Transport coefficients of Quark-Gluon Plasma in a Kinetic Theory approach

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Outline

1. Introduction to QGP
   - QGP in UrHIC e Elliptic Flow
2. Transport Theory
   - Numerical Implementation
   - Transport Coefficients: Green-Kubo
3. Shear Viscosity
4. Electric Conductivity

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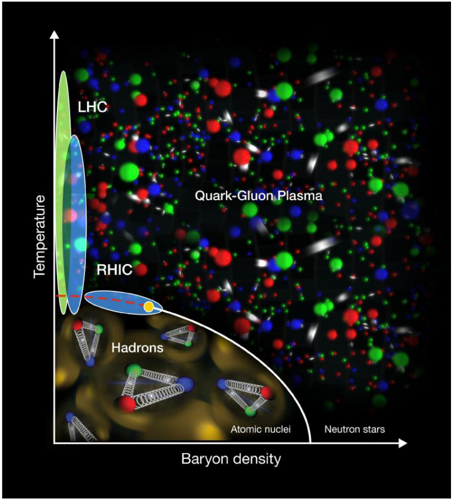
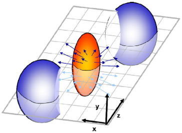
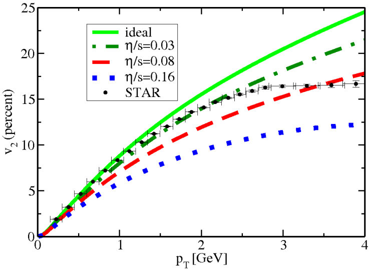
QCD phase diagram

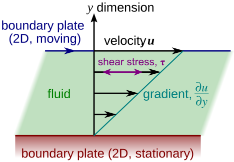


Elliptic Flow

$$v_2(p_T, b) = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$





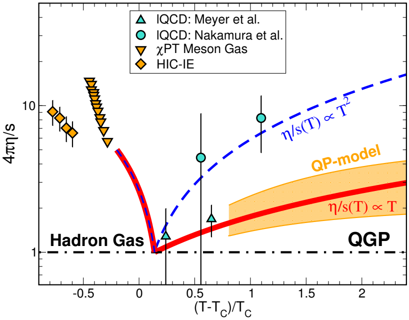


Shear Stress

$$\tau = \frac{F_x}{A_{yz}} = -\eta \frac{\partial u_x}{\partial y}$$

fluid	$P\ [Pa]$	$T\ [K]$	$\eta\ [Pa \cdot s]$	$\eta/s\ [\hbar/k_B]$
H_2O	$0.1 \cdot 10^6$	370	$2.9 \cdot 10^{-4}$	8.2
^4He	$0.1 \cdot 10^6$	2.0	$1.2 \cdot 10^{-6}$	1.9
H_2O	$22.6 \cdot 10^6$	650	$6.0 \cdot 10^{-5}$	2.0
^4He	$0.22 \cdot 10^6$	5.1	$1.7 \cdot 10^{-6}$	0.7
QGP	$88 \cdot 10^{33}$	$2 \cdot 10^{12}$	$\leq 5 \cdot 10^{11}$	≤ 0.4

T.Schäfer, D. Teaney, Rep. Prog. Phys **72** (2009) 126001



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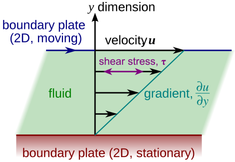
Plumari et al: arXiv:1304.6566v1

- QM: $\eta/s \simeq \frac{4}{15}\langle p \rangle \tau \Rightarrow \eta/s > \frac{1}{15}$
- AdS/CFT: $\eta/s = \frac{1}{4\pi}$



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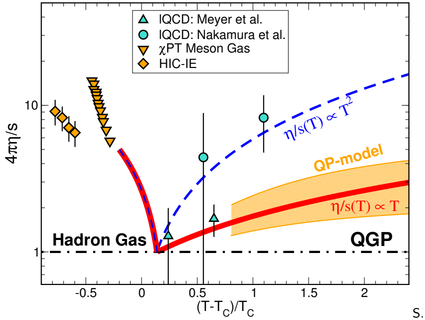
Viscosity $\longleftrightarrow$ microscopic details ?

$$\eta \rightarrow \lambda, \sigma, \langle p \rangle \ldots$$



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Quantum-Relativistic Transport Theory

General Transport Equation

$$\left(p^\mu \partial_\mu + (m^*(x)\partial_\mu m^*(x))\partial_p^\mu\right) f(x,p) = \mathcal{C}[f](x,p)$$

- free-streaming
- Mean Field: long range interactions $\rightarrow \epsilon - 3P \neq 0$
- Collisions: short range interactions $\rightarrow \eta, \sigma_{el}, \cdots$

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$$\mathcal{C}_{22} = \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p_1'}{(2\pi)^3 2E_1'}\frac{d^3p_2'}{(2\pi)^3 2E_2'} f_1' f_2' \times |\mathcal{M}_{1'2'\rightarrow 12}|^2 (2\pi)^4 \delta^{(4)}(p_1' + p_2' - p_1 - p_2)$$

gain

$$-\frac{1}{2E_1} \times \int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p_1'}{(2\pi)^3 2E_1'}\frac{d^3p_2'}{(2\pi)^3 2E_2'} f_1 f_2 \times |\mathcal{M}_{12\rightarrow 1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_1' - p_2')$$

loss

Numerical Implementation

Transport Equation

$$p^{\mu}\partial_{\mu}f(x,p) = \mathcal{C} = \mathcal{C}_{22} + \dots$$

- Test-Particles Method:

$$f(x,p) = \sum_{i=1}^{N} \delta^4(x_i(t)-x)\delta^4(p_i(t)-p)$$

$$N = N_{real} \times N_{test} \quad , \quad \sigma \to \sigma/N_{test}$$

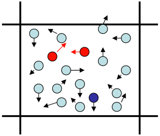
- Sthocastic Method:

$$P_{22} = \frac{\Delta N_{coll}^{2\to 2}}{\Delta N_1 \Delta N_2} = v_{rel}\frac{\sigma_{22}}{N_{test}}\frac{\Delta t}{\Delta^3 x}$$

if $P_{22} > rand()$ collision takes place;

$v_{rel} = \frac{\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}}{E_1 E_2}$

exact solutions in the limit $\Delta t \to 0, \Delta^3 x \to 0$



Δx

Transport Coefficients: Green-Kubo

Transport coefficients: $\eta, \zeta, \chi, D, \sigma_{el}$ characterize non-equilibrium behaviour of a system.

Fluctuation-Dissipation Theorem

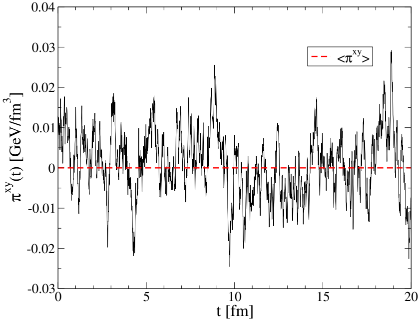
It establishes a relation between equilibrium fluctuations of a physical observable and a dissipative process that takes place when the system is perturbed from equilibrium.

Shear Viscosity

$$J \equiv \pi^{xy} = -\eta \frac{\partial u_x}{\partial y}$$

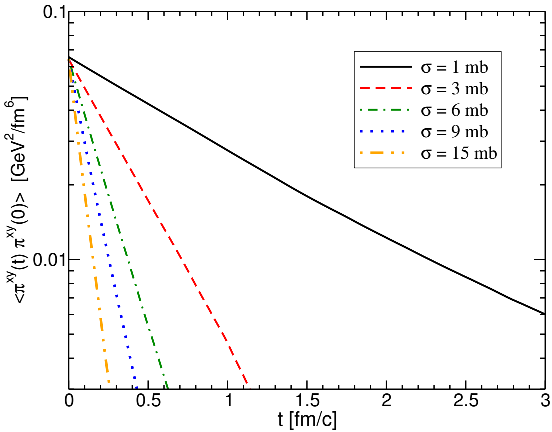
Green-Kubo Relation

$$\eta = \frac{V}{T} \int_0^\infty dt \, \langle \pi^{xy}(t) \pi^{xy}(0) \rangle$$



Transport Coefficients: Green-Kubo

$$\langle \pi^{xy}(t)\pi^{xy}(0)\rangle = \langle \pi^{xy}(0)^2\rangle e^{-t/\tau} \Longrightarrow \eta = \frac{V}{T}\int_0^\infty dt\, \langle \pi^{xy}(t)\pi^{xy}(0)\rangle = \frac{V}{T}\langle \pi^{xy}(0)^2\rangle \tau$$



Box setup

$T = 0.4\ GeV$

$V = 3^3\ fm^3$

$\Delta x \sim 0.1\ fm$

$\Delta t = 0.01\ fm/c$

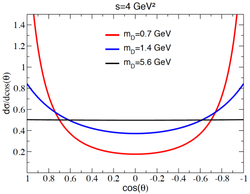
$T_{max} = 100\ fm/c$

$N_{test} = 1000$

Shear Viscosity: analytic formulas

pQCD angular dependence

$$\frac{d\sigma^{gg\to gg}}{d\Omega} \propto \frac{\alpha_s^2}{\left(q^2(\theta)+m_D^2\right)^2} \to \sigma_{tot} = \frac{9\pi\alpha_s^2}{2m_D^2}$$



Relaxation Time Approxiamtion (RTA)

$$p^\mu \partial_\mu f = C[f] \approx \frac{f - f^{eq}}{\tau} \Rightarrow \eta = \frac{4}{15}\rho\langle p\rangle\tau_{tr} = \frac{4}{15}\rho\langle p\rangle\frac{1}{\langle\rho\sigma_{tr}v_{rel}\rangle} = \frac{4}{15}\frac{\langle p\rangle}{\langle h(a)\rangle\sigma_{tot}}$$

$\sigma_{tr} = \int d\sigma^{gg\to gg}\sin^2\theta_{cm} = \sigma_{tot}h(a) < \frac{2}{3}\sigma_{tot}$

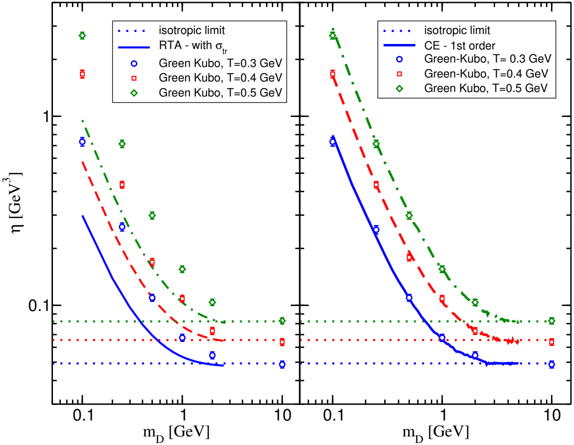
with $h(a) = 4a(a+1)[(2a+1)ln(1+a^{-1}) - 2]$ $\quad a = mD^2/s,$ $\quad h(m_D \to \infty) = 2/3$

Chapman-Enskog (CE)

$$\eta^{1st} = \frac{4}{15}\rho\langle p\rangle\tau_\eta = \frac{4}{15}\frac{\langle p\rangle}{g(a)\sigma_{tot}} \quad with \quad g(a) = \frac{1}{50}\int dy\, y^6\left[(y^2+\frac{1}{3})K_3(2y) - yK_2(2y)\right] h\left(\frac{a^2}{y^2}\right)$$

$a = m_D/2T, \qquad g(m_D \to \infty) = 2/3$

Shear Viscosity: anisotropic σ.



RTA

$$\eta = \frac{4}{5} \frac{T}{\langle h(a) \rangle \sigma_{tot}}$$

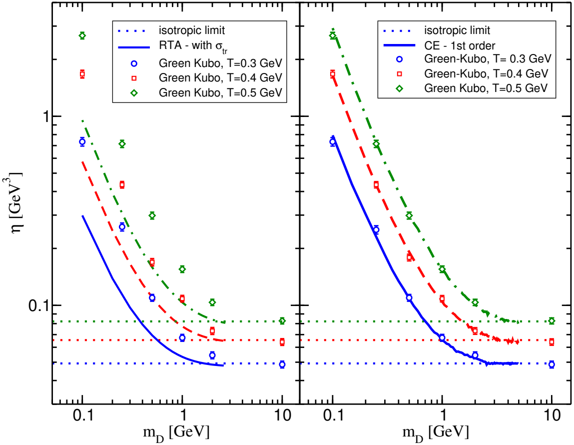
CE

$$\eta^{1st}_{CE} = \frac{4}{5} \frac{T}{g(a) \sigma_{tot}}$$

RTA cs CE: anisotropic case $\rightarrow$ factor ~ 2

However: RTA vs CE: isotropic limit $\rightarrow$ ok

Shear Viscosity: anisotropic σ.



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η^{1st}_{CE} *OK*! in all m_D range.

RTA cs CE: anisotropic case $\rightarrow$ factor ~ 2

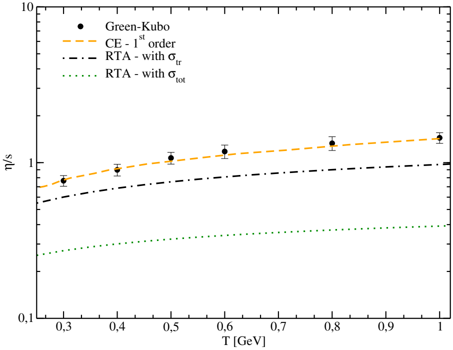
However: RTA vs CE: isotropic limit $\rightarrow$ ok

η/s of a gluon plasma

pQCD: anisotropic and energy-dependent cross-section

$$\frac{d\sigma^{gg\to gg}}{d\Omega} \propto \frac{\alpha_s^2}{\left(q^2(\theta)+m_D^2\right)^2} \quad \to \quad \sigma_{tot} = \frac{9\pi\alpha_s^2}{2m_D}\frac{s}{s+m_D^2}$$

$$m_D(T) = T\sqrt{4\pi\alpha_s} \qquad \alpha_s(T) = \frac{4\pi}{11\ln\left(\frac{2\pi T}{\Lambda}\right)^2}, \qquad \Lambda = 200\ MeV$$



$$(\eta/s)_{RTA} = \frac{1}{5}\frac{T}{\langle h(a)\rangle \rho\sigma_{tot}}$$

$$(\eta/s)^{1st}_{CE} = \frac{1}{5}\frac{T}{g(a)\rho\sigma_{tot}}$$

RTA often used in literature: factor ~ 1.5

Green-Kubo vs CE: 3%

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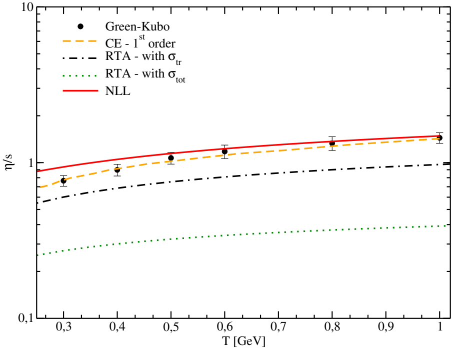

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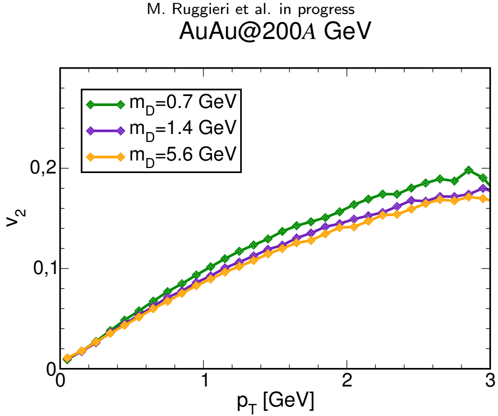
Green-Kubo vs CE: 3%

NLL P. Arnold et al: JHEP05(2003)051

$$\eta \sim \frac{T^3}{g^4 \ln g^{-1}}$$

Elliptic flow



$$v_2(p_T, b) = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

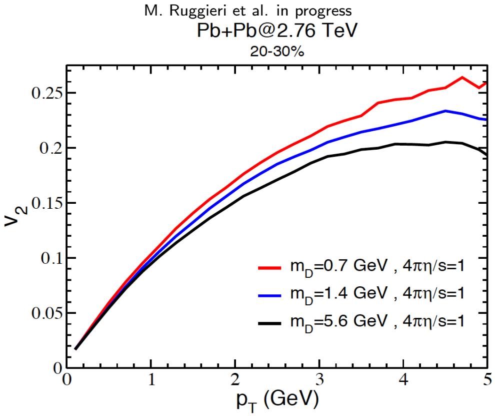
How to fix η?

$$\sigma_{tot}(\rho(\mathbf{x}), T) = \frac{1}{5} \frac{T}{g(a)} \frac{1}{\eta/s}$$

- σ locally computed
- $\eta = cost$ changing σ

- $v_2(p_T < 1.5\ GeV)$: η/s
- $v_2(p_T > 1.5\ GeV)$: sensitive to microscopic details of interaction

Elliptic flow



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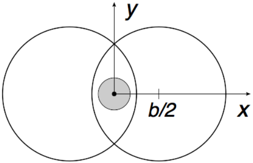
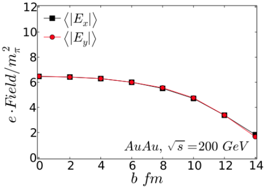
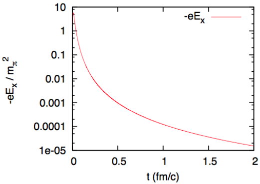
Electric Conductivity

$$j = \sigma_{el} E$$







K. Tuchin, arXiv: 1301.0099 (2013)   Y. Hirano, M. Hongo, T. Hirano, arXiv:1211.1114 (2012)

Observable

Electric Field $\Rightarrow$ effects on v_1:

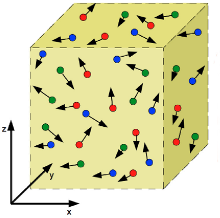
$$v_1 = \left\langle \frac{p_x}{p_T} \right\rangle$$

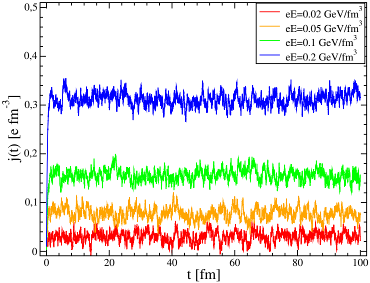
Electric Conductivity

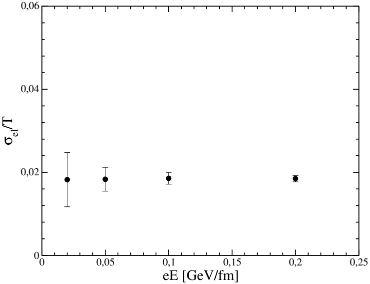
$$j_z = \sigma_{el} E_z$$

$$\frac{d}{dt} p_z^i = q_i e E_z$$

$$j_z(t) = \frac{1}{V} \sum_i e q_i \frac{p_z^i(t)}{m_j} \quad \Rightarrow \quad \sigma_{el} = \frac{j_z^{eq}}{E_z}$$







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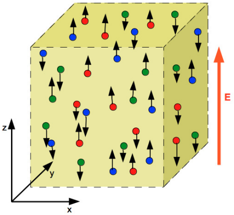
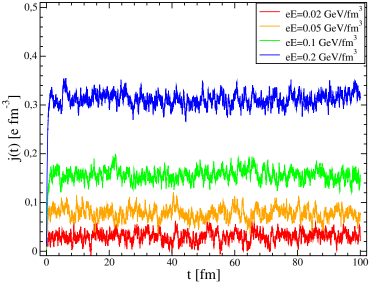
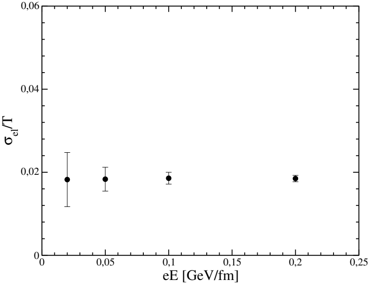




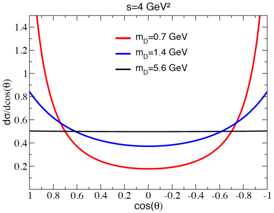


Electric Condictivity: Anisotropic cross-section

pQCD angular dependence

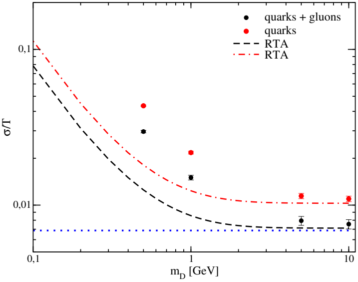
$$\frac{d\sigma^{gg\rightarrow gg}}{d\Omega} \propto \frac{\alpha_s^2}{\left(q^2(\theta)+m_D^2\right)^2} \rightarrow \sigma_{tot} = \frac{9\pi\alpha_s^2}{2m_D^2}$$

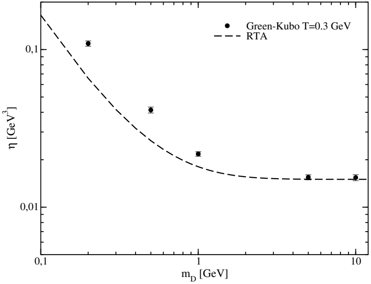
Box Setup: $m = 0.4\ GeV$, $\sigma_{tot} = 10\ mb$



RTA

$$\frac{\sigma_{el}}{T} = \frac{1}{T}\sum_q \frac{q^2\rho_q\tau_q}{m_q}$$





Electric Conductivity: Quasi-Particle Model

Quasi-Particle Model

- Description of Lattice results in terms of quasiparticles quarks and gluons.
- interaction generates quasiparticle mass $m(T)$

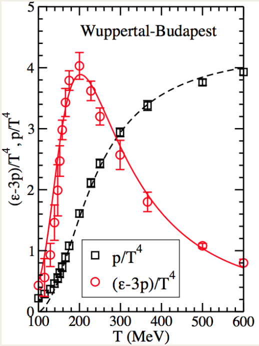
$$E = \sqrt{p^2 + m^2(T)}$$

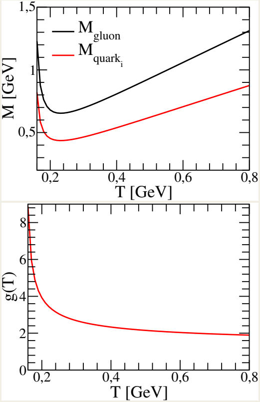
- $\Rightarrow$ quasiparticle are weakly interacting.

$$g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f)\ln\left[\lambda\left(\frac{T}{T_c} - \frac{T_s}{T_c}\right)\right]^2}$$

$$m_g^2(T) = \frac{1}{6} g^2 \left(N_c + \frac{1}{2} N_f\right) T^2$$

$$m_q^2(T) = \frac{N_c^2 - 1}{8N_c} g^2 T^2$$





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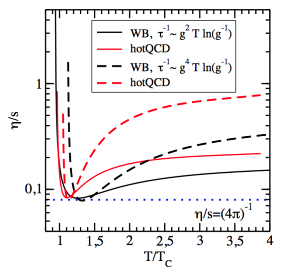
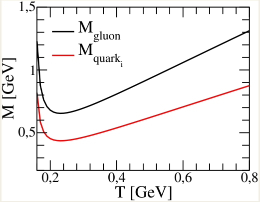
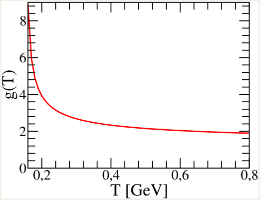
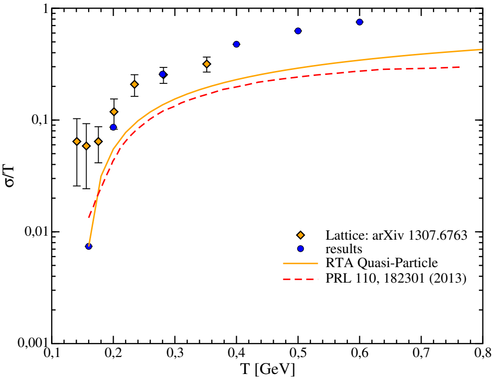


S. Plumari et all. Phys. Rev. D **84**, 094004 (2011)





Electric Conductivity: Quasi-Particle Model



$$\frac{\sigma_{el}}{T} = \frac{1}{T}\sum_q \frac{q^2 \rho_q \tau_q}{m_q}$$

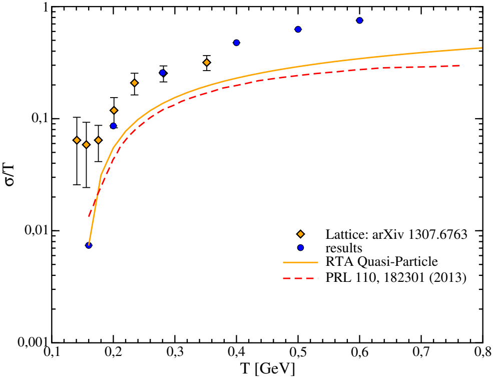
$$\tau_q = \sum_j \langle \rho_j \sigma^{qj} v_{rel}^{qj} \rangle$$

$$\sigma_{tot}^{ij}(s) \sim \beta^{ij} \frac{\alpha_s^2}{m_D^2(T)} \frac{s}{(s + m_D^2(T))}$$

$$m_D^2(T) = 4\pi\alpha_s T^2$$

	β
$qq \to qq$	$2\,\frac{8\pi}{9}$
$q\bar{q} \to q\bar{q}$	$\frac{8\pi}{9}$
$qg \to qg$	2π
$gg \to gg$	9π

Electric Conductivity: Quasi-Particle Model



agreement with Lattice data above T_c

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Conclusions

Results

Shear Viscosity:

- Relaxation-Time-Approximation (RTA) is in general in disagreement with Green-Kubo results.
- Chapman-Enskog is in agreement with Green-Kubo results at 3% of accuracy.

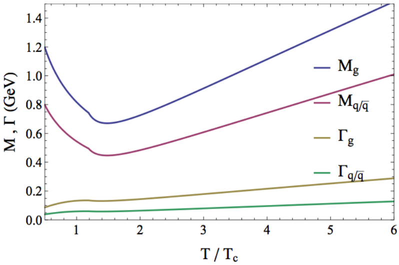
Electric Conductivity:

- RTA reproduces isotropic limit
- RTA is not in agreement in the anisotropic case
- agreement with Lattice data above T_c.

Outlooks

- Transport code development to simulate high energy collisions (RHIC, LHC) with fixed η/s (extension to v_n).
- Calculation of η of a mixture.
- Calculation of bulk viscosity ζ.

Dynamical Quasi-Particle Model



$$M_g^2(T) = \frac{g^2(T/T_c)}{6}\left(\left(N_c + \frac{1}{2}N_f\right)T^2 + \frac{N_c}{2}\sum_q \frac{\mu_q^2}{\pi^2}\right) ,$$

$$M_{q(\bar{q})}^2(T) = \frac{N_c^2 - 1}{8N_c} g^2 \left(T^2 + \frac{\mu_q^2}{\pi^2}\right) ,$$

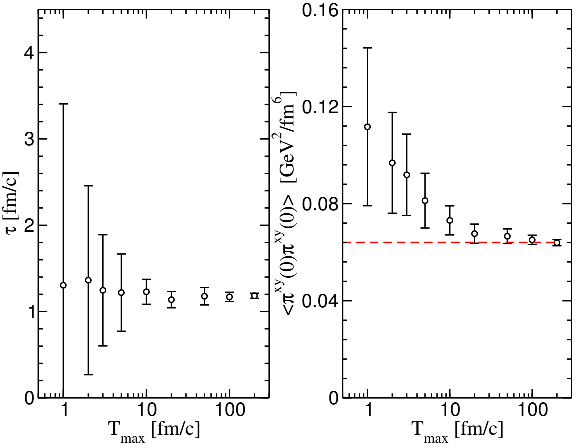
$$\Gamma_g(T) = \frac{1}{3} N_c \frac{g^2 T}{8\pi} \ln\left(\frac{2c}{g^2} + 1\right)$$

$$\Gamma_{q(\bar{q})}(T) = \frac{1}{3}\frac{N_c^2 - 1}{2N_c}\frac{g^2 T}{8\pi} \ln\left(\frac{2c}{g^2} + 1\right) ,$$

R. Marty, E. Bratkovskaya, W. Cassing, J. Aichelin, and H. Berrehrah

arXiv: 1305.7180v1

Correlator Convergence: T_{max}

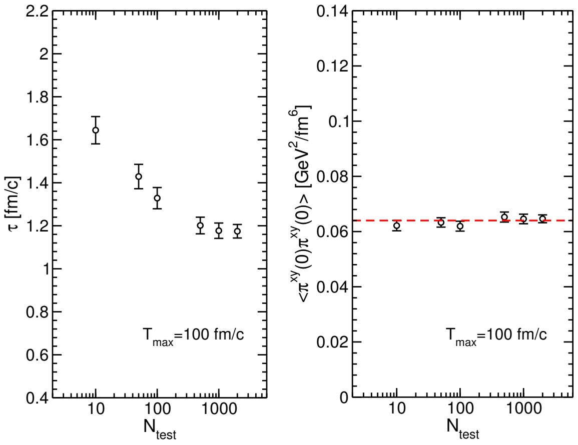


$$\langle \pi^{xy}(0)^2 \rangle = \frac{4}{15}\frac{\epsilon T}{V}$$

$$\eta = \frac{V}{T}\langle \pi^{xy}(0)\pi^{xy}(0)\rangle \tau$$

$$T_{max} = 100\ fm/c$$

Correlator Convergence: N_{test}



$$\langle \pi^{xy}(0)^2 \rangle = \frac{4}{15} \frac{\epsilon T}{V}$$

$$\eta = \frac{V}{T} \langle \pi^{xy}(0) \pi^{xy}(0) \rangle \, \tau$$

$N_{test} = 1000$

Shear Viscosity: massive case

Why consider $m \neq 0$?

Quasi-particle model: $M(T)$, reproduce E.O.S from Lattice QCD.

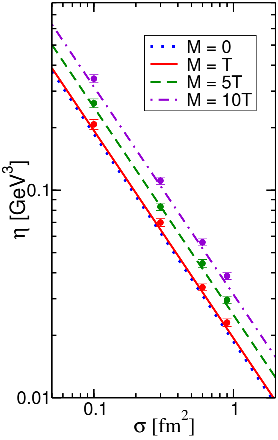
Chapman-Enskog: first order

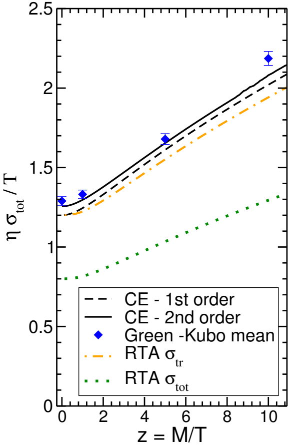
$$\eta = \frac{T}{\sigma_{tot}} h(z) \qquad z = \frac{M}{T}$$

$$h(z) = \frac{15}{16} \frac{z^4 \left[K_3(z)\right]^2}{(15z^2+2)K_2(2z) + (3z^3+49z)K_3(2z)}$$

N. Moroz, arXiv:1112.0277 1 dicembre (2011)
Wiranata, Prakash, arXiv:1203.0281 1 marzo (2012)

Shear Viscosity: massive case





CE first order

$$\eta = \frac{T}{\sigma_{tot}} h(z)$$

Green-Kubo

$$\eta = \frac{V}{T} \langle \pi^{xy}(0)^2 \rangle \tau$$

Chapman-Enskog

Chapman-Enskog: first order

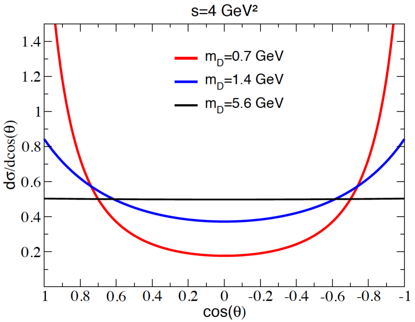
$$\eta_{CE}^{1st} = 10T\left[\frac{K_3(z)}{K_2(z)}\right]^2 \frac{1}{c_{00}} = g(m_D, T)\frac{T}{\sigma_{tot}}$$

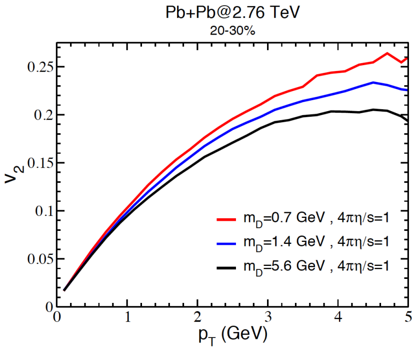
$$c_{00} = 16\left[\omega_2^{(2)} - z^{-1}\omega_1^{(2)} + (3z^2)^{-1}\omega_0^{(2)}\right]$$

$$\omega_i^{(2)} = \frac{z^3}{[K_2(z)]^2}\int_1^\infty dy(y^2-1)^7 y^i K_j(2zy)\,\sigma_{tr}$$

$j = \frac{5}{2} + \frac{1}{2}(-1)^i, \quad y = \frac{\sqrt{s}}{2M}$
$\sigma_{tr} \int d\Omega\, \sigma(s,\Theta)\sin^2\Theta$

v2 m_D





anisotropic cross-section

microscopic details become relevant at higher p_T

$$\frac{d\sigma^{gg\to gg}}{d\Omega} = \frac{9\pi\alpha_s^2}{2}\frac{1}{\left(q^2(\theta)+m_D^2\right)^2}\left(1+\frac{m_D^2}{s}\right)$$