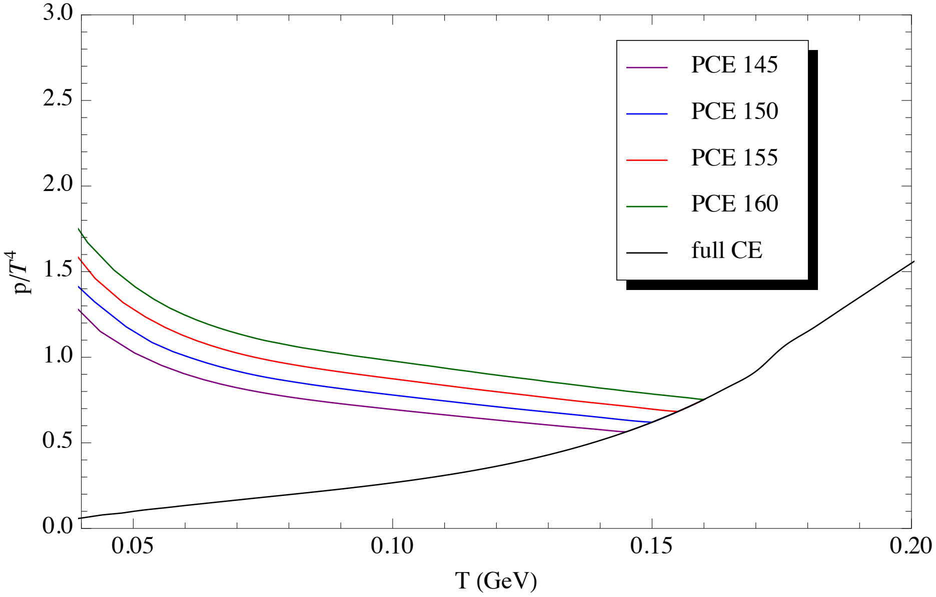
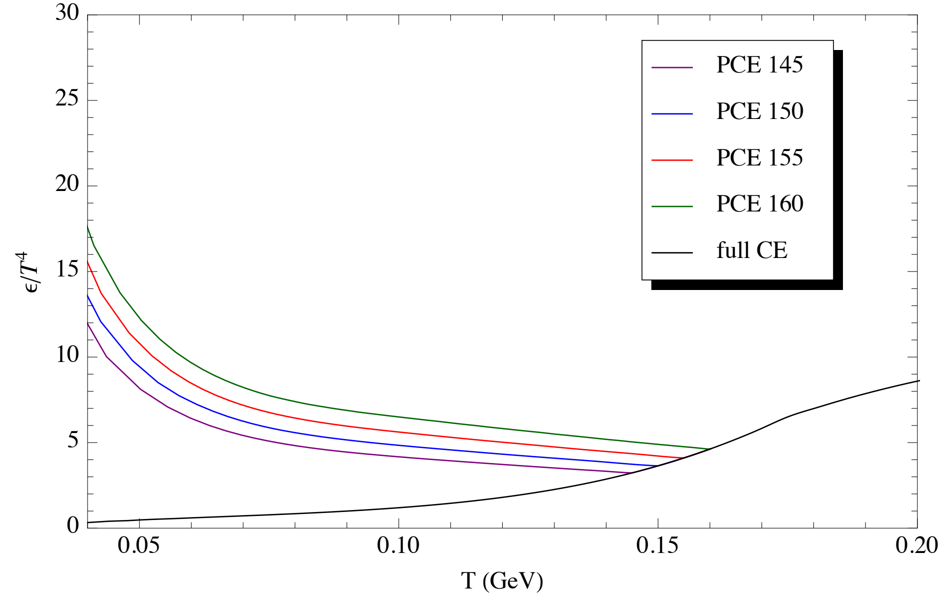
$$\frac{\bar{n}_{i1}(T,\{\mu_i\})}{\bar{n}_{i2}(T,\{\mu_i\})} = \frac{\bar{n}_{i1}(T,\{0\})}{\bar{n}_{i2}(T,\{0\})}$$

In particular this entails that particle ratio are fixed at T_{ch}.

Results– n_B=0

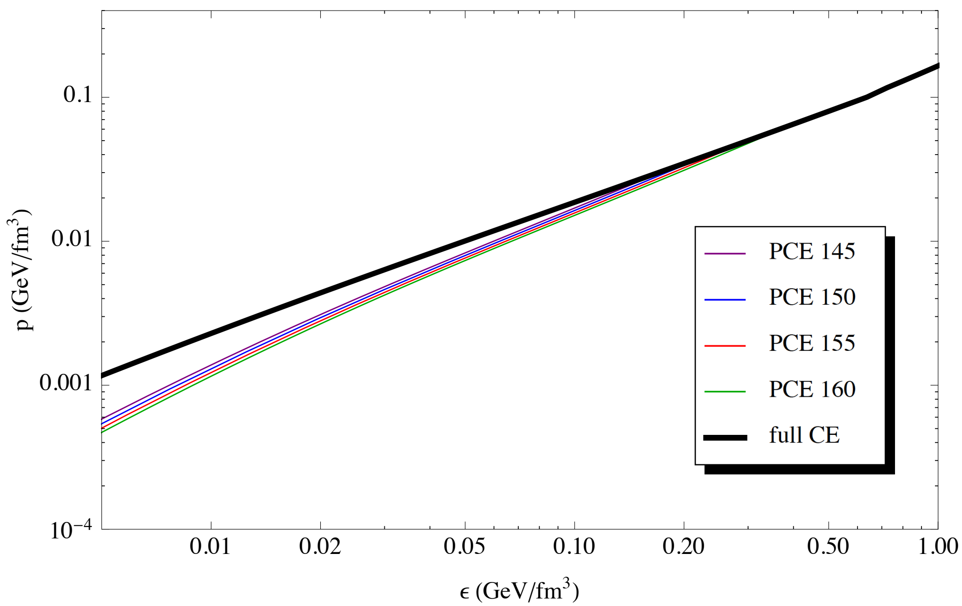
Below I show the results for the pressure and energy density, both scaled by T^4. Due to the present uncertainty on T_{ch} we consider 4 different values for it.

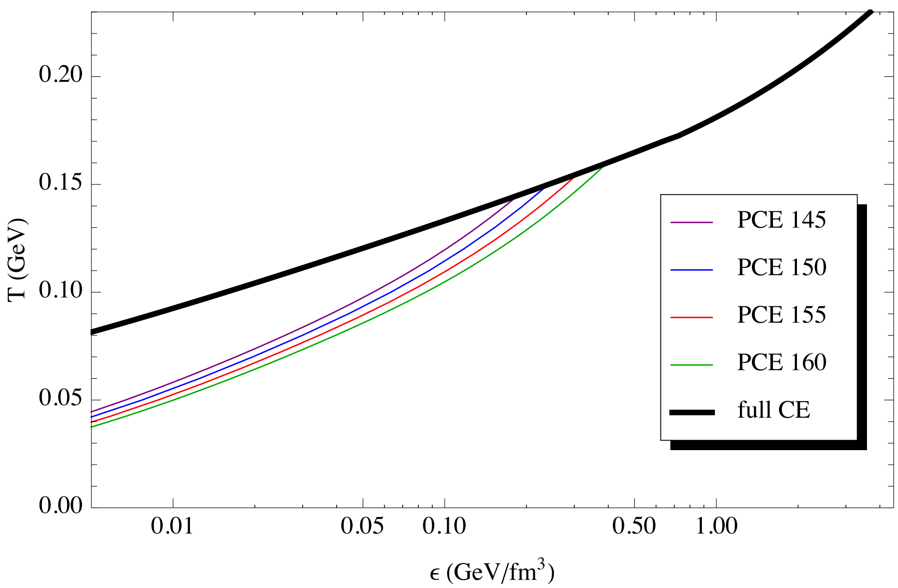




Results– n_B=0

Below I show the results for the pressure and temperature as functions of the energy density. This two quantities are used in hydrodynamics codes.

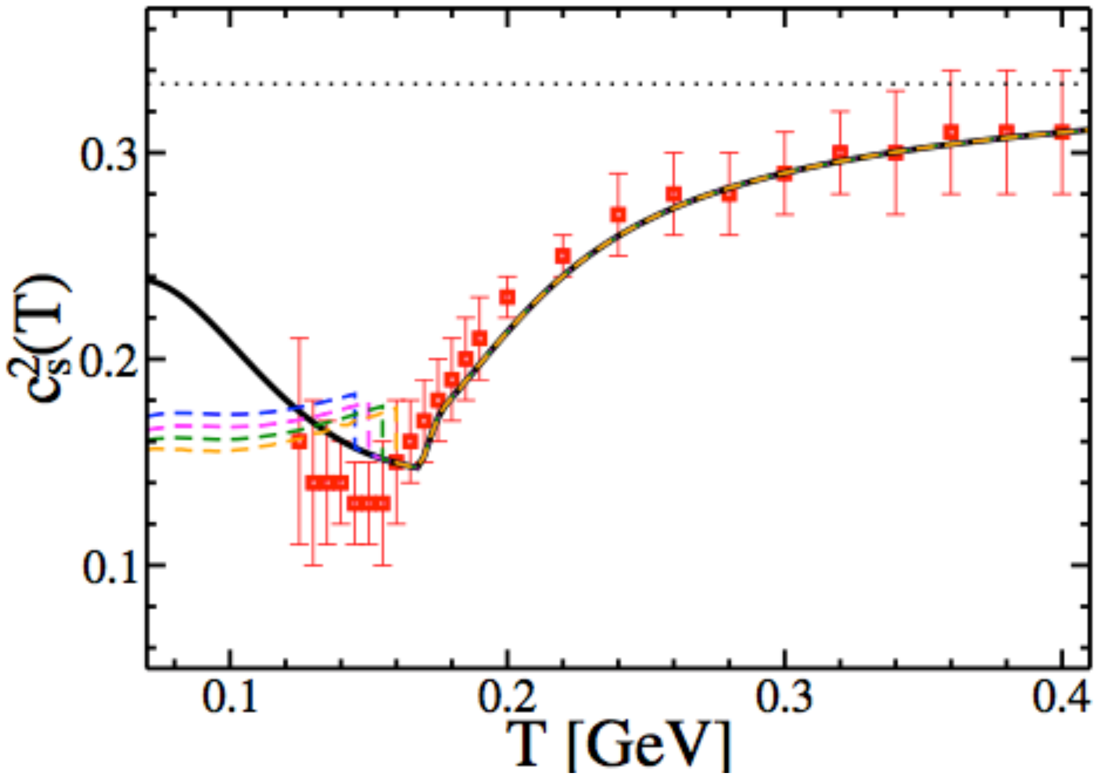




Results– n_B=0

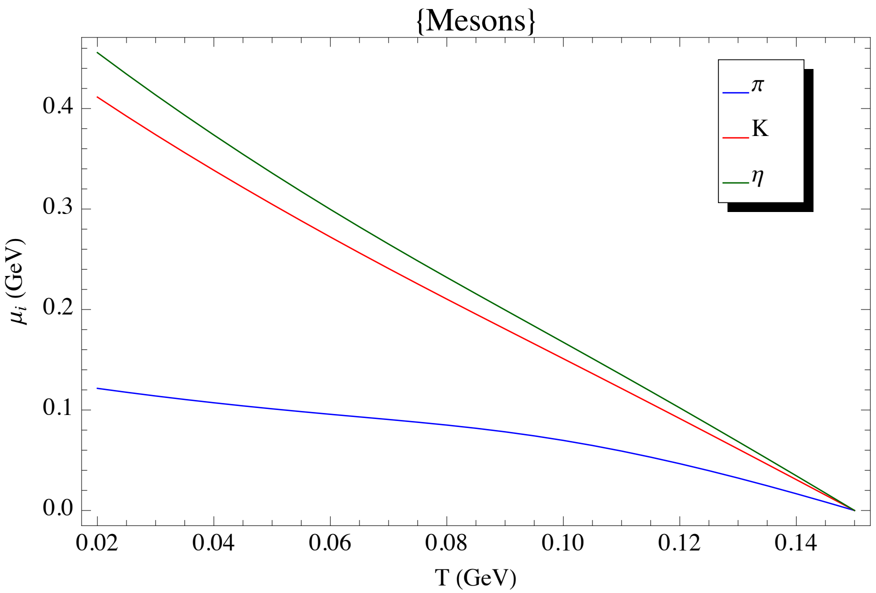
This is the result for the speed of sound, which is fundamental for the hydrodynamic codes.

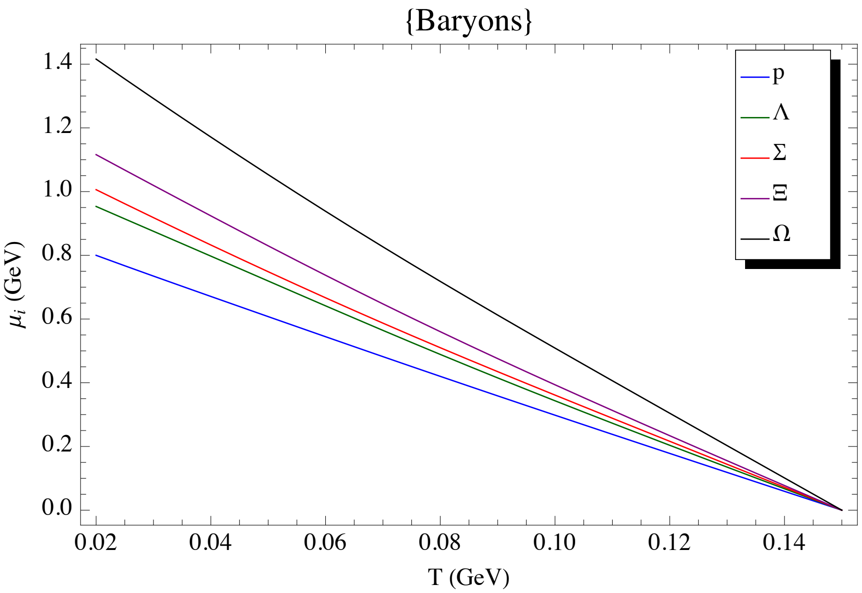
$$c_s^2 = \frac{\partial p}{\partial \epsilon}$$



Results– n_B=0

These are the chemical potentials obtained at T_{ch}=150 MeV. At vanishing net-baryon density, particles and anti-particles get the same chemical potential.
These effective chemical potentials are important to determine the final state multiplicities.





Conclusions

- We studied the EoS for the thermodynamics of QCD, in which we used:
  - Lattice data for $T>T^*$;
  - HRG in CE for $T_{ch}<T<T^*$;
  - HRG in PCE, with resonance decays into fundamental states, for $T<T_{ch}$.

- We obtained results at $n_B=0$, useful for the description of HIC at LHC and at the highest RHIC energies.

- Free download at http://personalpages.to.infn.it/~ratti/EoS/Equation_of_State/Home.html

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- We studied the EoS for the thermodynamics of QCD, in which we used:
  - Lattice data for $T>T^*$;
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Outlook

- We are already studying the case of finite density, in order to provide a realistic EoS for the RHIC low energy runs and for the future FAIR experiment.

Thanks for the attention

Hadron-Resonance Gas model

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$$n_k(T,\mu_k) = \left(\frac{\partial p}{\partial \mu_k}\right)_T = \frac{d_k}{(2\pi)^3} \int d^3\vec{p} \frac{1}{(-1)^{B_k+1} + e^{(\sqrt{\vec{p}^{\,2}+m_k{}^2}-\mu_k)/T}}$$

Where B_k is the baryon number of the particle k, d_k its degeneracy and m_k its mass, while its chemical potential in full chemical equilibrium is given by:

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$$n_k(T, \mu_k) = \left(\frac{\partial p}{\partial \mu_k}\right)_T = \frac{d_k}{(2\pi)^3} \int d^3\vec{p} \frac{1}{e^{(\sqrt{\vec{p}^2 + m_k{}^2} - \mu_k)/T} \mp 1}$$

Where d_k, m_k and μ_k denote respectively the degeneracy, the mass and the chemical potential of the particle k.

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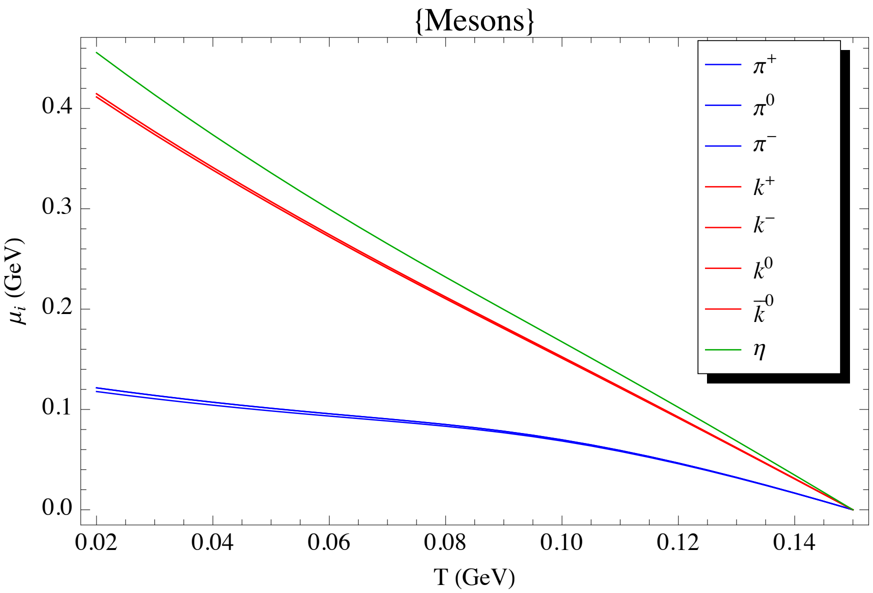
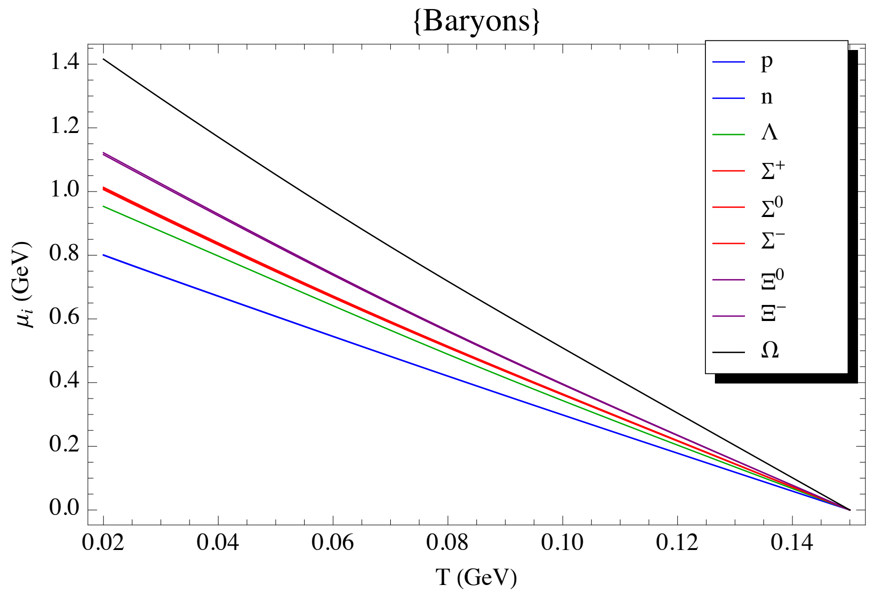




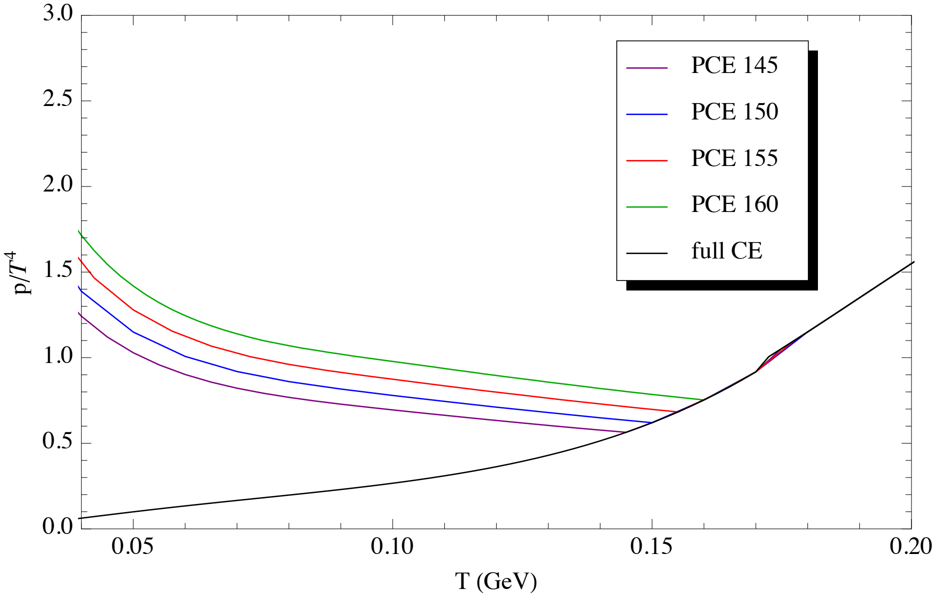
PCE – 2 bis

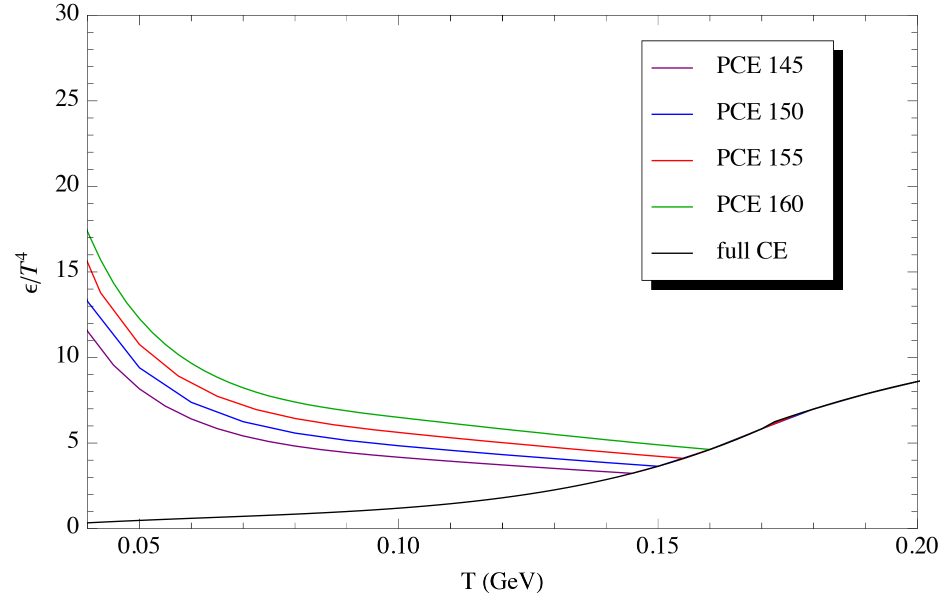
Ho da risolvere un sistema altamente non lineare di n equazioni in n incognite (che sono i potenziali chimici efficaci per le particelle stabili).

Notiamo anche come attraverso questa prescrizione anche il numero relativo fra le specie adroniche sia fissato durante tutta l'evoluzione del sistema.

Results– n_B=0

Below I show the results for the pressure and energy density, both scaled by T^4. The difference between the full CE and PCE case is clear. Due to the present uncertainty on T_{ch} we consider 4 different values for it.





PCE - 3

Le specie adroniche considerate stabili sono le seguenti

Ed i corrispondenti anti-barioni.

Nel caso a densità barionica nulla (conseguentemente anche muB=muQ=muS=0) il numero di anti-particelle eguaglia quello delle particelle.
In generale le varie specie (in particolare le diverse parti dei multipletti) si distinguono solo nel caso di densità finita.
Per le collisioni al LHC e per le top beam energies al RHIC siamo praticamente nel caso a densità nulla, ma abbiamo sviluppato anche il caso a densità finita per analizzare i dati di scan a basse energie effettuate al RHIC

Results– n_B=0

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