CORTONA 2013
XIV CONVEGNO su PROBLEMI
di FISICA NUCLEARE
TEORICA
29–31 OTTOBRE 2013

Marco Radici
INFN - Pavia

Nucleon Tomography

INFN
Istituto Nazionale
di Fisica Nucleare

why the Nucleon ?

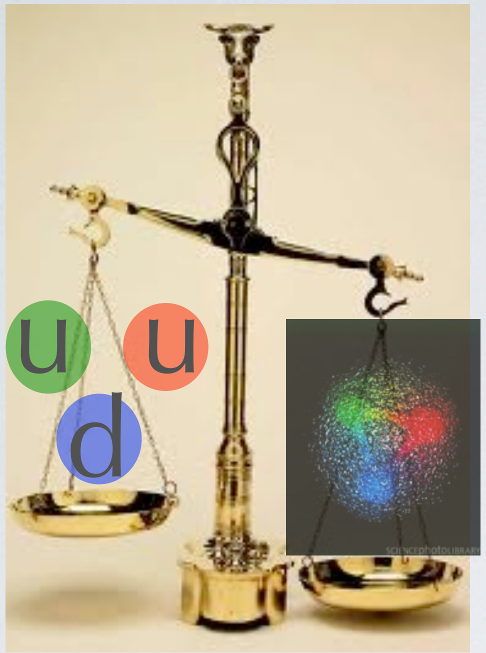
It makes up 99% of visible universe..

yet, we don’t know how its structure comes about!

🟦 $q - \langle \Phi_{Higgs} \rangle$

“Higgs” (current) mass

~ 9 MeV



938 MeV

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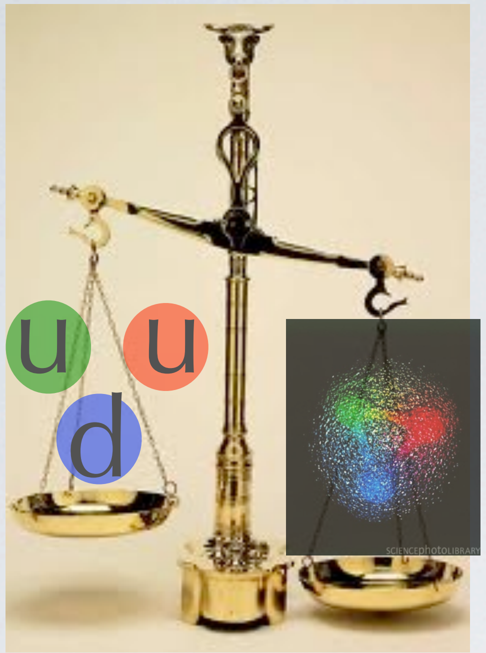

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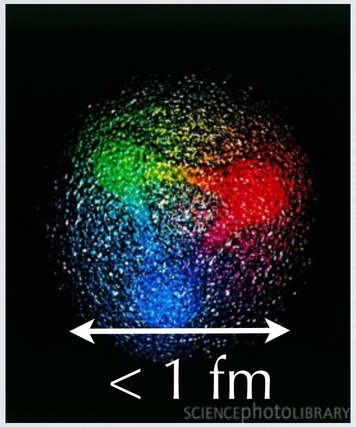


spontaneous breaking of
QCD chiral symmetry

🟦 $q - \langle \Phi_{Higgs} \rangle$

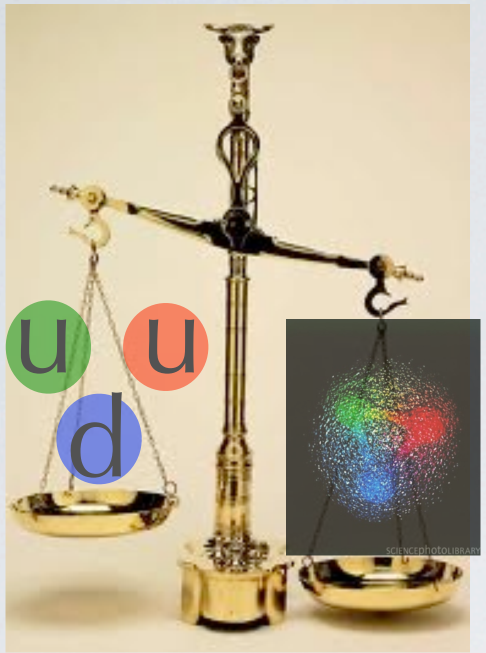
"Higgs" (current) mass

🟧 $q - \langle \bar{q}q \rangle$

$\langle g^2 F_{\mu\nu} F^{\mu\nu} \rangle$

"QCD" mass (dressing)

~ 9 MeV



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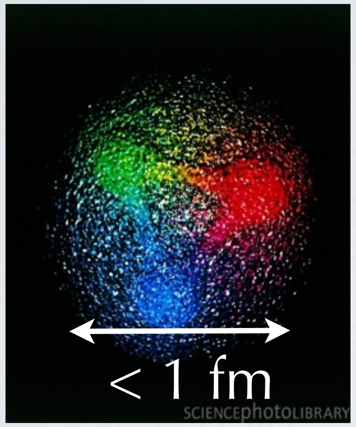

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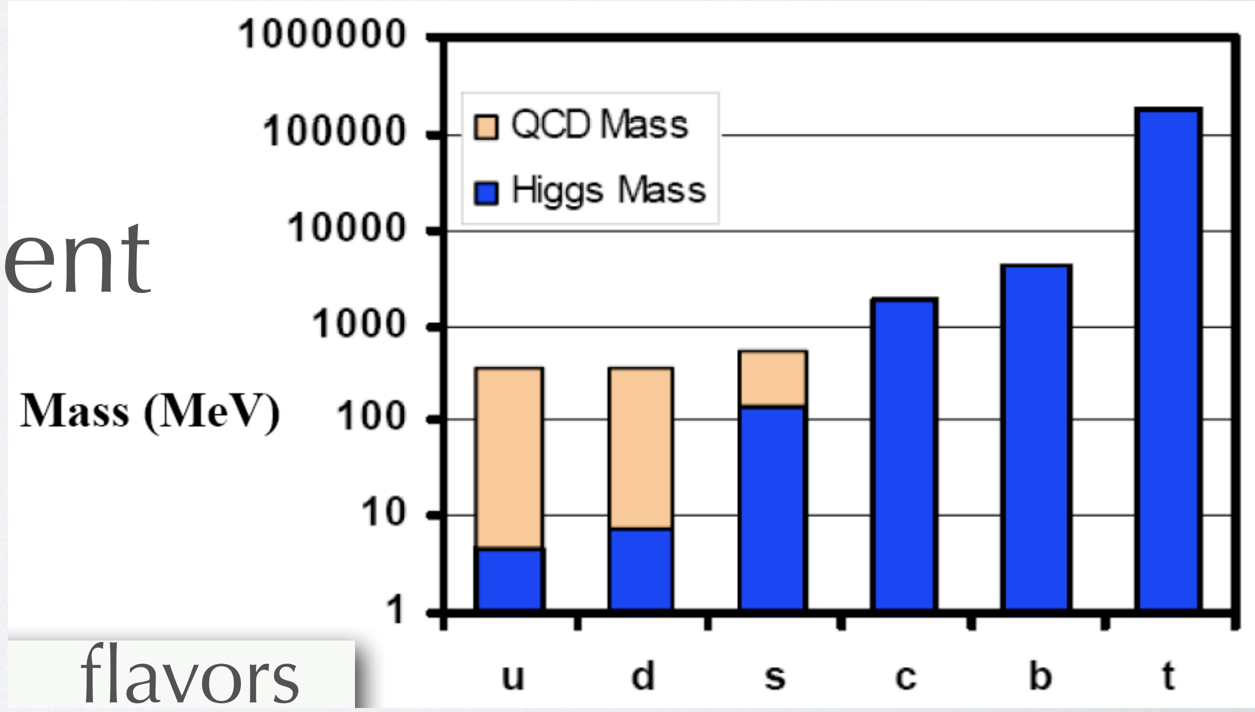
+ } constituent mass

*B. Mueller, NP**A750** (05)*



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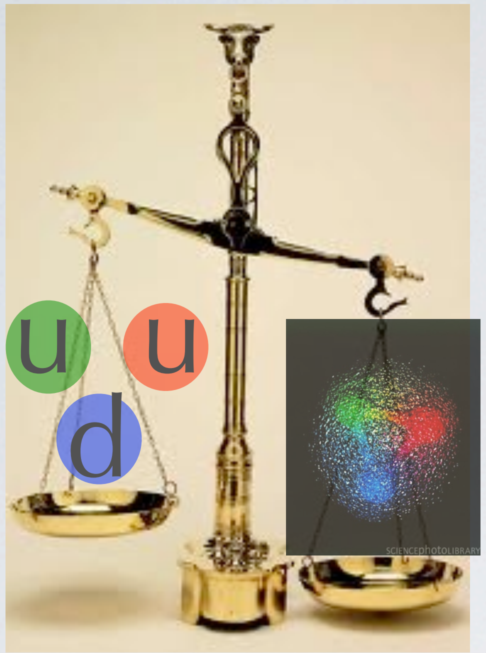

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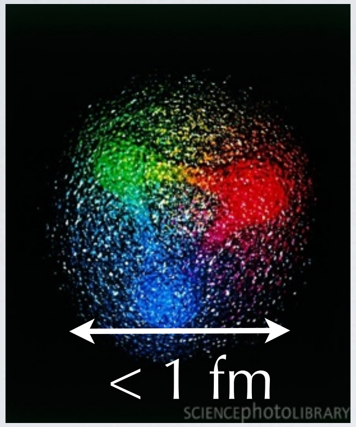
visible mass generated by dynamics of QCD confinement



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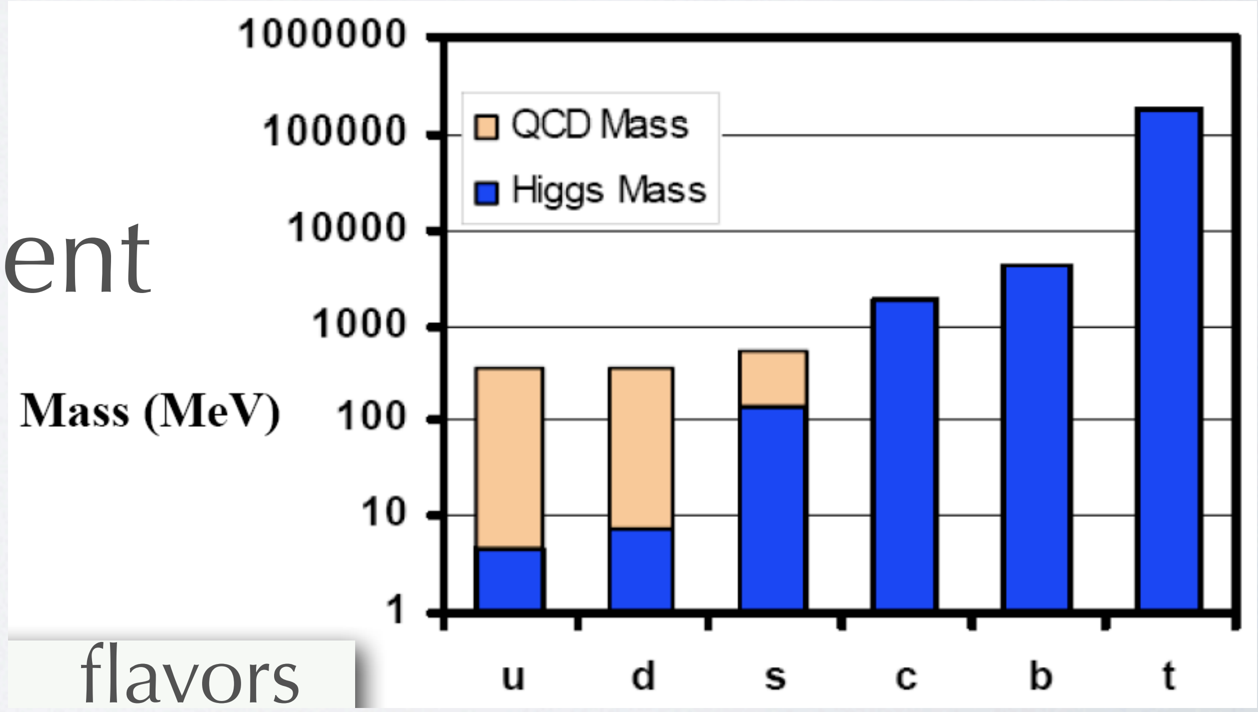
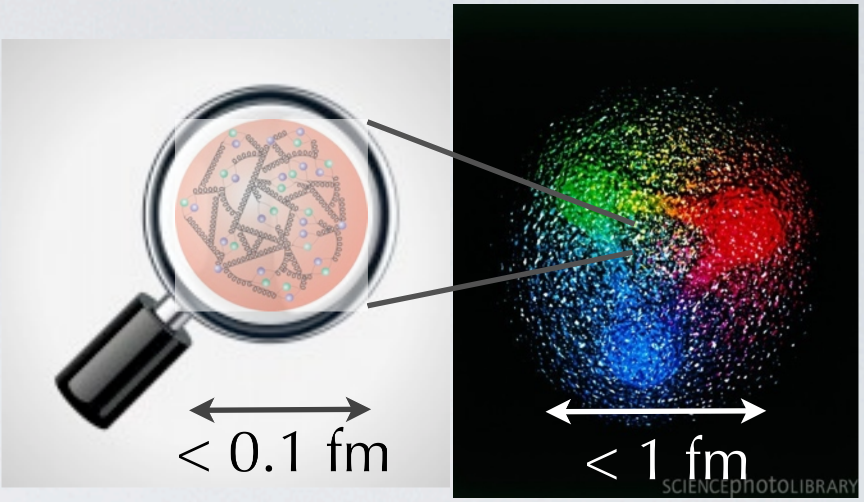


understanding confinement ⇒ high-energy probe



Ex. : Deep-Inelastic Scattering

understanding confinement
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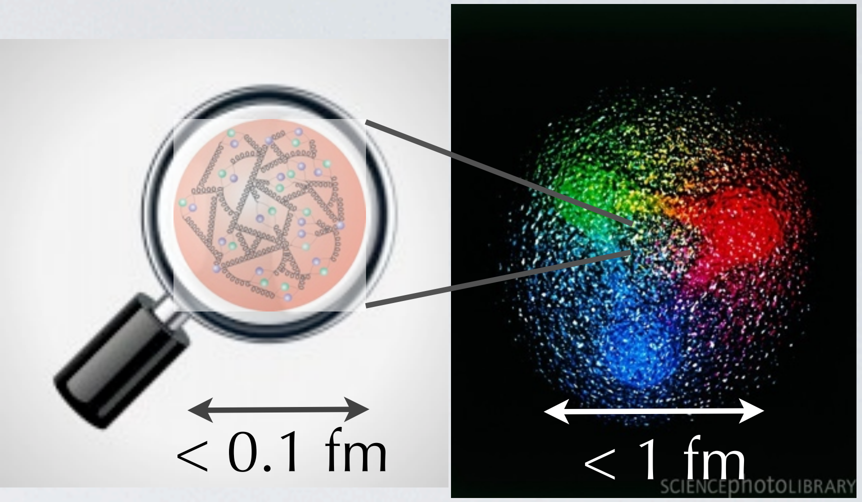

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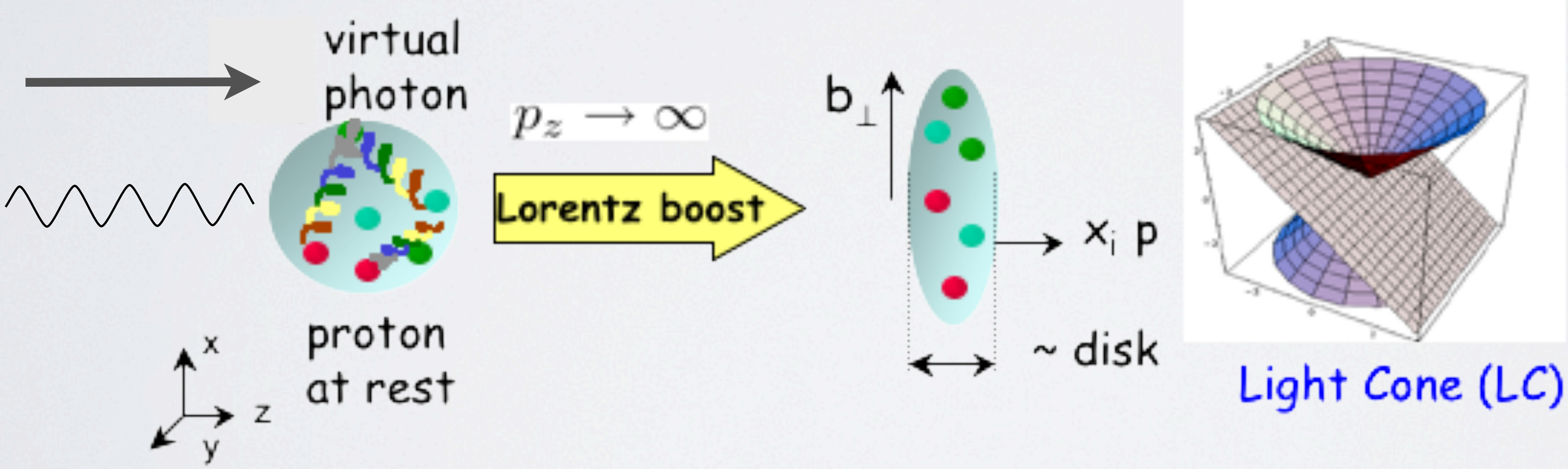
DIS regime



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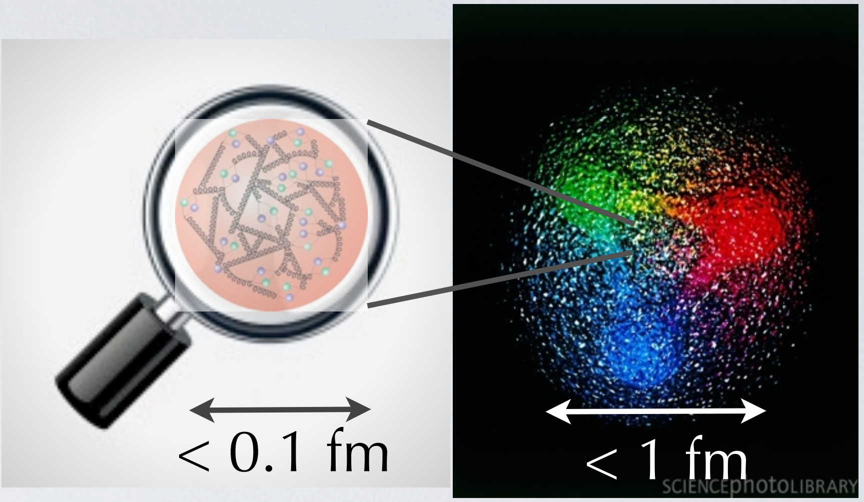
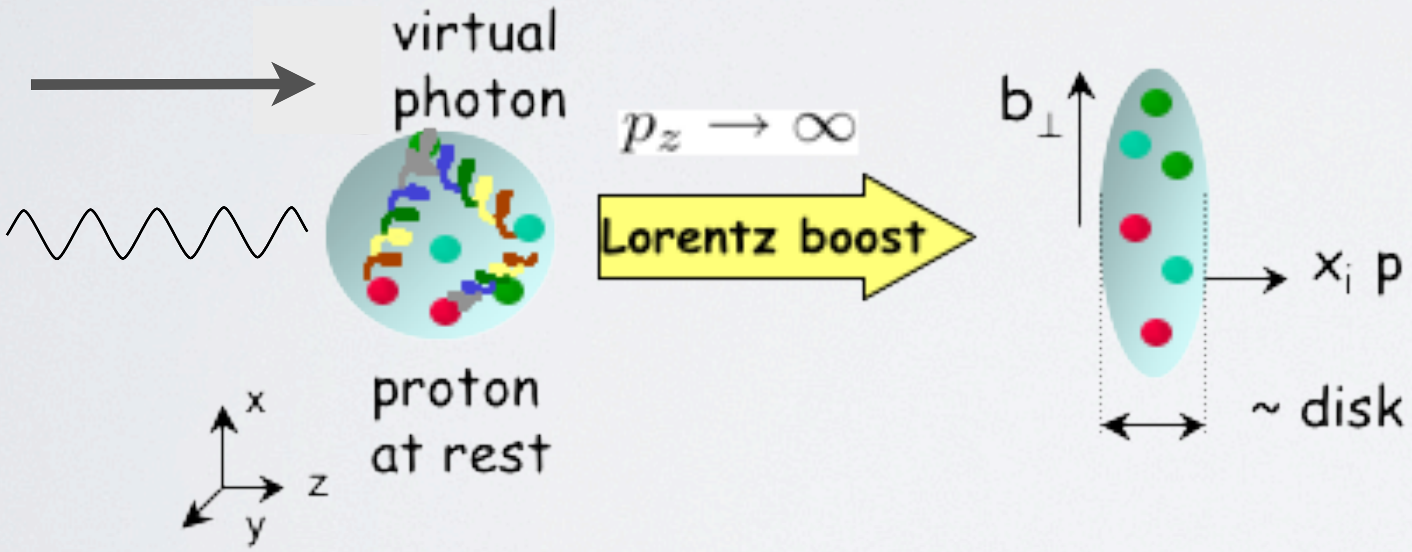
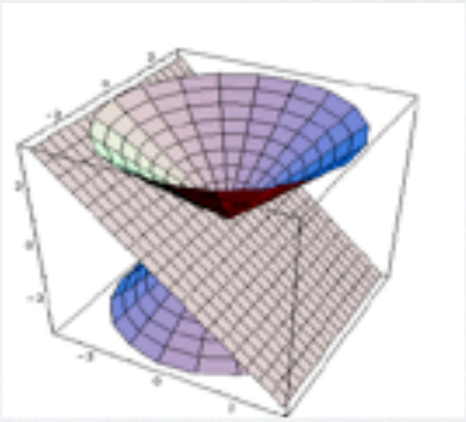


Ex. : Deep-Inelastic Scattering

Infinite Momentum Frame
(IMF)

DIS regime





Light Cone (LC)

understanding confinement
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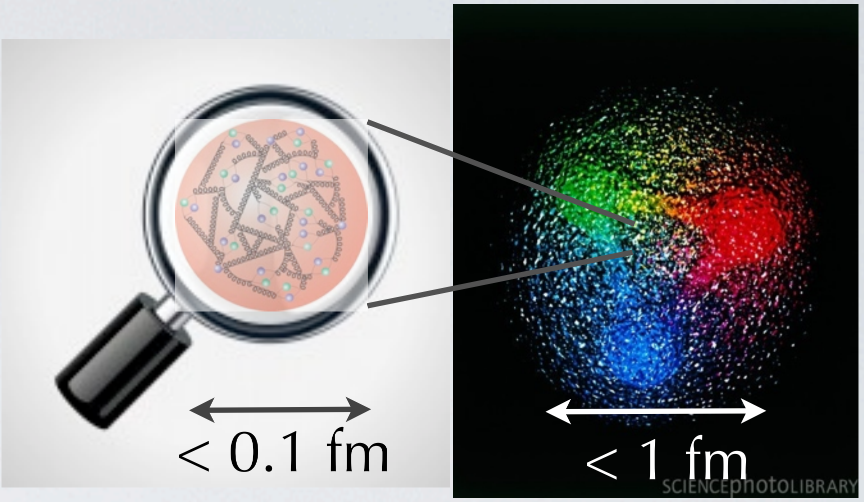
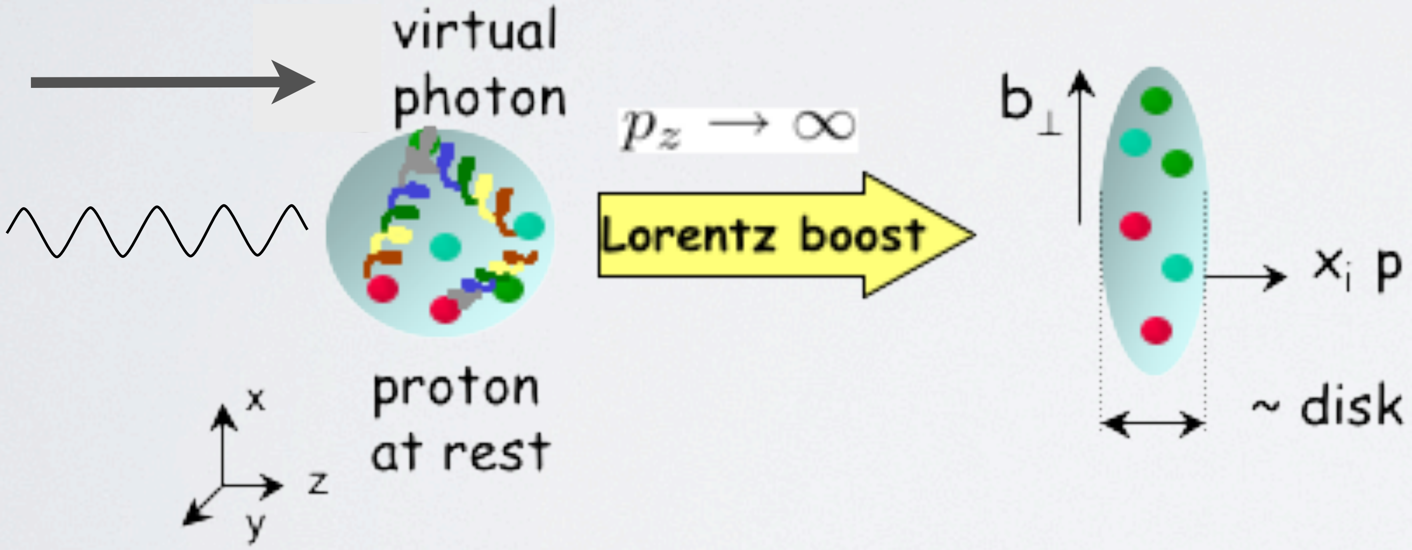
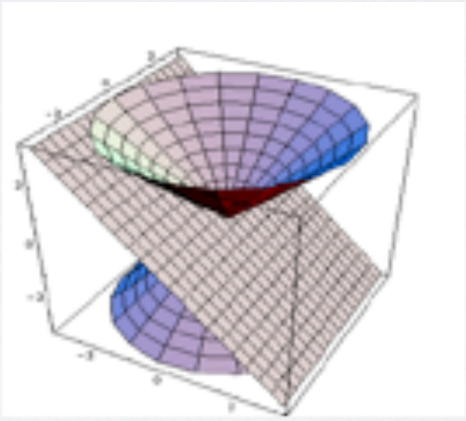


Ex. : Deep-Inelastic Scattering

Infinite Momentum Frame <=> Light-Cone kin.
(IMF) (LC)

DIS regime





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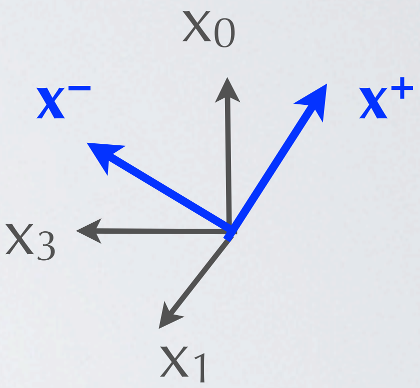

$$x^\pm = \frac{x_0 \pm x_3}{\sqrt{2}}$$

quantization at "LC time" $x^+=0$

→ $x^- = -x_3\sqrt{2}$ new longitudinal variable

→ $p^+ = x\,P^+$ $p_\perp \ll p^+$ collinear kin.

understanding confinement
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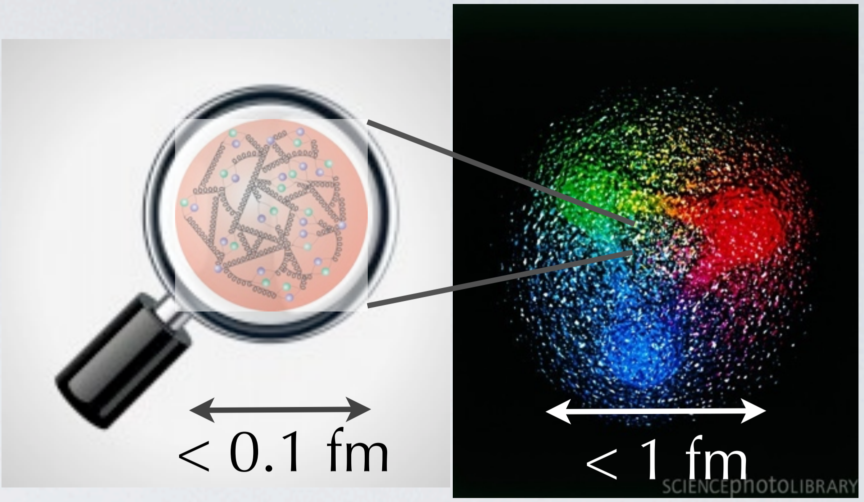

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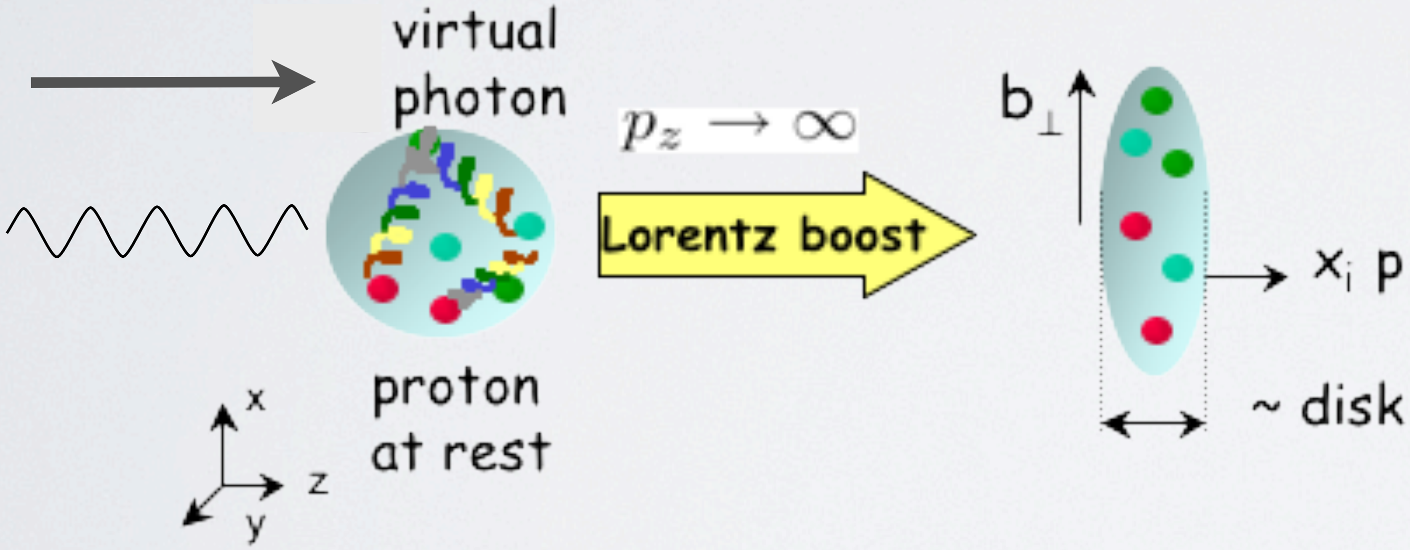

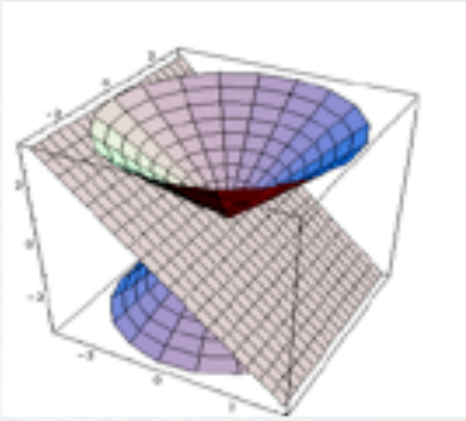

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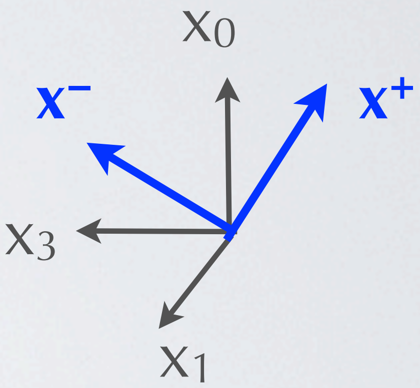

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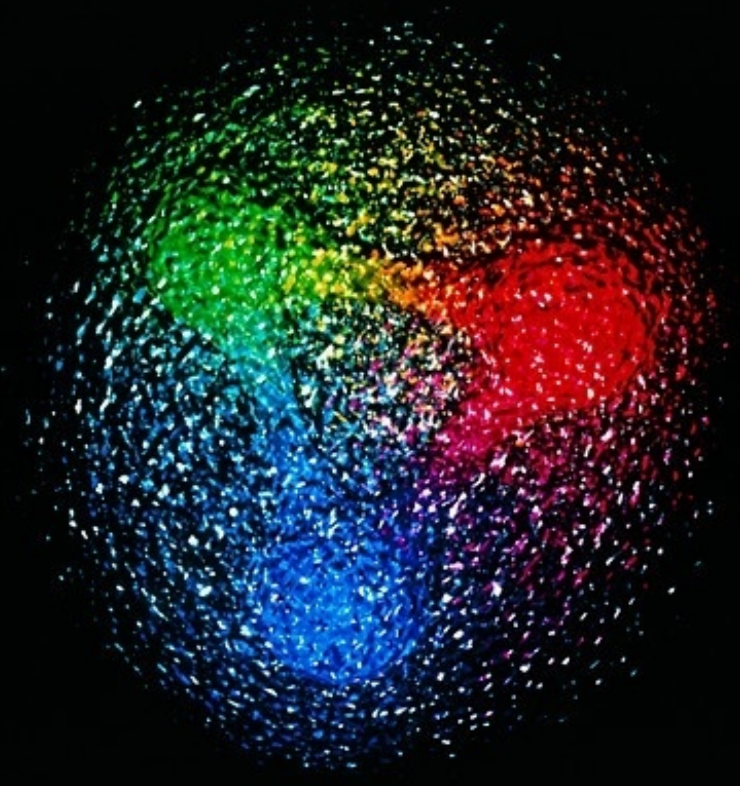
Need IFM

If we want to extract information on the distribution of charge and magnetization of partons inside the nucleon from G_E,G_M (or F_1,F_2), it can be done rigorously only in the IMF

G.A. Miller,
Annu. Rev. Nucl. Part. Sci. ***60*** *(10) 1*
and references therein



e.m. form factors of N are extracted from

$$\langle N(P',S')|J^\mu(0)|N(P,S)\rangle = \bar{u}(P'S')\left[\gamma^\mu F_1(Q^2) + iF_2(Q^2)\frac{\sigma^{\mu\nu}q_\nu}{2M}\right]u(P,S)$$

$q = P' - P \ , \ Q^2 = -q^2 \geq 0$

$F_1(0) = e_N \ , \ F_2(0) = \kappa_N$

in Breit frame **P′** = −**P** = **q** /2
<N′|J$^{\mu}$(0)|N> involves the Sachs form factors

$$\begin{aligned} G_E(Q^2) &= F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2) \\ G_M(Q^2) &= F_1(Q^2) + F_2(Q^2) \end{aligned}$$

such that, e.g., G$_E$ ∝ <N′|J^0(0)|N> ; then

$$\rho(r) = \frac{2}{\pi}\int_0^\infty dQQ^2\, j_0(Qr)\, G_E(Q^2)$$

$$\rho(r) = \frac{2}{\pi}\int_0^\infty dQ\,Q^2\, j_0(Qr)\, G_E(Q^2)$$

but $\rho = |\psi|^2$
is a static density
in rest frame

Breit frame changing with $Q^2=\mathbf{q}^2$
rel. w.f. |N(P)>
boost → |N′(P′)> ≠ |N(P)>
density interpretation
in principle is lost

The interpretation of $\rho \leftrightarrow G_E$ works only in
the nonrelativistic limit:

$$\left\{ \begin{array}{lcl} x = \dfrac{p_0 + p_3}{P_0 + P_3} & \approx & \dfrac{1}{3} + \dfrac{p_3}{3m} \\ M - 3m & \ll & 3m \end{array} \right.$$

Solution

$$\begin{aligned}
\langle N'|J^+(0)|N\rangle \quad &= \bar{u}_{N'}\gamma^+ u_N\, F_1(t) + \bar{u}_{N'}\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u_N\, F_2(t) \\
= \langle N'|\bar{q}(0)\gamma^+ q(0)|N\rangle \quad &= \int dx \int \frac{dx^-}{2\pi} e^{ixP^+x^-}\langle N'|\bar{q}(-\frac{x^-}{2},0,\mathbf{0})\,\gamma^+\, q(\frac{x^-}{2},0,\mathbf{0})|N\rangle \\
&= \bar{u}_{N'}\gamma^+ u_N \int dx H(x,\xi,t) + \bar{u}_{N'}\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u_N \int dx E(x,\xi,t)
\end{aligned}$$

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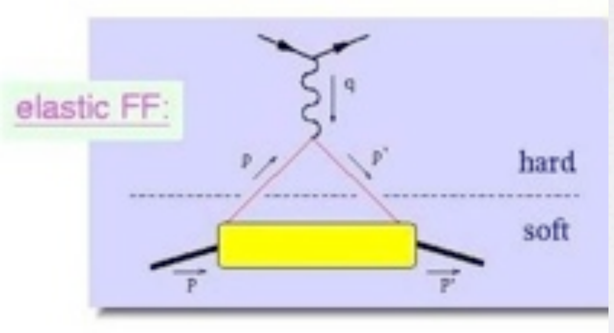
local



t = (P′-P)^2

Solution

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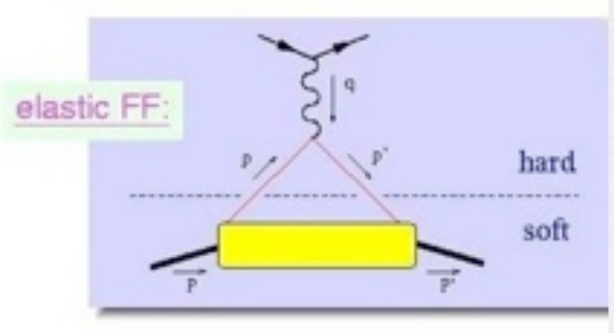

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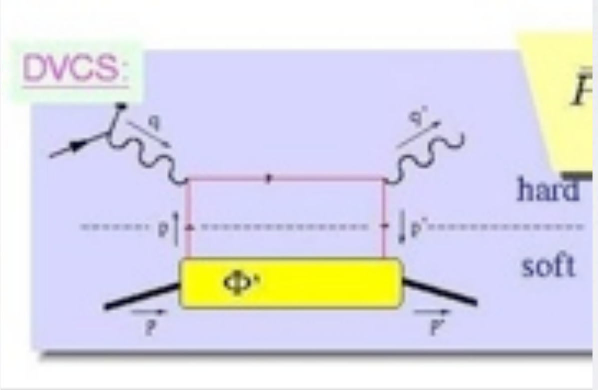

nonlocal along LC "–" direction

in gauge A$^+$=0

Solution

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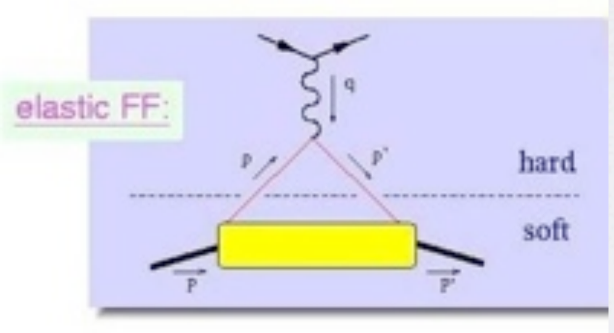

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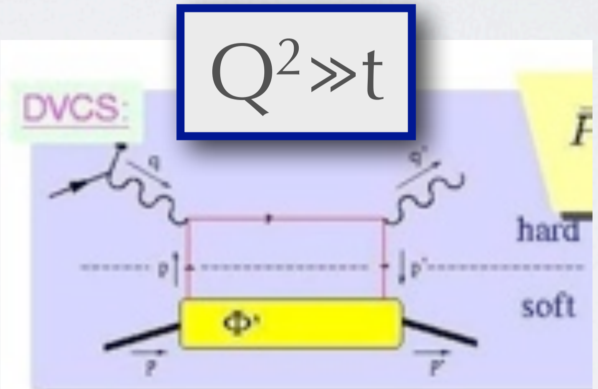

$Q^2 \gg t$

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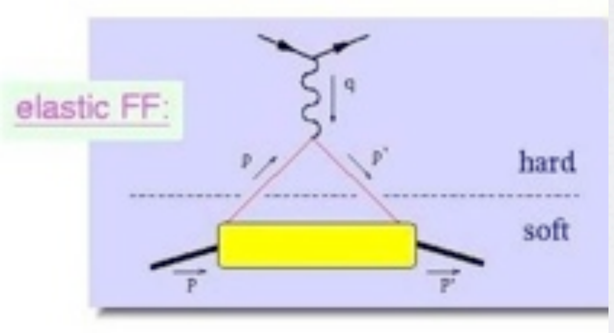

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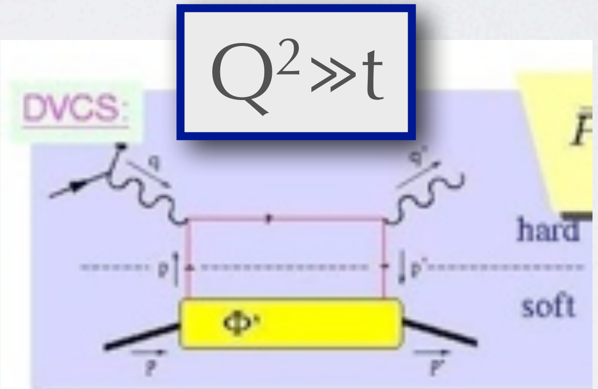

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from

Form Factors

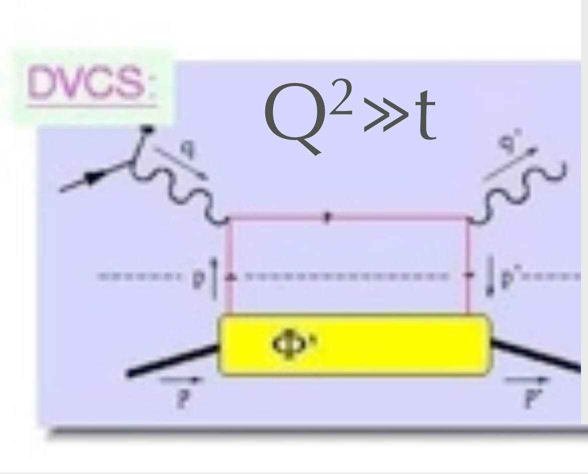
$F_1(t)$, $F_2(t)$

to

Generalized Parton Distributions (GPD)

$H(x,\xi,t)$, $E(x,\xi,t)$

in IFM , GPD depend on invariants



$$\begin{aligned} x &= \frac{(p+p')^+}{(P+P')^+} \\ \xi &= \frac{(P-P')^+}{(P+P')^+} \\ t &= (P'-P)^2 = \Delta^2 \end{aligned}$$

average longitudinal parton momentum

change in N longitudinal momentum

global change in N momentum

(and also on Q^2, omitted for brevity)

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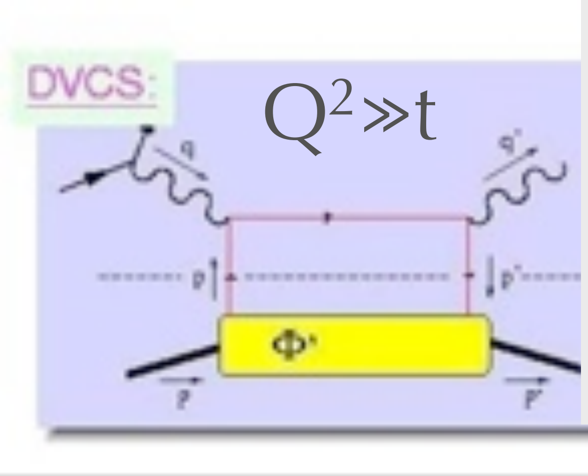

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In the limit $\xi \to 0$ no long. change $P'^+ = P^+$ ($P^+ \gg |\mathbf{P}_\perp|$ IFM)

$t \neq 0$ but change in $\mathbf{P}'_\perp \neq \mathbf{P}_\perp$

$$H(x,0,t=-(\mathbf{P}'-\mathbf{P})^2) = \sum_\lambda \int \frac{dx^-}{2\pi} e^{ixP^+x^-} \langle P^+, \mathbf{P}'_\perp, \lambda |\, \bar{q}(-\frac{x^-}{2}, 0, \mathbf{0})\, \gamma^+\, q(\frac{x^-}{2}, 0, \mathbf{0})\, | P^+, \mathbf{P}_\perp, \lambda \rangle$$

$$q(x,\mathbf{b}) = \int \frac{d\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}}\, H(x,0,t=-\mathbf{q}^2)$$

λ=±1/2 N helicity

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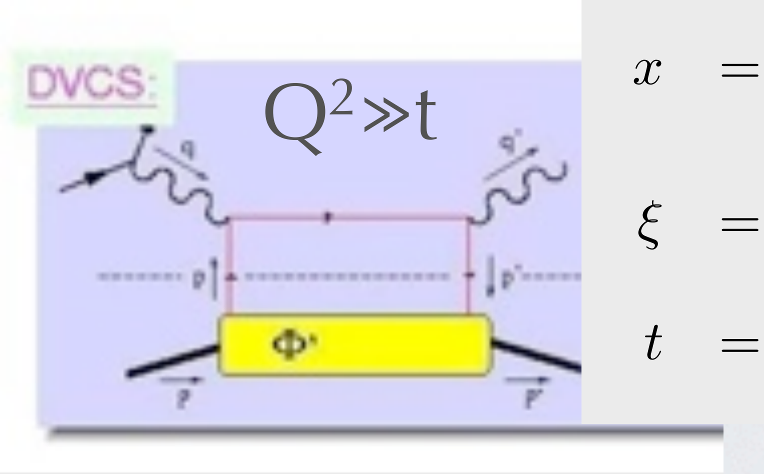

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q(x,**b**) is a density in impact parameter **b** ↔ $\mathbf{q} = \mathbf{P}'_\perp - \mathbf{P}_\perp$

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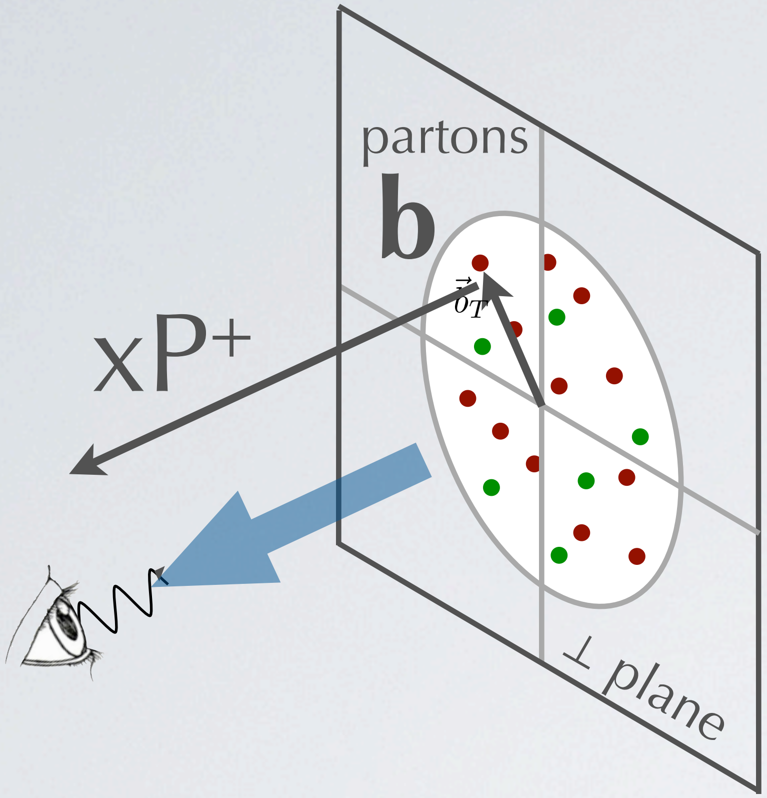

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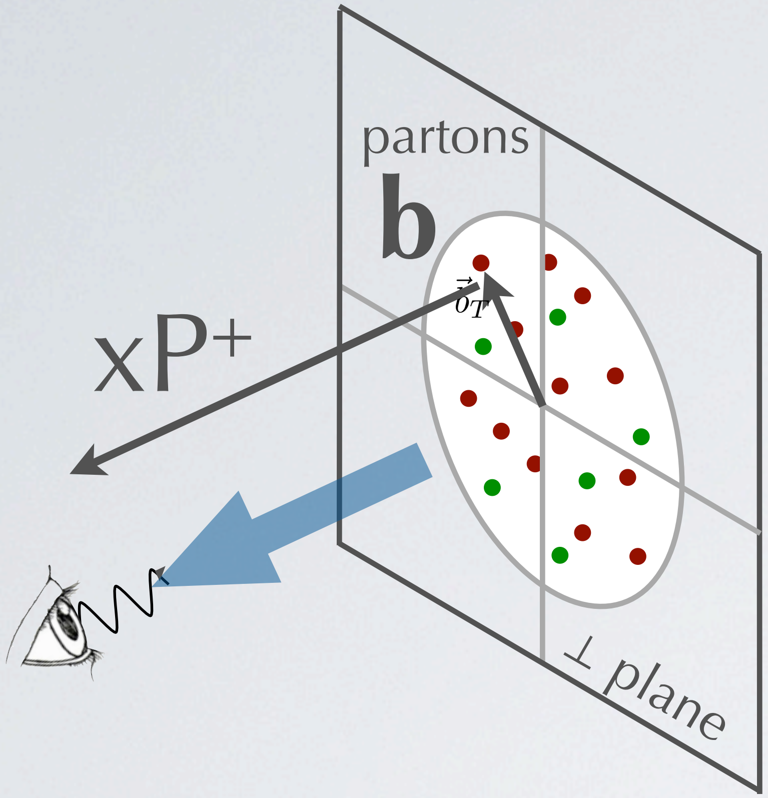

IMF + non-collinear kin.

snapshot of N in ⊥ plane

at each x

⇒ tomography of N

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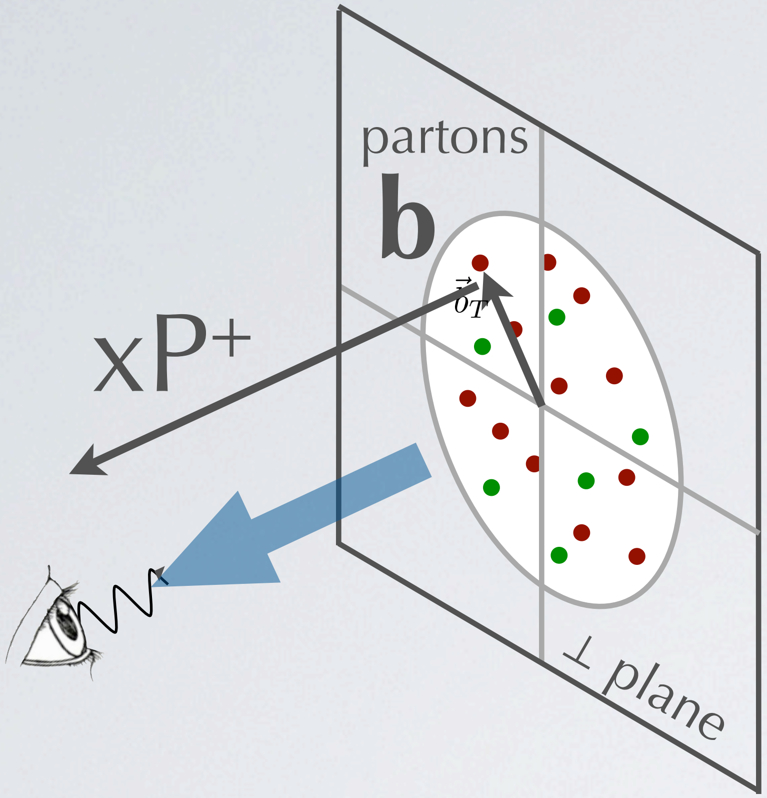

IMF + non-collinear kin.

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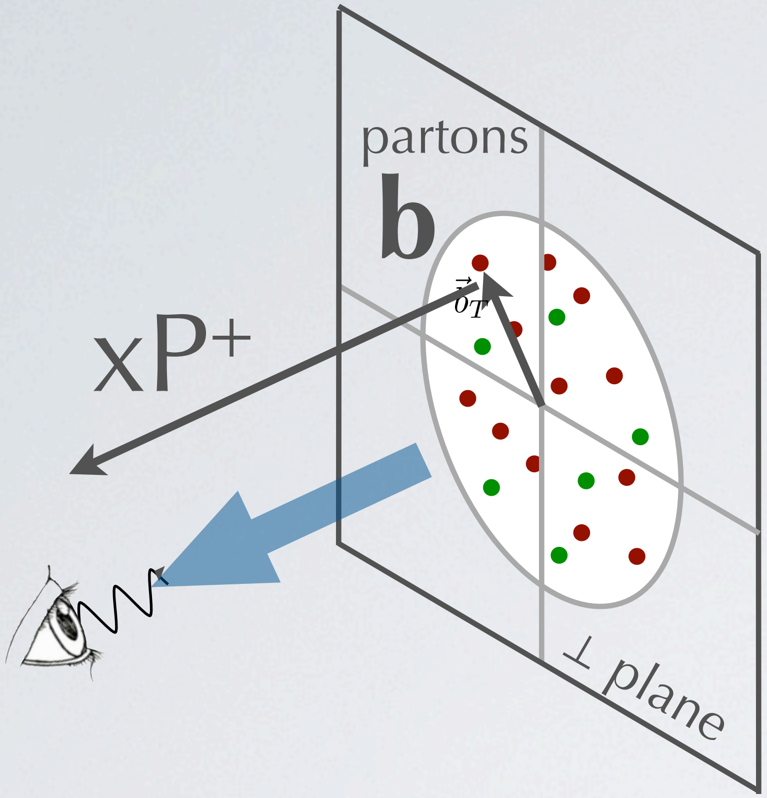
⇒ tomography of N

valid for all x ⇒ integrate in x ⇒ ⊥ charge density ρ(**b**)

$$\begin{aligned}\rho^0(\mathbf{b}) &= \int dx \int \frac{d\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}}\, H(x,0,t=-\mathbf{q}^2) \\ &= \int \frac{d\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}}\, F_1(Q^2=\mathbf{q}^2)\end{aligned}$$

q(x,**b**) is a density in impact parameter **b** ↔ **q**=**P′**$_\perp$-**P**$_\perp$



IMF + non-collinear kin.

snapshot of N in ⊥ plane

at each x

⇒ tomography of N

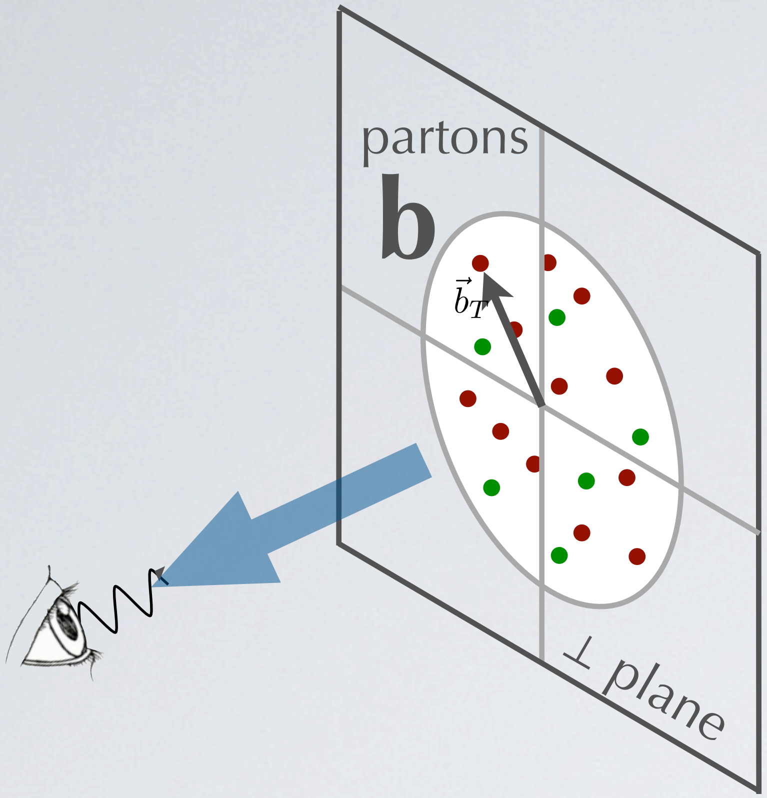
valid for all x ⇒ integrate in x ⇒ ⊥ charge density ρ(**b**)

$$\rho^0(\mathbf{b}) \quad = \quad \int dx \int \frac{d\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}}\, H(x,0,t=-\mathbf{q}^2)$$

$$= \quad \int \frac{d\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}}\, F_1(Q^2=\mathbf{q}^2)$$

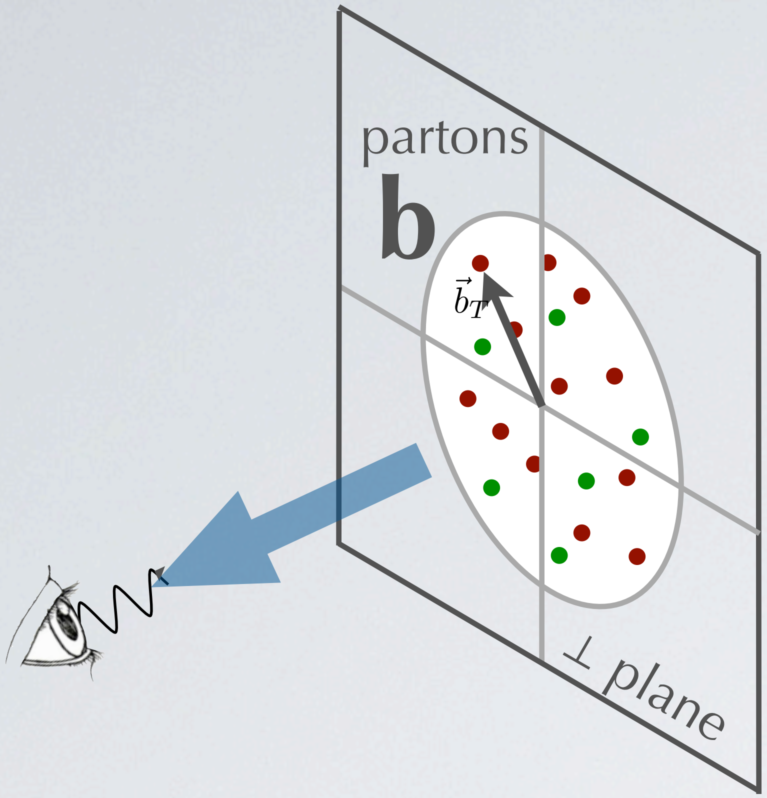
Breit <J^0> ↔ G$_E$ ↔ ρ(r)	IMF <J$^+$> ↔ F$_1$ ↔ ρ^0(**b**)
non relativistic	rigorous

⊥ charge density $\boldsymbol{\rho}^0(\mathbf{b})$



$$\rho^0(\mathbf{b}) \quad = \quad \int \frac{d\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}}\, F_1(Q^2 = \mathbf{q}^2)$$

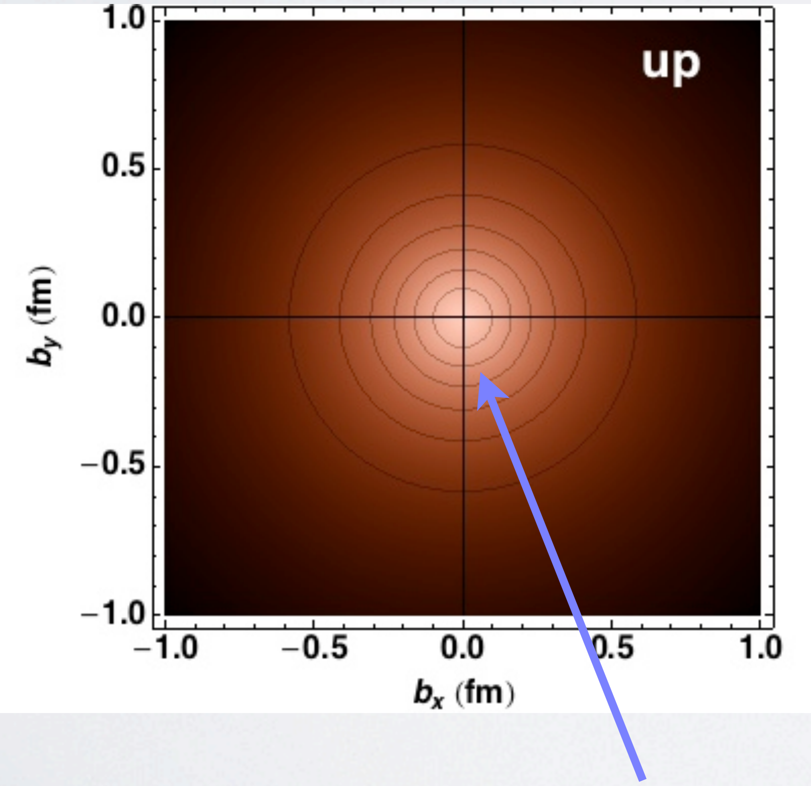
⊥ charge density $\rho^0(\mathbf{b})$

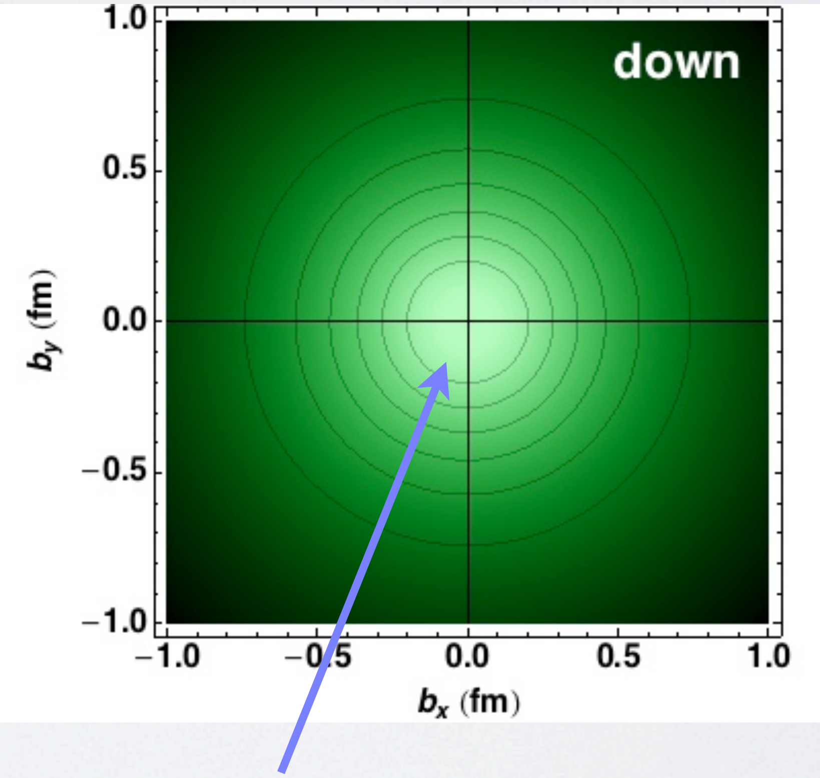


$$\rho^0(\mathbf{b}) \;=\; \int \frac{d\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}}\, F_1(Q^2=\mathbf{q}^2)$$

inside proton

Bacchetta & Contalbrigo, *The proton in 3D*
Il Nuovo Saggiatore **28** (12) n.1,2

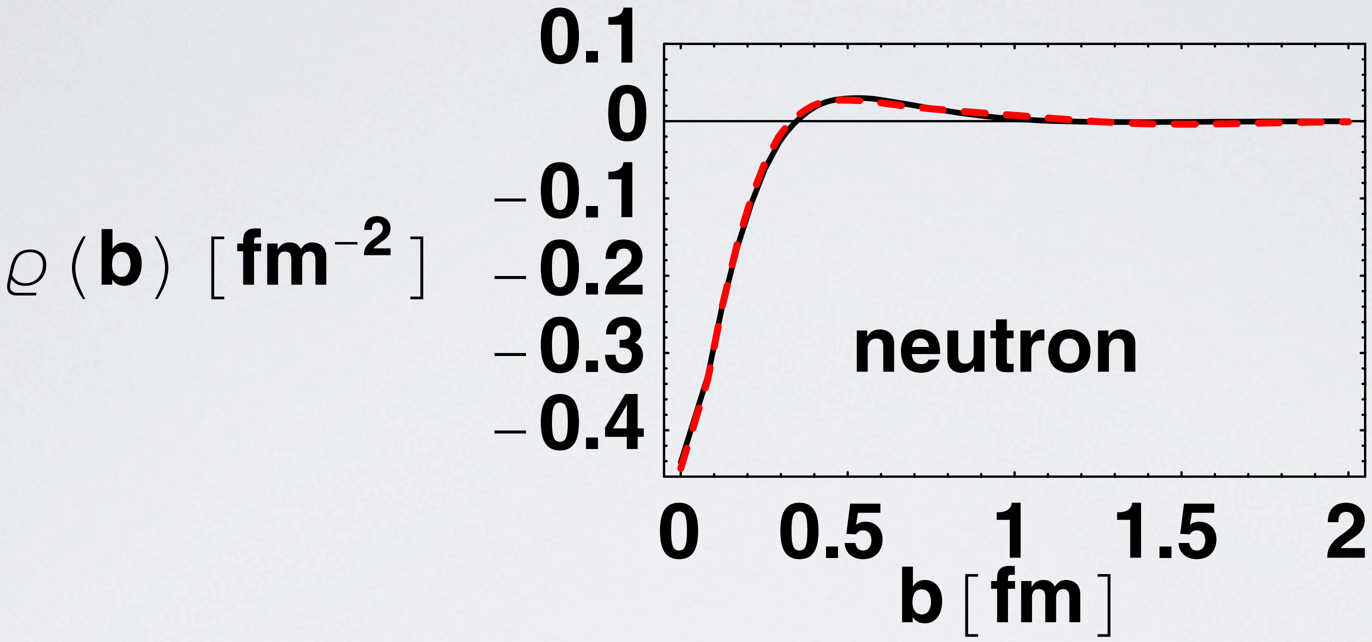




up

down is 30% larger

inside neutron



*G.A. Miller, PRL**99** (07) 112001*

neutron core with charge <0 !
then oscillations because of π cloud

N$^\uparrow$ polarized along S$_x$:

$$\gamma^\mu H(x,0,t) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} E(x,0,t) = \int \frac{dx^-}{2\pi} e^{ixP^+x^-} \langle P^+, \mathbf{P'}_\perp, S_x |\, \bar{q}(-\frac{x^-}{2})\, \gamma^+\, q(\frac{x^-}{2})\, | P^+, \mathbf{P}_\perp, S_x \rangle$$

$$\rho(\mathbf{b}) = \rho^0(\mathbf{b}) + \sin\phi_b \int_0^\infty \frac{d|\mathbf{q}|}{2\pi} \frac{\mathbf{q}^2}{2M} J_1(|\mathbf{q}|b)\, F_2(\mathbf{q}^2)$$

$N^{\uparrow}$ polarized along S_x :

$$\frac{1}{\sqrt{2}}\,\left[\langle P^+,\mathbf{P}_\perp,\uparrow| + \langle P^+,\mathbf{P}_\perp,\downarrow|\right]$$

$$\gamma^\mu H(x,0,t) + \frac{i\sigma^{\mu\nu}q_\nu}{2M}E(x,0,t) = \int \frac{dx^-}{2\pi}e^{ixP^+x^-}\,\langle P^+,\mathbf{P'}_\perp,S_x|\;\bar{q}(-\frac{x^-}{2})\,\gamma^+\,q(\frac{x^-}{2})\;|P^+,\mathbf{P}_\perp,S_x\rangle$$

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$$\frac{1}{\sqrt{2}}\left[|P^+,\mathbf{P}_\perp,\uparrow\rangle + |P^+,\mathbf{P}_\perp,\downarrow\rangle\right]$$

non spin-flip

spin-flip

$$\rho(\mathbf{b}) = \rho^0(\mathbf{b}) + \sin\phi_b \int_0^\infty \frac{d|\mathbf{q}|}{2\pi}\,\frac{\mathbf{q}^2}{2M}\,J_1(|\mathbf{q}|b)\,F_2(\mathbf{q}^2)$$

$\perp$ charge density “deformed” as sinΦ$_b$

with **b** = |**b**|(cosΦ$_b$, sinΦ$_b$)

intensity $\propto$ F$_2$(0)=κ

$N^\uparrow$ polarized along S_x :

$$\frac{1}{\sqrt{2}}\left[\langle P^+,\mathbf{P}_\perp,\uparrow| + \langle P^+,\mathbf{P}_\perp,\downarrow|\right]$$

$$\gamma^\mu H(x,0,t) + \frac{i\sigma^{\mu\nu}q_\nu}{2M}E(x,0,t) = \int \frac{dx^-}{2\pi} e^{ixP^+x^-} \langle P^+,\mathbf{P'}_\perp,S_x|\,\bar{q}(-\frac{x^-}{2})\,\gamma^+\,q(\frac{x^-}{2})\,|P^+,\mathbf{P}_\perp,S_x\rangle$$

$$\frac{1}{\sqrt{2}}\left[|P^+,\mathbf{P}_\perp,\uparrow\rangle + |P^+,\mathbf{P}_\perp,\downarrow\rangle\right]$$

non spin-flip

spin-flip

$$\rho(\mathbf{b}) = \rho^0(\mathbf{b}) + \sin\phi_b \int_0^\infty \frac{d|\mathbf{q}|}{2\pi}\,\frac{\mathbf{q}^2}{2M}\,J_1(|\mathbf{q}|b)\,F_2(\mathbf{q}^2)$$

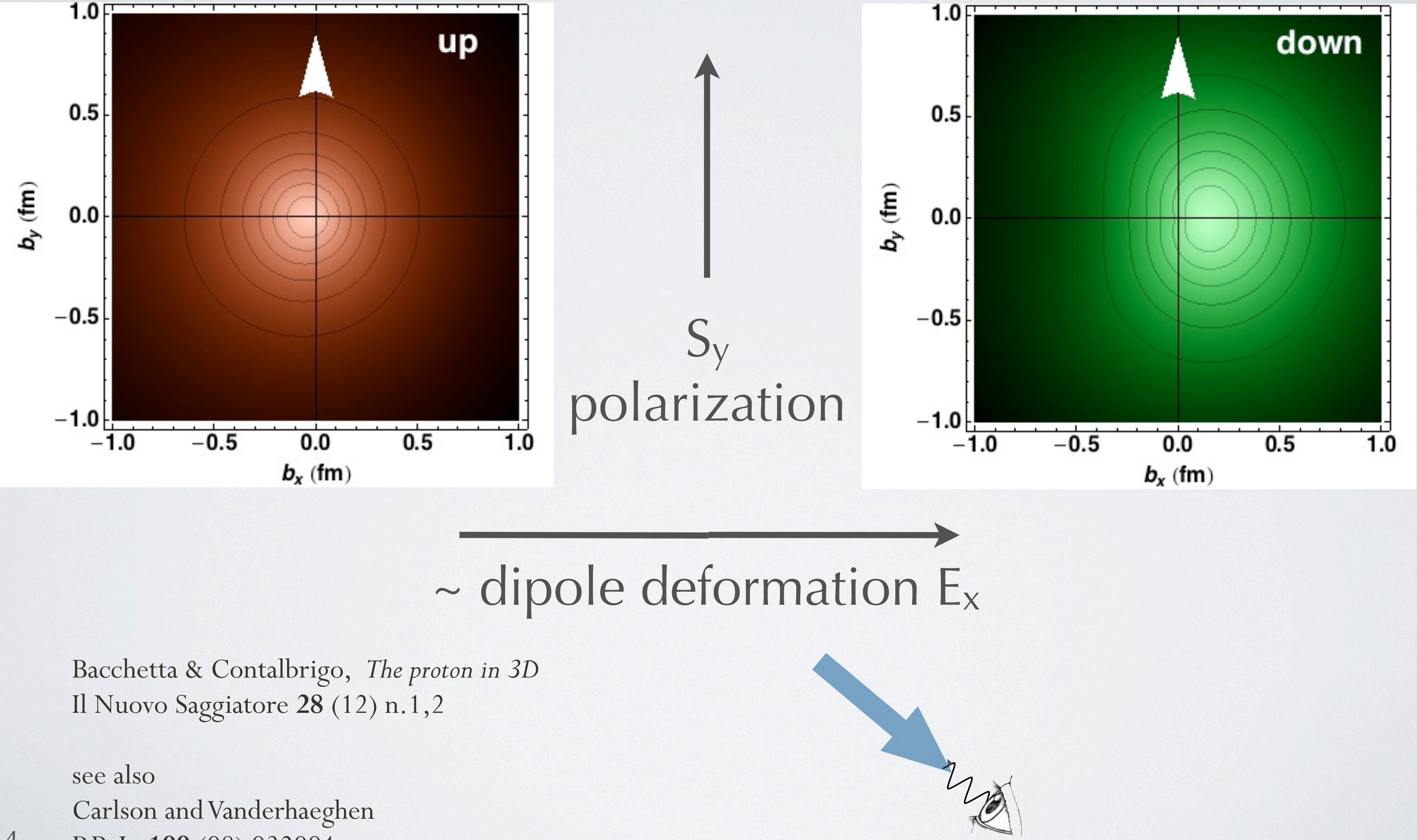
⊥ charge density “deformed” as $\sin\Phi_b$

with $\mathbf{b} = |\mathbf{b}|(\cos\Phi_b, \sin\Phi_b)$

intensity $\propto F_2(0)=\kappa$

polarization S_x ⇒ dipole E_y

Flavor separation of "deformation" proton

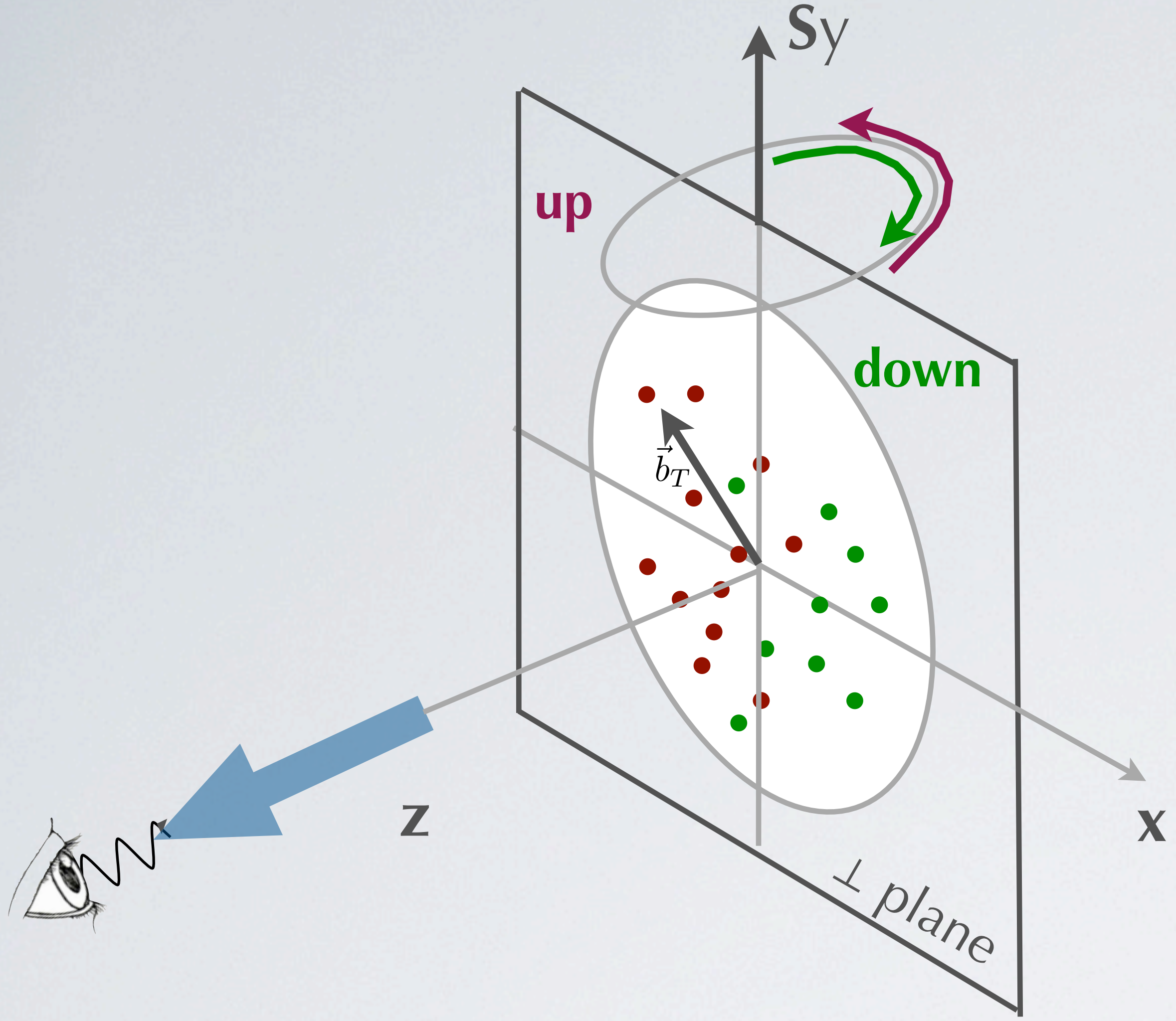


S_y
polarization

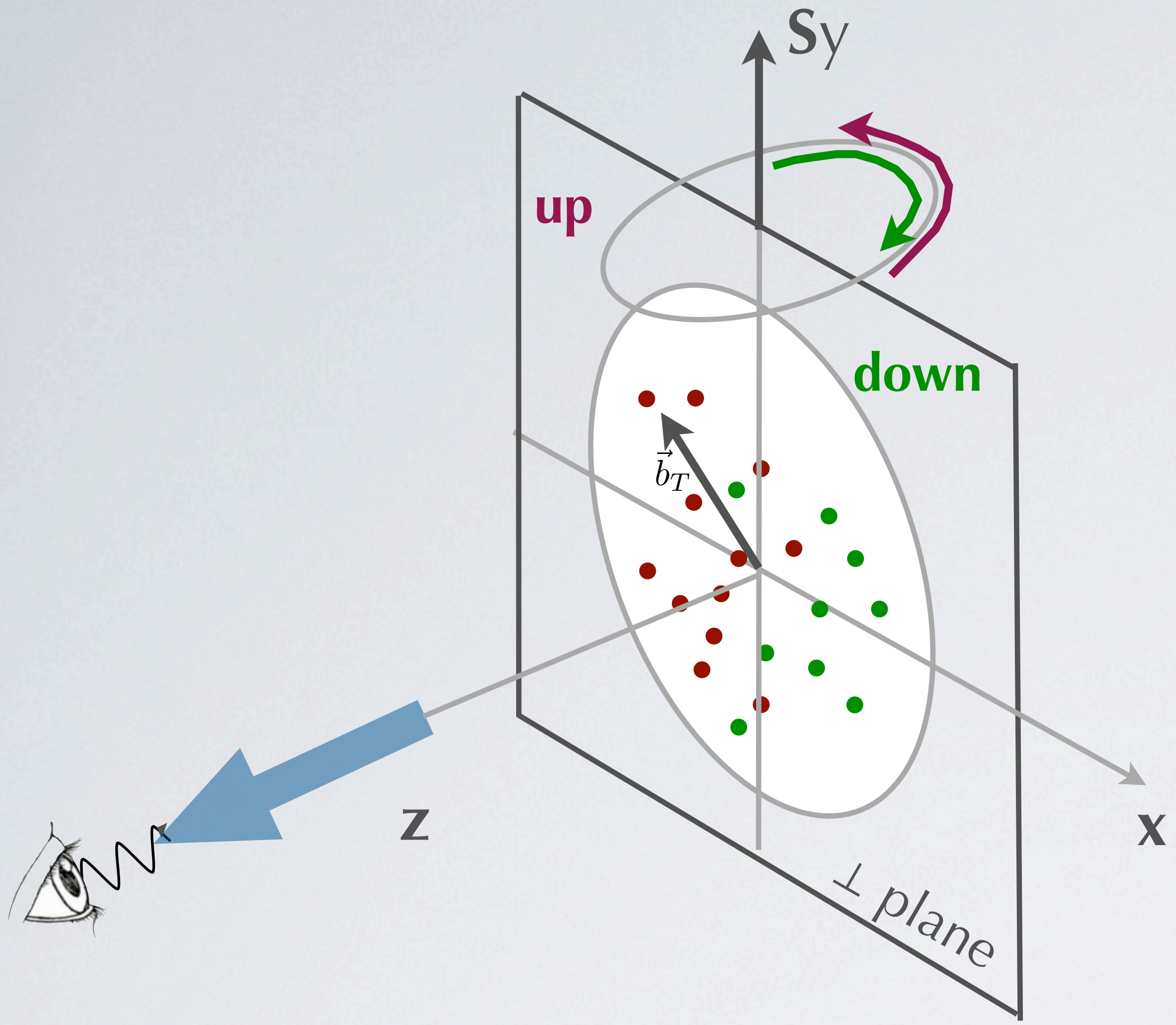
~ dipole deformation E_x

Bacchetta & Contalbrigo, *The proton in 3D*
Il Nuovo Saggiatore **28** (12) n.1,2

see also
Carlson and Vanderhaeghen
P.R.L. **100** (08) 032004



N polarization along **y**
gives a twist along **x**
to parton distributions
because of their
Orbital Angular Momentum
(OAM)

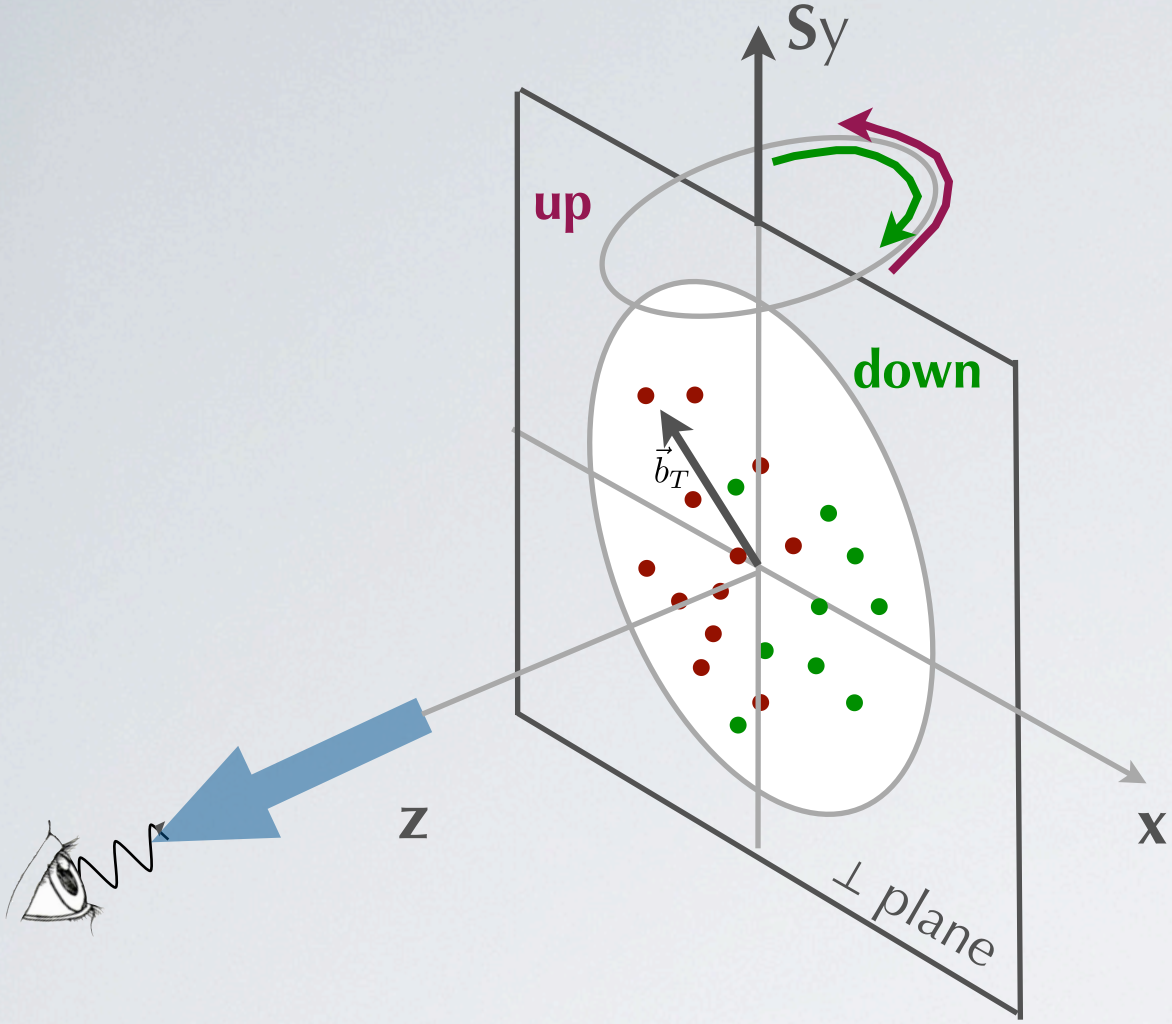


N polarization along **y** gives a twist along **x** to parton distributions because of their **Orbital Angular Momentum (OAM)**

Can we access parton **OAM** from GPD ?

Not yet

issues with unique gauge-inv. definition (common problem to gauge field theories)



N polarization along **y**
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Orbital Angular Momentum
(OAM)

Can we access parton **OAM** from GPD ?

Not yet

issues with unique
gauge-inv. definition
(common problem to
gauge field theories)

Can we access parton **total J** from GPD ?

Yes

Ji's sum rule

*X. Ji, PRL**78** (97)*

parton J

GPD

$$J^q(Q^2) = \frac{1}{2}\int_0^1 dx\, x\left(H^q(x,0,0;Q^2) + E^q(x,0,0;Q^2)\right)$$

Ji's sum rule

*X. Ji, PRL**78** (97)*

parton J

GPD

$$J^q(Q^2) = \frac{1}{2}\int_0^1 dx\, x\left(H^q(x,0,0;Q^2) + E^q(x,0,0;Q^2)\right)$$

↙

parton momentum
distribution $f_1{}^q(x)$
well known

Ji's sum rule

X. Ji, PRL ***78*** *(97)*

parton J

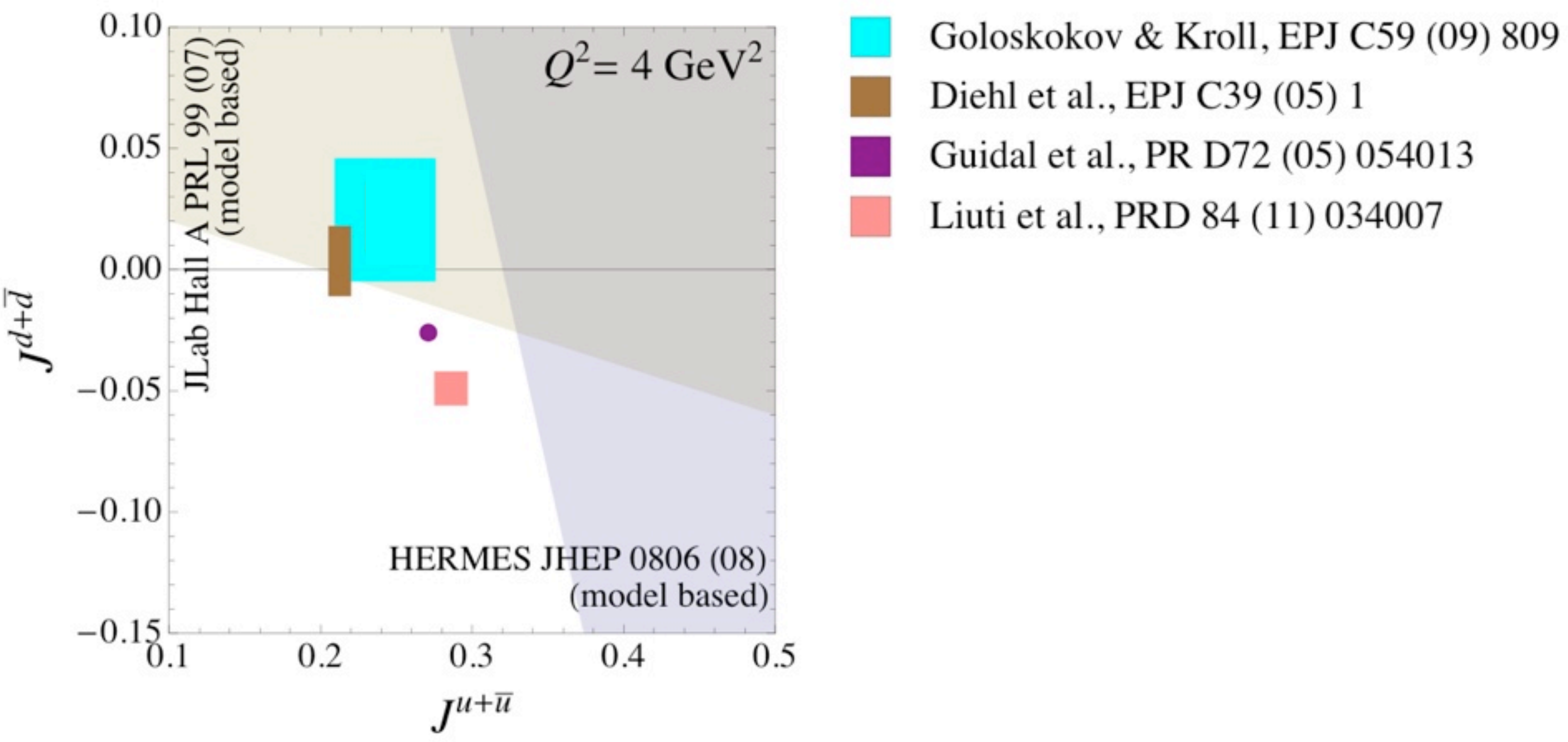
GPD

$$J^q(Q^2) = \frac{1}{2}\int_0^1 dx\, x \left(H^q(x,0,0;Q^2) + E^q(x,0,0;Q^2) \right)$$

parton momentum
distribution $f_1{}^q(x)$
well known

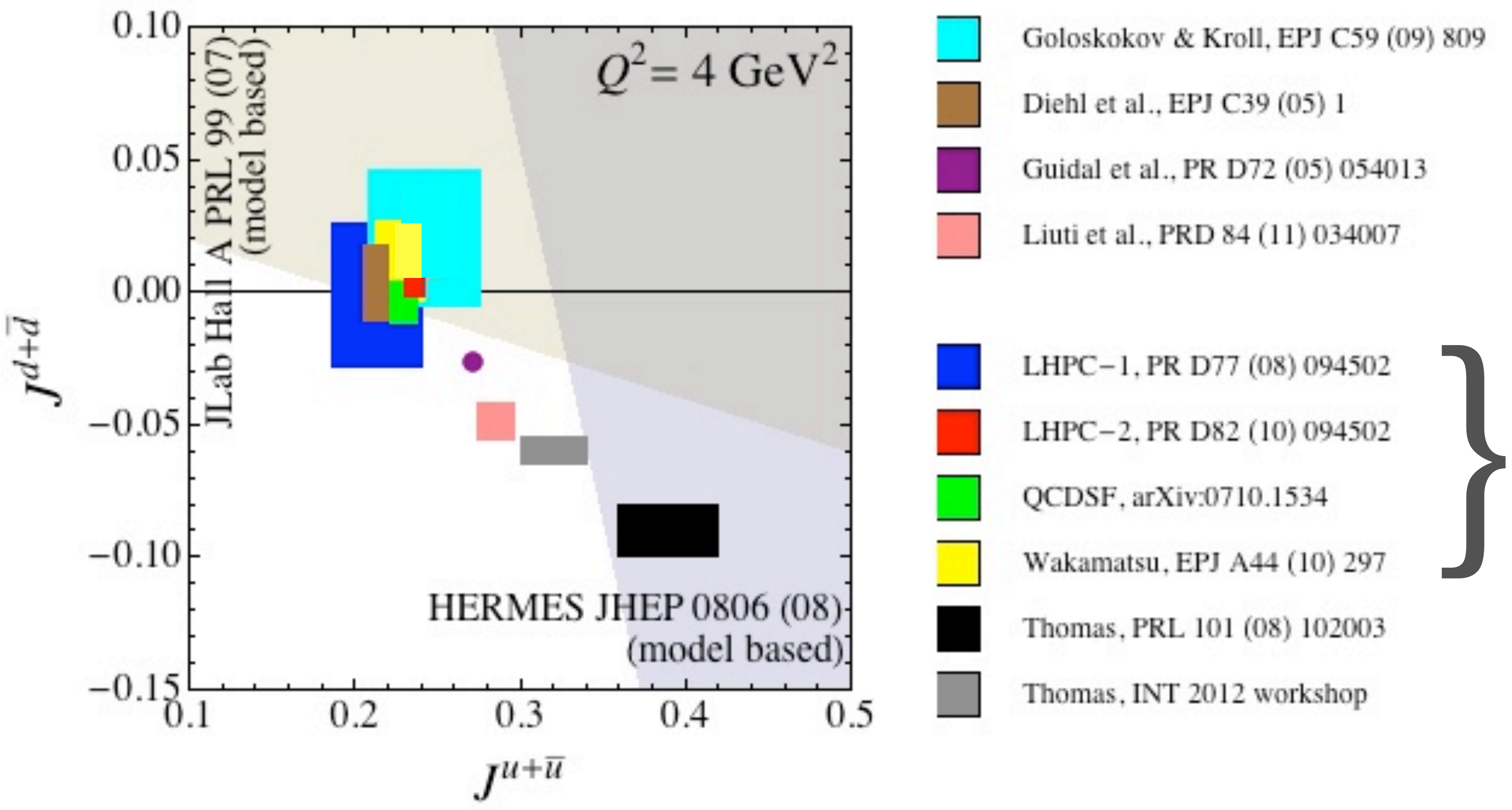
not directly accessible
(E^q → N spin flip)
need model
extrapolation

results on **parton J** from (model) parametrizations of **GPD**



Bacchetta, Radici, arXiv:1206.2565 [hep-ph]
"Physics Opportunities with the 12 GeV Upgrade at Jefferson Lab", arXiv:1208.1244 [hep-ex]

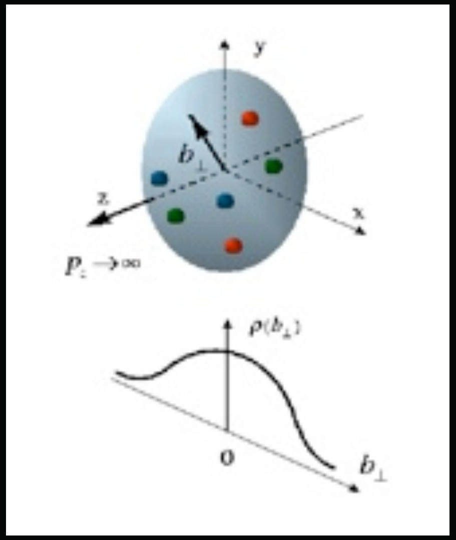
comparison with lattice QCD



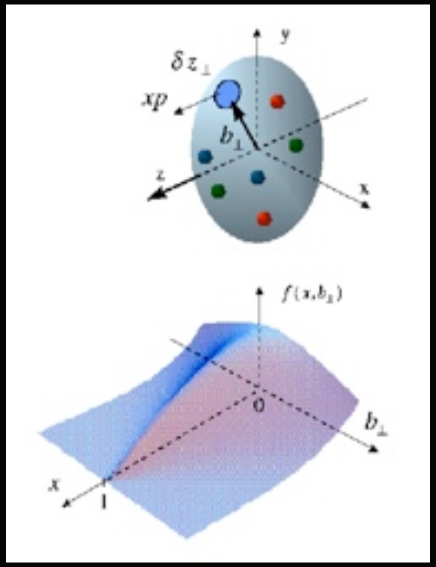
lattice

Bacchetta, Radici, arXiv:1206.2565 [hep-ph]
"Physics Opportunities with the 12 GeV Upgrade at Jefferson Lab", arXiv:1208.1244 [hep-ex]

From Form Factors to Generalized Form Factors
elastic scattering of GPD in IMF

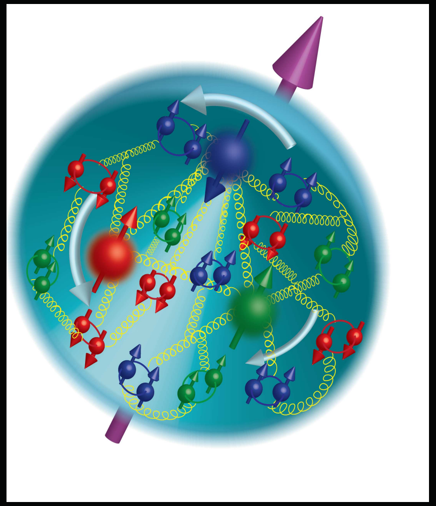


increase the number of investigated dimensions in the structure of N



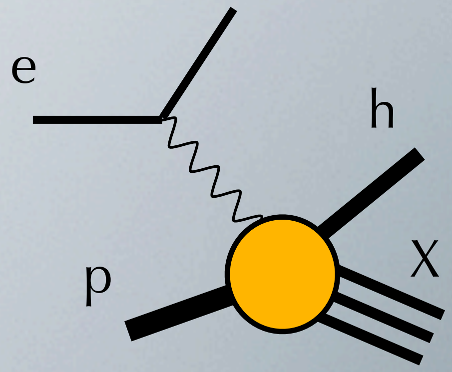
<u>First gain</u>:

partonic decomposition of N spin
in terms of
GPD in the collinear limit



density $q(x,\mathbf{b})$ → $q(x,\mathbf{k}_\perp)$

density in k space:
Transv. Mom. Distribution
(TMD)



Semi-inclusive DIS
(SIDIS)

density $q(x,\mathbf{b}) \rightarrow q(x,\mathbf{k}_\perp)$

density in k space:
Transv. Mom. Distribution
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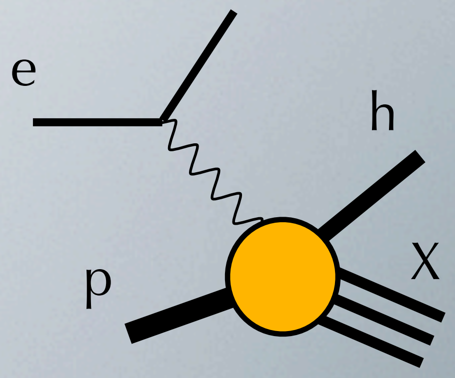


Semi-inclusive DIS
(SIDIS)

SIDIS cross section @leading twist: 8 TMD

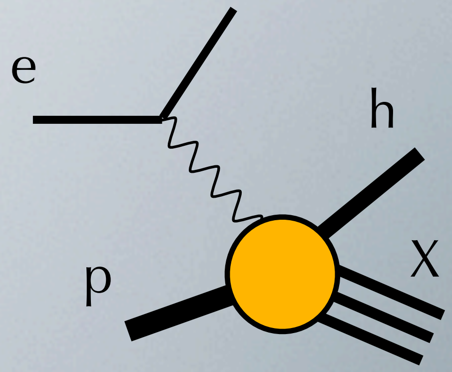
quark pol.

		U	L	T
nucleon pol.	U	f_1		$h_1^\perp$
	L		g_{1L}	$h_{1L}^\perp$
	T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

Twist-2 TMDs

density $q(x,\mathbf{b}) \rightarrow q(x,\mathbf{k}_\perp)$

density in k space:
Transv. Mom. Distribution
(TMD)



Semi-inclusive DIS
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nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

** Anselmino et al., N.P.Proc.Suppl. **191** (09) 98*
** Bacchetta, Courtoy, Radici, JHEP **03** (13) 119*

$$\int d\mathbf{k}_\perp \, \mathrm{TMD}(x,\mathbf{k}_\perp) \rightarrow \mathbf{PDF}(x)$$

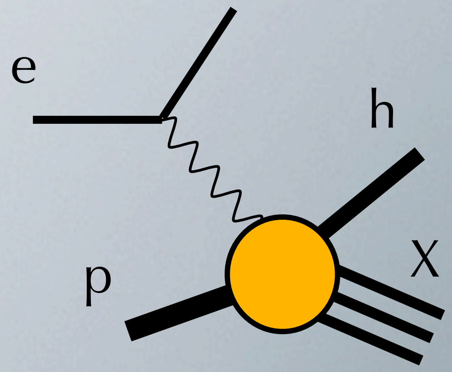
We know the integrated PDF (almost) very well.
We know the TMD still poorly.

density $q(x,\mathbf{b})$ → $q(x,\mathbf{k}_\perp)$

density in k space:

Transv. Mom. Distribution

(TMD)



Semi-inclusive DIS (SIDIS)

SIDIS cross section @leading twist: 8 TMD

Unpolarized distribution

Signori, Bacchetta, Radici, Schnell, arXiv:1309.3507 [hep-ph]

quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

** Anselmino et al., N.P.Proc.Suppl. **191** (09) 98*
** Bacchetta, Courtoy, Radici, JHEP **03** (13) 119*

$\int d\mathbf{k}_\perp$ TMD$(x,\mathbf{k}_\perp)$ → **PDF**(x)

We know the integrated PDF (almost) very well.
We know the TMD still poorly.

quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

quark pol.

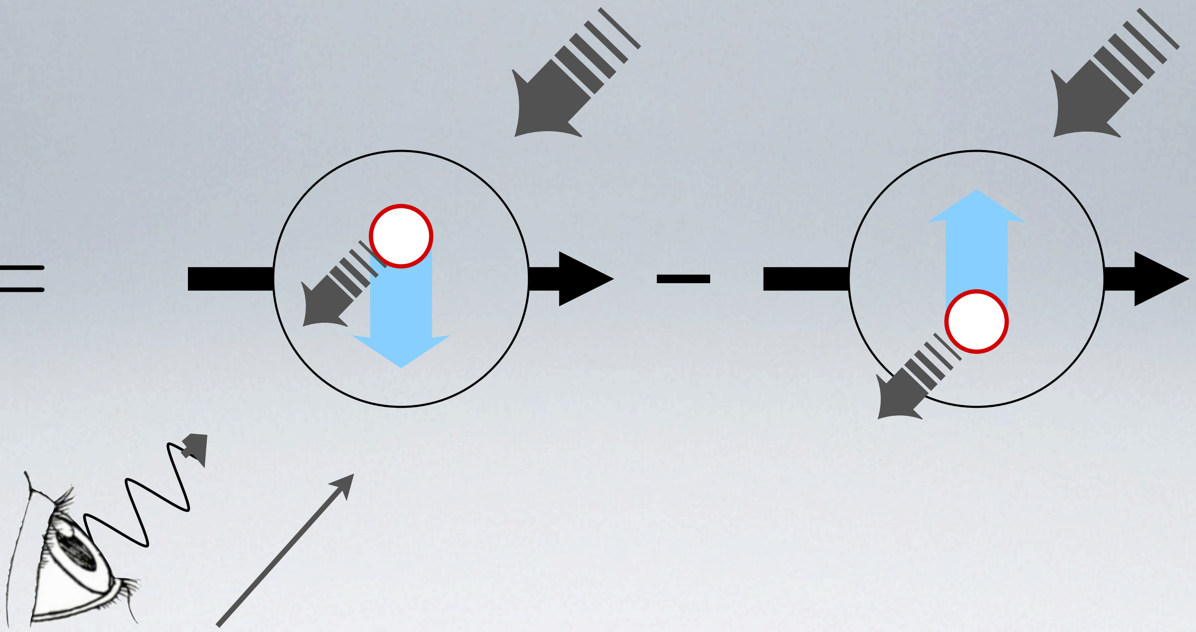
nucleon pol.

	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1, $h_{1T}^{\perp}$

Twist-2 TMDs

Sivers function

$f_{1T}^{\perp} =$



		quark pol.		
		U	L	T
nucleon pol.	U	f_1		$h_1^{\perp}$
	L		g_{1L}	$h_{1L}^{\perp}$
	T	$f_{1T}^{\perp}$	g_{1T}	h_1, $h_{1T}^{\perp}$

Twist-2 TMDs

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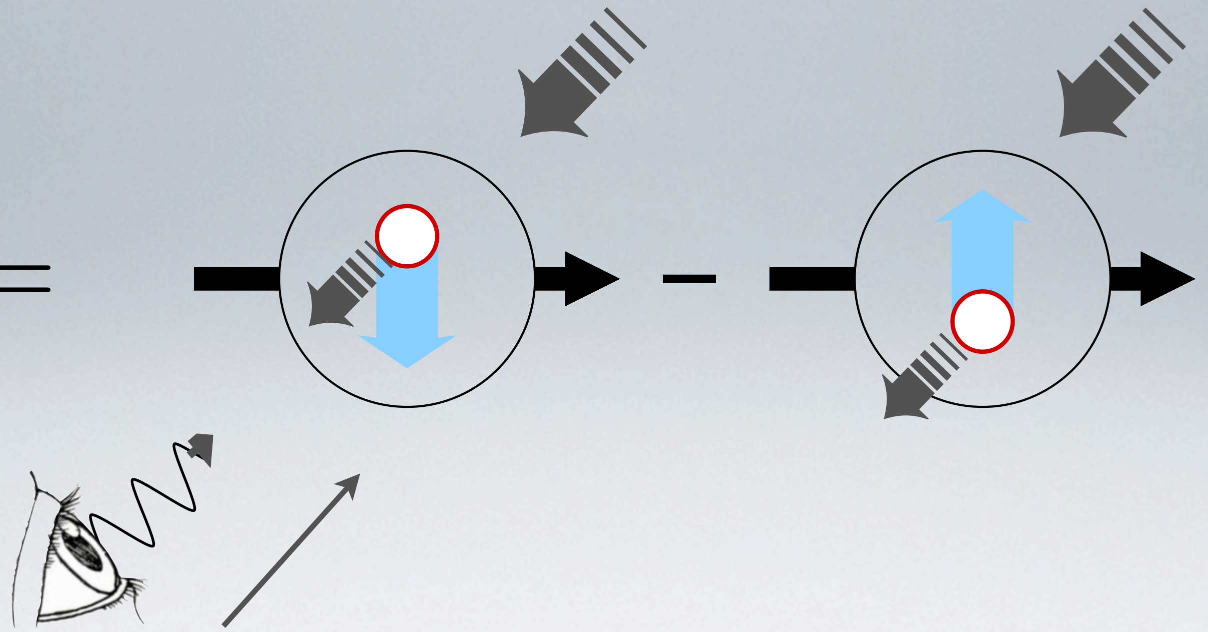
$f_{1T}^{\perp} =$

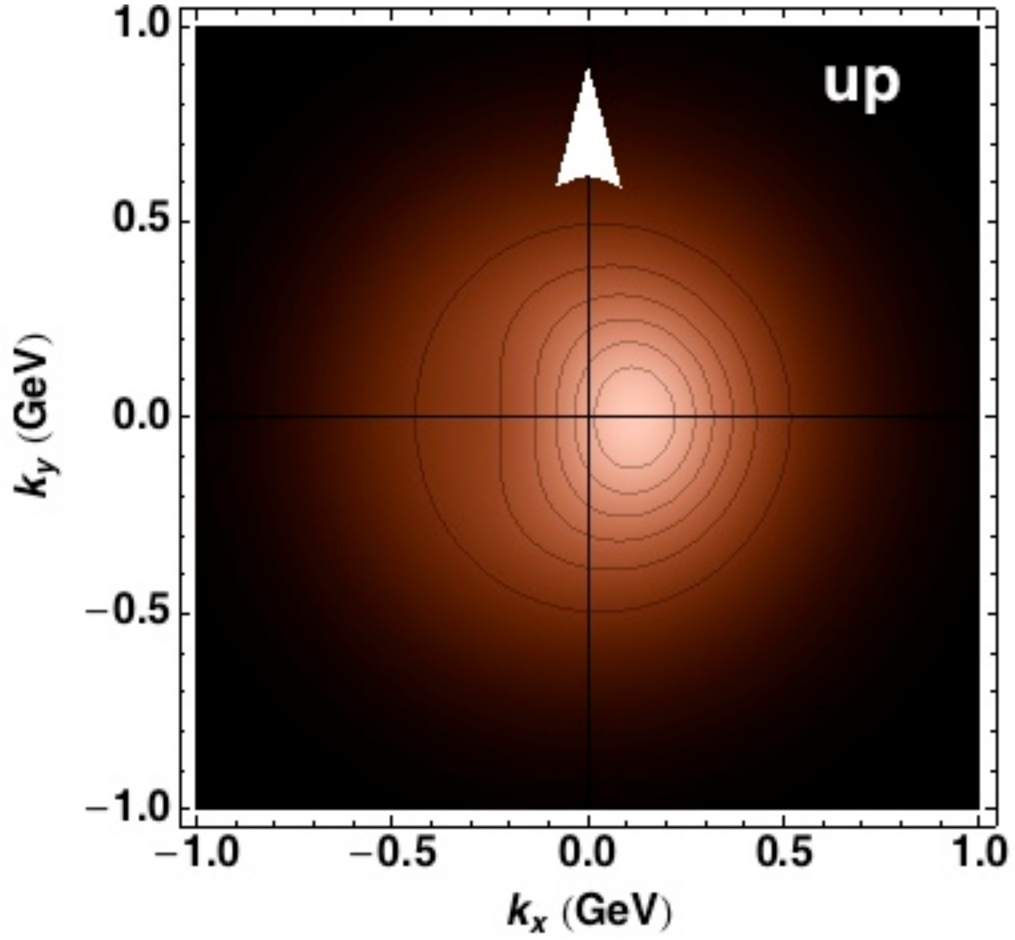


distortion of quark distribution
because of $N^{\uparrow}$ polarization

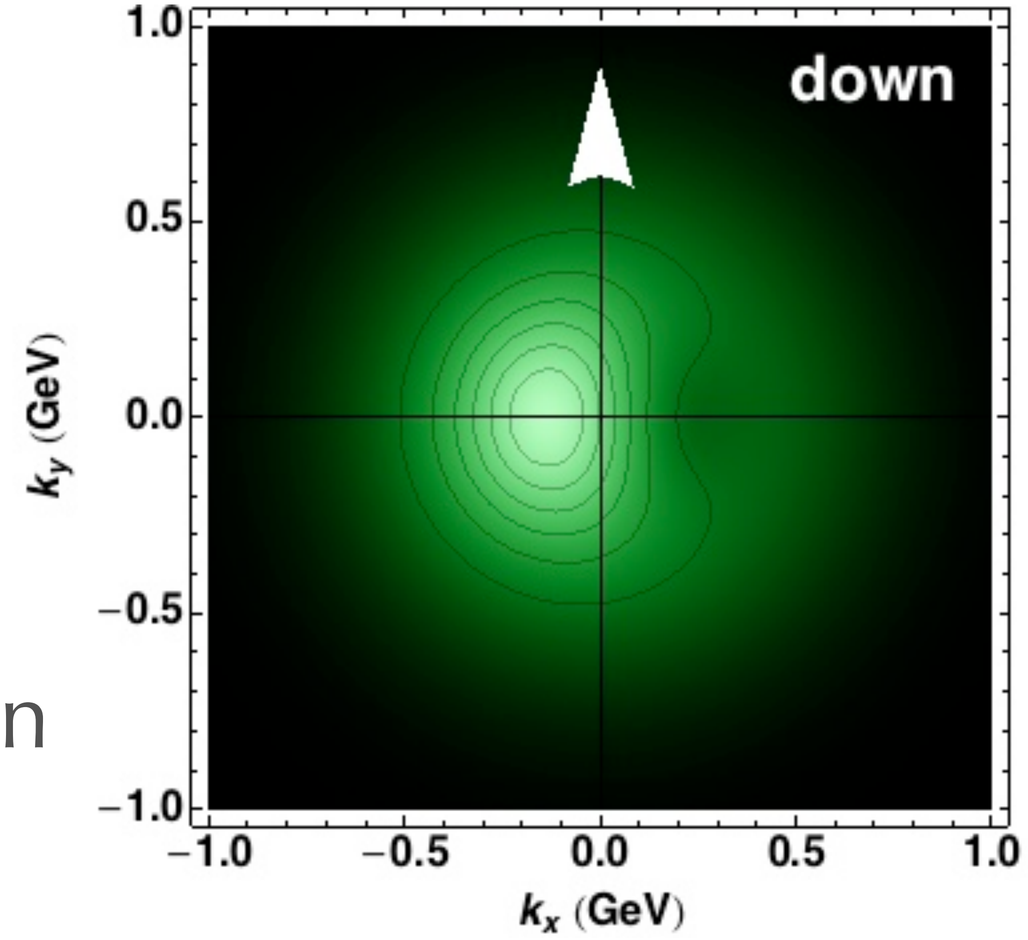
*Sivers, PRD **41** (90) 83*

distribution of unpolarized q in polarized $p^\uparrow$

$$f_{q/p^\uparrow}(x, \mathbf{k}_\perp) = f_1^q(x, \mathbf{k}_\perp^2) - f_{1T}^{\perp q}(x, \mathbf{k}_\perp^2)\, \frac{(\hat{\mathbf{P}} \times \mathbf{k}_\perp) \cdot \mathbf{S}}{M}$$



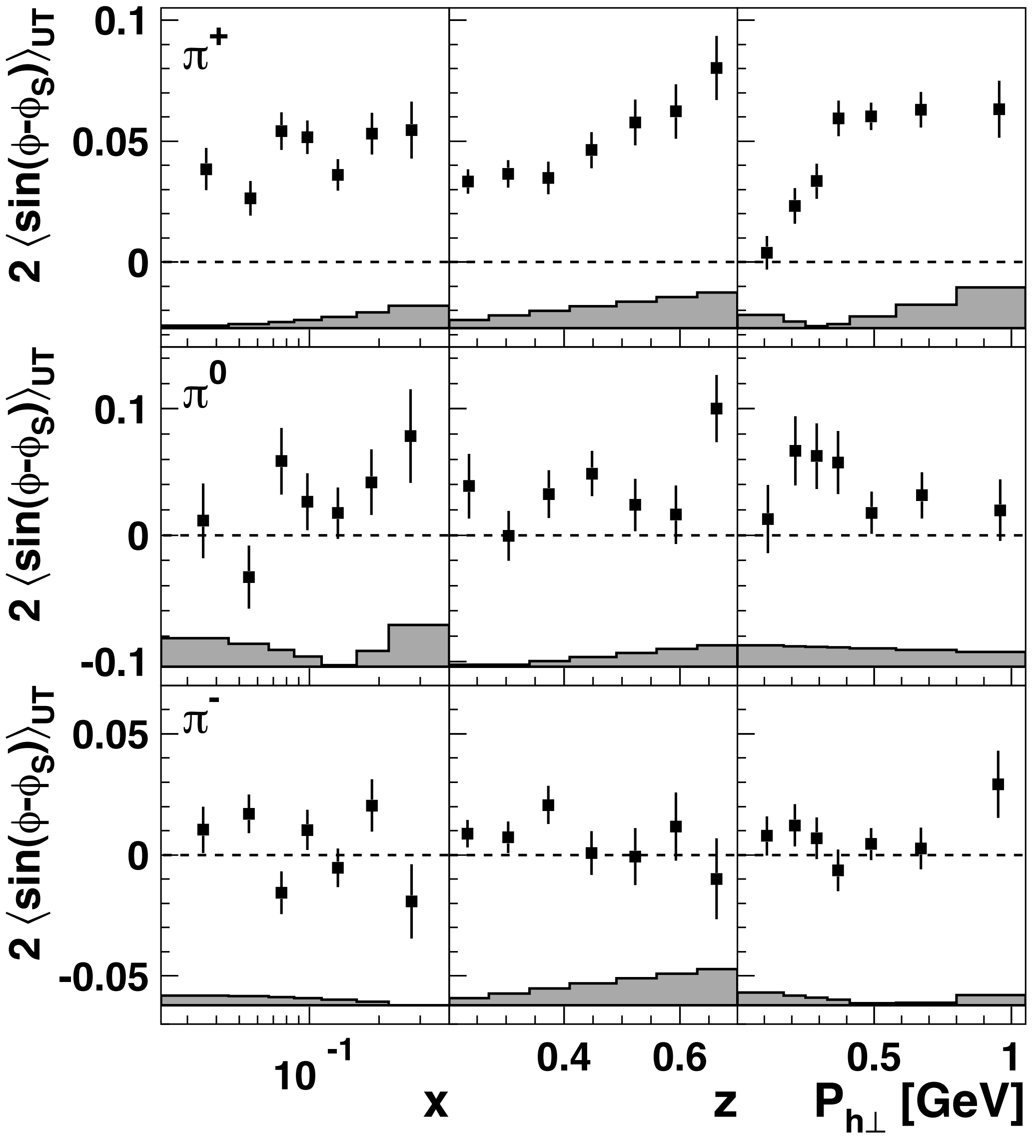
S_y
polarization



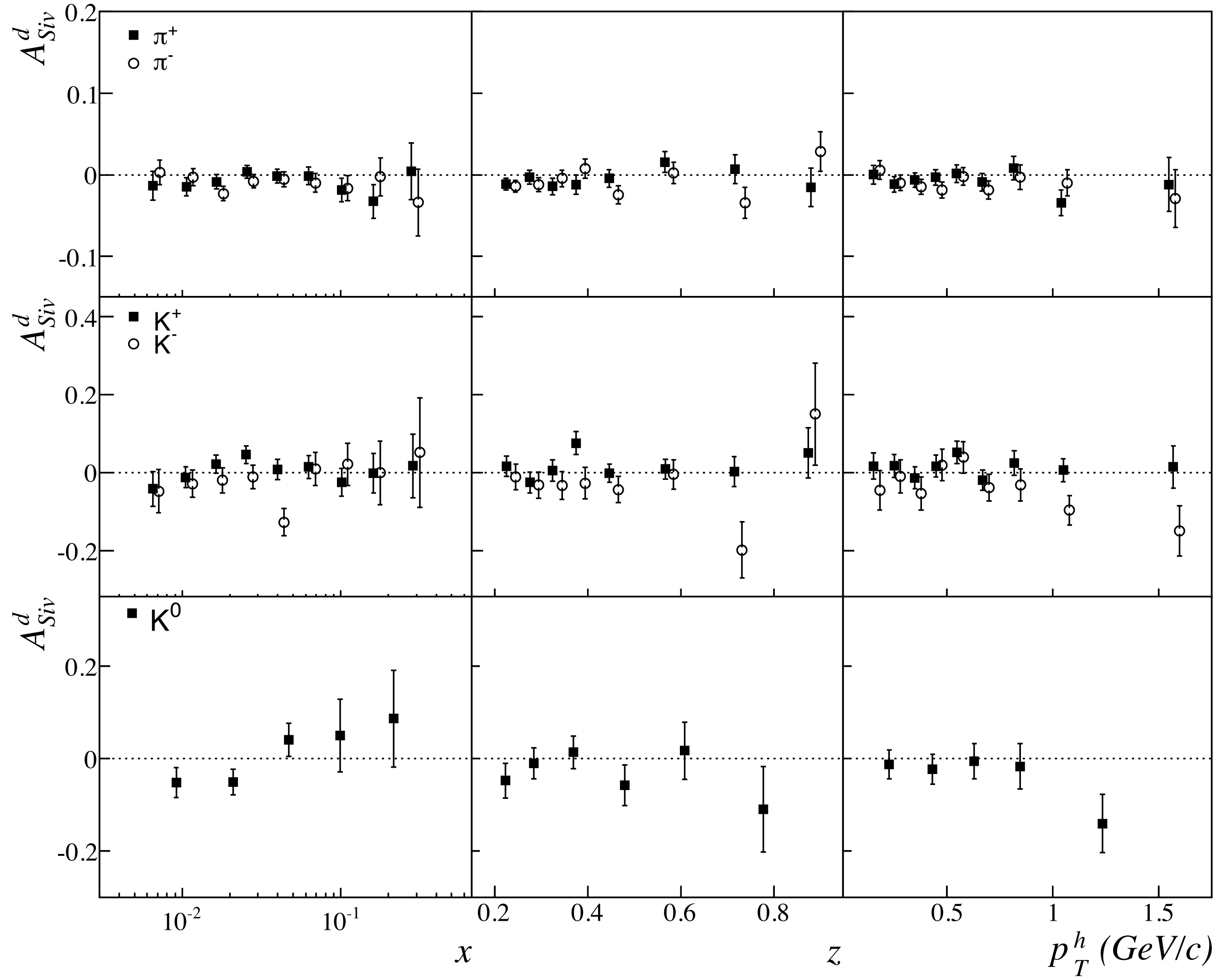
deformation induced by Sivers function

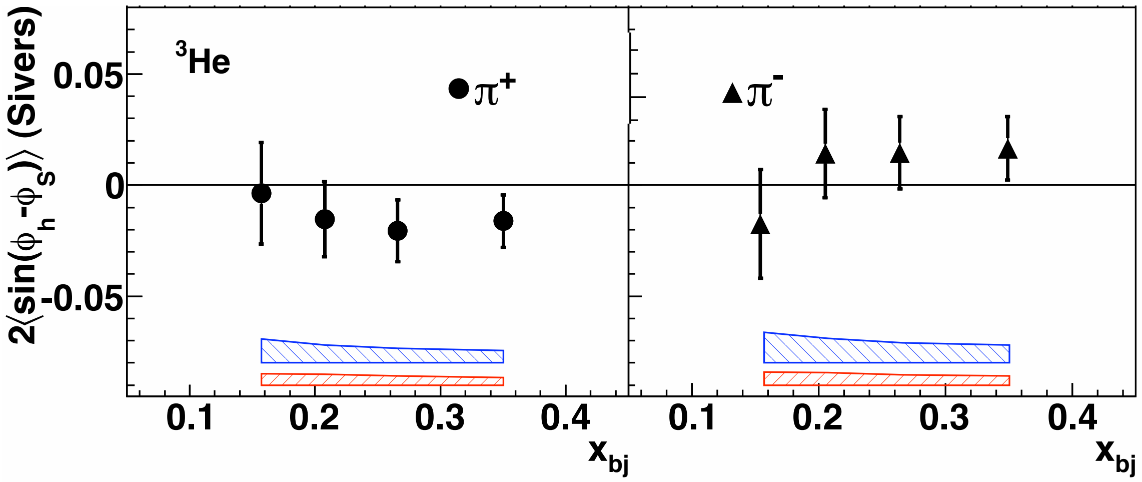
Bacchetta & Contalbrigo,
The proton in 3D
Il Nuovo Saggiatore **28** (12) n.1,2

*PRL**103** (09) 152002*



*PL B**673** (09) 127*





Jefferson Lab

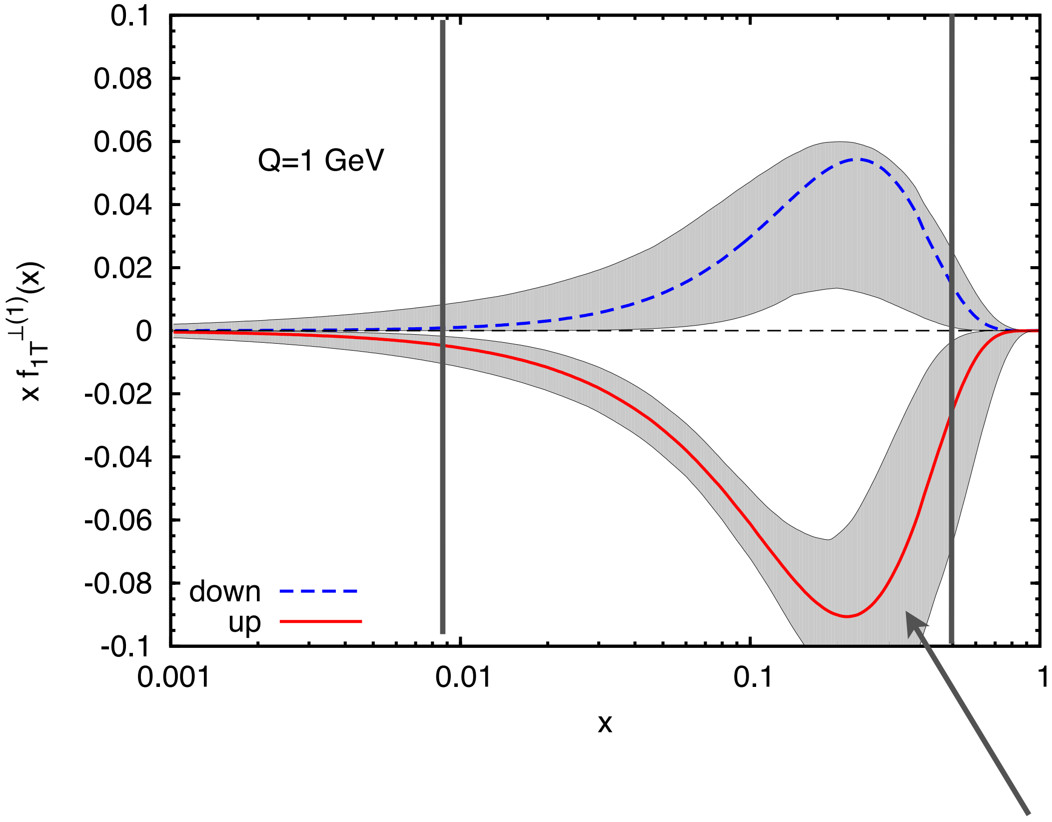
Hall A

*PRL**107** (11) 072003*

Sivers effect has been measured in $p^{\uparrow}(e,e'\pi)$!

Sivers function has been extracted

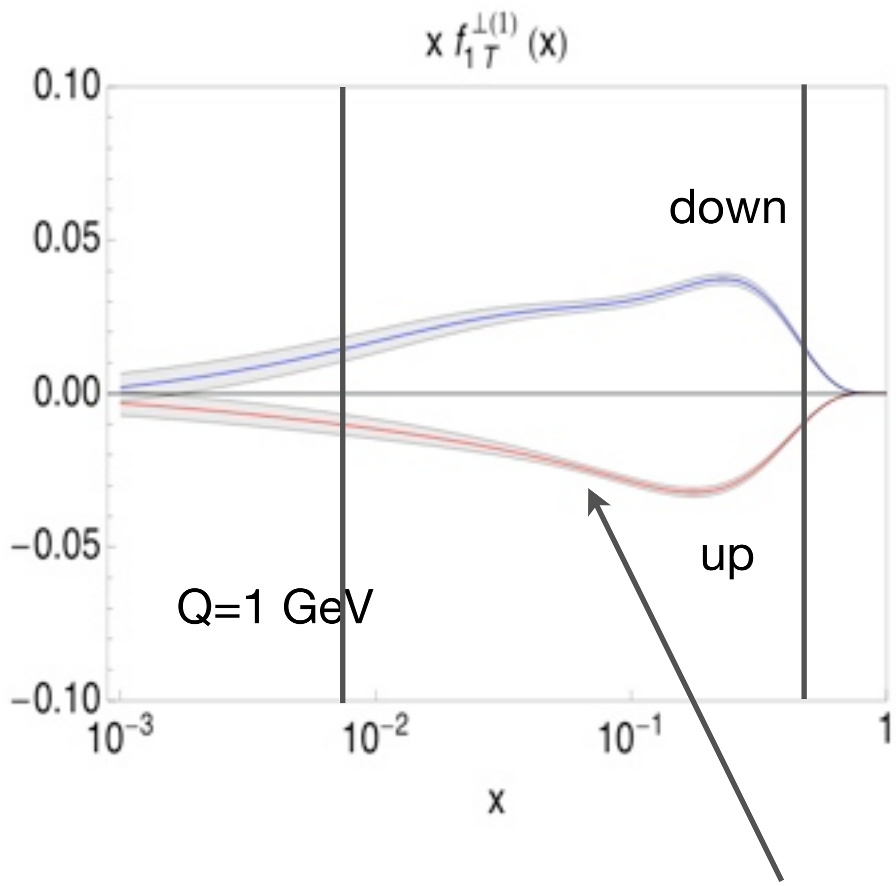
Torino 2012 update



$\Delta\chi^2\simeq15$

adapted by Stefano Melis from
*Anselmino et al., PRD**86** (12) 014028*
*older extraction: E.P.J. **A39** (09)*

Pavia 2011



$\Delta\chi^2=1$

*Bacchetta, Radici, PRL**107** (11)*

Reason #1 for studying the Sivers function

Let's recall the Ji's sum rule

non spin-flip spin-flip in N

$$J^q(Q^2) = \frac{1}{2}\int_0^1 dx\, x \Big(H^q(x,0,0;Q^2) + E^q(x,0,0;Q^2) \Big)$$

Reason #1 for studying the Sivers function

Let's recall the $\boxed{\text{Ji's sum rule}}$

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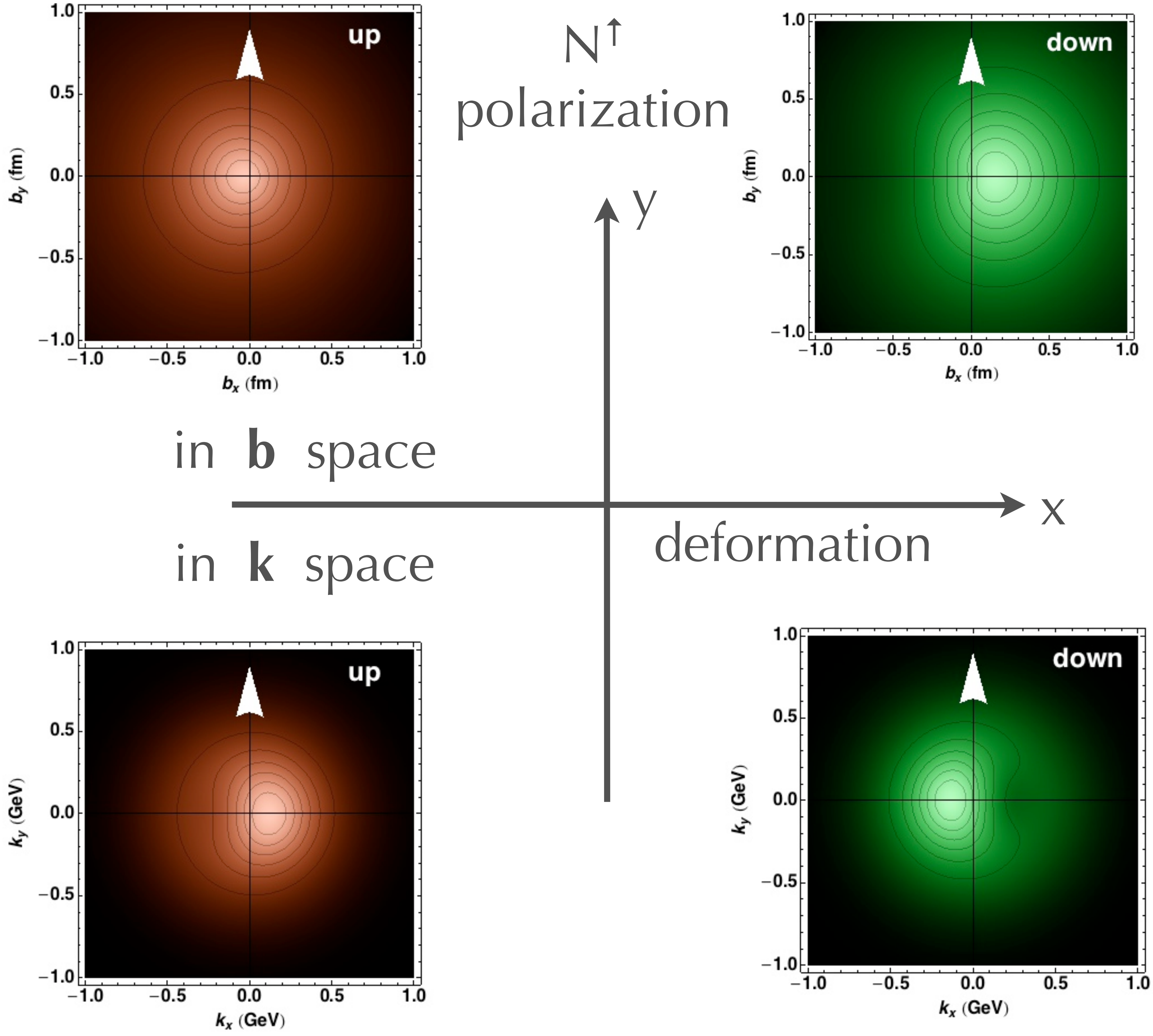
↙ ↘

unpolarized PDF $f_1^q(x)$
well kown PDF

not directly accessible

GPD E



$N^{\uparrow}$
polarization

in **b** space

in **k** space

deformation

TMD $f_{1T}^{\perp}$
Sivers
effect

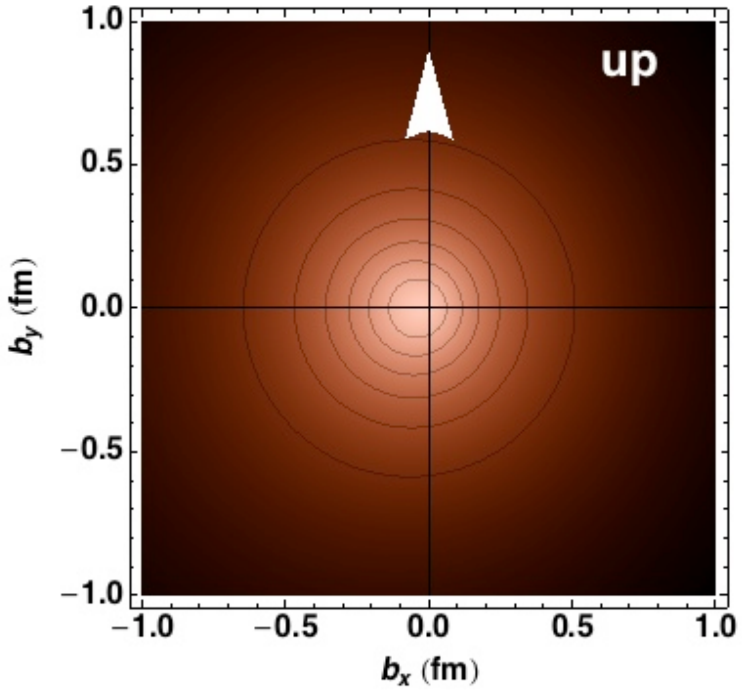
GPD E

lensing function

Burkardt, P.R.D **66** (02) 114005

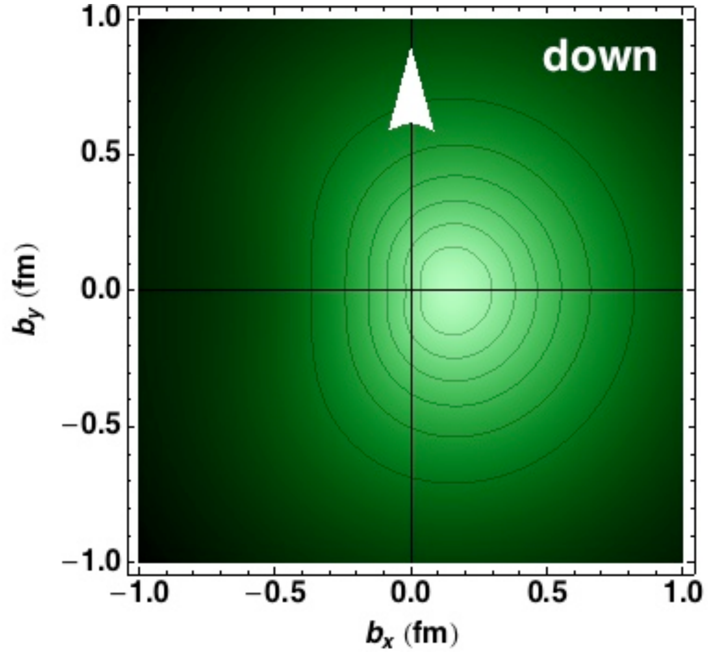
TMD $f_{1T}^{\perp}$

Sivers effect



$N^{\uparrow}$
polarization

y

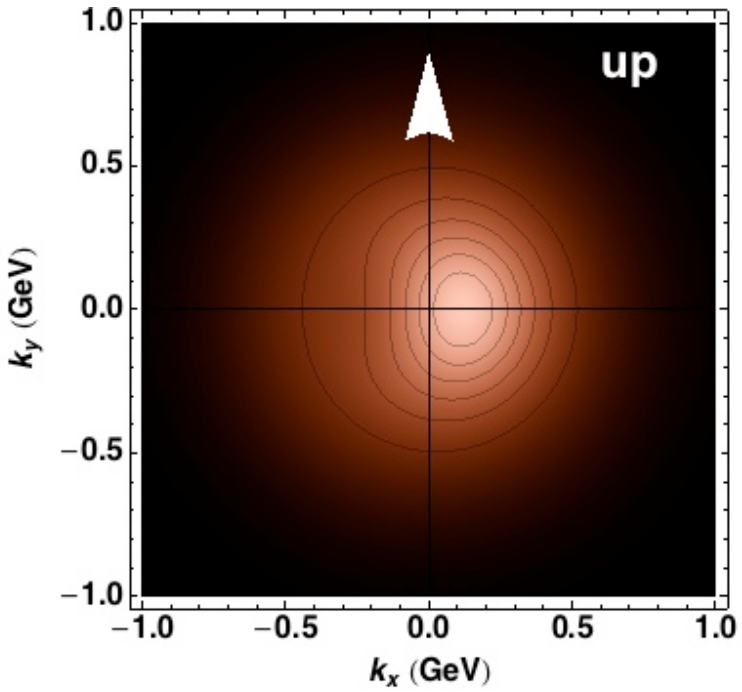


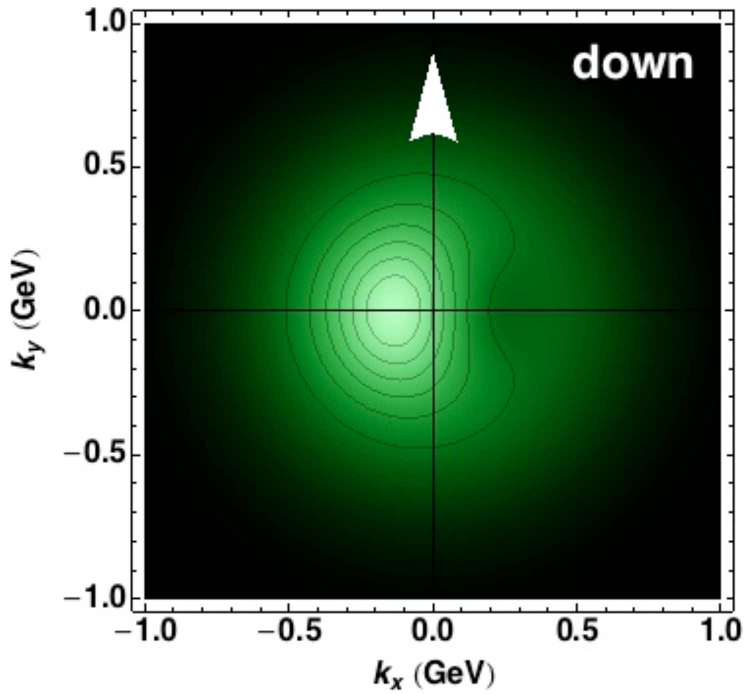
in **b** space

x

deformation

in **k** space





Ji's sum rule

$$J^q(Q^2) = \frac{1}{2}\int_0^1 dx\, x \Big(H^q(x,0,0;Q^2) + E^q(x,0,0;Q^2) \Big)$$

assumption

(at some Q_L)

$$f_{1T}^{\perp(0)q}(x;Q_L^2) = -L(x) E^q(x,0,0;Q_L^2)$$

k moment of Sivers

lensing function

Bacchetta, Conti, M.R.
P.R.D**78** (08) 074010

Ji's sum rule

$$J^q(Q^2) = \frac{1}{2}\int_0^1 dx\, x \left(H^q(x,0,0;Q^2) + E^q(x,0,0;Q^2)\right)$$

assumption

(at some Q$_L$)

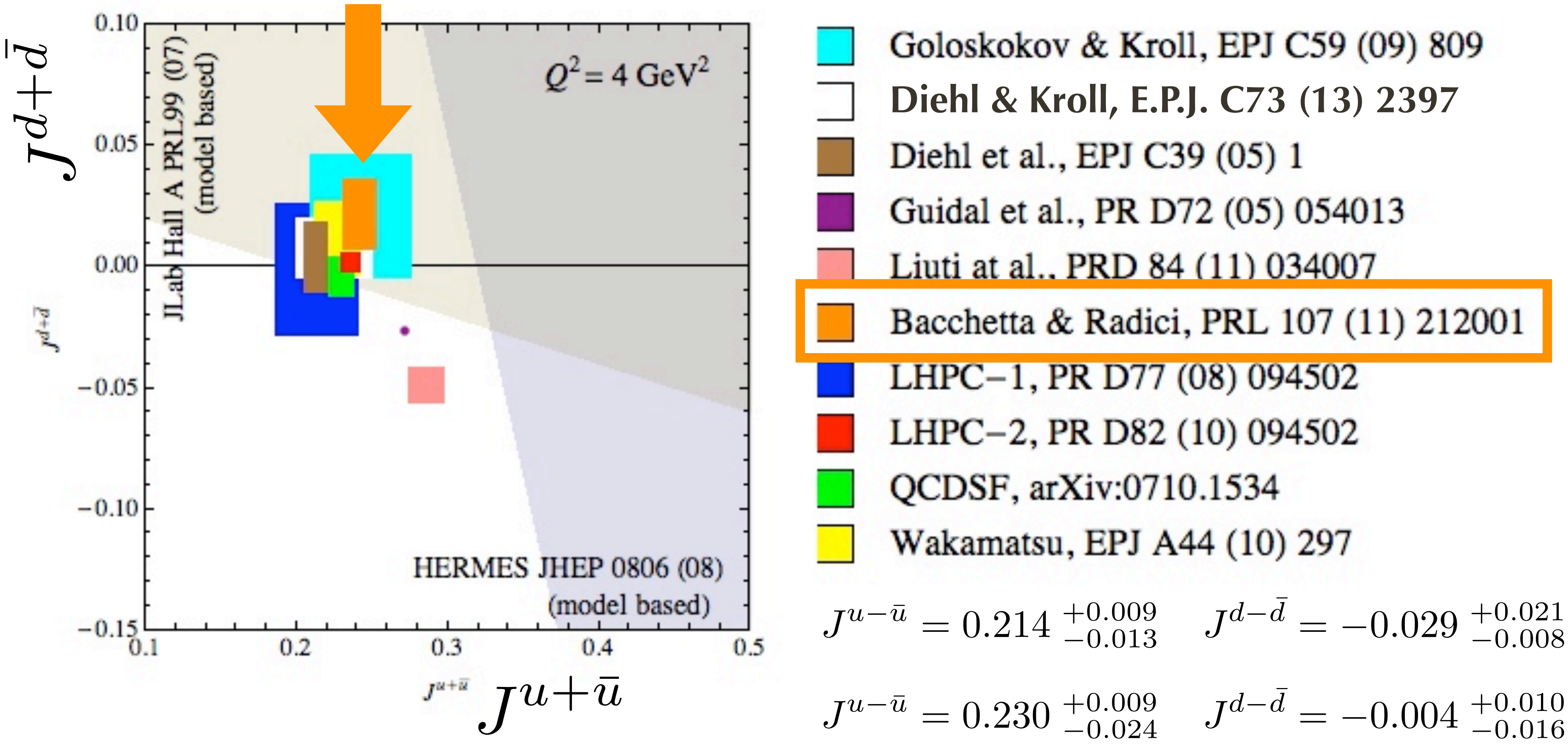
$$f_{1T}^{\perp(0)q}(x;Q_L^2) = -L(x)E^q(x,0,0;Q_L^2)$$

k moment of Sivers

lensing function

Bacchetta, Conti, M.R.
P.R.D**78** (08) 074010

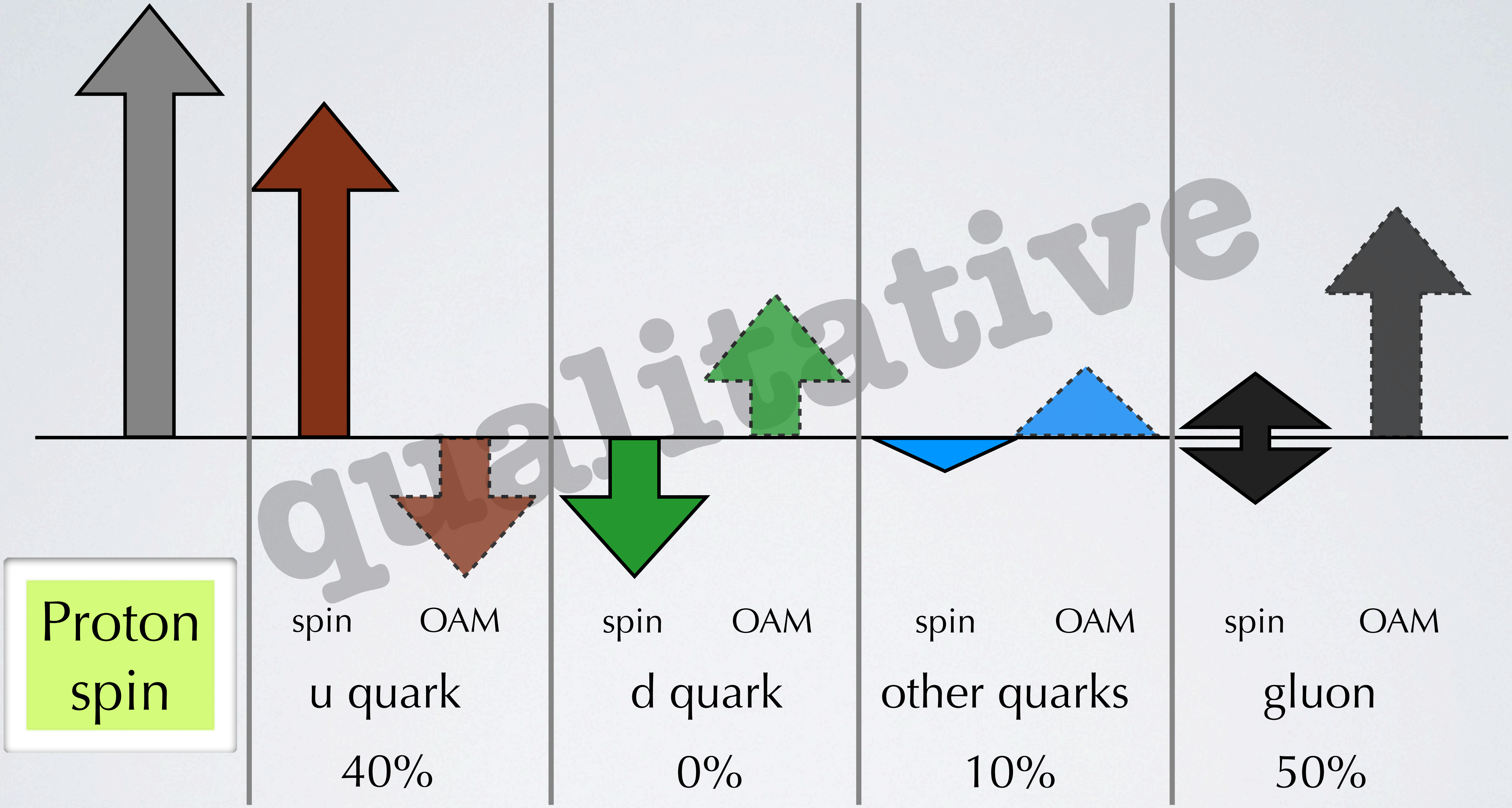
comparison with other GPD extractions and lattice results



$J^{u-\bar{u}} = 0.214\,^{+0.009}_{-0.013}$ $J^{d-\bar{d}} = -0.029\,^{+0.021}_{-0.008}$ 🟧

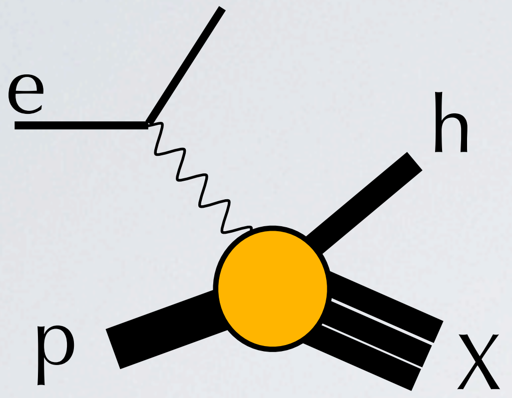
$J^{u-\bar{u}} = 0.230\,^{+0.009}_{-0.024}$ $J^{d-\bar{d}} = -0.004\,^{+0.010}_{-0.016}$ ☐

plausible scenario



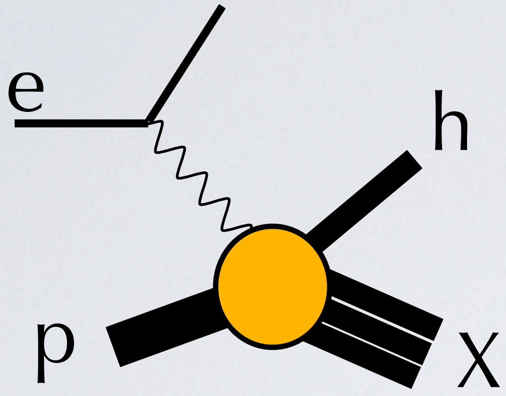
with many uncertainties,
particularly on sea quarks and gluons

Reason #2 for studying the Sivers function



SIDIS

Reason #2 for studying the Sivers function



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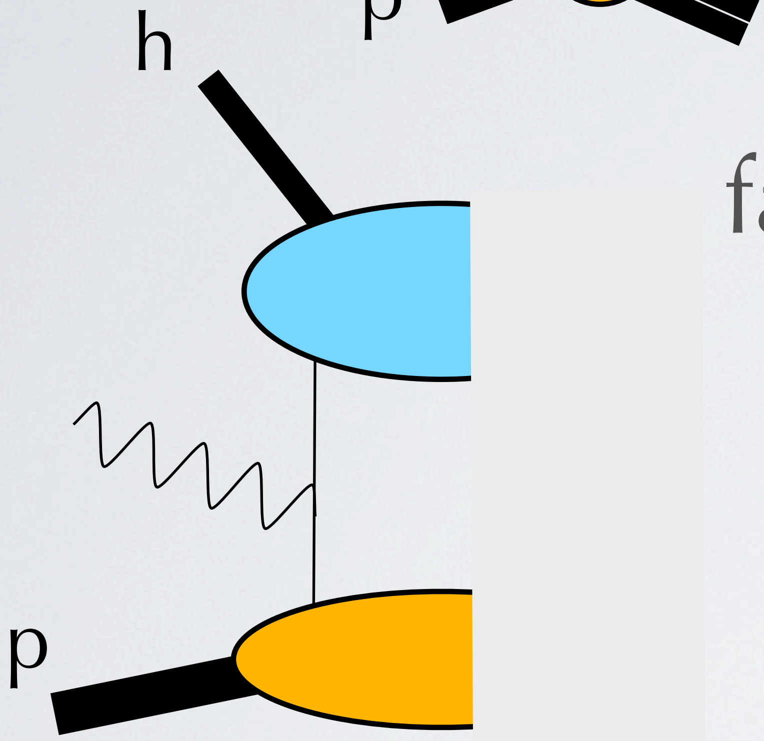
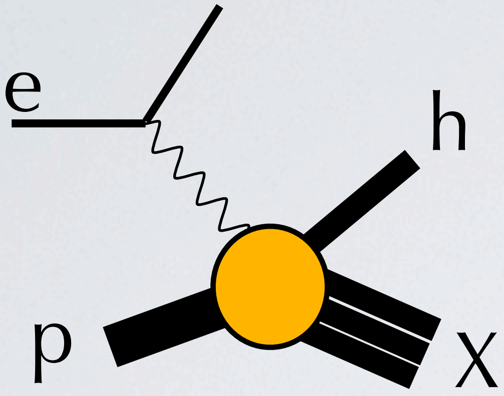


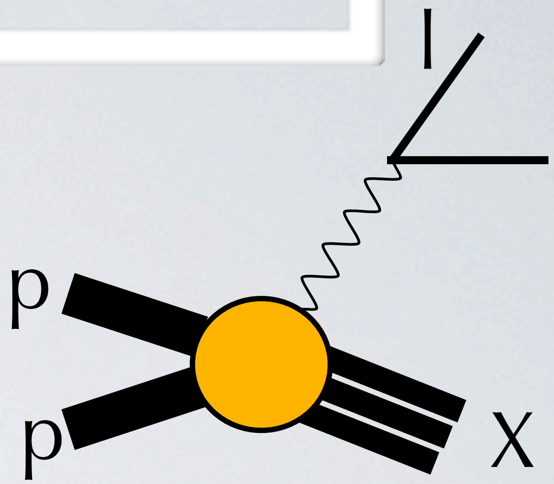
factorization theorems

Reason #2 for studying the Sivers function

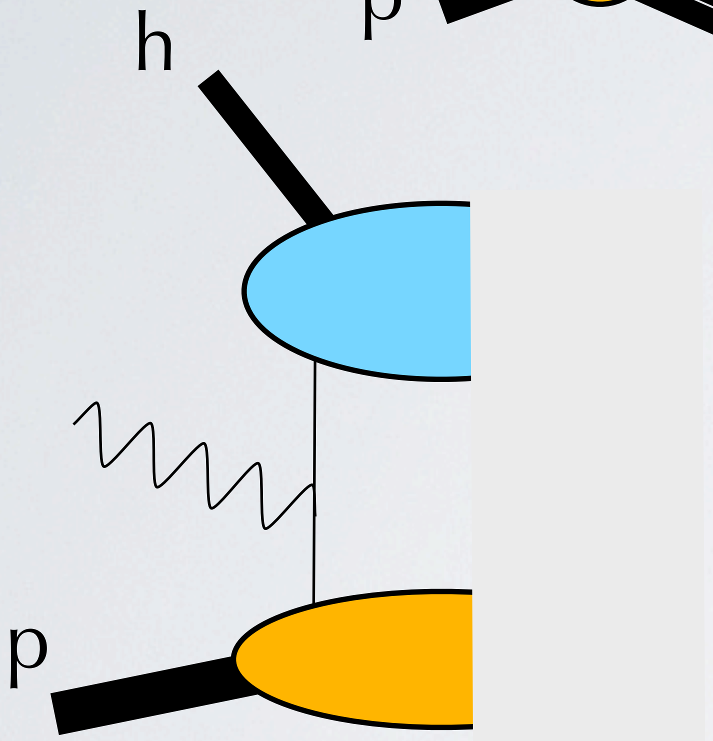


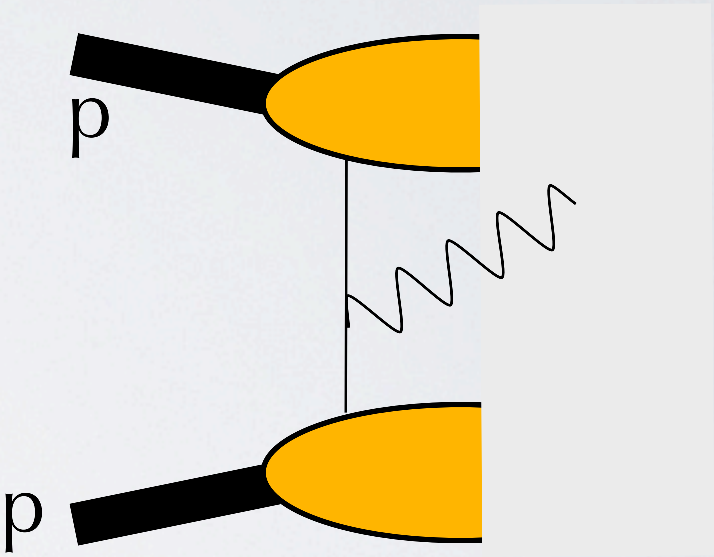
SIDIS

Drell-Yan

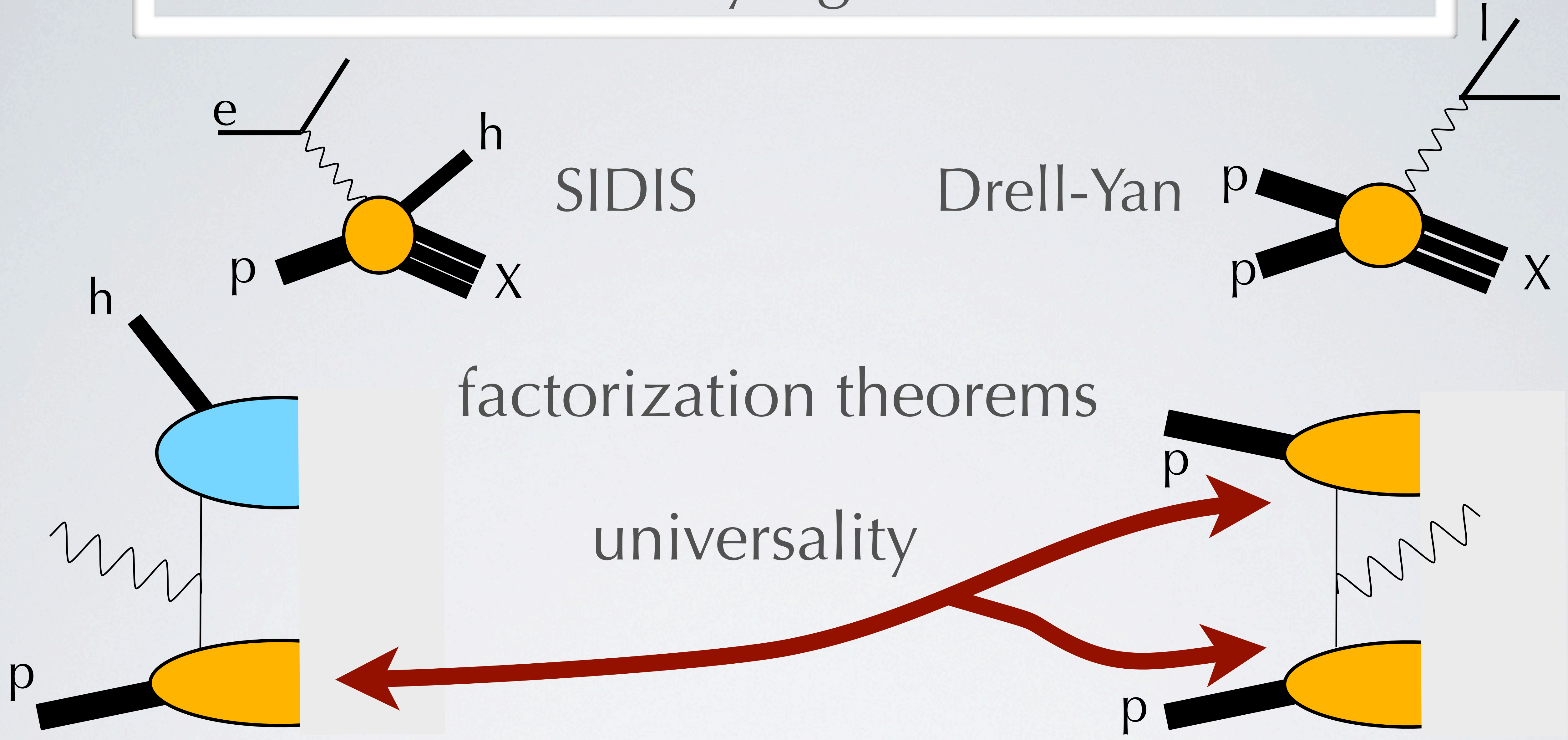


factorization theorems





Reason #2 for studying the Sivers function



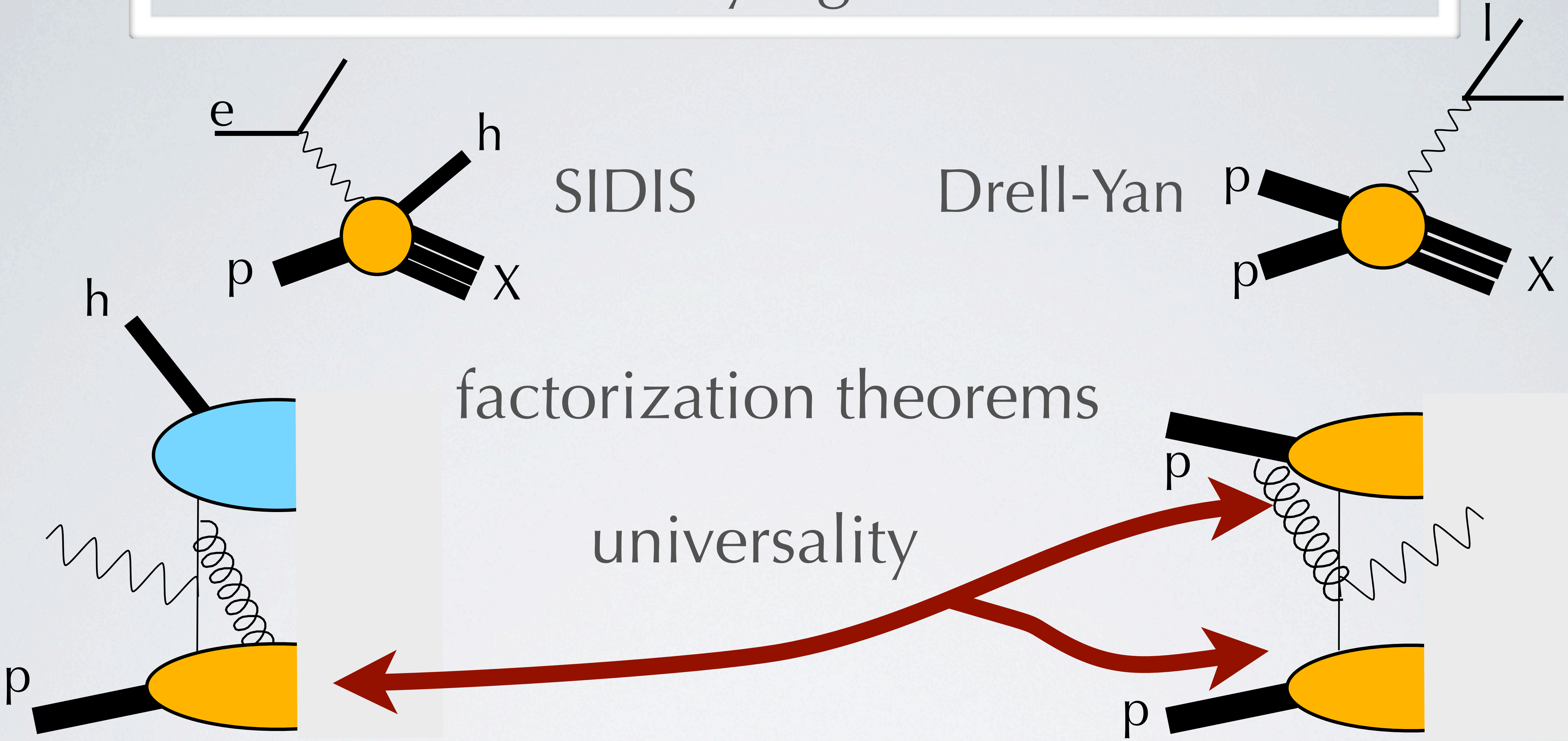
SIDIS

Drell-Yan

factorization theorems

universality

Reason #2 for studying the Sivers function



SIDIS

Drell-Yan

factorization theorems

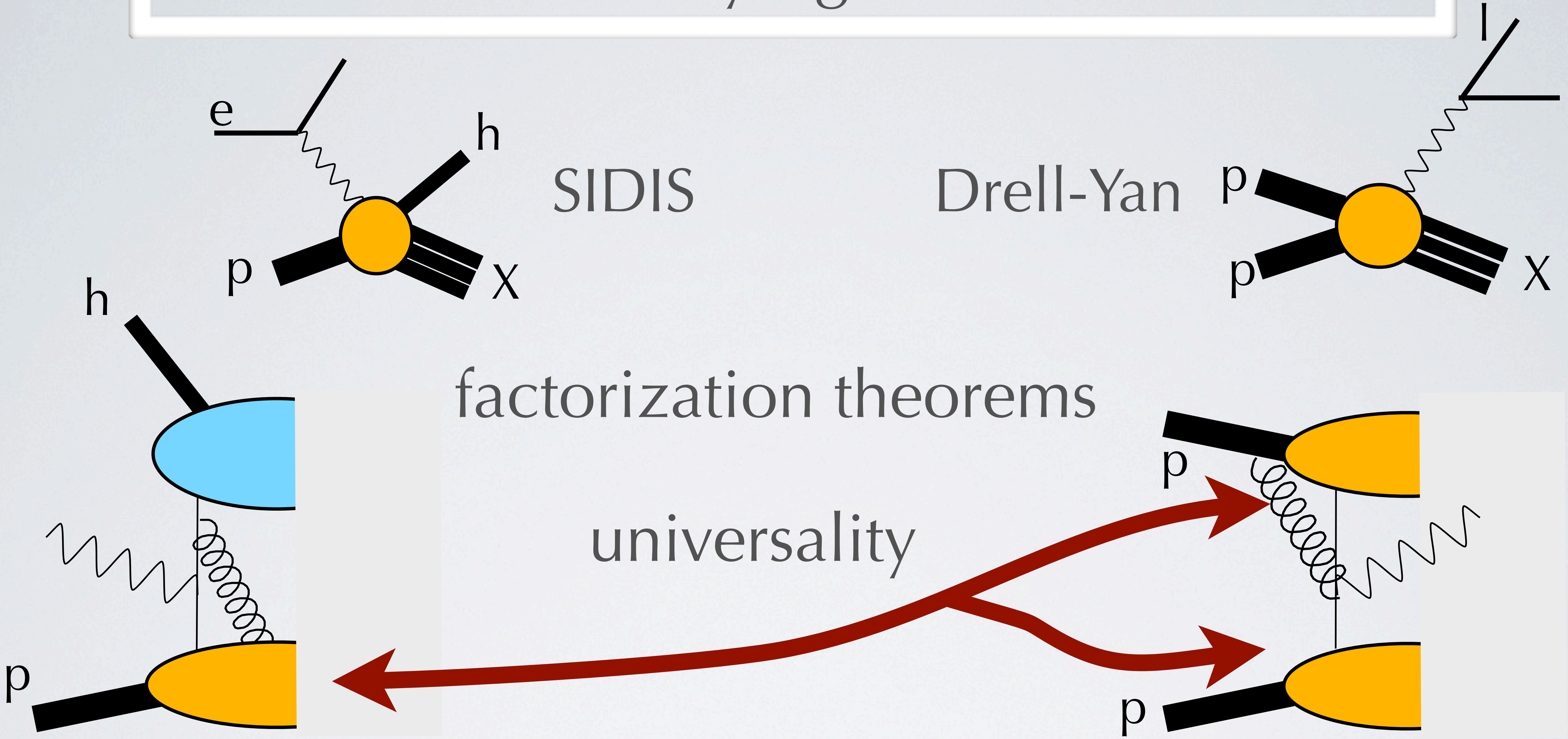
universality

“Final”

residual color interactions

“Initial”

Reason #2 for studying the Sivers function



SIDIS

Drell-Yan

factorization theorems

universality

“Final”

residual color interactions

“Initial”

<u>QCD prediction to be tested:</u> $\text{Sivers}\Big|_{\text{SIDIS}} = -\text{Sivers}\Big|_{\text{D-Y}}$

*Collins, PL B**536** (02)*

> “With 3D projections, we will be entering a new age. Something which was never technically possible before: a stunning visual experience which ‘turbocharges’ the viewing.”
>
> James Cameron