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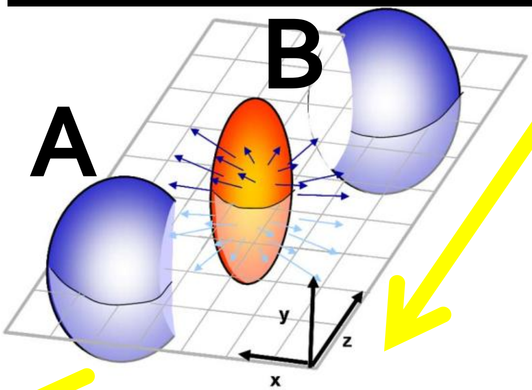
THERMALIZATION, ISOTROPIZATION AND ELLIPTIC FLOW OF QGP

Based on collaboration with:
V. Greco, S. Plumari and F. Scardina

Cortona, 2013 October 30

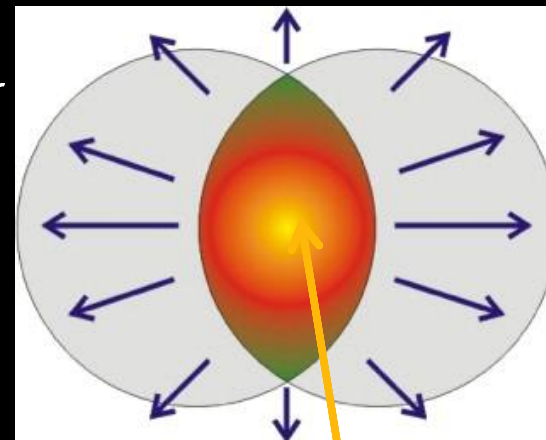
Heavy Ion Collisions



Impact parameter direction

Collision (flight) direction

Impact parameter direction



Collision direction

FIREBALL:
Hot and dense expanding ***parton mixture***
QUARK-GLUON-PLASMA (QGP)
T about 10^{12} K, t about 10^{-23} seconds

A,B: Cu, Au (RHIC@BNL)
Pb (LHC@CERN).

$$\sqrt{s} \;\; \text{up to} \;\; 200 \times A \; \text{GeV} \, , \qquad \text{RHIC}$$

$$\sqrt{s} \;\; \text{up to} \;\; 2.76 \times A \; \text{TeV} \, , \qquad \text{LHC}$$

Megyn Kelly, anchor of Fox News Channel

More in A. Puglisi's talk

Boltzmann equation and QGP

In order to ***simulate*** the temporal evolution of the fireball we solve the ***Boltzmann equation*** for the parton distribution function ***f***:

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{\nabla}\right) f(\boldsymbol{x},\boldsymbol{p},t) = C[f]$$

L. Boltzmann, 1872

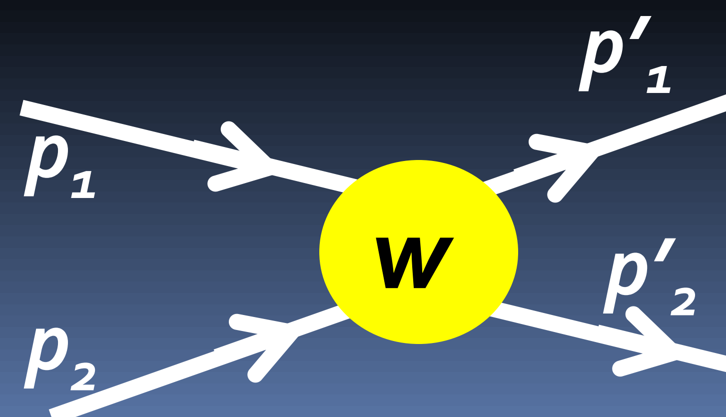
Drift term ***Collision integral***

Drift term: change of ***f*** due to particles flowing into and out of the phase space volume centered at (***x***,***p***).

Collision integral: change of ***f*** due to collision processes in the phase space volume centered at (***x***,***p***).

For the case of 12->1'2' processes:

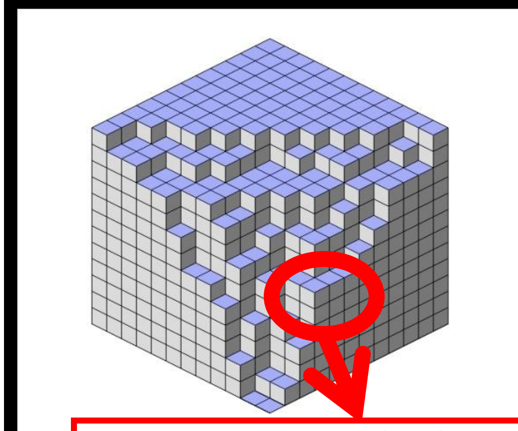
$$C[f] = \frac{1}{2}\int d\boldsymbol{p}_2 \int d\boldsymbol{p}_1' \int d\boldsymbol{p}_2'\, w(12\rightarrow 1'2') \times [f(\boldsymbol{x},\boldsymbol{p}_1',t)f(\boldsymbol{x},\boldsymbol{p}_2',t) - f(\boldsymbol{x},\boldsymbol{p}_1,t)f(\boldsymbol{x},\boldsymbol{p}_2,t)]$$



More in A. Puglisi's talk

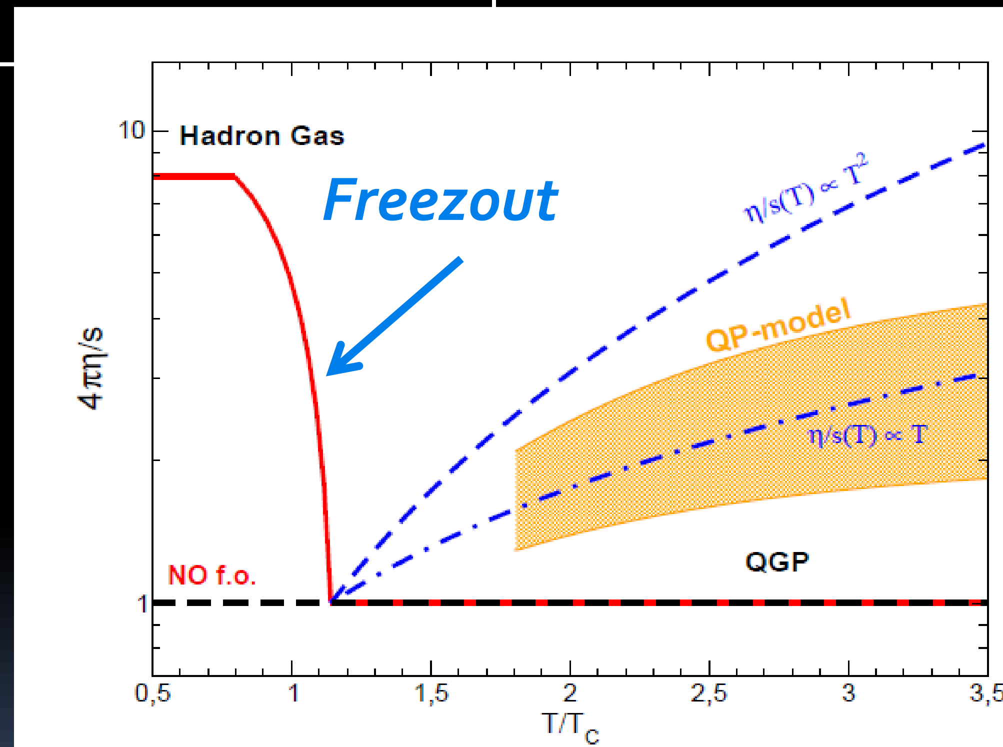
eta/s: hydro "by" transport

We use ***Boltzmann equation*** to simulate a fluid at ***fixed eta/s***. ***Cross section*** is ***computed*** in ***each configuration space cell*** according to ***Chapman-Enskog equation*** to give the ***wished value of eta/s at local T***.



CE

$$\frac{\eta}{s} = \frac{\langle p \rangle}{g(m_D)\rho} \frac{1}{\sigma}$$

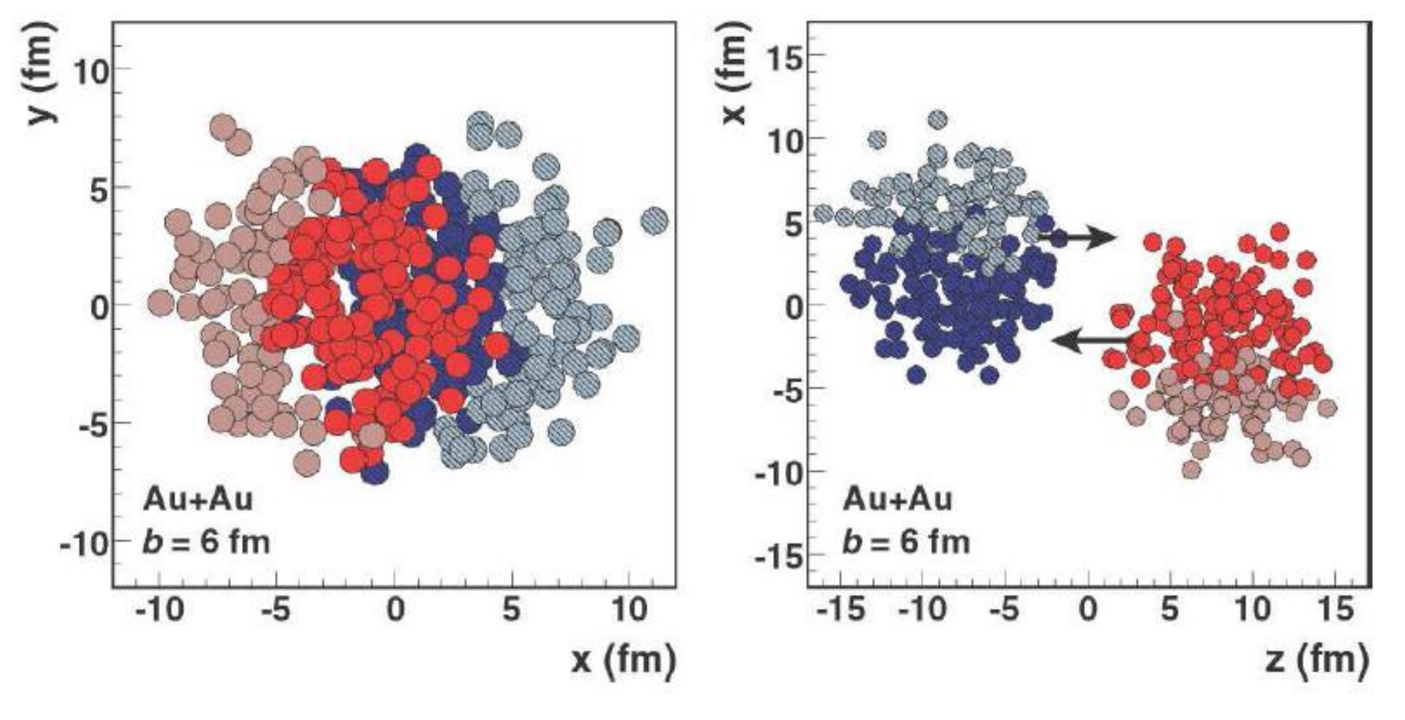


For ***small transverse momenta***, this approach is meaningful since ***this momentum domain*** corresponds to the ***hydro domain***, where ***microscopic details are not important*** and only ***eta/s*** is relevant.

Plumari *et al.*, Phys. Rev. C86 (2012).
Greco *et al.*, Phys. Lett. B670 (2009).
Plumari *et al.*, J.Phys.Conf.Ser. 420 (2013).

Initial condition: Glauber

(Almost) Geometrical description of the fireball:

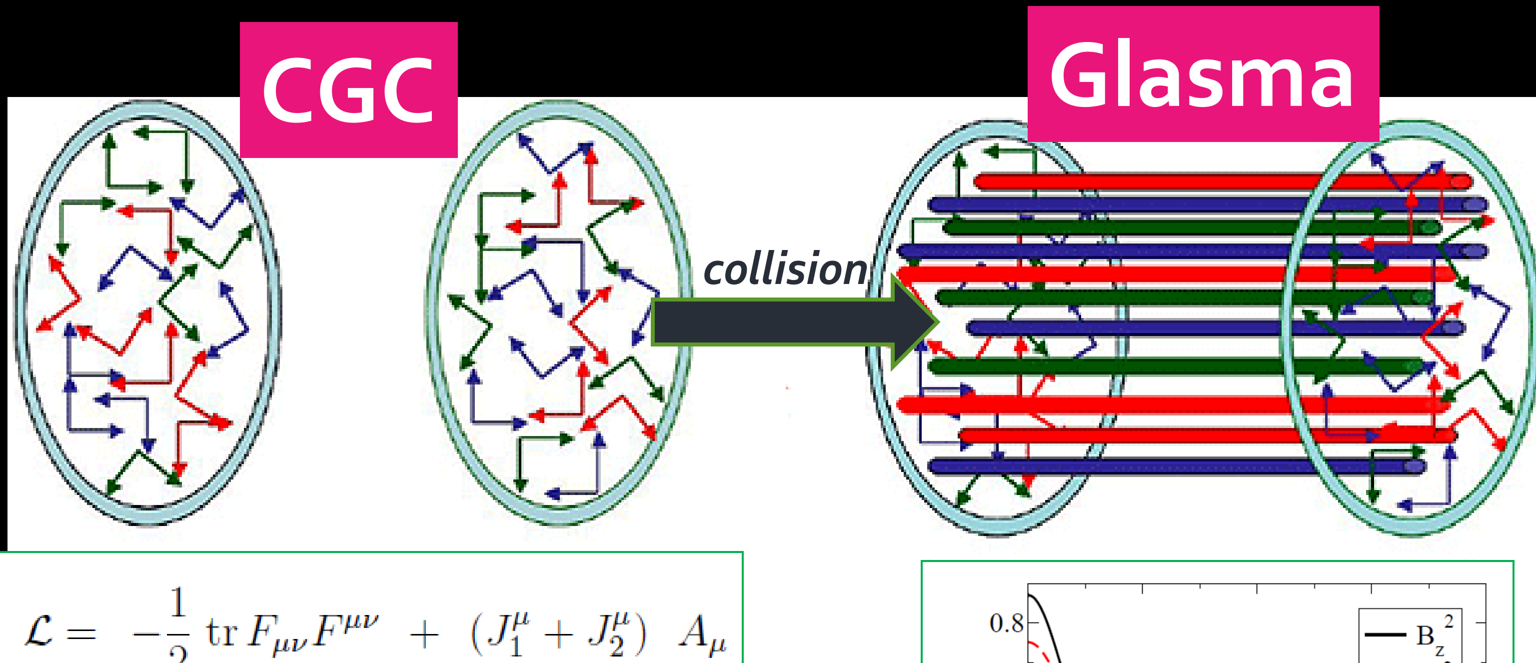


Assuming a nucleon distribution in the parents nuclei (typically a ***Woods-Saxon***), one counts ***how many particles*** from each nucleus are present in the ***overlap region***; among them, the ***participants*** are the nucleons that effectively can have an interaction (in fact, the particles that *are in the overlap region* but *do not interact*, are not considered).

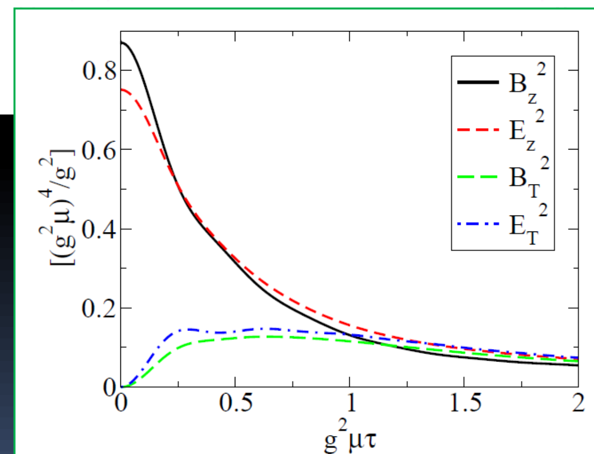
For a review see: Miller *et al.*, Ann.Rev.Nucl.Part.Sci. **57, 205 (2007)**

McLerran and Venugopalan, PRD **49**, 2233 (1994)
McLerran and Venugopalan, PRD **49**, 3352 (1994)

Initial condition: Glasma



$$\mathcal{L} = \underbrace{-\frac{1}{2}\,\mathrm{tr}\,F_{\mu\nu}F^{\mu\nu}}_{\text{gluon dynamics}} + \underbrace{(J_1^\mu + J_2^\mu)}_{\text{fast partons}}\, A_\mu$$



Lappi and McLerran, Nucl. Phys. **A772** (2006)

Reviews/Lectures

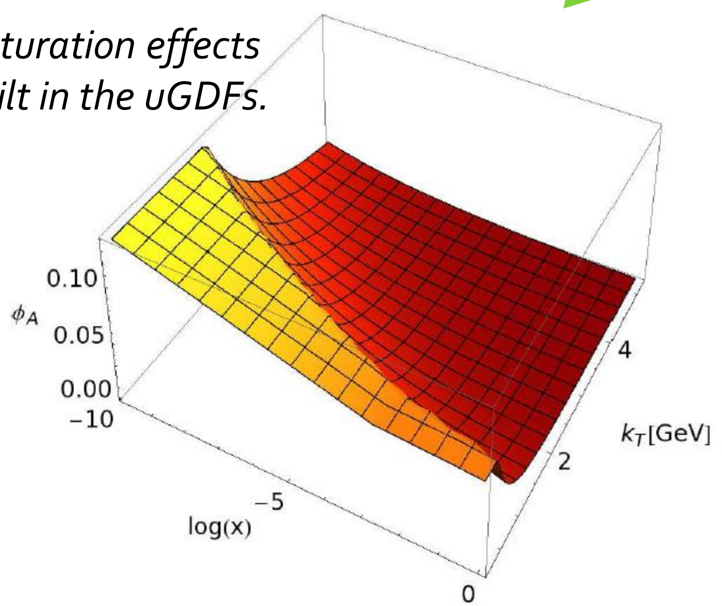
McLerran, 2011
Iancu, 2009
McLerran, 2009
Lappi, 2010
Gelis, 2010
Fukushima, 2011

Initial condition: fKLN-Glasma

(f)KLN spectrum

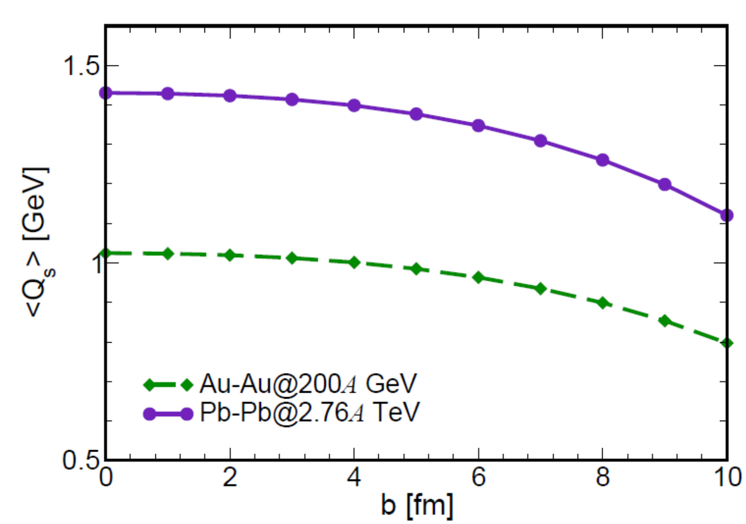
$$\frac{dN_g}{d^2x_\perp dy} \propto \int \frac{d^2p_T}{p_T^2} \int_0^{p_T} d^2k_T \alpha_s(Q^2) \times \phi_A\left(x_A, \frac{(p_T+k_T)^2}{4}; \boldsymbol{x}_\perp\right) \times \phi_B\left(x_B, \frac{(p_T-k_T)^2}{4}; \boldsymbol{x}_\perp\right)$$

Saturation effects built in the uGDFs.



Nardi *et al.*, Nucl. Phys. A**747**, 609 (2005)
Kharzeev *et al.*, Phys. Lett. B**561**, 93 (2003)
Nardi *et al.*, Phys. Lett. B**507**, 121 (2001)
Drescher and Nara, PRC**75**, 034905 (2007)
Hirano and Nara, PRC**79**, 064904 (2009)
Hirano and Nara, Nucl. Phys. A**743**, 305 (2004)
Albacete and Dumitru, arXiv:1011.5161[hep-ph]
Albacete *et al.*, arXiv:1106.0978 [nucl-th]

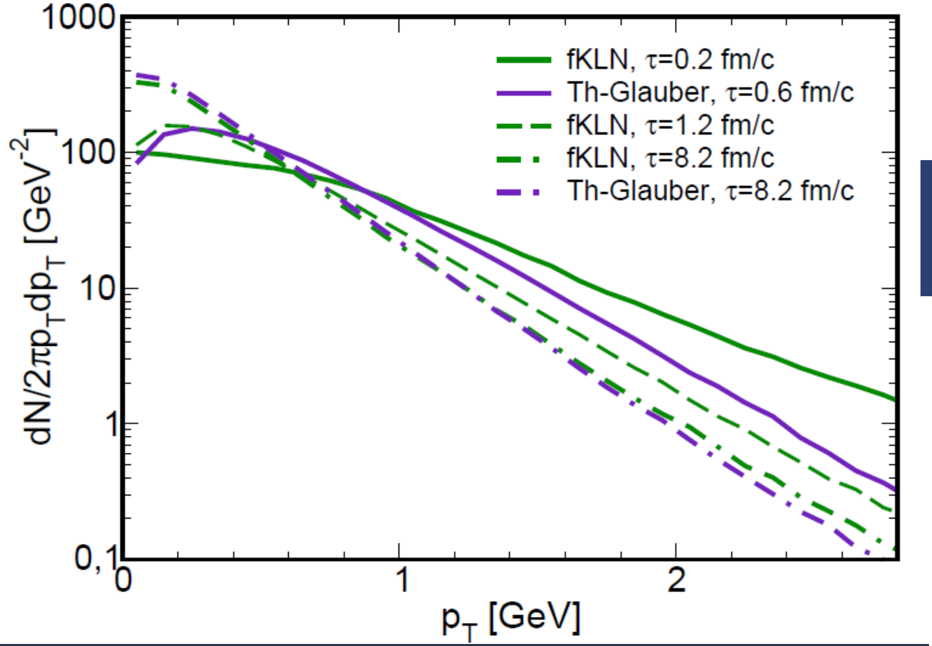
Saturation scale ***Qs*** depends on:
1.) ***position in transverse plane***;
2.) ***gluon rapidity***.



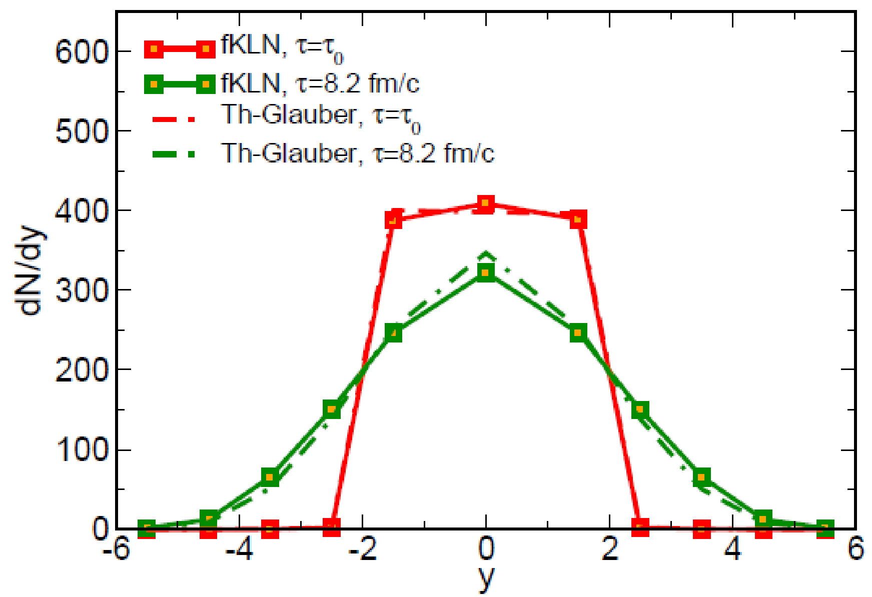
M. R. *et al.*, 1303.3178 [nucl-th]

Initial condition and thermalization

AuAu@200A GeV Spectra



AuAu@200A GeV Multiplicity

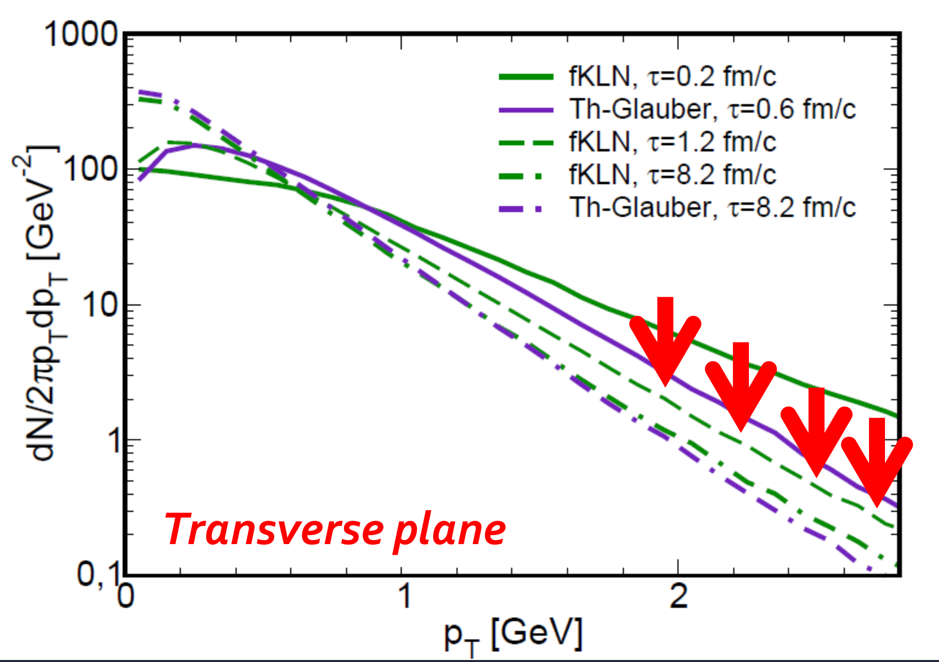


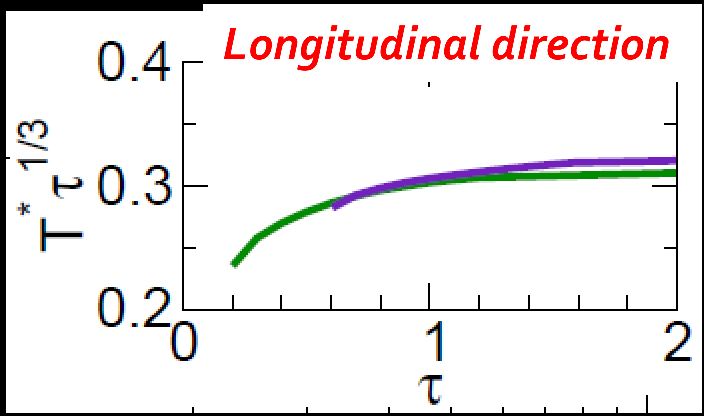
M. R. *et al.*, 1303.3178 [nucl-th]

Initial condition and thermalization

Thermalization in less than 1 fm/c,
in agreement with:
Greiner *et al.*, Nucl. Phys. A**806**, 287 (2008).

AuAu@200A GeV





$$\sigma_{tot} = \frac{\langle p \rangle}{\rho\, g(a)} \frac{1}{\eta/s}$$

Not so surprising:
Because eta/s is small, large cross sections naturally lead to fast thermalization.
However, interesting:
We have dynamics in the early stages of the simulation, which prepares the momentum distribution to build up the elliptic flow.

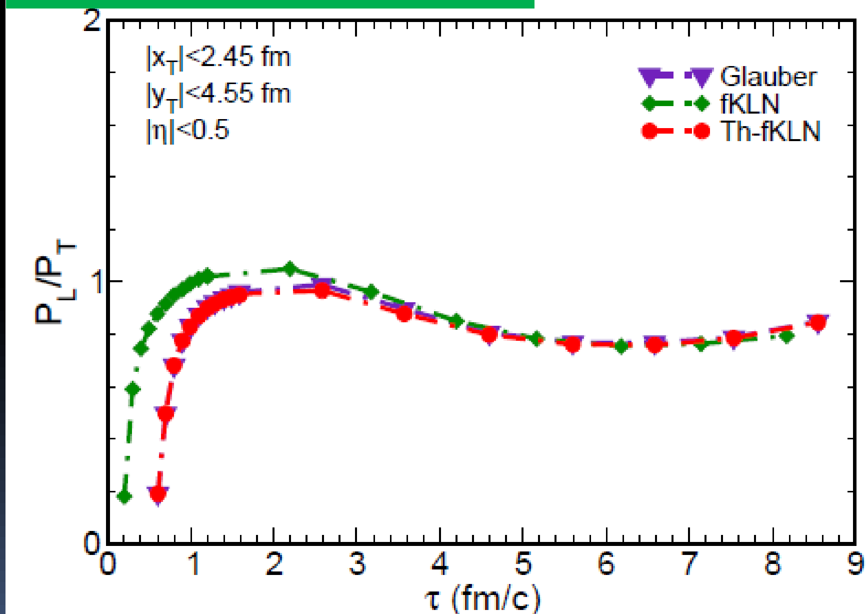
M. R. *et al., work in progress*

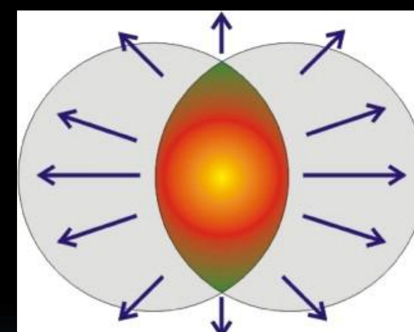
Fireball Isotropization

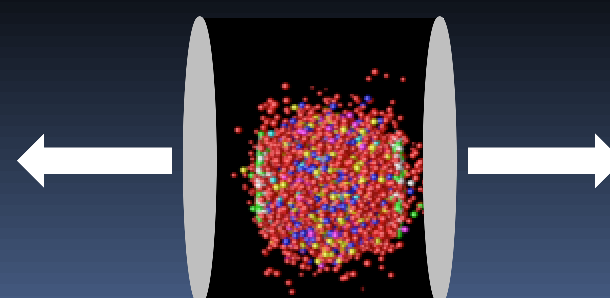
$$T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{E} f(x,p)$$

$$P_T = \frac{1}{V} \int_\Omega d^2x_\perp d\eta \, \frac{T_{xx} + T_{yy}}{2} \, ,$$
$$P_L = \frac{1}{V} \int_\Omega d^2x_\perp d\eta \, T_{zz} \, ,$$

AuAu@200A GeV





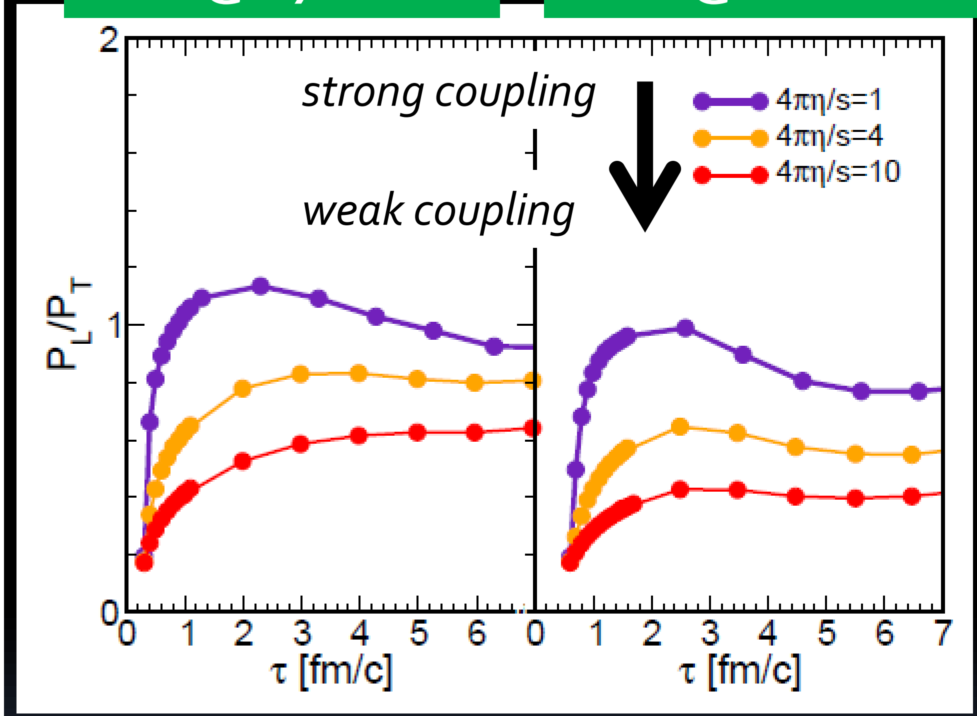


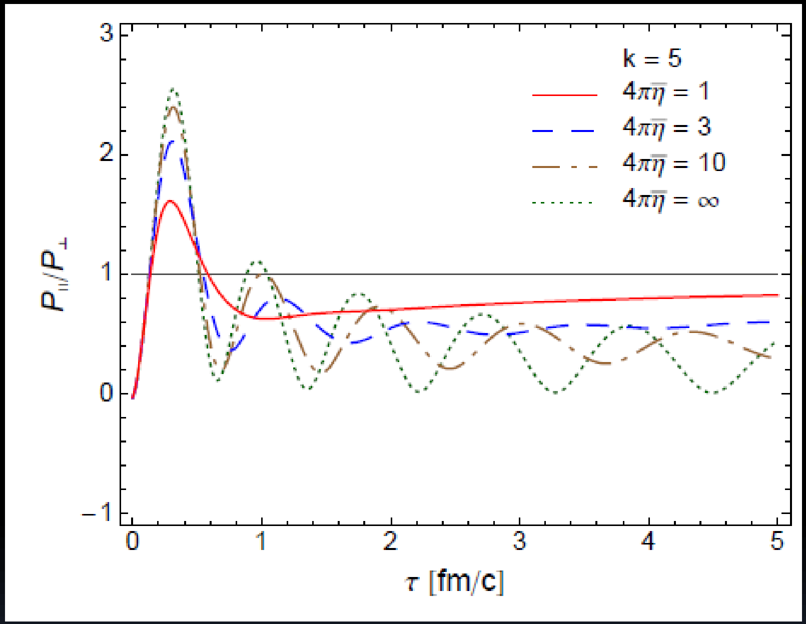
Complete isotropization in strong coupling

M. R. *et al.*, *work in progress*

Fireball Isotropization

PbPb@2.76A TeV *AuAu@200A GeV*





Ryblewski and Florkowski, PRD88 (2013)

InComplete isotropization in weak coupling

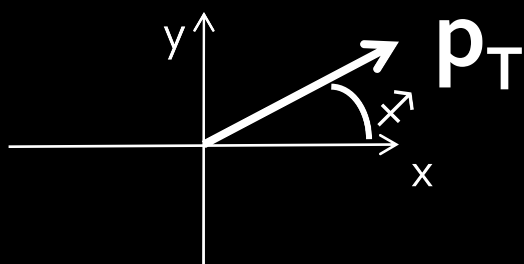
Elliptic flow in RHICs

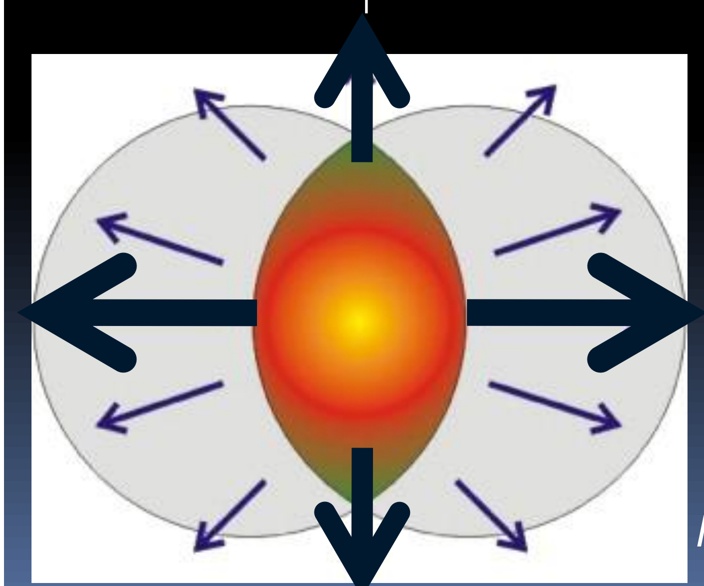
Particle multiplicity in momentum space

$$\frac{d^3N}{dy p_T dp_T d\phi} = \frac{1}{2\pi}\frac{d^2N}{dy p_T dp_T}\left[1 + 2v_2(y, p_T)\cos 2\phi\right]$$

Elliptic flow:
leading contribution to anisotropy in momentum space

$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle = \left\langle \frac{p_x^2 - p_y^2}{p_T^2} \right\rangle$$





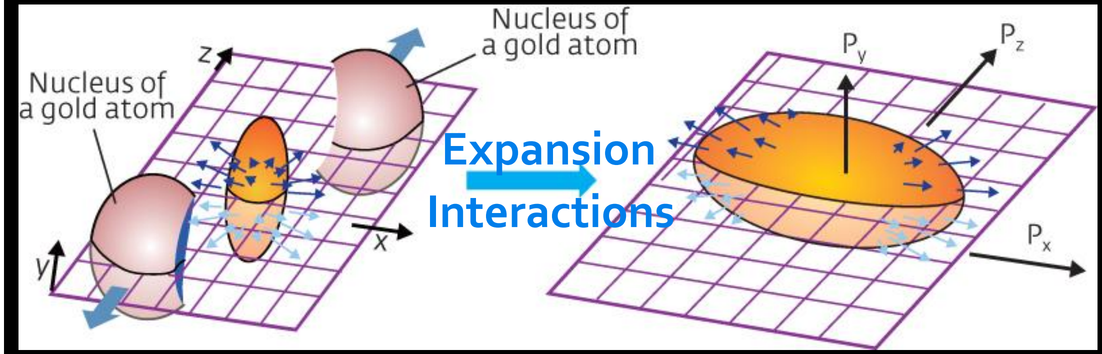
*Immediately after the collision, **pressure gradient** along **X** is larger than that along **Y**.*
*As a consequence, **the medium expands preferentially along the short axis of the ellipse,** creating a **flow.***

Collision direction

Impact parameter direction

M. R. *et al., work in progress*

Elliptic flow in RHICs

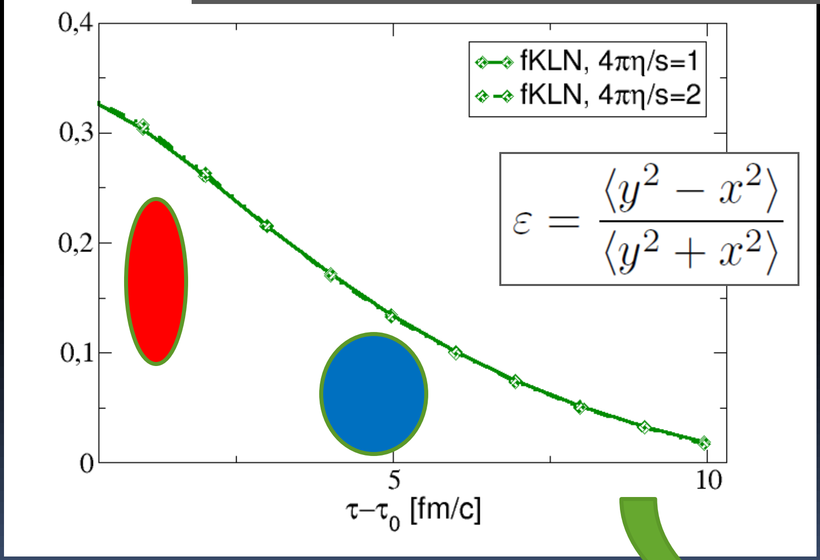


Collective vs Thermal:

v2 measures how efficiently the anisotropy of configuration in the initial state is transmitted to momenta in final states

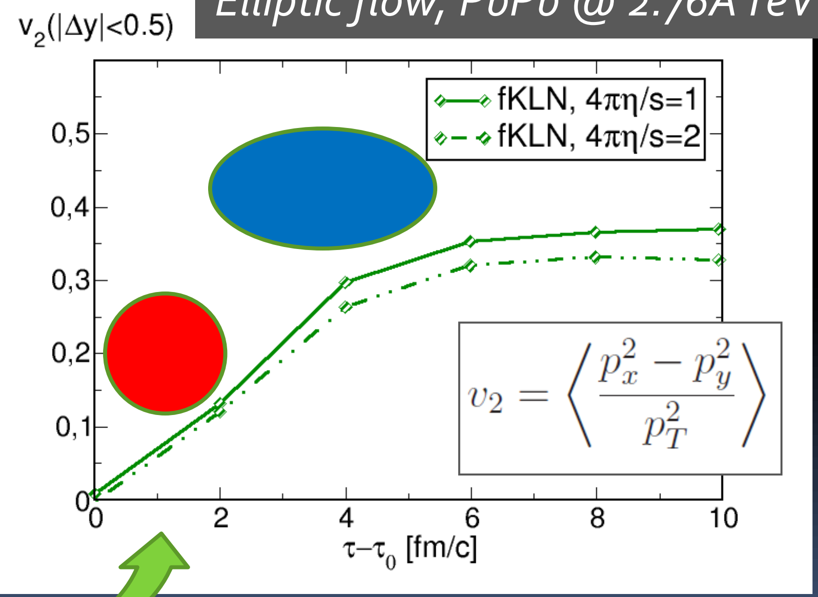
[J. Y. Ollitraut, PRD46 (1992)]

Eccentricity, PbPb @ 2.76A TeV



$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$

Elliptic flow, PbPb @ 2.76A TeV



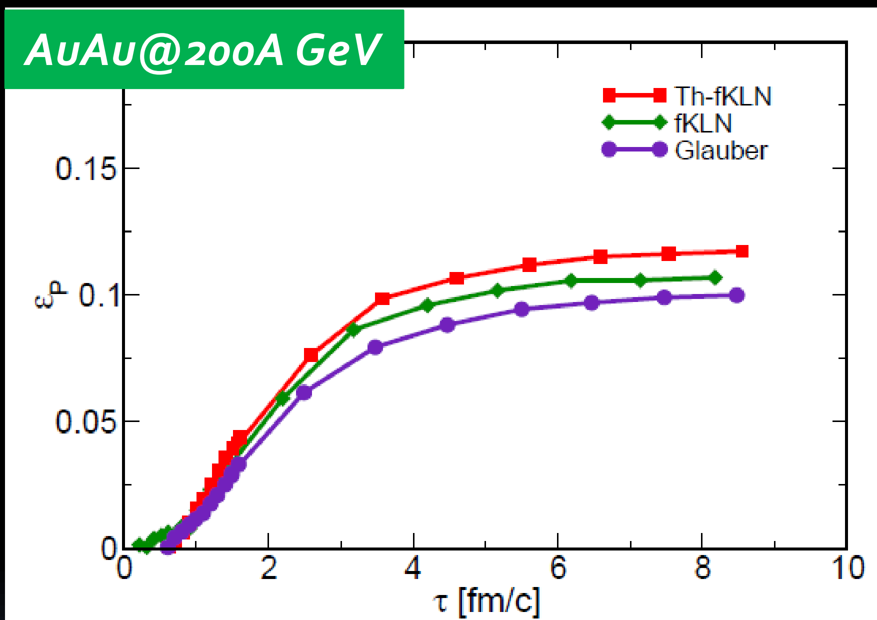
$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_T^2} \right\rangle$$

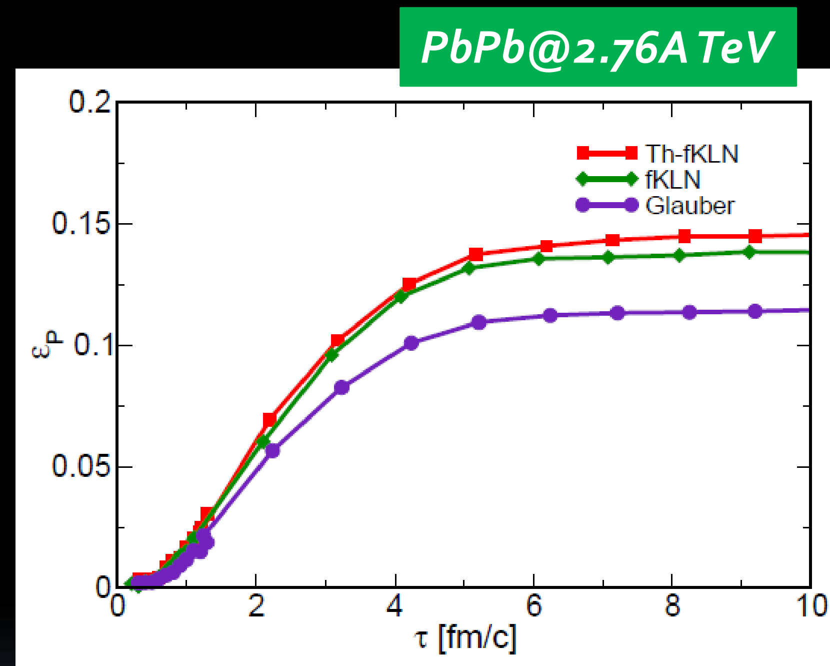
Transfer of anisotropy

M. R. *et al.*, *work in progress*

Elliptic flow in RHICs

Elliptic flow is mostly generated early in the nucleus-nucleus collision, and is present at the ***partonic level*** before partons hadronize.





Elliptic flow is sensitive to the properties of the hot and dense state of partonic matter created after the collision.

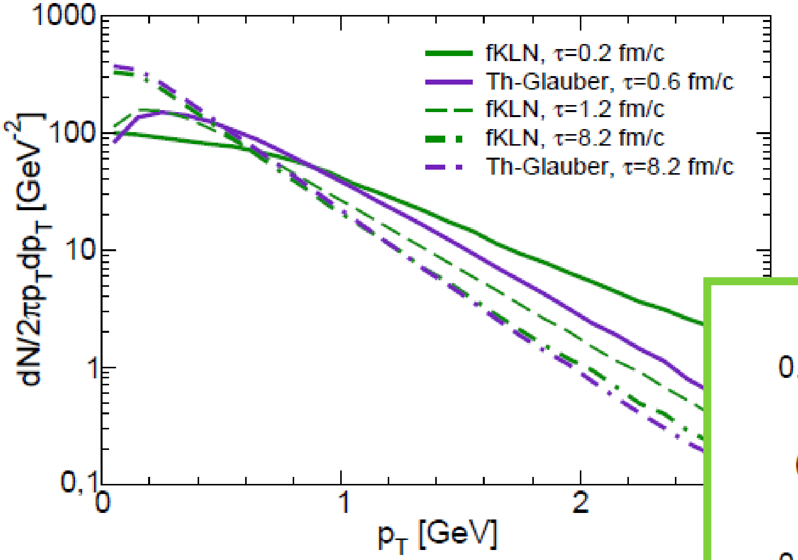
Scenario in agreement with:

Csernai *et al.*, PRL **97**, 152303 (2006)
Greco *et al.*, PRC **68**, 034904 (2003)
Peschanski and Saridakis, PRC **80** (2009)
Huovinen and Petreczky, NPA **837** (2010)
Zhang et al., PLB 99 (1998)
Heinz and Kolbe, **QGP3**

M. R. *et al.*, 1303.3178 [nucl-th]

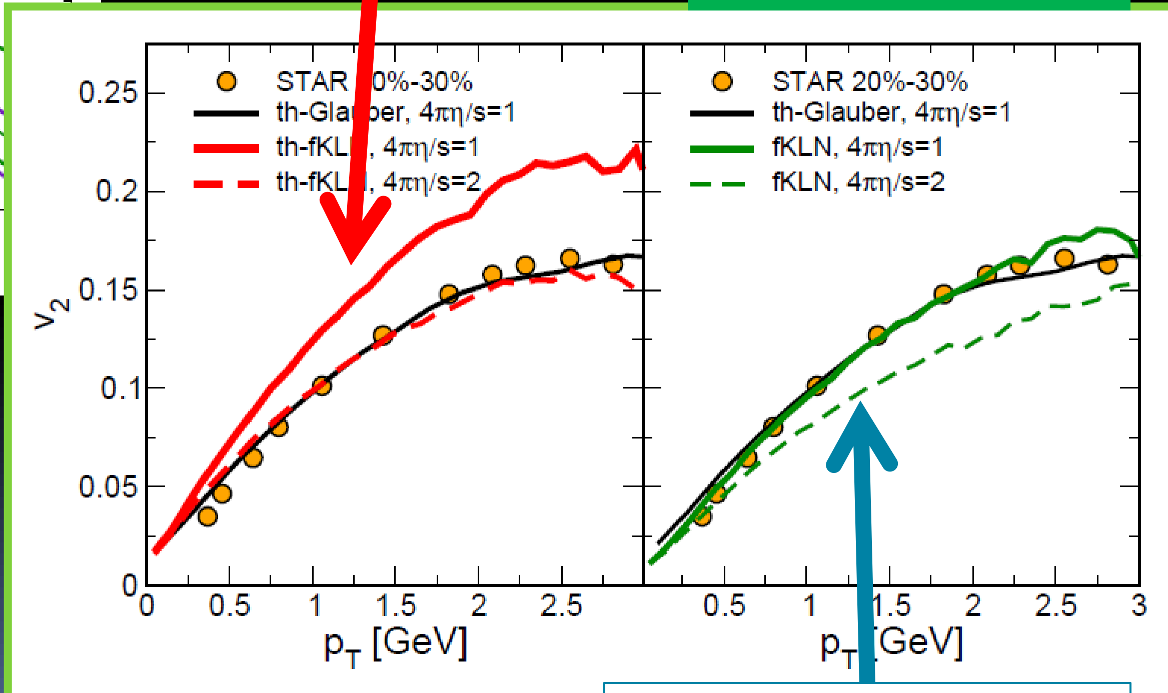
Elliptic flow from Transport

In agreement with:
Heinz *et al.*, PRC **83**, 054910 (2011)



Hydro initial condition

AuAu@200A GeV
20%-30% centrality

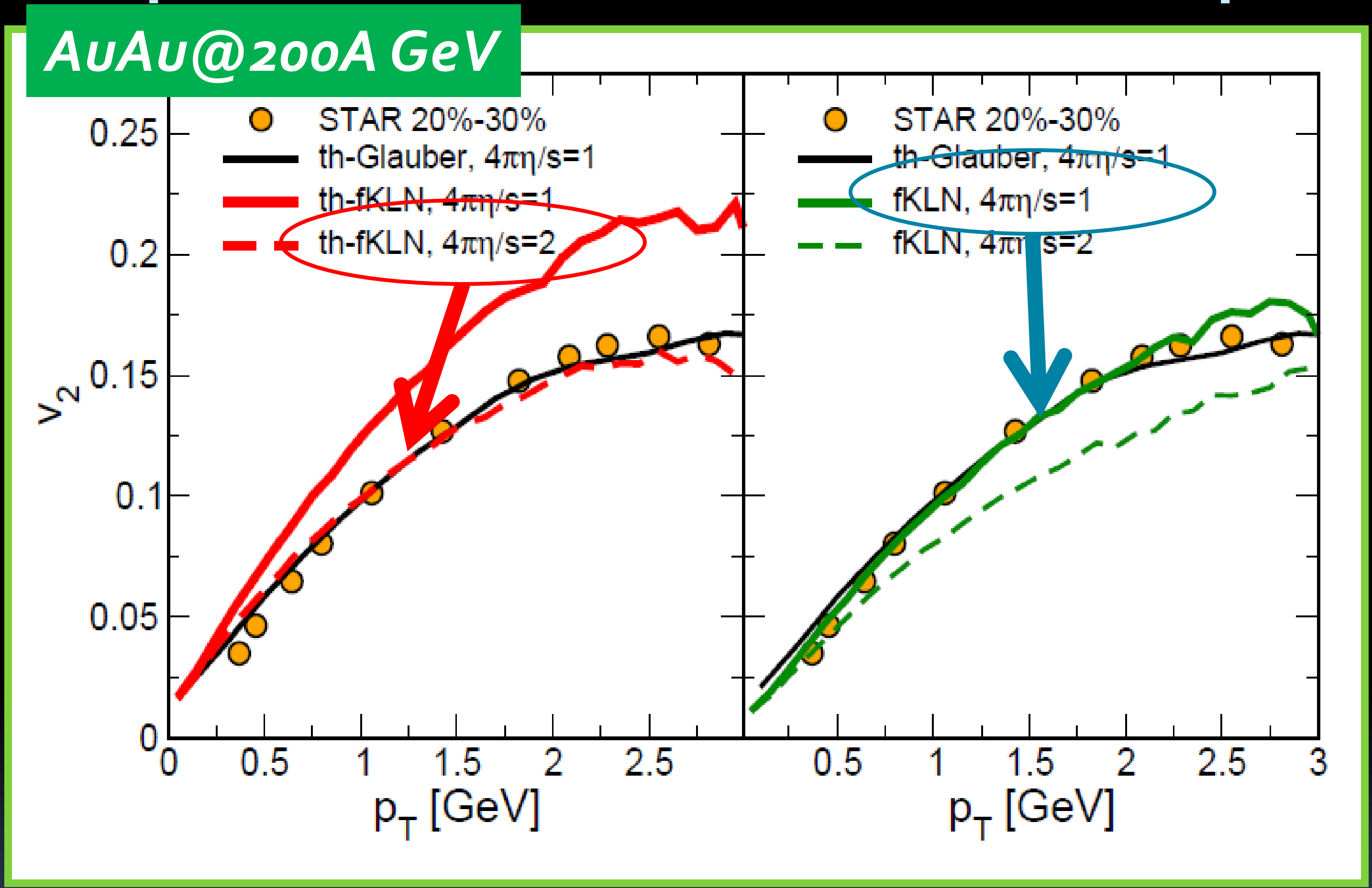


Glasma initial condition

***Hydro*:**
large initial eccentricity
is balanced by larger viscosity
which dumps the flow.

M. R. *et al.*, 1303.3178 [nucl-th]

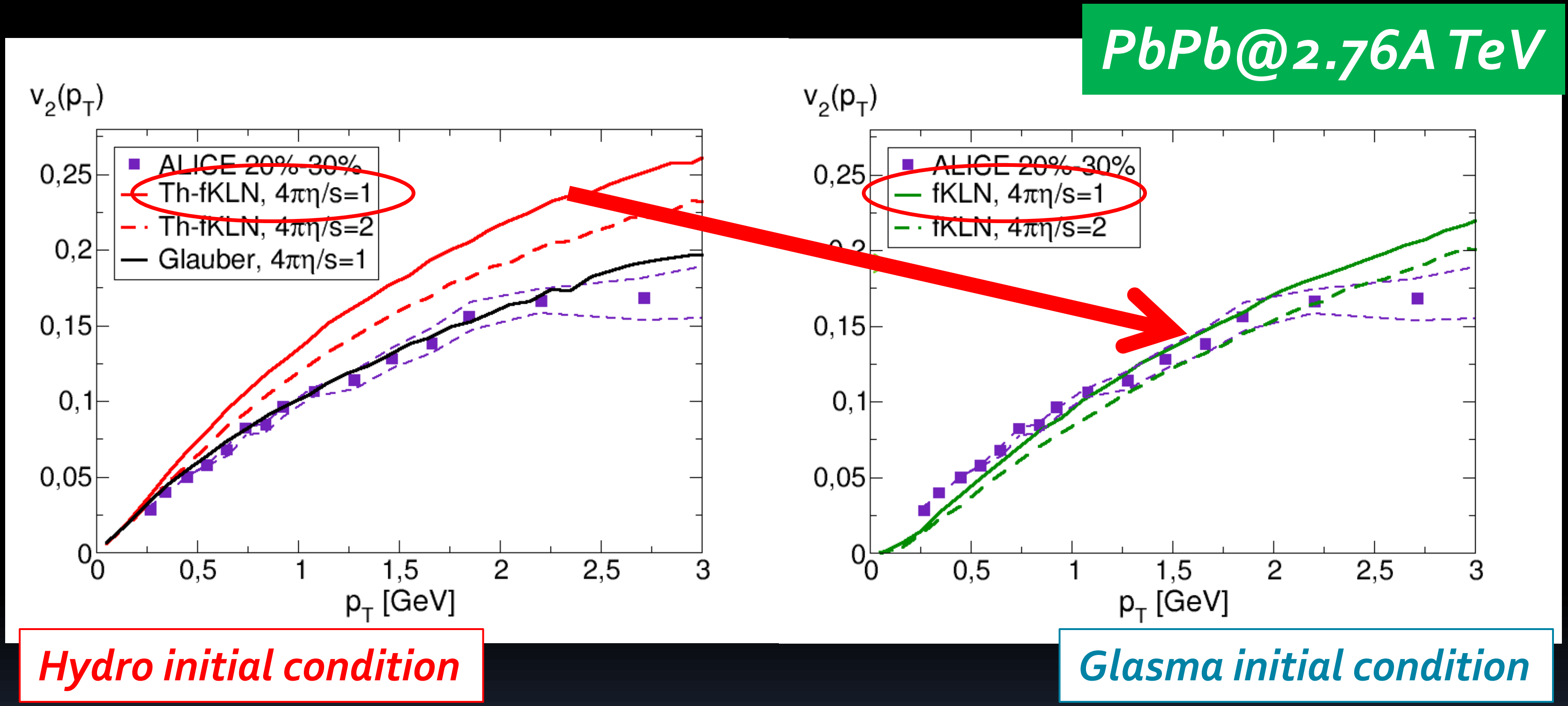
Elliptic flow from Transport



Implementing the proper initial condition in momentum space, as well as in configuration space, leads to the ***estimate of eta/s in agreement with the Glauber initial condition.***

M. R. *et al., work in progress*

Elliptic flow from Transport



*Implementing the proper initial condition in momentum space, as well as in configuration space, leads to the **estimate of eta/s in agreement with the Glauber initial condition.***

Conclusions and Outlook

- We used ***Kinetic Theory*** to compute the ***elliptic flow*** of plasma produced in heavy ion collisions, at ***both RHIC and LHC energies,*** as well as its ***thermalization times*** and ***isotropization efficiency***.
- ***Initial distribution in momentum space affects the flow and the building up of momentum anisotropy.***
- ***Microscopic details have a very little relevance for the theoretical computation of the elliptic flow by transport theory.***

Outlook

(.)*Bose-Einstein condensate in the initial stage*

(.)Initial conditions from classical field dynamics

(.)"Cosmology" of HICs (initial state fluctuations)

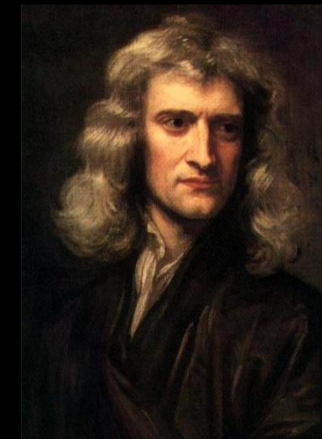
THANK YOU FOR YOUR ATTENTION

I acknowledge:
(.) Dr. Hiroaki Abuki
(.) Dr. Santosh Kumar Das
(.) Prof. Kenji Fukushima
(.) Prof. Tetsufumi Hirano
(.) Prof. Akira Ohnishi
for many discussions about the topics discussed in this talk.

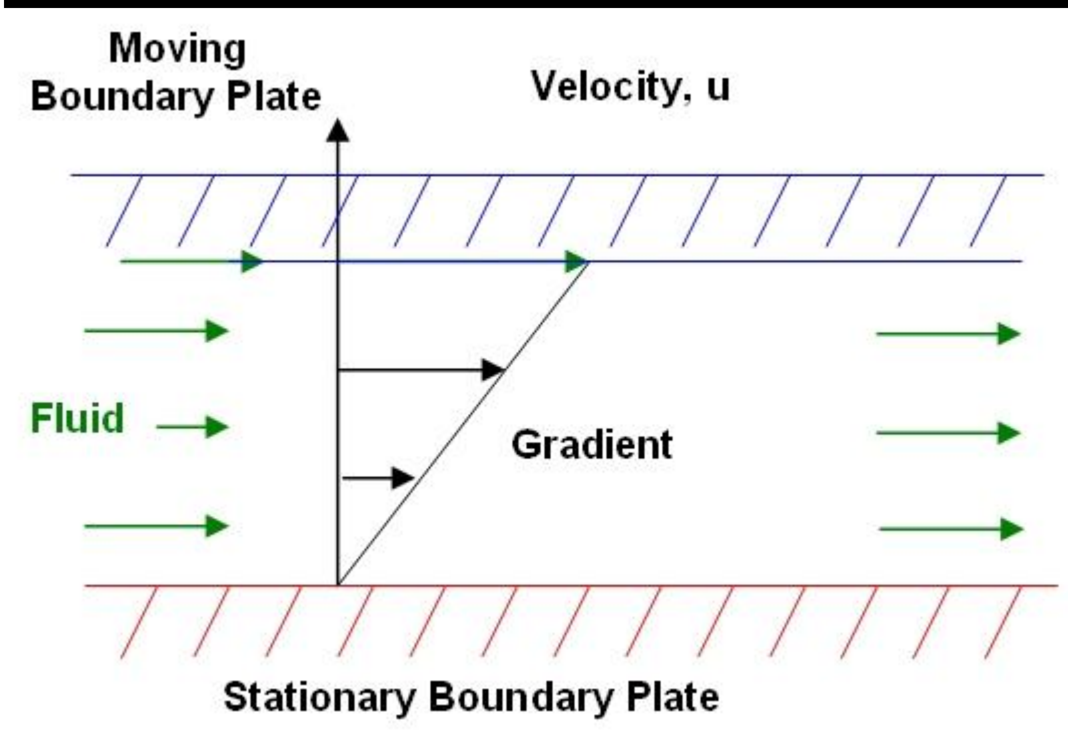
There are two breeds of fools: those who do not doubt anything, those who doubt everything.
(Charles-Joseph de Ligne)

Appendices

Shear viscosity in a nutshell



Operative definition of ***shear viscosity***:

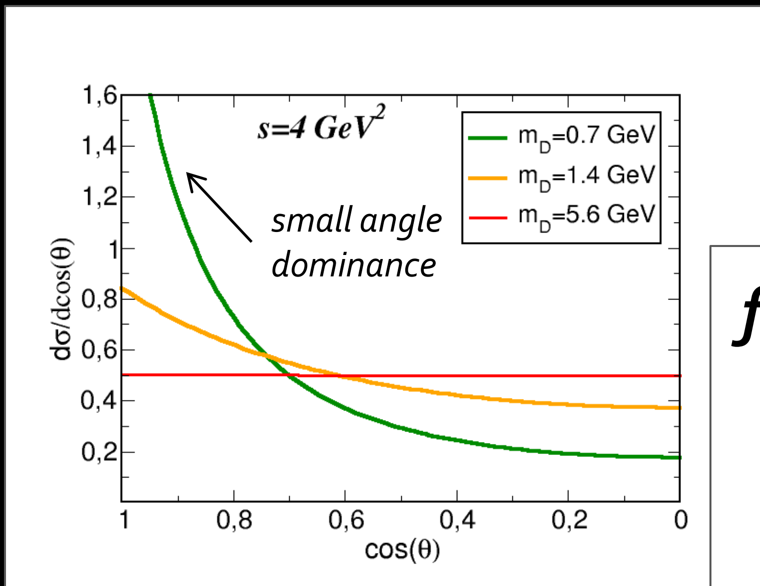


Because of friction, a force F is necessary to have a constant velocity for the upper plane:

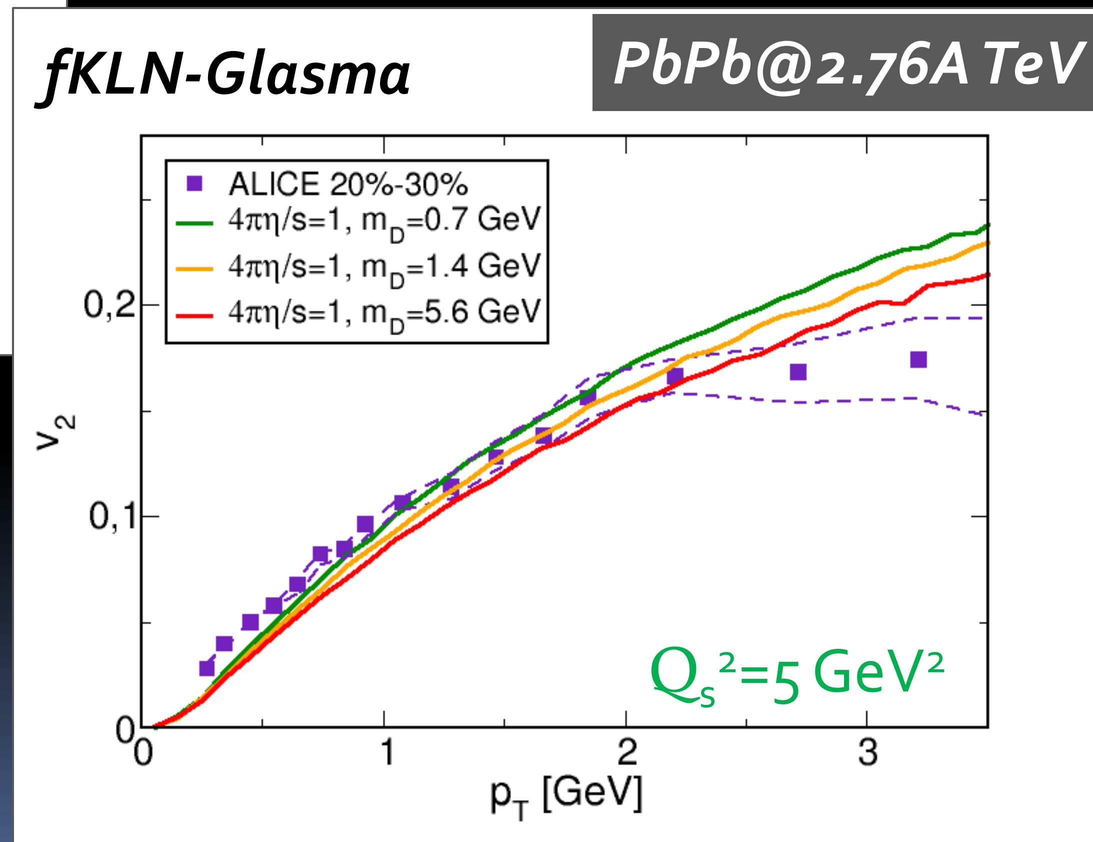
$$\frac{F}{A} = \eta \frac{\partial u}{\partial y}$$

In kinetic theory, the viscosity is described in terms of momentum transfer between different layers of the fluid.

Are micro-details important?



$$\frac{d\sigma_{gg\to gg}}{dt} = \frac{9\pi^2\alpha_s^2}{2}\frac{1}{(t-m_D^2)^2}\left(1+\frac{m_D^2}{s}\right)$$



Same cross section used in:

Zhang *et al.*, PLB 455 (1999)

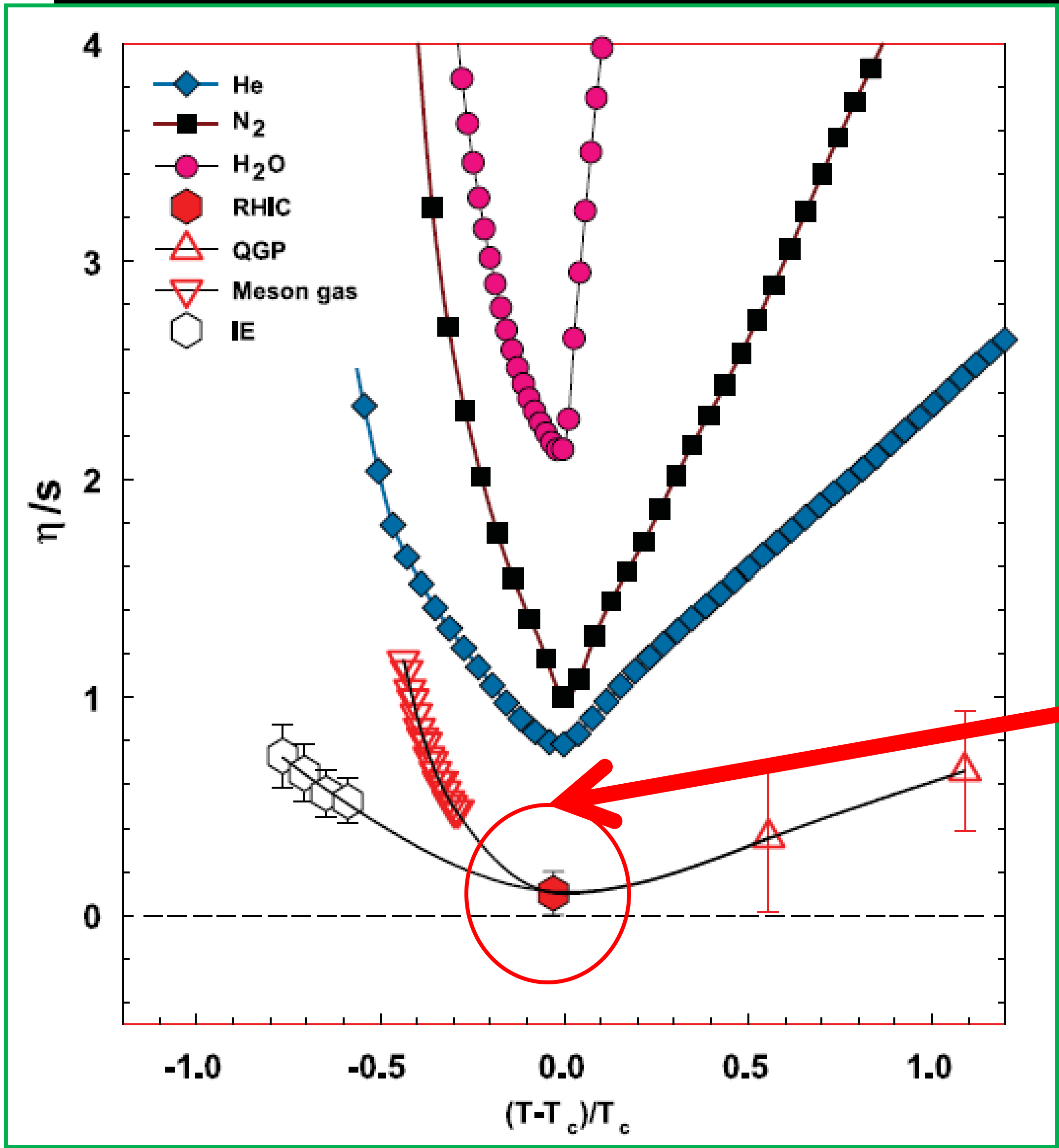
Molnar and Gyulassy, NPA 697 (2002)

Greco et al., PLB 670 (2009)

Increasing m_D makes the cross section isotropic. However:
Strong change of the cross section does not result in a strong change of the elliptic flow.

M. R. *et al.*, *work in progress*

eta/s of QGP



Lacey *et al.*, PRL **98**, 092301 (2007)

Also (almost) perfect liquid

Similar figure in:
Kovtun *et al.* in PRL **94**, 111691 (2005)
Csernai *et al.*, PRL **97**, 152303 (2006)

Quark-Gluon-Plasma

Perfect quantum fluid, according to:
Policastro et al., **PRL 87 (2001)**

Lattice computations

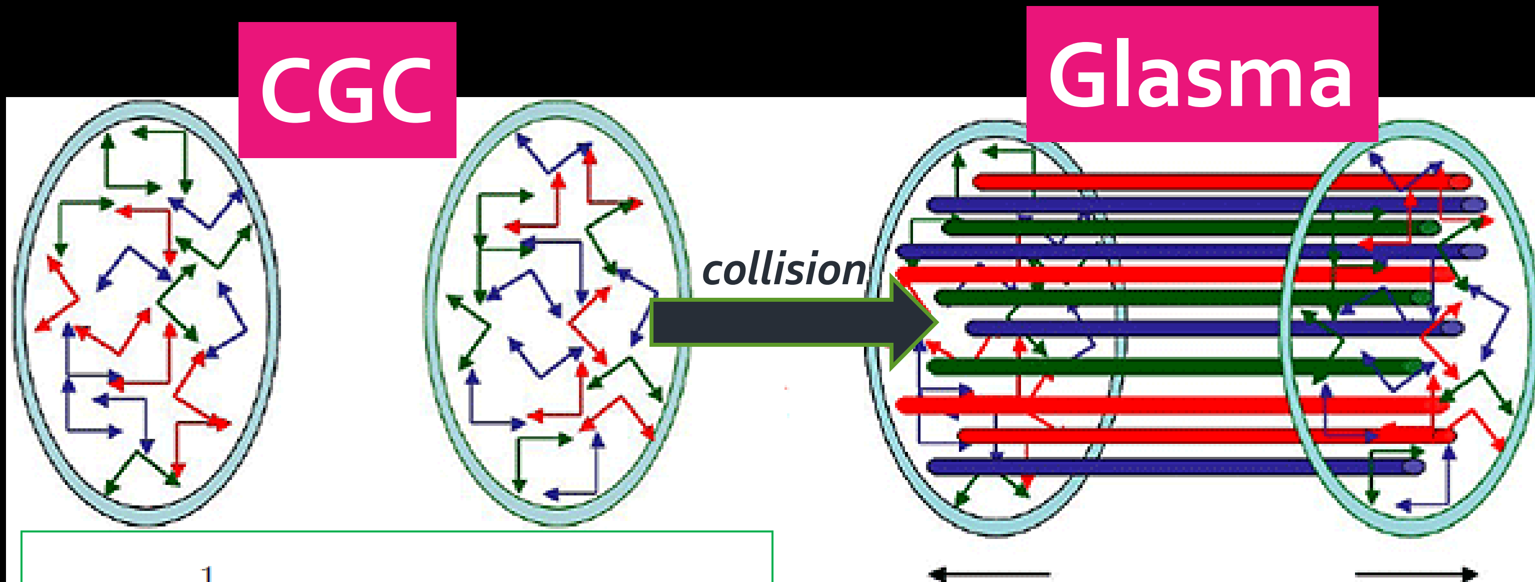
Nakamura and Sakai, **2005**
Abuki *et al.*, **2010**

Meson gas computation

Chen and Nakano, **2006**

McLerran and Venugopalan, PRD **49**, 2233 (1994)
McLerran and Venugopalan, PRD **49**, 3352 (1994)

Initial condition: CYM-Glasma



$$\mathcal{L} = \underbrace{-\frac{1}{2}\,\mathrm{tr}\,F_{\mu\nu}F^{\mu\nu}}_{\text{gluon dynamics}} + \underbrace{(J_1^\mu + J_2^\mu)}_{\text{fast partons}} A_\mu$$

$$[D_\mu, F^{\mu\nu}] = J^\nu$$

Reviews/Lectures

McLerran, 2011
Iancu, 2009
McLerran, 2009
Lappi, 2010
Gelis, 2010
Fukushima, 2011

Classical Yang-Mills spectrum

Mehtar-Tani *et al.*, NPA**846**, (2010)

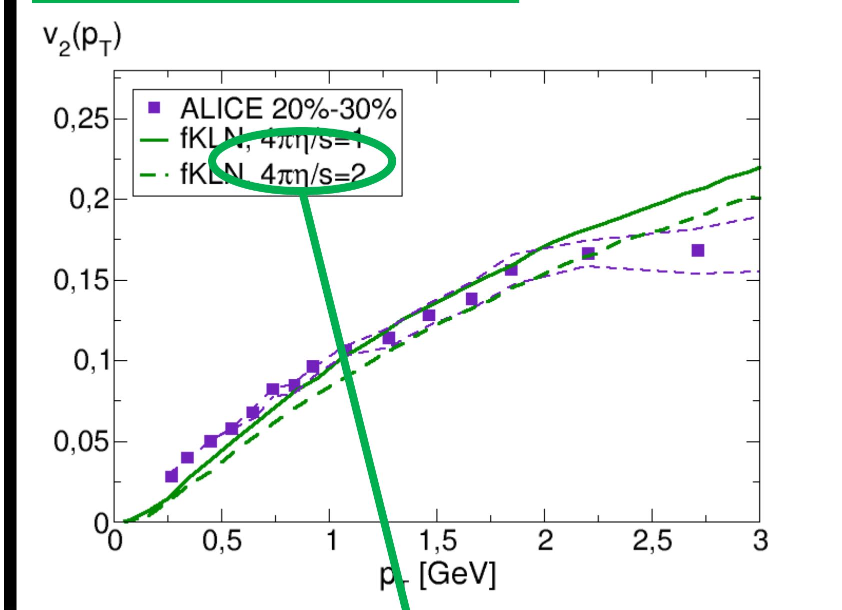
Exempli gratia:
Mode decomposition, assuming free dispersion law

$$\frac{dN}{dy d^2\boldsymbol{k}_T} = \frac{1}{(2\pi)^2}\frac{1}{|\boldsymbol{k}_T|}\left[\frac{1}{\tau}\boldsymbol{E}_a(\boldsymbol{k}_T)\cdot\boldsymbol{E}_a(-\boldsymbol{k}_T) + \tau\pi_a(\boldsymbol{k}_T)\pi_a(-\boldsymbol{k}_T)\right]$$

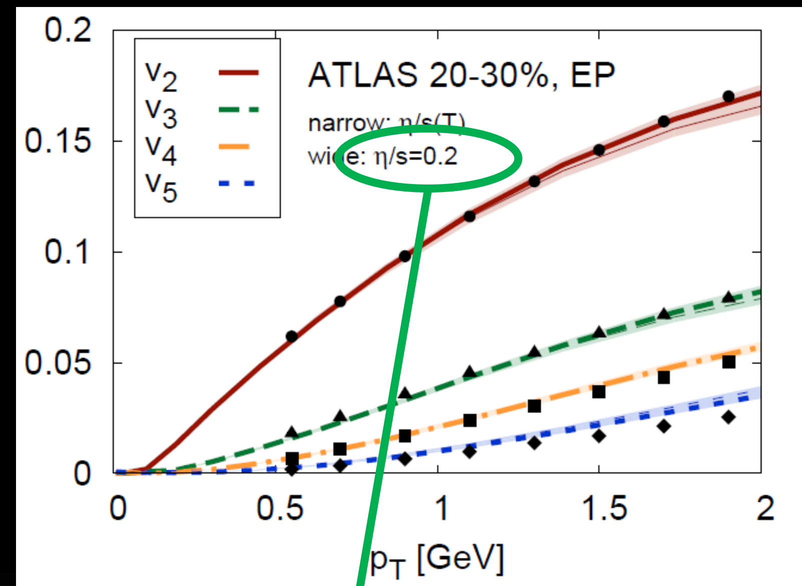
M. R. *et al.*, *work in progress*

Comparison with CYM

PbPb@2.76 TeV



Gale *et al.*, PRL 110, 012302 (2013)

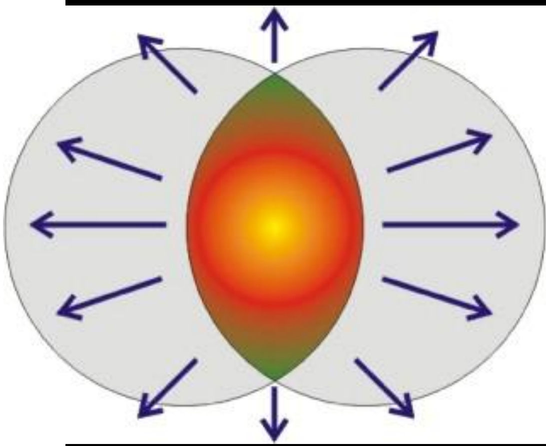


Corresponds to eta/s between 1 and 2

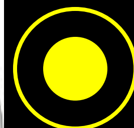
Corresponds to eta/s=2.5

Implementing the proper initial condition in momentum space, as well as in configuration space, leads to a smaller v2 and to a different ***estimate of eta/s.***

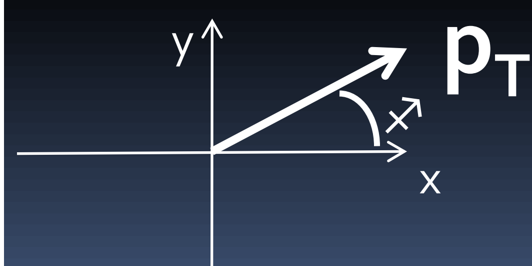
Elliptic flow in RHICs, 1



Collision direction



Impact parameter direction

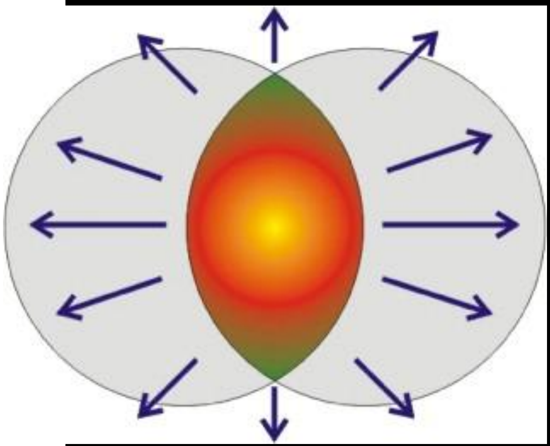


Particle multiplicity in momentum space

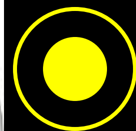
$$\frac{d^3N}{dy p_T dp_T d\phi} = \frac{1}{2\pi}\frac{d^2N}{dy p_T dp_T}\left[1 + 2v_1(y, p_T)\cos\phi + 2v_2(y, p_T)\cos 2\phi + \ldots\right]$$

Fourier decomposition in terms of harmonics in the transverse plane.

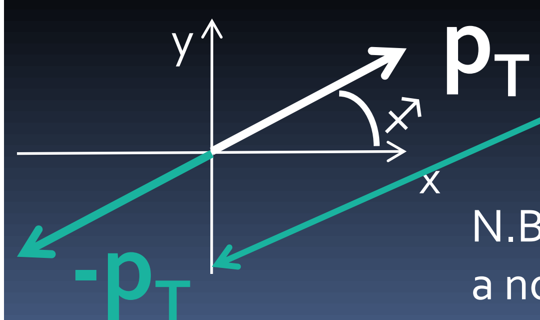
Elliptic flow in RHICs, 1



Collision direction



Impact parameter direction



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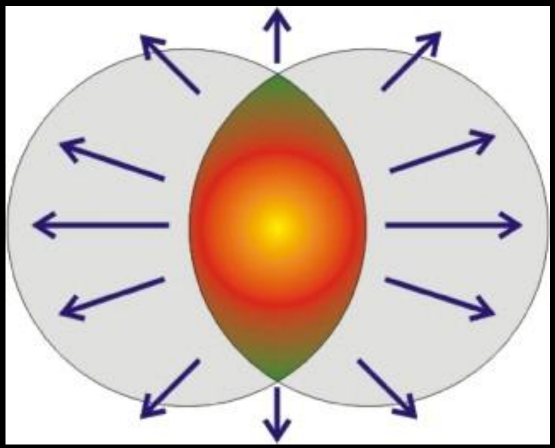
<u>It vanishes for symmetry reasons</u>:
The distribution would be different for the transformation p_T-> - p_T, in disagreement with the symmetry of the problem.

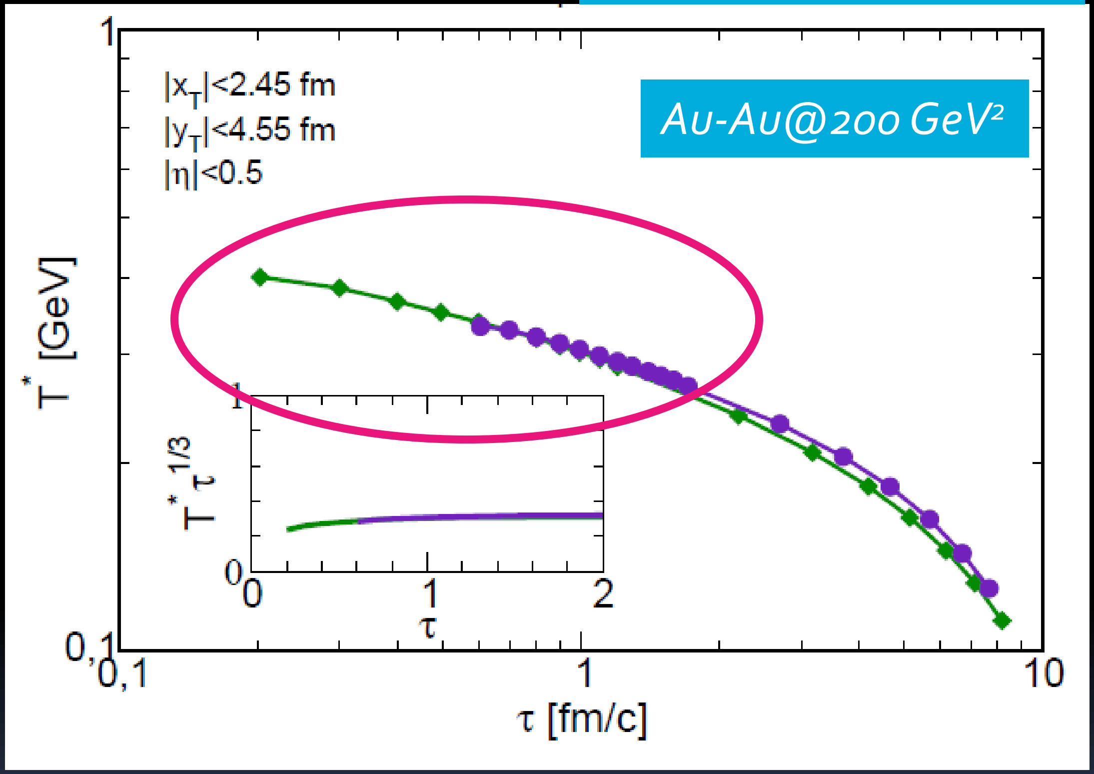
N.B.: both STAR and ALICE collaborations report a nonvanishing measured v1, which is however due to *initial state fluctuations* which we neglect, see e.g.
G. Eyyubova, Acta Phys. Pol. B Proceedings Supplement 5 (2012)

M. R. *et al.*, *work in progress*

Heavy Ion Collisions

Temperature evolution





Initial temperature: ***0.34 GeV***

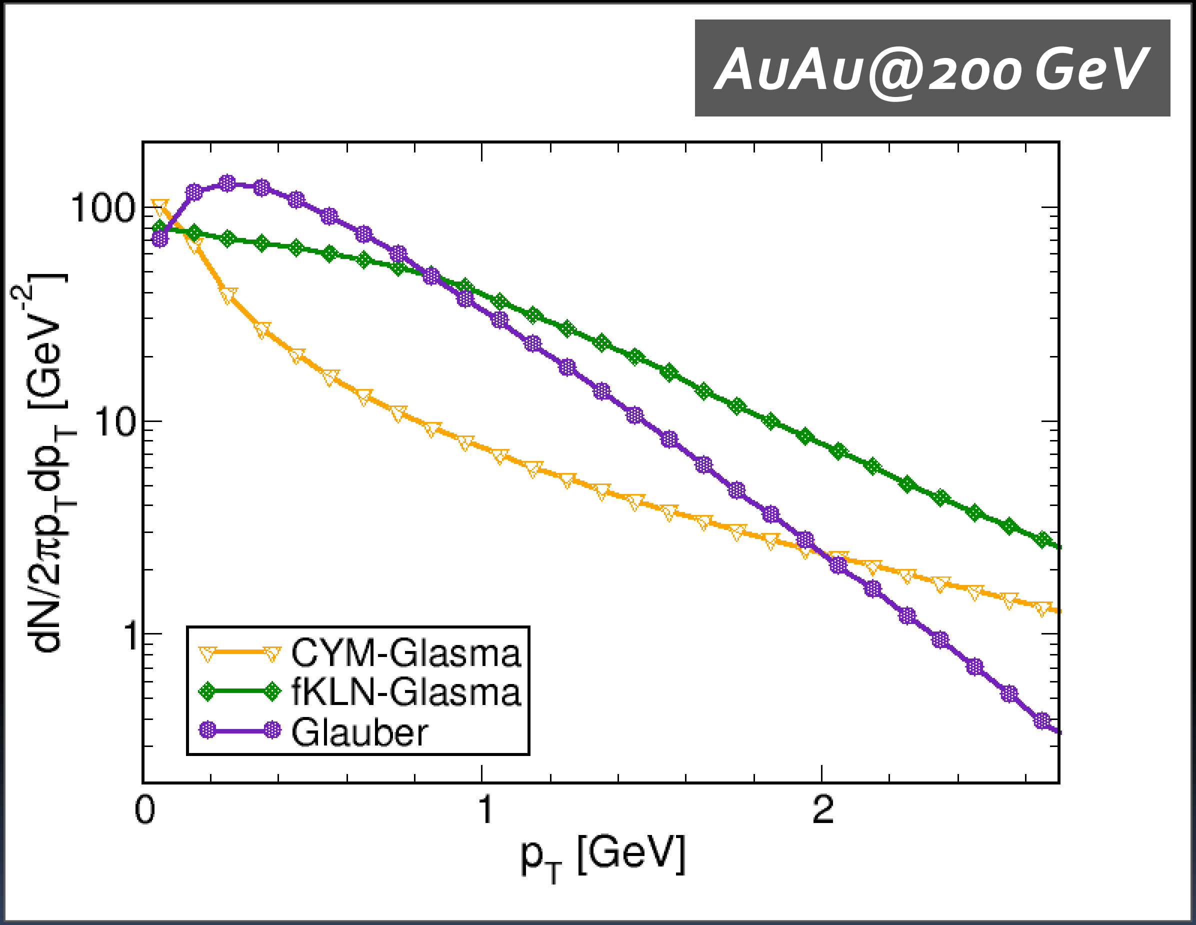
to be compared with

QCD pseudo-critical Tc: ***0.15 GeV*** [Y. Aoki et al., Nature 443 (2006)]

Given the large temperature involved, a description in terms of partons rather than hadrons is appropriated.

M. R. *et al., work in progress*

Initial spectra: summary

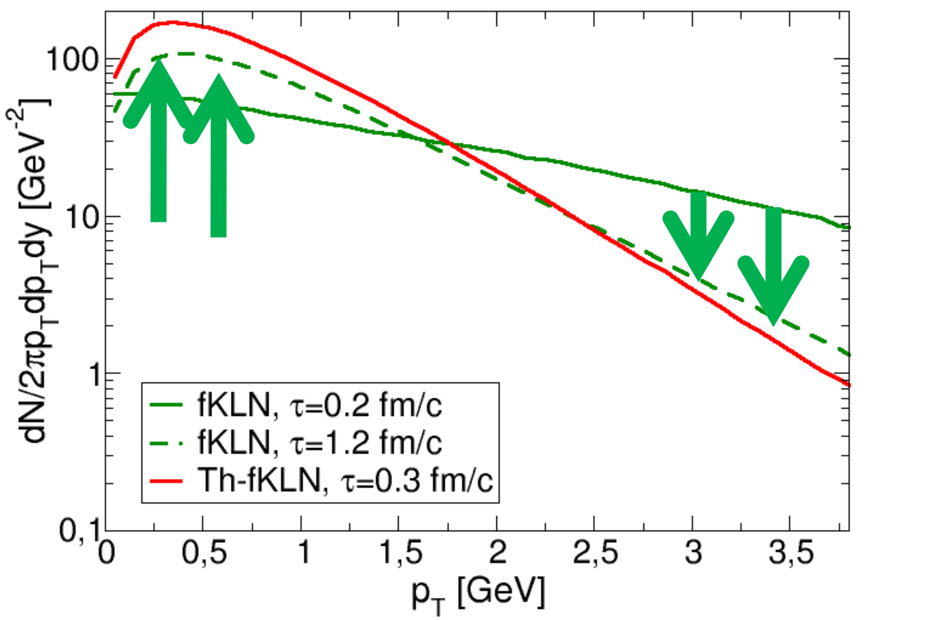


CYM data taken from:
Mehtar-Tani *et al.*, NPA**846**, (2010).
See also: Lappi, PLB**703**, (2011).

M. R. *et al.*, *work in progress*

Initial condition and thermalization

PbPb@2.76 TeV



We use Transport to simulate a fluid at ***fixed*** eta/s; the ***cross section*** is ***computed*** in ***each phase space cell*** to give the wished value of eta/s.

Fast thermalization

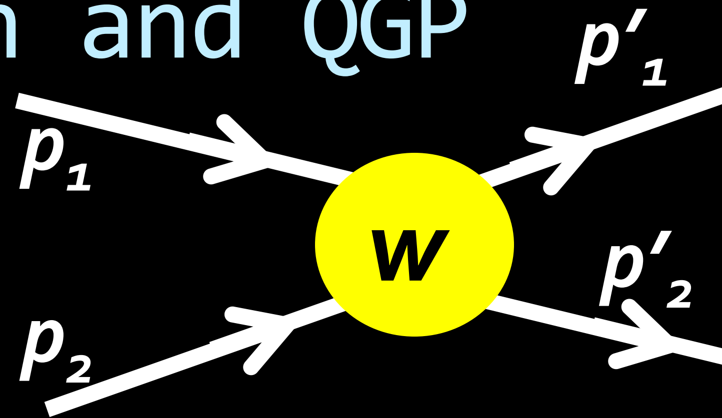
$$\sigma_{tot} = \frac{\langle p \rangle}{\rho\, g(a)} \frac{1}{\eta/s}$$

Not so surprising:
Because eta/s is fixed, there are large cross sections which naturally lead to fast thermalizatio

However, interesting:
We have dynamics in the early stages of the simulation, which prepares the momentum distribution to build up the elliptic flow.

Boltzmann equation and QGP

$$C[f] = \frac{1}{2}\int d\boldsymbol{p}_2 \int d\boldsymbol{p}'_1 \int d\boldsymbol{p}'_2 \; w(12 \to 1'2') \times [f(\boldsymbol{x},\boldsymbol{p}'_1,t)f(\boldsymbol{x},\boldsymbol{p}'_2,t) - f(\boldsymbol{x},\boldsymbol{p}_1,t)f(\boldsymbol{x},\boldsymbol{p}_2,t)]$$

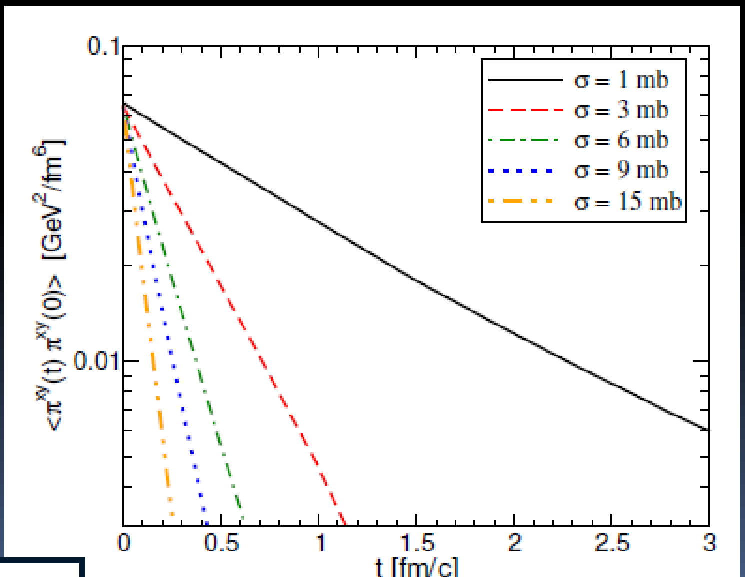


Details about the microscopic processes leading to dissipation and local equilibration enter into the equation only via w(12->1'2').

Common use of kinetic theory:
(.) fix a microscopic process;
(.) compute its rate;
(.) insert the latter into C[f];
(.) compute the evolution of ***f***.

$$\eta = \frac{1}{T}\int_0^\infty dt \int_V d^3x \, \langle \pi^{xy}(\mathbf{x},t)\pi^{xy}(\mathbf{0},t)\rangle$$

$$\langle \pi^{xy}(t)\pi^{xy}(0)\rangle = \langle \pi^{xy}(0)\pi^{xy}(0)\rangle e^{-t/\tau}$$



Plumari *et al.*, Phys. Rev. C86 (2012).

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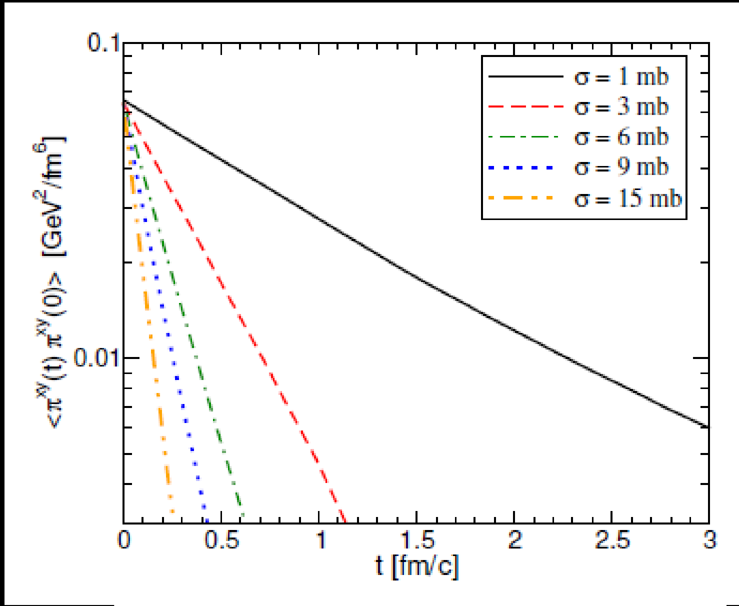
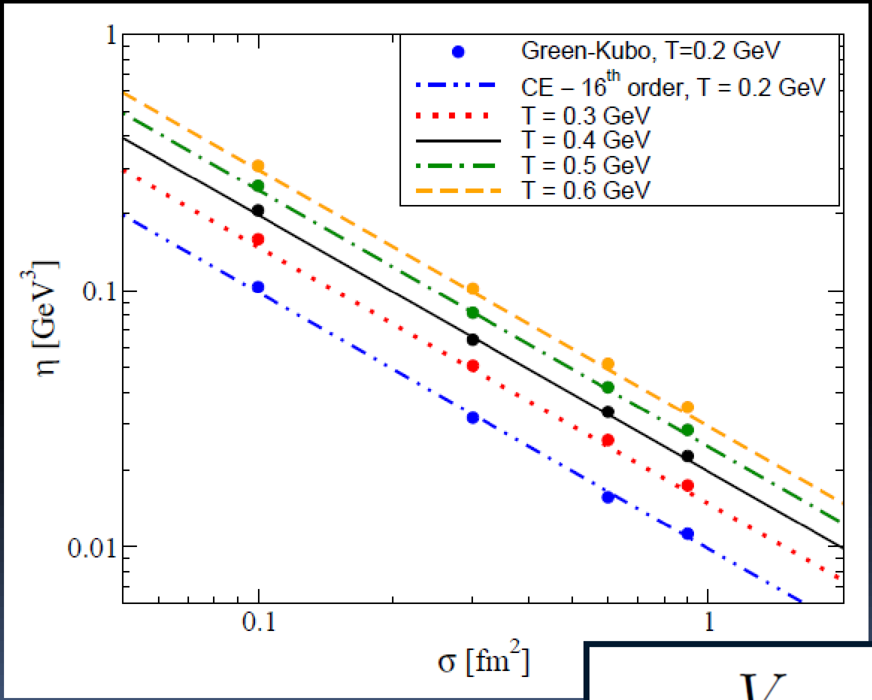


Plumari *et al.*, Phys. Rev. C86 (2012).

These results lead to:



$$\eta = \frac{V}{T}\langle \pi^{xy}(0)\pi^{xy}(0)\rangle \tau$$

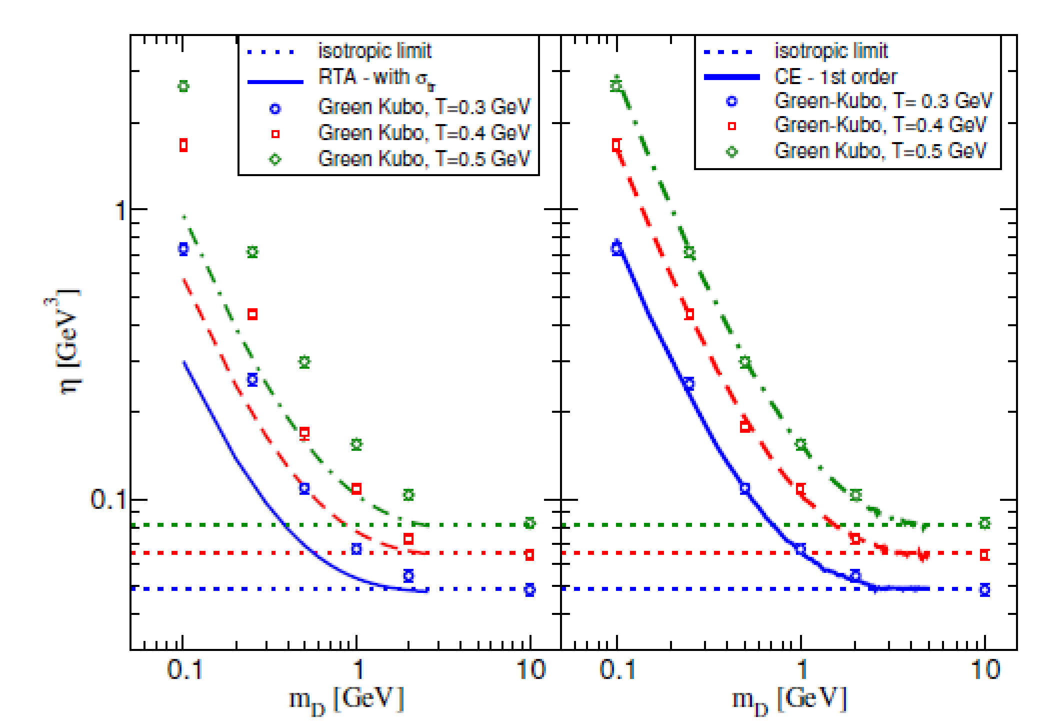
Boltzmann equation and QGP

Viscosity of a gluon plasma

Plumari *et al.*, Phys. Rev. C86 (2012).

$$\frac{d\sigma^{gg\to gg}}{dq^2} = \frac{9\pi\alpha_s^2}{2}\frac{1}{(q^2+m_D^2)^2}$$

depends on the angle between ingoing and outgoing momenta



Boltzmann equation and QGP

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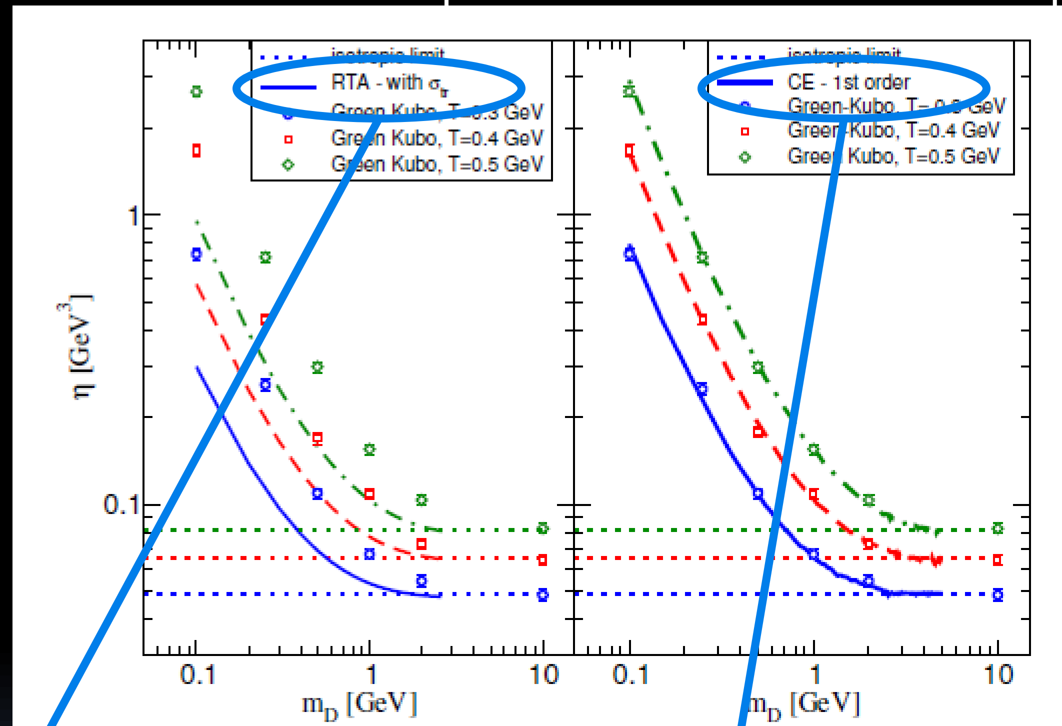
depends on the angle between ingoing and outgoing momenta

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{\nabla}\right) f(\boldsymbol{x},\boldsymbol{p},t) = C[f]$$

Plumari *et al.*, Phys. Rev. C86 (2012).



RTA

$$C[f] \approx -\frac{f - f_{eq}}{\tau}$$

***R**elaxation **T**ime **A**pproximation*

CE

$$f = f_{eq} + \delta f \Rightarrow C[f_{eq} + \delta f]$$

***C**hapman-**E**nskog*

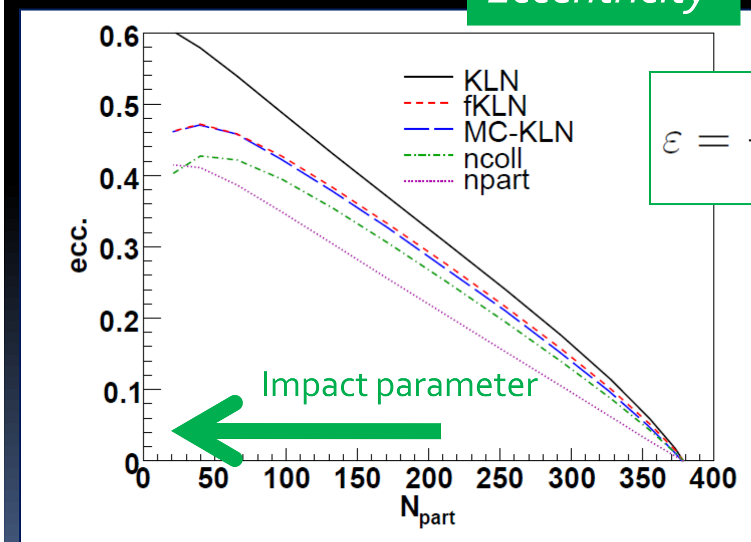
Elliptic flow from Hydro

Glauber: $\eta/s \approx \frac{1}{4\pi}$

CGC: $\eta/s \approx \frac{2}{4\pi}$

The difference can be understood in terms of different initial eccentricity

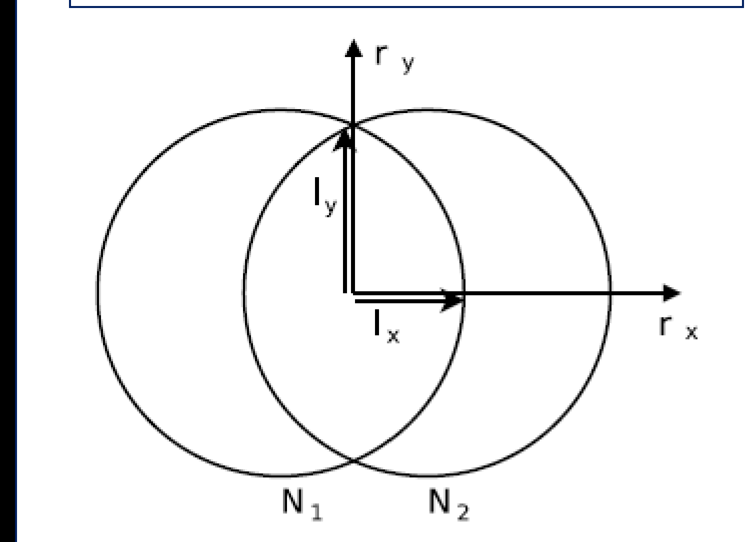
Eccentricity



$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$

Drescher and Nara, PRC **75** (2007) 034905
Adil *et al.*, nucl-th/0605012

$$\frac{dN}{d^2 r_\perp dy} \sim \min(Q^2_{s,1}(y, r_\perp), Q^2_{s,2}(y, r_\perp))$$



l_x $\rho_{\rm CGC}(r_x, 0) \sim Q^2_{s,1}(r_x, 0) \sim n_{\rm part,1}(r_x, 0)$

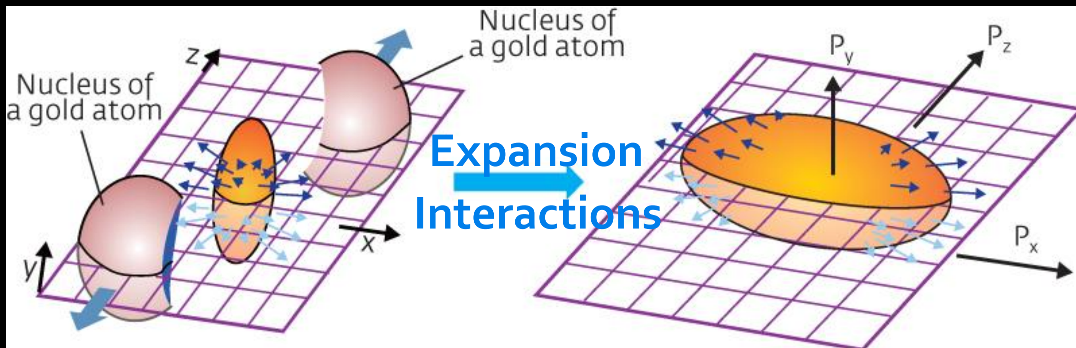
$-l_x$ $\rho_{\rm CGC}(r_x, 0) \sim Q^2_{s,2}(r_x, 0) \sim n_{\rm part,2}(r_x, 0)$

$l_x, -l_x$ $\rho_{\rm Glauber}(r_x, 0) \sim n_{\rm part,1}(r_x, 0) + n_{\rm part,2}(r_x, 0)$

l_y
$$\rho_{\rm CGC}(0, r_y) \sim (n_{\rm part,1}(0, r_y) + n_{\rm part,2}(0, r_y))/2 \sim \rho_{\rm Glauber}(0, r_y) .$$

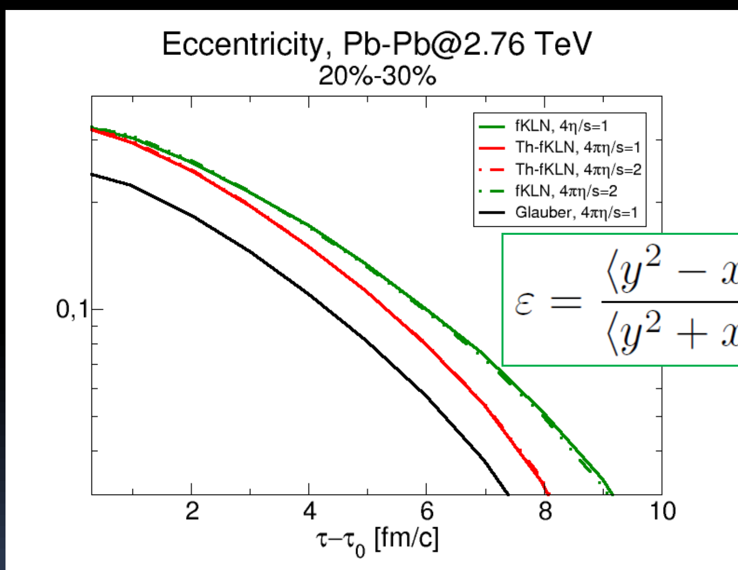
M. R. *et al.*, *work in progress*

Understanding flow

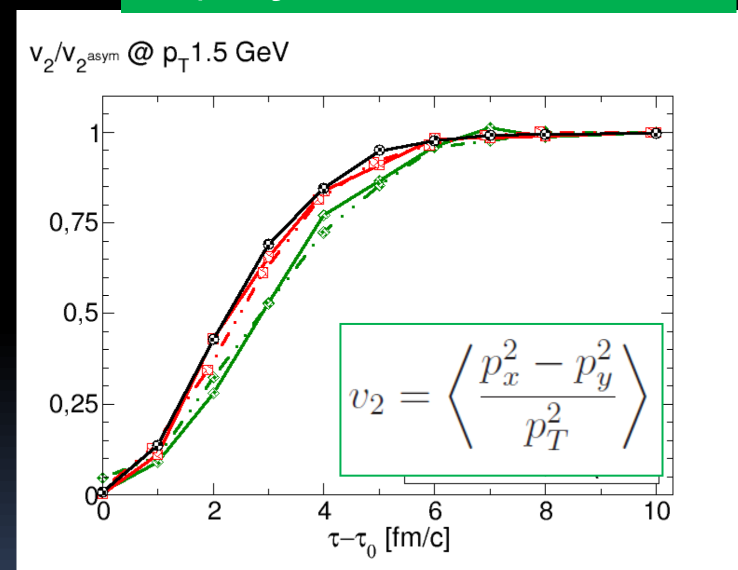


***Flow** equivalent to **transfer of anisotropy**, from transverse coordinate space to momentum space*

Elliptic flow, PbPb @ 2.76 TeV



$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$



$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_T^2} \right\rangle$$

Larger eccentricity at $t-t_0>0$ implies less flow, by definition, hence a smaller v_2.

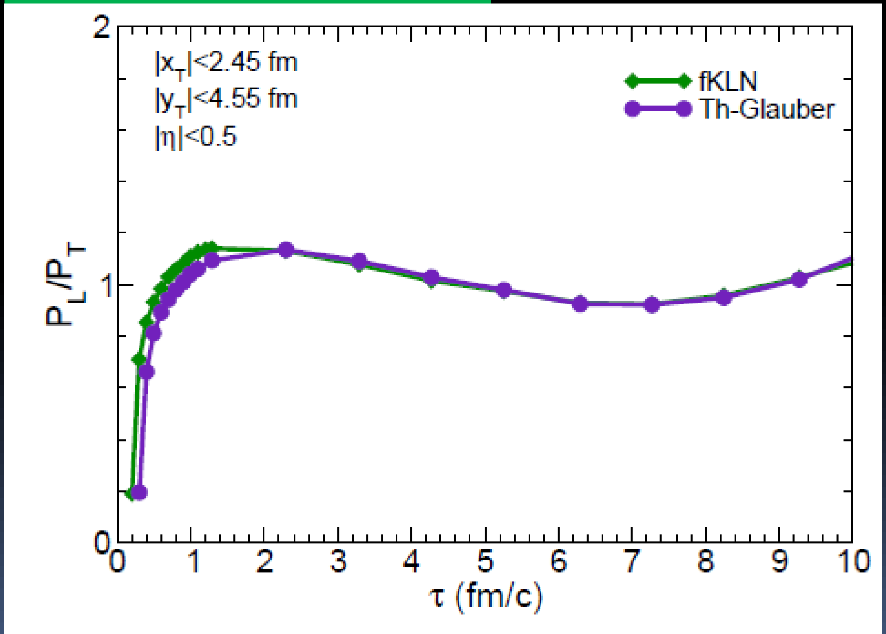
M. R. *et al., work in progress*

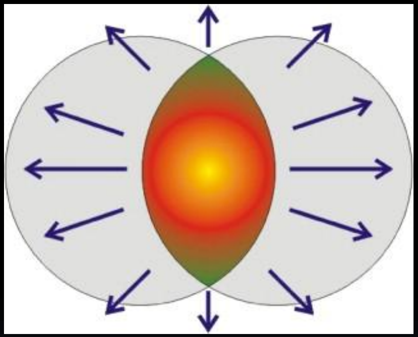
Fireball Isotropization

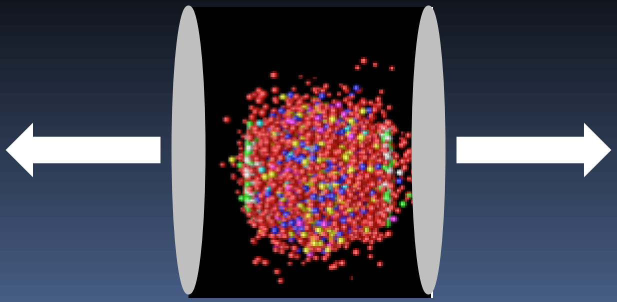
$$T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{E} f(x,p)$$

$$P_T = \frac{1}{V} \int_\Omega d^2x_\perp d\eta \, \frac{T_{xx}+T_{yy}}{2} \; ,$$
$$P_L = \frac{1}{V} \int_\Omega d^2x_\perp d\eta \, T_{zz} \; ,$$

PbPb@2.76 TeV

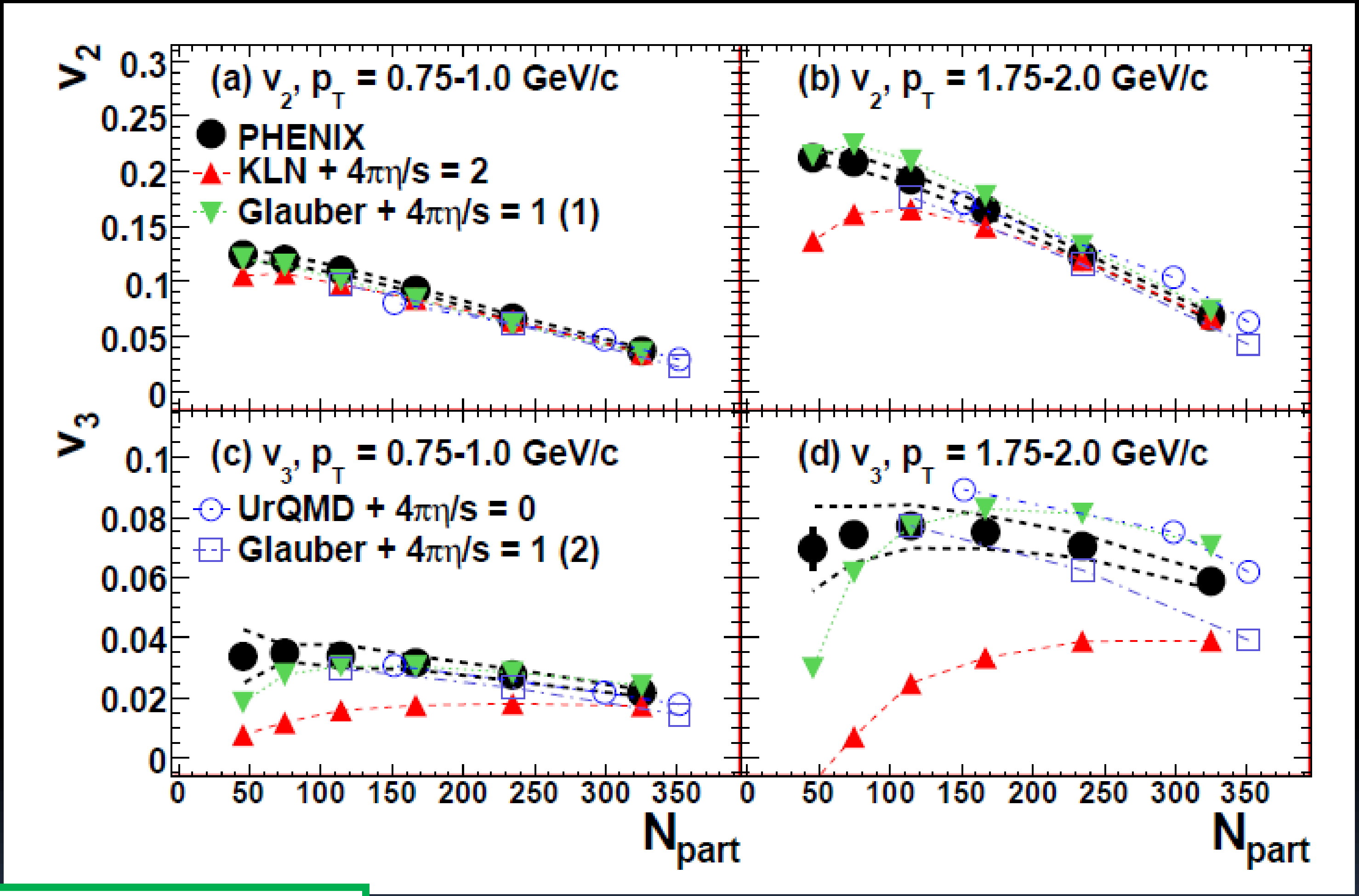






Complete isotropization in strong coupling

Why eta/s is important?



Glauber: $\eta/s \approx \dfrac{1}{4\pi}$

CGC: $\eta/s \approx \dfrac{2}{4\pi}$

Adare *et al.*, [PHENIX Collaboration], PRL **107**, 252301 (2011).

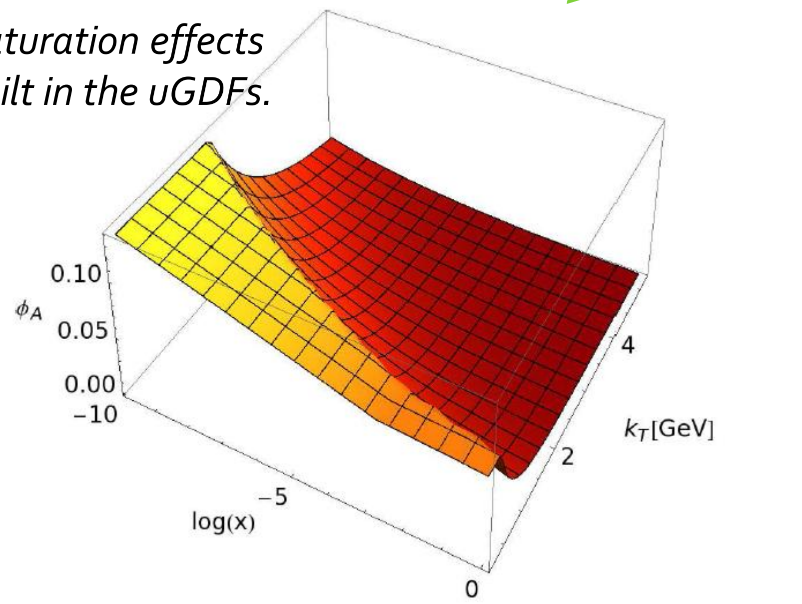
The value of eta/s affects the theoretical estimate of the higher harmonics.

Initial condition: fKLN-Glasma

(f)KLN spectrum

$$\frac{dN_g}{d^2x_\perp dy} \propto \int \frac{d^2p_T}{p_T^2}\int_0^{p_T} d^2k_T \alpha_s(Q^2) \times\phi_A\left(x_A, \frac{(p_T+k_T)^2}{4}; \boldsymbol{x}_\perp\right) \times\phi_B\left(x_B, \frac{(p_T-k_T)^2}{4}; \boldsymbol{x}_\perp\right)$$

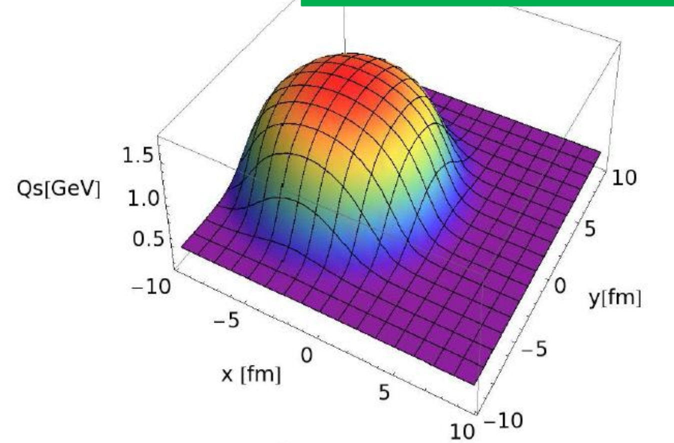
Saturation effects built in the uGDFs.



Nardi *et al.*, Nucl. Phys. A**747**, 609 (2005)
Kharzeev *et al.*, Phys. Lett. B**561**, 93 (2003)
Nardi *et al.*, Phys. Lett. B**507**, 121 (2001)
Drescher and Nara, PRC**75**, 034905 (2007)
Hirano and Nara, PRC**79**, 064904 (2009)
Hirano and Nara, Nucl. Phys. A**743**, 305 (2004)
Albacete and Dumitru, arXiv:1011.5161[hep-ph]
Albacete *et al.*, arXiv:1106.0978 [nucl-th]

Saturation scale ***Qs*** depends on:
1.) ***position in transverse plane***;
2.) ***gluon rapidity***.

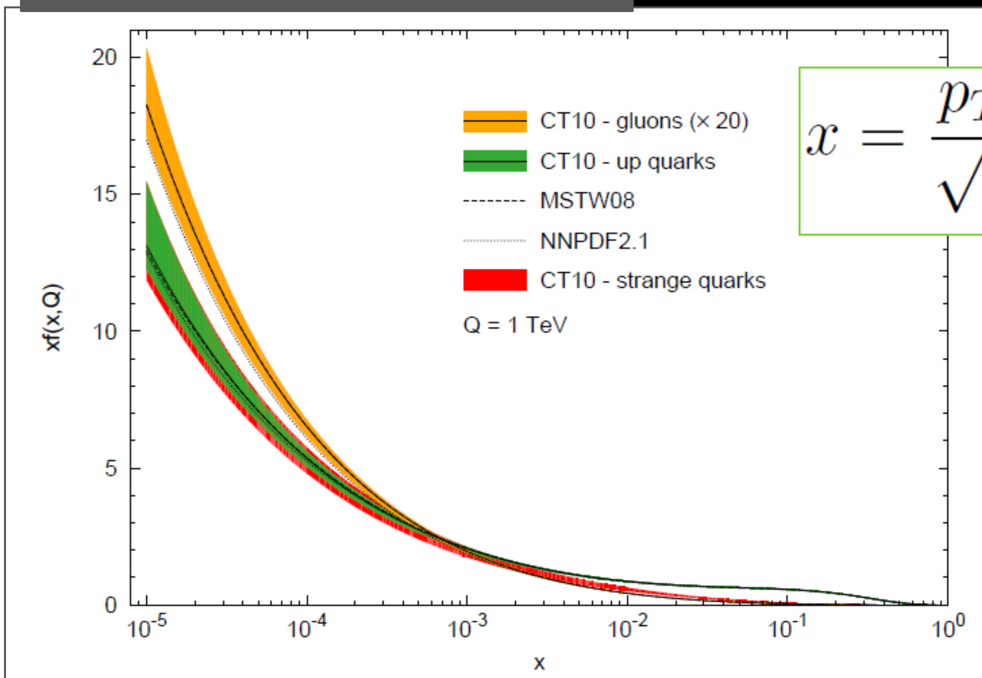
PbPb@2.76A TeV



McLerran and Venugopalan, PRD **49**, 2233 (1994)
McLerran and Venugopalan, PRD **49**, 3352 (1994)

Saturation in a nutshell

Parton distribution functions

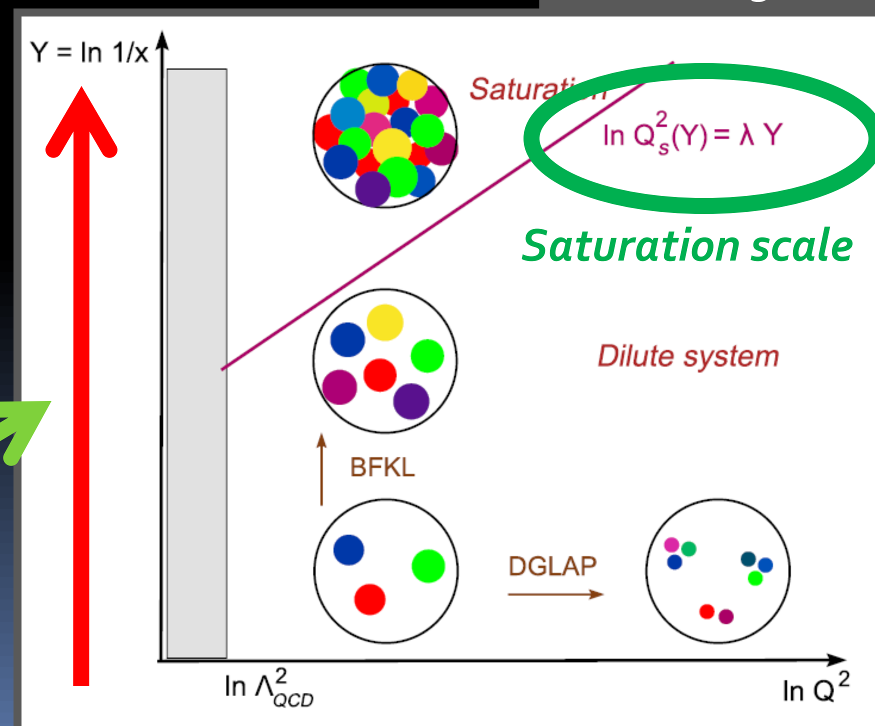


$$x = \frac{p_T}{\sqrt{s}}$$

Brandt and Klasen,arXiv:1305.5677

*Direction of **decreasing** transverse partons area ($1/Q^2$)*

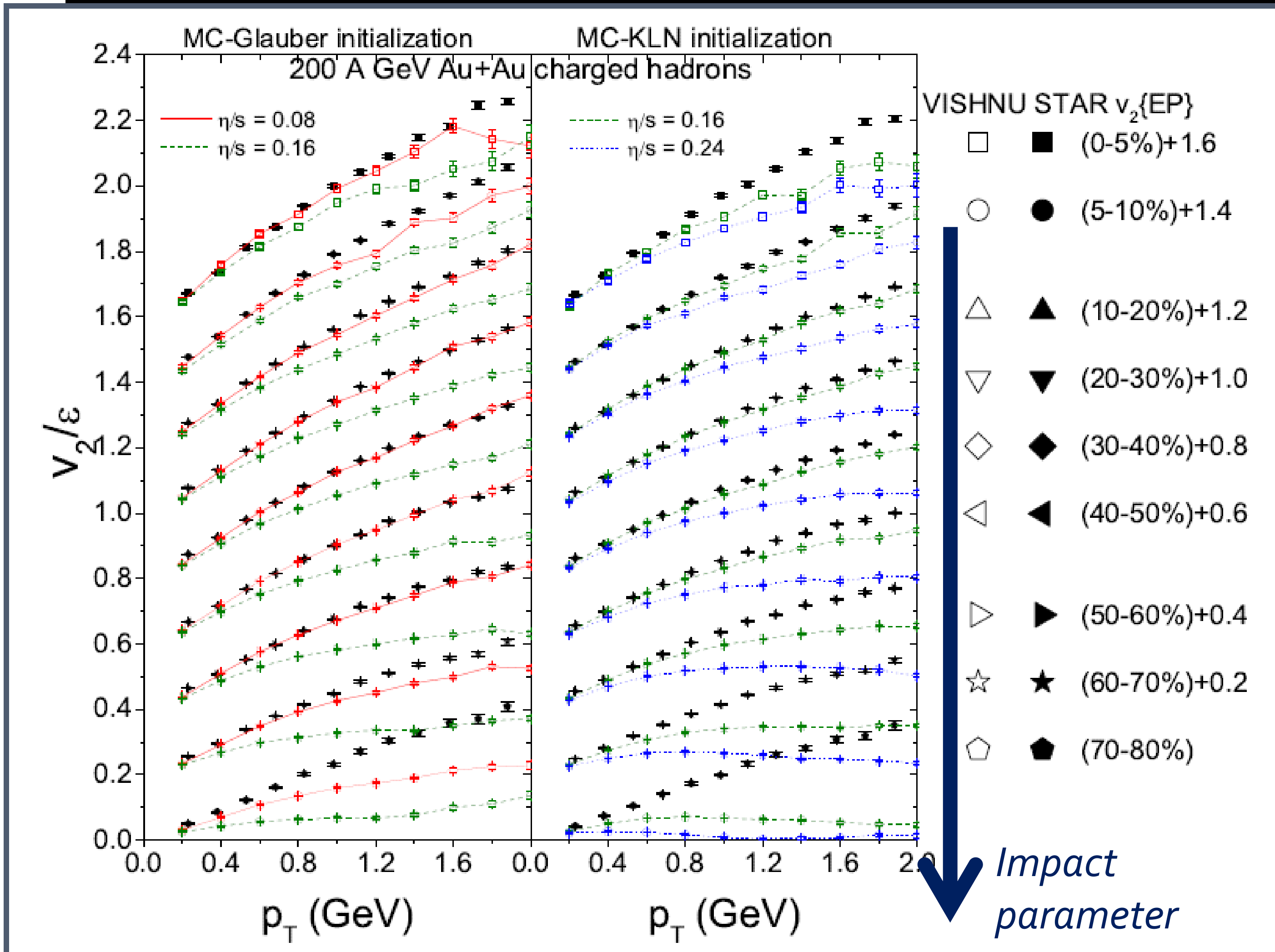
Phase Diagram



*Direction of **increasing** gluon number*

The system evolves to a dense state, characterized by a large occupation number: Color Glass Condensate (CGC)

Elliptic flow from Hydro



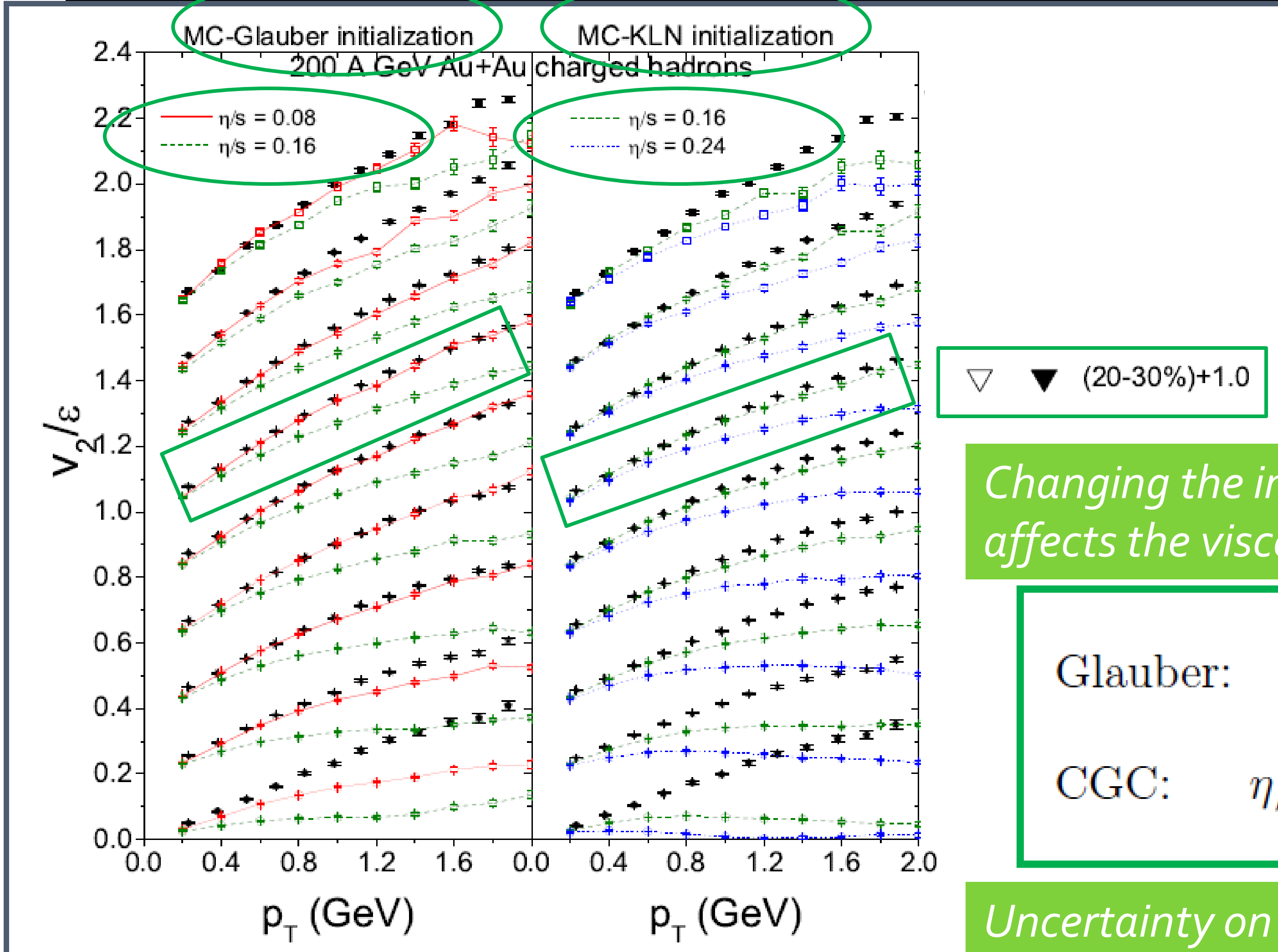
Heinz *et al.*, PRC **83**, 054910 (2011)

Hydro simulations: fireball treated as a ***fluid*** with a given ***shear viscosity.***

ASSUMPTION:
fluid is thermalized in both cases.
Free streaming up to thermalization time.

Elliptic flow data *are useful to estimate the* ***shear viscosity*** *of the QGP.*

Elliptic flow from Hydro



Heinz *et al.*, PRC **83**, 054910 (2011)

Hydro simulations: fireball treated as a ***fluid*** with a given ***shear viscosity.***

Changing the initial condition affects the viscosity of the fireball:

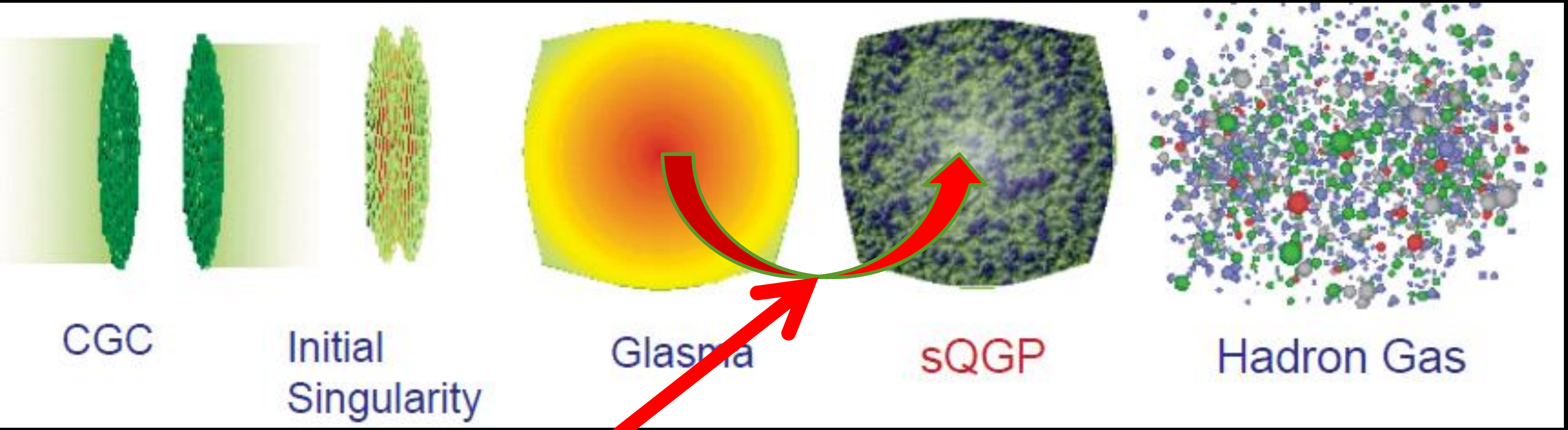
$$\text{Glauber:} \quad \eta/s \approx \frac{1}{4\pi}$$

$$\text{CGC:} \quad \eta/s \approx \frac{2}{4\pi}$$

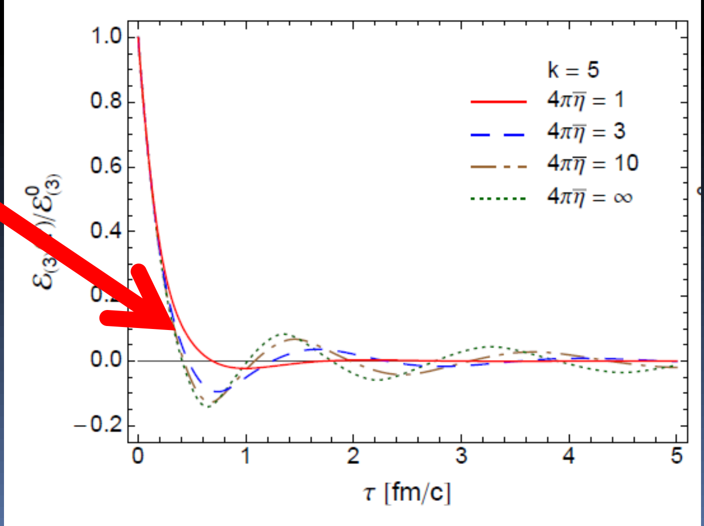
Uncertainty on the initial condition implies uncertainty on the ratio eta/s of the produced quark-gluon plasma.

“Standard Model” for HICs

L. McLerran, 1011.3204 [hep-th]



Ryblewski and Florkowski, PRD88 (2013)

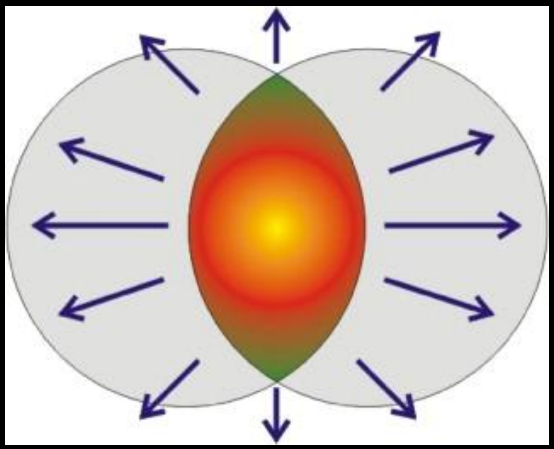


Fast Glasma decay to parton liquid

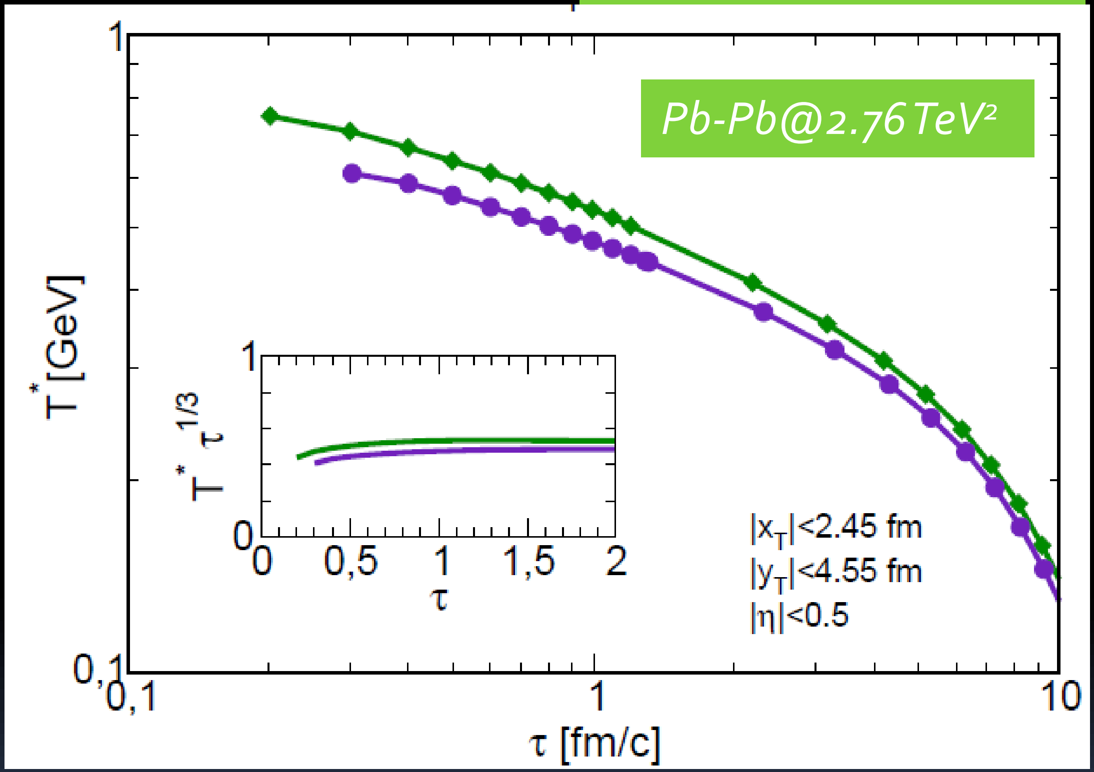
Initial time for the kinetic evolution

M. R. *et al.*, *work in progress*

Heavy Ion Collisions

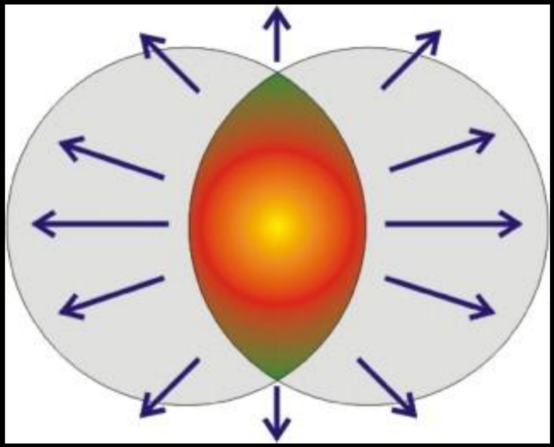


Temperature evolution



M. R. *et al., work in progress*

Heavy Ion Collisions



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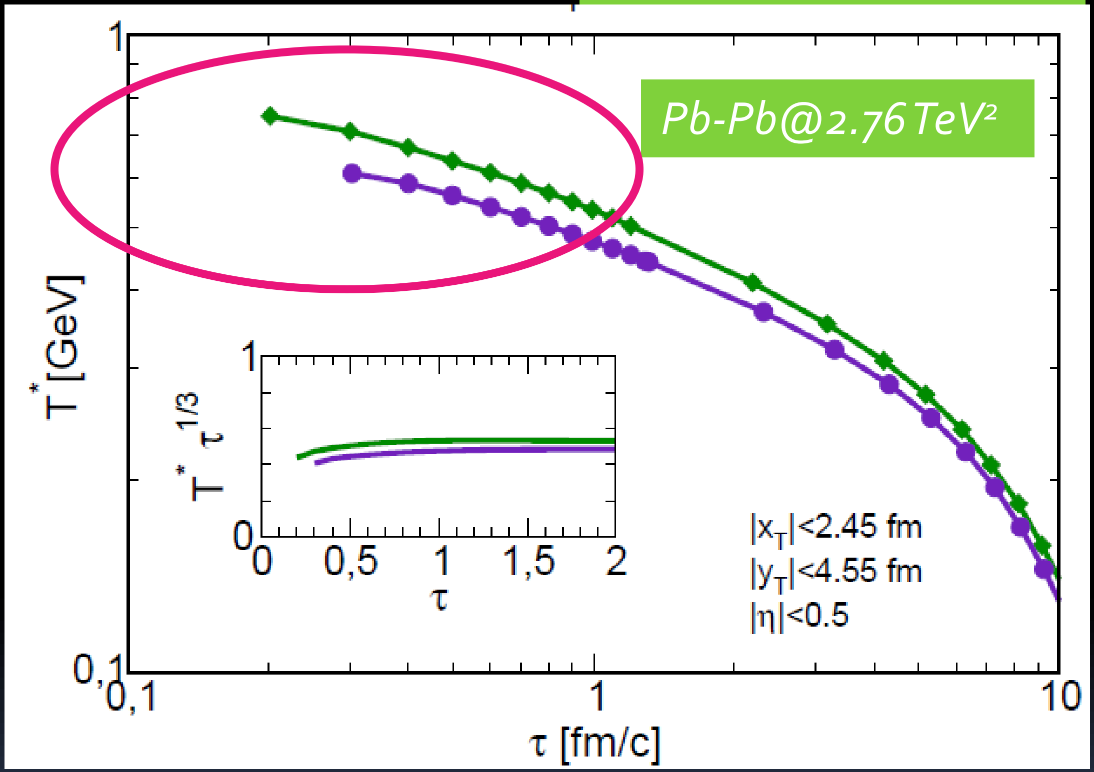

Initial temperature: ***0.55 GeV***

to be compared with

QCD pseudo-critical Tc: ***0.15 GeV*** [Y. Aoki et al., Nature 443 (2006)]

Given the large temperature involved, a description in terms of partons rather than hadrons is appropriated.

Boltzmann equation and QGP

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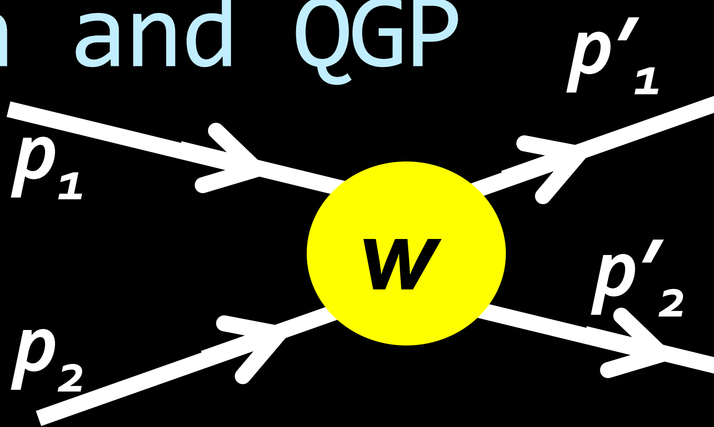


Details about the microscopic processes leading to dissipation and local equilibration enter into the equation only via w(12->1'2').

Common use of kinetic theory:

(.) fix a microscopic process;

(.) compute its rate;

(.) insert the latter into C[f];

(.) compute the evolution of ***f***.

Example: computation of shear viscosity by means of Green-Kubo relation:

$$\eta = \frac{1}{T}\int_0^\infty dt \int_V d^3x \, \langle \pi^{xy}(\mathbf{x},t)\pi^{xy}(\mathbf{0},t)\rangle$$

M. S. Green, 1954.
R. Kubo, 1957.

Boltzmann equation and QGP

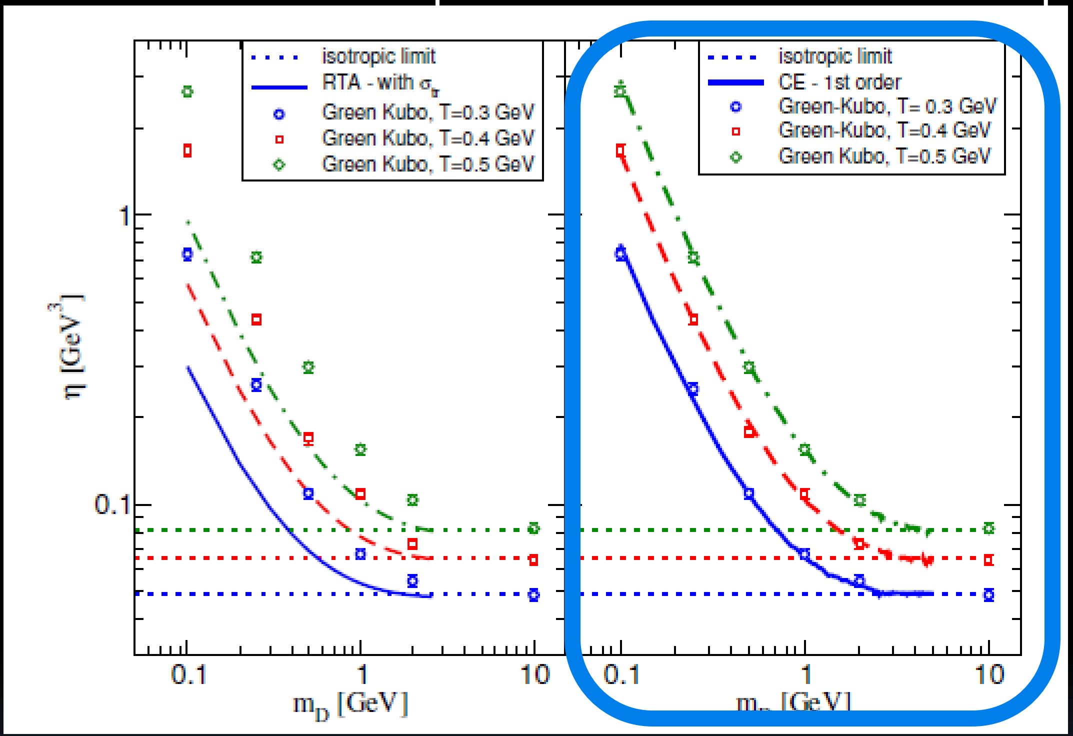
Viscosity of a gluon plasma

Plumari *et al.*, Phys. Rev. C86 (2012).

$$\frac{d\sigma^{gg\to gg}}{dq^2} = \frac{9\pi\alpha_s^2}{2}\frac{1}{(q^2+m_D^2)^2}$$

depends on the angle between ingoing and outgoing momenta

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{\nabla}\right) f(\boldsymbol{x},\boldsymbol{p},t) = C[f]$$



CE

$$f = f_{eq} + \delta f \Rightarrow C[f_{eq} + \delta f]$$

Chapman-Enskog

CE is a better approximation to the Green-Kubo result. This is a useful observation, since CE offers analytical tool to relate eta to sigma which we use in our transport code.

Initial conditions:summary

IC@RHIC	ecc	Coordinate	Momenta	to (fm/c)
Glauber	0.282	0.85Npart+0.15Ncoll	Thermal	0.6
Th-fKLN (Hydro)	0.326	fKLN	Thermal	0.6
CYM	0.288	Ncoll	CYM	0.1-0.2
fKLN	0.336	fKLN	fKLN	0.1-0.2

IC@LHC	ecc	Coordinate	Momenta	to (fm/c)
Glauber	0.282	0.85Npart+0.15Ncoll	Thermal	0.3
Th-fKLN (Hydro)	0.326	fKLN	Thermal	0.3
CYM	0.288	Ncoll	CYM	0.1-0.2
fKLN	0.336	fKLN	fKLN	0.1-0.2

Initial eccentricities in agreement with previous estimates:

Drescher and Nara, PRC **75** (2007) 034905
Adil *et al.*, nucl-th/0605012
Gale *et al.*, 1209.6330 [nucl-th]
Schenke *et al.*, 1206.6805 [hep-ph]

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CYM	0.288	Ncoll	CYM	0.1-0.2
fKLN	0.336	fKLN	fKLN	0.1-0.2

IC@LHC	ecc	Coordinate	Momenta	to (fm/c)
Glauber	0.282	0.85Npart+0.15Ncoll	Thermal	0.3
Th-fKLN (Hydro)	0.326	fKLN	Thermal	0.3
CYM	0.288	Ncoll	CYM	0.1-0.2
fKLN	0.336	fKLN	fKLN	0.1-0.2

Initial eccentricities in agreement with previous estimates:

Drescher and Nara, PRC **75** (2007) 034905
Adil *et al.*, nucl-th/0605012
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