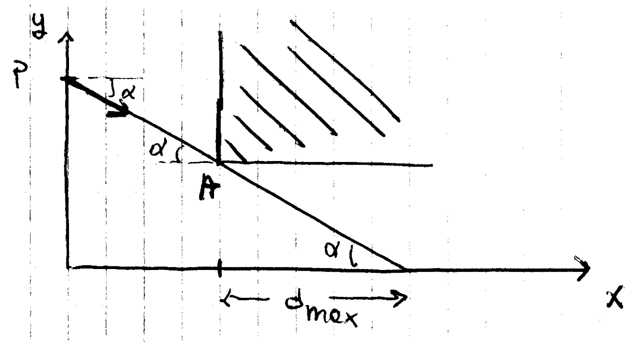
Esercizio n° 1



Eq. traiettoria

$$y = y_P + tg\alpha\, x - \frac{g x^2}{2 v_0^2}(1+tg^2\alpha)$$

Gittata max per

$$\left.\frac{dy}{dx}\right|_{x_A, y_A} = \max =$$

$$\left.\frac{dy}{dx}\right|_A = tg\alpha - \frac{g \cancel{2}}{\cancel{2} v_0^2} x_A (1+tg^2\alpha) = tg\alpha \text{ se } v_0 \to \infty$$

1) Massima distanza si ha per v_0 infinita e moto sulla retta tratteggiata in figura.

$$\frac{d_{max} + x_A}{y_P} = \frac{1}{tg\alpha} \qquad tg\alpha = \frac{y_P}{d_{max} + x_A}$$

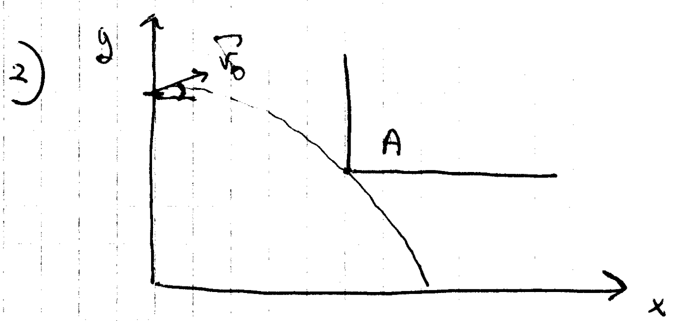
$$tg\alpha = \frac{y_P - y_A}{x_A} \Rightarrow$$

$$\frac{y_P}{d_{max} + x_A} = \frac{y_P - y_A}{x_A}$$

$$d_{max} + x_A = \frac{y_P x_A}{y_P - y_A} \qquad d_{max} = x_A\left(\frac{y_P}{y_P - y_A} - 1\right)$$

$$= x_A \frac{y_A}{y_P - y_A} = 30\,m$$

2)



v_0 data

$$\begin{cases} y(t) = y_P + v_0 t \sin\alpha - \frac{1}{2} g t^2 \\ x(t) = v_0 t \cos\alpha \end{cases}$$

Devo "toccare" A $\Rightarrow$

$$\begin{cases} y_A = y_P + v_0 t \sin\alpha - \frac{1}{2} g t^2 \\ x_A = v_0 t \cos\alpha \end{cases} \Rightarrow \quad t = \frac{x_A}{v_0 \cos\alpha}$$

$$y_A = y_P + x_A \,tg\alpha - \frac{1}{2} g \frac{x_A^2}{v_0^2 \cos^2\alpha} = y_P + x_A \,tg\alpha - \frac{g\, x_A^2}{2 v_0^2} (1 + tg^2\alpha)$$

Risolvo per $tg\alpha$:

$$\frac{g\, x_A^2}{2 v_0^2} tg^2\alpha - x_A \,tg\alpha + \frac{g\, x_A^2}{2 v_0^2} - y_P + y_A = 0$$

$$\Delta = x_A^2 - 4 \frac{g x_A^2}{2 v_0^2} \left(\frac{g x_A^2}{2 v_0^2} - \underbrace{(y_P - y_A)}_{h} \right)$$

$$= x_A^2 - \frac{g^2 x_A^4}{v_0^4} + \frac{2 g x_A^2 h}{v_0^2} > 0 \quad (= 23.56\ m^2)$$

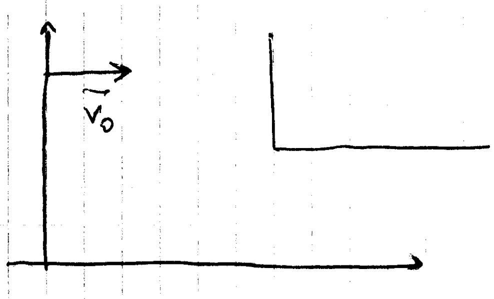
NO: se α è maggiore, la parabola è più piccola $\Rightarrow$ distanza minore

$$tg\alpha = \frac{x_A \pm \sqrt{\Delta}}{\cancel{2} \frac{g x_A^2}{\cancel{2} v_0^2}} = \frac{v_0^2}{g x_A^2} \left(x_A \pm \sqrt{\Delta} \right)$$

$$= 0.525$$

$$\Rightarrow \quad \alpha = tg^{-1}\, 0.525 = 0.484 \text{ rad}$$

3)



$$\begin{cases} y(t) = y_P - \frac{1}{2} g t^2 \\ x(t) = v_0 t \end{cases}$$

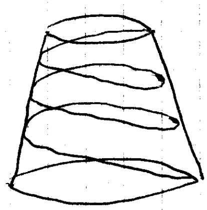
$$x(t) = v_0 \sqrt{\frac{2y_P}{g}} < x_4$$

tempo in cui il proiettile tocca terra: $y(t) = 0 \Rightarrow t = \sqrt{\frac{2y_P}{g}}$

$$\begin{cases} \dot{x}(t) = v_0 \\ \dot{y}(t) = -gt \end{cases} \Rightarrow \vec{V}_f = \left(v_0, -\sqrt{2 g y_P}\right) = (10, -8.85)\ m/s$$

Esercizio n° 2

$$\begin{cases} r(\theta) = R + \frac{b}{2\pi}\theta \\ z(t) = h - v_z t \end{cases}$$



1) $\vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta - v_z\,\hat{z}$ $\qquad \dot{r} = \frac{b}{2\pi}\dot{\theta}$

$$|\vec{v}|^2 = v_0^2 = \frac{b^2}{4\pi^2}\dot{\theta}^2 + \left(R + \frac{b}{2\pi}\theta\right)^2 \dot{\theta}^2 + v_z^2$$

$$\Rightarrow \dot{\theta}^{P} = \sqrt{\frac{v_0^2 - v_z^2}{\frac{b^2}{4\pi^2} + \left(R + \frac{b\theta}{2\pi}\right)^2}} \underset{\theta = 2\pi n}{=} \sqrt{\frac{v_0^2 - v_z^2}{\frac{b^2}{4\pi^2} + (R + nb)^2}} = 0.305\ rad/s$$

2) $\theta(t) = \omega t$

$$\vec{v} = v\,\hat{T}$$

$$v = \sqrt{\left(R + \frac{b\omega t}{2\pi}\right)^2 \omega^2 + \frac{b^2\omega^2}{4\pi^2} + v_z^2} = 0.354\ m/s$$

$$\vec{v} = \frac{b\omega}{2\pi}\,\hat{e}_r + \left(R + \frac{b\omega t}{2\pi}\right)\omega\,\hat{e}_\theta - v_z\,\hat{z}$$

$$= \sqrt{\left(R + \frac{b\omega t}{2\pi}\right)^2 \omega^2 + \frac{b^2\omega^2}{4\pi^2} + v_z^2}\;\hat{T}$$

$$\Rightarrow \quad \hat{T}_r = \frac{b\omega}{2\pi v} = 0.252$$

3) $\vec{a} = \dot{v}\,\hat{T} + \frac{v^2}{R}\,\hat{N} \qquad \hat{v} = \hat{T} \quad \Rightarrow \quad a_{\parallel} = \dot{v}$

$$a_{\parallel} = \frac{\cancel{2}\left(R + \frac{b\omega t}{2\pi}\right)\omega^2\,\frac{b\omega}{2\pi}}{\cancel{2}v} = \frac{b\omega^3}{2\pi v}\left(R + \frac{b\omega t}{2\pi}\right)$$

$$= 0.0577 \ m/s^2$$

Altro modo di trovare $a_{\parallel}$ è

$$a_{\parallel} = \frac{\vec{a}\cdot\vec{v}}{|\vec{v}|}$$

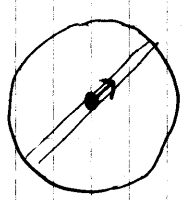
$$\vec{a} = (\cancel{\ddot{r}} - r\dot{\theta}^2)\,\hat{e}_r + (\cancel{r\ddot{\theta}} + 2\dot{r}\dot{\theta})\,\hat{e}_\theta + \cancel{\ddot{z}}\,\hat{z}$$

$$= -\left(R + \frac{b\omega t}{2\pi}\right)\omega^2\,\hat{e}_r + \cancel{2}\,\frac{b\omega^2}{\cancel{2}\pi}\,\hat{e}_\theta$$

$$\frac{\vec{a}\cdot\vec{v}}{v} = \frac{1}{v}\left[-\frac{b\omega^3}{2\pi}\left(R + \frac{b\omega t}{2\pi}\right) + \frac{b\omega^3}{\pi}\left(R + \frac{b\omega t}{2\pi}\right)\right]$$

$$= \frac{b\omega^3}{2\pi v}\left(R + \frac{b\omega t}{2\pi}\right)$$

Esercizio n° 3



$$v' = ct$$

1) SR assoluto, scelgo coord. polari

$$\begin{cases} r(t) = \frac{1}{2} c t^2 \\ \theta(t) = \omega t \end{cases}$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

$$= \left(c - \frac{1}{2} c t^2 \omega^2\right)\hat{e}_r + 2c\omega t\ \hat{e}_\theta$$

$$\Rightarrow \quad \vec{a} = -111.5\ m/s^2\ \hat{e}_r + 30\ m/s^2\ \hat{e}_\theta$$

2) $$\vec{a} = \vec{a}' + \vec{\omega} \times (\vec{\omega} \times \vec{r}) + 2(\vec{\omega} \times \vec{v}')$$

Devo trovare $\vec{a}'$

$$\vec{a} = \vec{a}' - \omega^2 \vec{r} + 2\omega \dot{r}\ \hat{e}_\theta \quad \Rightarrow \quad \vec{a}' = c\ \hat{e}_r$$

Ovvio anche dal fatto che $\vec{a}' = \dot{\vec{v}}'$