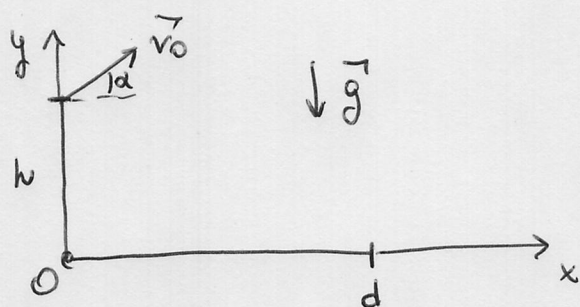


Problema 1



$$1) \begin{cases} x(t) = v_0 \cos \alpha t \\ y(t) = h + v_0 \sin \alpha t - \frac{1}{2} g t^2 \end{cases} \quad (1)$$

Per trovare v_0 impongo che esista $\bar{t}$ t.c. $\begin{cases} x(\bar{t}) = d \\ y(\bar{t}) = 0 \end{cases} \Rightarrow$

$$\begin{cases} d = v_0 \cos \alpha \bar{t} \\ 0 = h + v_0 \sin \alpha \bar{t} - \frac{1}{2} g \bar{t}^2 \end{cases} \Rightarrow \bar{t} = \frac{d}{v_0 \cos \alpha}$$

$$0 = h + v_0 \sin \alpha \frac{d}{v_0 \cos \alpha} - \frac{1}{2} g \frac{d^2}{v_0^2 \cos^2 \alpha}$$

$$\frac{g d^2}{2 v_0^2 \cos^2 \alpha} = h + d \frac{\sin \alpha}{\cos \alpha} \Rightarrow$$

$$v_0 = \sqrt{\frac{g d^2}{2 \cos^2 \alpha} \frac{1}{h + d \frac{\sin \alpha}{\cos \alpha}}}$$

2) Ora conosco v_0 . Eq. (1) è ancora valide. In più

$$\begin{cases} v_x(t) = v_0 \cos \alpha \\ v_y(t) = v_0 \sin \alpha - g t \end{cases}$$

Nel punto più alto della traiettoria $v_y(t) = 0 \Rightarrow$

$$|\vec{v}| = v_x = v_0 \cos \alpha$$

3) Nel punto più alto $v_0 \sin \alpha - g t_{\max} = 0 \Rightarrow t_{\max} = \frac{v_0 \sin \alpha}{g}$

$$\Rightarrow x_{\max} = x(t_{\max}) = \frac{v_0^2 \cos \alpha \sin \alpha}{g}$$

$$4) \quad \vec{v}(t) = v_x(t) \hat{x} + v_y(t) \hat{y} =$$

$$= v_0 \cos \alpha \hat{x} + (v_0 \sin \alpha - gt) \hat{y}$$

$$|\vec{v}| = \sqrt{(v_0 \cos \alpha)^2 + (v_0 \sin \alpha - gt)^2} = \sqrt{v_0^2 + g^2 t^2 - 2v_0 \sin \alpha gt}$$

Per $y_{\max}$ $|\vec{v}| = v_0 \cos \alpha$ come dal punto 2) o

dalle formule pre. con $t_{\max} = \frac{v_0 \sin \alpha}{g}$

$$\Rightarrow \vec{v} = v \hat{T} \Rightarrow \hat{T} = \hat{x} + \frac{v_0 \sin \alpha - gt_{\max}}{v_0 \cos \alpha} \hat{y} = \hat{x}$$

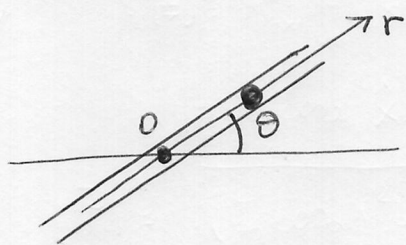
$$\Rightarrow \hat{N} = -\hat{y}$$

$$\vec{a} = \dot{v} \hat{T} + \frac{v^2}{R} \hat{N} = -g \hat{y}$$

$$\dot{v} = \frac{1}{\cancel{\sqrt{v_0^2 + g^2 t^2 - 2v_0 \sin \alpha gt}}} \cdot [\cancel{2g^2 t} - \cancel{2v_0 \sin \alpha g}] =$$

$$= \frac{g[gt - v_0 \sin \alpha]}{\sqrt{v_0^2 + g^2 t^2 - 2v_0 \sin \alpha gt}} \stackrel{\text{per } t = t_{\max}}{=} 0$$

$$\Rightarrow \frac{v^2}{R} (-\hat{y}) = -g \hat{y} \Rightarrow \frac{v_0^2 \cos^2 \alpha}{R} = g \Rightarrow R = \frac{v_0^2 \cos^2 \alpha}{g}$$



$$\begin{cases} r(t) = v_0 t \\ \theta(t) = \omega_0 t \end{cases}$$

$$\begin{cases} \dot{r} = v_0 \\ \dot{\theta} = \omega_0 \end{cases} \quad \begin{cases} \ddot{r} = 0 \\ \ddot{\theta} = 0 \end{cases}$$

$$1) \quad \vec{v} = \dot{r} \hat{e}_r + r \dot{\theta} \hat{e}_\theta = v_0 \hat{e}_r + (v_0 \omega_0 t) \hat{e}_\theta$$

$$|\vec{v}| = \sqrt{v_0^2 + (v_0 \omega_0 t)^2} = v_0 \sqrt{1 + \omega_0^2 t^2}$$

$$2) \quad \vec{a} = (\ddot{r} - r \dot{\theta}^2) \hat{e}_r + (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) \hat{e}_\theta = -v_0 \omega_0^2 t \hat{e}_r + 2 v_0 \omega_0 \hat{e}_\theta$$

$$|\vec{a}| = \sqrt{4 v_0^2 \omega_0^2 + v_0^2 \omega_0^4 t^2} = v_0 \omega_0 \sqrt{4 + \omega_0^2 t^2}$$

3) SR assoluto vs. SR scianaleture

$$\vec{v} = \vec{v}_{tr} + \vec{v}'$$

$$\vec{v}' = v_0 \hat{e}_r$$

$$\vec{v} = v_0 \hat{e}_r + (v_0 \omega_0 t) \hat{e}_\theta$$

$$\Rightarrow \vec{v}_{tr} = \vec{v} - \vec{v}' = (v_0 \omega_0 t) \hat{e}_\theta$$

$$\Rightarrow |\vec{v}_{tr}| = v_0 \omega_0 t$$

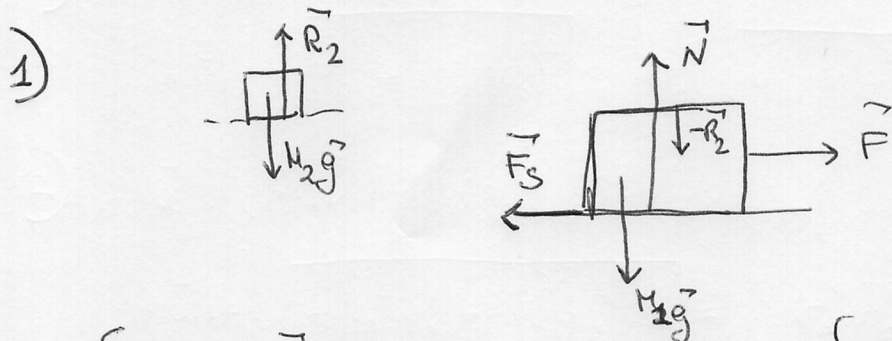
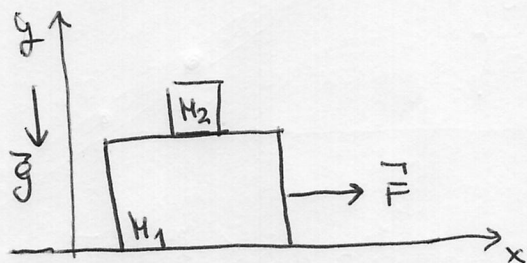
$$4) \quad \vec{a} = \underbrace{\vec{a}'}_0 + \underbrace{\vec{\omega} \times (\vec{\omega} \times \vec{r}')}_0 + \underbrace{\dot{\vec{\omega}} \times \vec{r}'}_0 + 2 \vec{\omega} \times \vec{v}'$$

$$= -\omega_0^2 \vec{r}' + 2 \vec{\omega} \times \vec{v}' = -\omega_0^2 r \hat{e}_r + \underbrace{2 \vec{\omega} \times \vec{v}'}_{\vec{a}_\omega}$$

$$\Rightarrow |\vec{a}_\omega| = |2 \vec{\omega} \times \vec{v}'| = 2 \omega_0 v_0 |\hat{z} \times \hat{e}_r| = 2 \omega_0 v_0$$

Problema 3

4



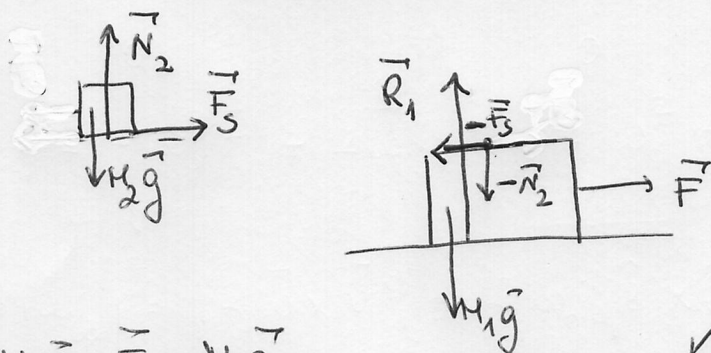
$$\begin{cases} M_2 \vec{g} + \vec{R}_2 = 0 \\ M_1 \vec{g} + \vec{N} - \vec{R}_2 + \vec{F} + \vec{F}_S = 0 \end{cases}$$

$$\begin{cases} -M_2 g + R_2 = 0 & R_2 = M_2 g \\ -M_1 g + N - R_2 = 0 & N = (M_1 + M_2) g \\ F - F_S = 0 & F = F_S \end{cases}$$

Legge dell'attrito statico

$$F = F_S \leq \mu_s N = \mu_s (M_1 + M_2) g \Rightarrow F_{\max} = \mu_s (M_1 + M_2) g$$

2) $\vec{F}$ nota



$$\begin{cases} \vec{N}_2 + M_2 \vec{g} + \vec{F}_S = M_2 \vec{a}_2 \\ \vec{F} - \vec{F}_S + \vec{R}_1 + M_1 \vec{g} - \vec{N}_2 = M_1 \vec{a}_1 \end{cases}$$

acc. di trasl. $\vec{a}_2 = \vec{a}_1 + \vec{a}_2'$
 acc. relative $= 0 \Rightarrow \vec{a}_1 = \vec{a}_2$

$$\begin{cases} +F_S = M_2 a_{1x} \\ N_2 - M_2 g = 0 \end{cases}$$

$$\begin{cases} F - F_S = M_1 a_{1x} \\ -M_1 g + R_1 - N_2 = 0 \end{cases}$$

$$N_2 = M_2 g$$

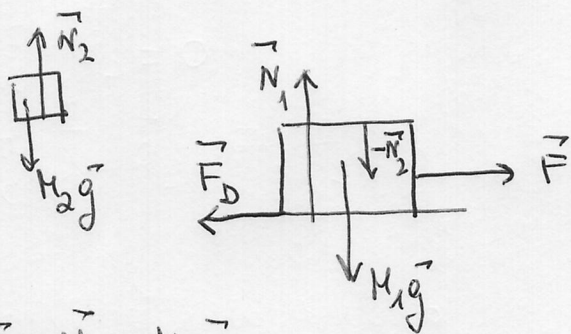
$$a_{1x} = \frac{1}{M_1} (F - F_S)$$

$$F_S = M_2 a_{1x} = \frac{M_2}{M_1} (F - F_S) \Rightarrow F_S \left(1 + \frac{M_2}{M_1}\right) = \frac{M_2}{M_1} F$$

$$F_S = \frac{M_2}{M_1 + M_2} F \leq \mu_s N_2$$

$$\Rightarrow \mu_s \geq \frac{M_2 F}{M_1 + M_2} \frac{1}{N_2} = \frac{M_2}{M_1 + M_2} \frac{F}{M_2 g} = \frac{F}{g(M_1 + M_2)}$$

3)



$$\begin{cases} M_2 \vec{g} + \vec{N}_2 = M_2 \vec{a}_2 \\ \vec{F}_D + \vec{N}_1 - \vec{N}_2 + M_1 \vec{g} + \vec{F} = M_1 \vec{a}_1 \end{cases}$$

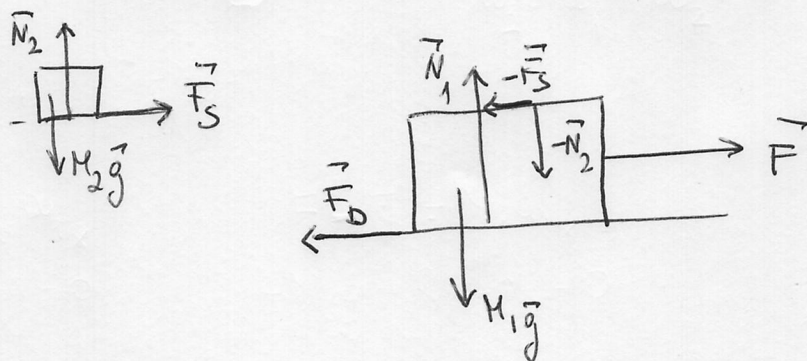
$$\begin{cases} -M_2 g + N_2 = 0 & N_2 = M_2 g \\ a_{2x} = 0 \\ -F_D + F = M_1 a_{1x} \\ N_1 - M_1 g - N_2 = 0 \end{cases}$$

$$\Rightarrow N_1 = (M_1 + M_2)g$$

$$M_1 a_{1x} = F - F_D = F - \mu_D (M_1 + M_2)g$$

$$a_{1x} = \frac{F - \mu_D (M_1 + M_2)g}{M_1}$$

4)



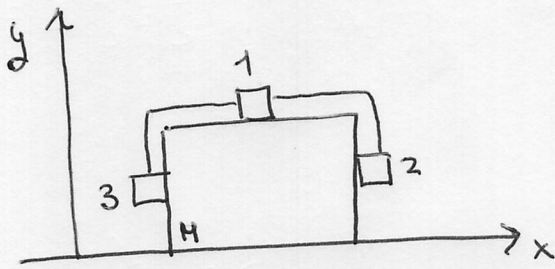
$$\begin{cases} \vec{N}_2 + M_2 \vec{g} + \vec{F}_S = M_2 \vec{a}_2 \\ \vec{F} + \vec{F}_D - \vec{F}_S + \vec{N}_1 - \vec{N}_2 + M_1 \vec{g} = M_1 \vec{a}_1 \end{cases}$$

$$\begin{cases} N_2 - M_2 g = M_2 a_{2y} = 0 \\ F_S = M_2 a_{2x} = M_2 a_{1x} \\ F - F_D - F_S = M_1 a_{1x} \\ N_1 - N_2 - M_1 g = 0 \end{cases}$$

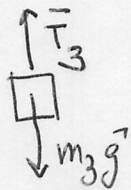
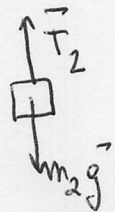
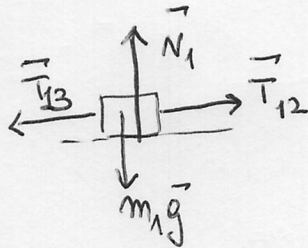
$$\Rightarrow \begin{cases} N_2 = M_2 g \\ F - F_D - M_2 a_{1x} = M_1 a_{1x} \\ N_1 = N_2 + M_1 g \end{cases} \Rightarrow \begin{cases} (M_1 + M_2) a_{1x} = F - F_D = F - \mu_D N_1 \\ N_1 = (M_1 + M_2) g \end{cases}$$

$$\Rightarrow a_{1x} = \frac{F - \mu_D (M_1 + M_2) g}{M_1 + M_2}$$

$$= \frac{F}{M_1 + M_2} - \mu_D g$$



1)



$$m_1 \vec{a}_1 = \vec{N}_1 + m_1 \vec{g} + \vec{T}_{12} + \vec{T}_{13}$$

$$m_2 \vec{a}_2 = m_2 \vec{g} + \vec{T}_2$$

$$m_3 \vec{a}_3 = m_3 \vec{g} + \vec{T}_3$$

$$\begin{cases} m_1 a_{1x} = T_{12} - T_{13} \\ 0 = N_1 - m_1 g \\ m_2 a_{2y} = T_2 - m_2 g \\ m_3 a_{3y} = T_3 - m_3 g \end{cases} \quad (1)$$

Richiedo $a_{1x} = a_{2y} = a_{3y} = 0$

$$\text{So } T_{12} = T_2 \quad \Rightarrow \quad T_{13} = T_3$$

$$\begin{cases} T_{12} - T_{13} = 0 \\ N_1 - m_1 g = 0 \\ T_2 - m_2 g = 0 \\ T_3 - m_3 g = 0 \end{cases}$$

$$T_2 = T_3$$

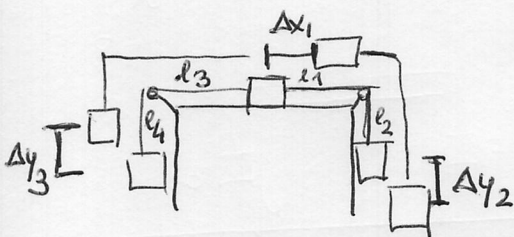


$$m_2 g = T_2 = T_3 = m_3 g$$

$$\Rightarrow m_2 = m_3$$

ora intuitivo!

2) Ritorno al sistema (1) e impongo il fatto che i fili sono inestensibili $\Rightarrow$



$$l_1 + l_2 = l_1 - \Delta x_1 + l_2 + \Delta y_2$$

$$\Rightarrow \Delta x_1 = \Delta y_2 \Rightarrow |v_{1x}| = |v_{2y}|$$

$$|a_{1x}| = |a_{2y}|$$

$$l_3 + l_4 = l_3 + \Delta x_1 + l_4 - \Delta y_3$$

$$\Rightarrow \Delta x_1 = \Delta y_3 \Rightarrow |v_{1x}| = |v_{3y}|$$

$$|a_{1x}| = |a_{3y}|$$

Per i segni, con la scelta degli assi, trovo

$$a_{1x} = a_{3y} = -a_{2y} \Rightarrow$$

$$\begin{cases} m_1 a_{1x} = T_2 - T_3 \\ -m_2 a_{1x} = T_2 - m_2 g \\ m_3 a_{1x} = T_3 - m_3 g \end{cases} \Rightarrow$$

$$(m_1 + m_2 + m_3) a_{1x} =$$

$$\cancel{T_2 - T_3} - \cancel{T_2} + m_2 g + \cancel{T_3} - m_3 g$$

$$\Rightarrow a_{1x} = \frac{(m_2 - m_3) g}{m_1 + m_2 + m_3}$$

$$\Rightarrow |a_{1x}| = \frac{|m_2 - m_3| g}{m_1 + m_2 + m_3}$$

$$3) T_2 = m_2 g - m_2 a_{1x} =$$

$$= m_2 g - m_2 \frac{m_2 - m_3}{m_1 + m_2 + m_3} g = \frac{m_1 m_2 + m_2 m_3 + m_2 m_3}{m_1 + m_2 + m_3} g$$

$$= \frac{(m_1 + 2m_3) m_2}{m_1 + m_2 + m_3} g$$

$$T_3 = m_3 g + m_3 a_{1x} =$$

$$= m_3 g + m_3 \frac{m_2 - m_3}{m_1 + m_2 + m_3} g = \frac{m_1 m_3 + m_2 m_3 + m_2 m_3}{m_1 + m_2 + m_3} g = \frac{(m_1 + 2m_2) m_3}{m_1 + m_2 + m_3} g$$

4)

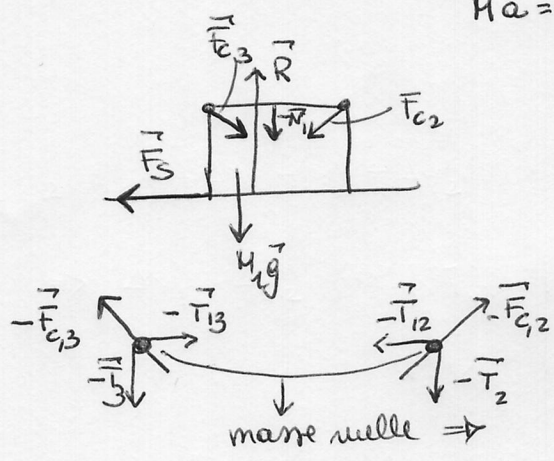
$$m_1 a_{1x} = T_2 - T_3$$

$$0 = N_1 - m_1 g$$

$$m_2 a_{2y} = T_2 - m_2 g$$

$$m_3 a_{3y} = T_3 - m_3 g$$

$$M \vec{a} = M \vec{g} + \vec{R} + \vec{F}_S + \vec{F}_{c2} + \vec{F}_{c3}$$



$$\vec{F}_{c3} = -\vec{T}_{13} - \vec{T}_3$$

$$\vec{F}_{c2} = -\vec{T}_{12} - \vec{T}_2$$

$$M a_x = -F_S + T_3 - T_2 = 0$$

$$F_S = T_3 - T_2 =$$

$$0 = -Mg + R - T_3 - T_2 - N_1$$

$$= \frac{(m_1 + 2m_2) m_3 g}{m_1 + m_2 + m_3} - \frac{(m_1 + 2m_3) m_2 g}{m_1 + m_2 + m_3}$$

$$F_S = \frac{m_1 m_3 + \cancel{2m_2 m_3} - m_1 m_2 - \cancel{2m_2 m_3}}{m_1 + m_2 + m_3} g = \frac{m_1 (m_3 - m_2)}{m_1 + m_2 + m_3} g$$

$$R = Mg + T_3 + T_2 + N_1 = Mg + \frac{(m_1 + 2m_2) m_3 g}{m_1 + m_2 + m_3} + \frac{(m_1 + 2m_3) m_2 g}{m_1 + m_2 + m_3} + N_1$$

$$= Mg + \frac{m_1 m_3 + \cancel{2m_2 m_3} + m_1 m_2 + \cancel{2m_2 m_3}}{m_1 + m_2 + m_3} g + m_1 g$$

$$= Mg + \frac{m_1 m_3 + m_1 m_2 + 4m_2 m_3}{m_1 + m_2 + m_3} g + m_1 g$$

$$= (M + m_1) + \frac{m_1 m_3 + m_1 m_2 + 4m_2 m_3}{m_1 + m_2 + m_3} g$$

$$F_S \leq \mu_s R \Rightarrow$$

$$\mu_s \geq \frac{\frac{m_1 (m_3 - m_2)}{m_1 + m_2 + m_3}}{(M + m_1) + \frac{m_1 m_3 + m_1 m_2 + 4m_2 m_3}{m_1 + m_2 + m_3}} = \frac{m_1 (m_3 - m_2)}{(M + m_1)(m_1 + m_2 + m_3) + m_1 m_3 + m_1 m_2 + 4m_2 m_3}$$