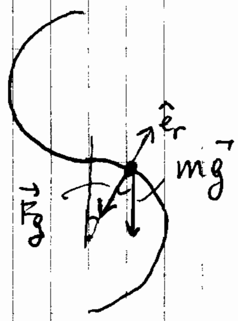
Esercizio n° 1

1) Forze che agiscono su m:



da $m\vec{g} + \vec{F_g} = m\vec{a}$

$$\vec{a} = -R\dot{\theta}^2 \hat{e}_r = -\frac{v^2}{R}\hat{e}_r$$

$$\Rightarrow -m\frac{v^2}{R} = -F_g - mg\cos\alpha$$

$$\Rightarrow F_g = m\left(\frac{v^2}{R} - g\cos\alpha\right)$$

Dalla conservazione dell'energia meccanica:

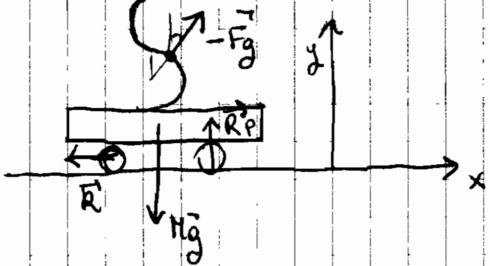
$$4R\,mg = \frac{1}{2}mv^2 + mg(R + R\cos\alpha)$$

$$\Rightarrow v = \sqrt{2gR(3-\cos\alpha)}$$

$$\Rightarrow F_g = m\left(2g\frac{R}{R}(3-\cos\alpha) - g\cos\alpha\right) =$$

$$= 3mg(2-\cos\alpha)$$

Forze su M:



$$M\vec{g} + \vec{R_p} + \vec{R} - \vec{F_g} = 0$$

Proiettando lungo x:

$$R = F_g \sin\alpha =$$

$$= 3mg\sin\alpha(2-\cos\alpha) =$$

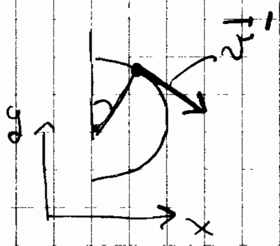
$$= 33.34\ N$$

2) Conservazione dell'en. meccanica di $m+M$:

$$\frac{1}{2} m v^2 + \frac{1}{2} M V^2 + mg R (1+\cos\alpha) = mg\, 4R$$

$$\frac{1}{2} m v^2 + \frac{1}{2} M V^2 = mg R (3-\cos\alpha)$$

$$\vec{v} = v_x \hat{x} + v_y \hat{y}$$

$$\begin{cases} v_x = v_x' + V \\ v_y = v_y' \end{cases}$$ con  $\Rightarrow \frac{v_y'}{v_x'} = -\operatorname{tg}\alpha$

Conservazione della q.d.m. lungo l'orizzontale

$$m v_x + M V = 0 \quad \Rightarrow \quad m v_x' + (m+M) V = 0$$

$$\Downarrow$$

$$V = -\frac{m}{m+M} v_x'$$

$$\Rightarrow \frac{1}{2} m (v_x^2 + v_y^2) + \frac{1}{2} M V^2 =$$

$$= \frac{1}{2} m (v_x' + V)^2 + \frac{1}{2} m \operatorname{tg}^2\alpha\, v_x'^2 + \frac{1}{2} M \frac{m^2}{(m+M)^2} v_x'^2$$

$$= \frac{1}{2} m v_x'^2 + \frac{1}{2} m \frac{m^2}{(m+M)^2} v_x'^2 + \frac{1}{2} m\, 2 v_x' \left(-\frac{m}{m+M} v_x'\right)$$

$$+ \frac{1}{2} m \operatorname{tg}^2\alpha\, v_x'^2 + \frac{1}{2} M \frac{m^2}{(m+M)^2} v_x'^2 =$$

$$= \frac{1}{2} m v_x'^2 \left[1 - 2\frac{m}{m+M} + \operatorname{tg}^2\alpha + \frac{m^2}{(m+M)^2} + \frac{mM}{(m+M)^2} \right]$$

$$= \frac{1}{2} m v_x'^2 \left[1 + \operatorname{tg}^2\alpha - 2\frac{m}{m+M} + \frac{m\,\cancel{(m+M)}}{(m+M)^{\cancel{2}}} \right]$$

$$= \frac{1}{2} m v_x'^2 \left[1 + \operatorname{tg}^2\alpha - \frac{m}{m+M} \right] = \frac{1}{2} m v_x'^2 \, \frac{M + (m+M)\operatorname{tg}^2\alpha}{m+M}$$

$$\Rightarrow \quad \frac{1}{2} \cancel{m}\, v_x'^2 \; \frac{M + (m+M)\, tg^2\alpha}{m+M} = \cancel{m}\, g\, R\,(3 - \cos\alpha)$$

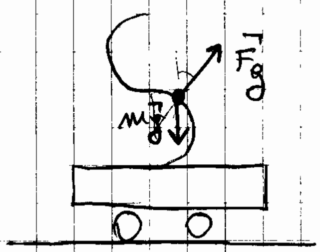
$$v_x' = \sqrt{\frac{[2gR(3-\cos\alpha)](m+M)}{M + (m+M)\, tg^2\alpha}}$$

scelgo la sol. positiva, in accordo con quanto mostrato prima in fig.

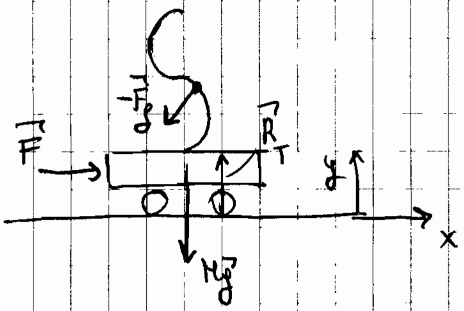
$$\Rightarrow \quad \vec{V} = - \; m \sqrt{\frac{2gR(3-\cos\alpha)}{(m+M)\,[M + (m+M)\, tg^2\alpha]}} \;\hat{x}$$

$$= - \; 0.859 \text{ m/s } \hat{x}$$

(3)



$$\vec{F}_g + m\vec{g} = m\vec{A}$$



$$\vec{R}_T + M\vec{g} + \vec{F} - \vec{F}_g = M\vec{A}$$

Poichè $\vec{A} = A\,\hat{x}$

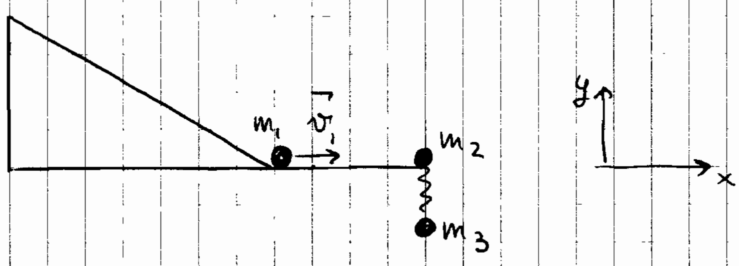
$$F = (m+M)\,A$$

Proiettando la prima eqn:

$$\begin{cases} F_g \sin\alpha = m A \\ F_g \cos\alpha = mg \end{cases} \Rightarrow \quad A = g\, tg\alpha, \qquad F_g = \frac{mg}{\cos\alpha}$$

$$\Rightarrow \quad F = (m+M)\, g\, tg\alpha = 56.58 \text{ N}$$

Esercizio n° 2



$m_1 = m_2 = m$

$m_3 = 2m$

1) Dalla conservazione dell'energia per m_1,

$$m_1 g h = \frac{1}{2} m_1 v_1^2 \Rightarrow \vec{v}_1 = \sqrt{2gh}\ \hat{x} = 7.67\ m/s\ \hat{x}$$

Essendo l'urto perfettamente anelastico,

$$(m_1+m_2)\vec{v}_{12} = m_1 \vec{v}_1 \Rightarrow \vec{v}_{12} = \frac{\vec{v}_1}{2} = 3.83\ m/s\ \hat{x}$$

Poichè la forza della molla non è impulsiva,

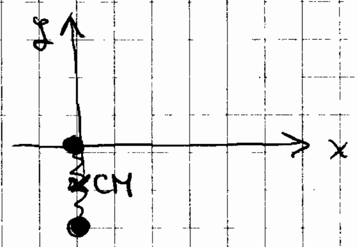
$$\vec{v}_3 = 0 \Rightarrow$$

$$\vec{v}_{CM} = \frac{(m_1+m_2)\vec{v}_{12}}{m_1+m_2+m_3} = \frac{2\ \vec{v}_{12}}{4} = \frac{\vec{v}_{12}}{2} = 1.92\ m/s\ \hat{x}$$

2) Dopo l'urto, si ha

$$M \vec{a}_{CM} = M\vec{g} \Rightarrow \vec{a}_{CM} = \vec{g} \Rightarrow \text{si ha un moto parabolico}$$

Da $\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{g} t^2$

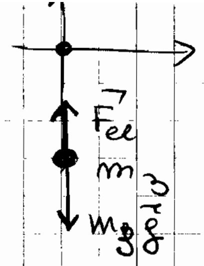


$$x_{CM}(t) = \frac{v_{12}}{2} t$$

$$y_{CM}(t) = y_{CM}(t=0) - \frac{1}{2} g t^2$$

$$y_{CM}(t=0) = \frac{m_3\, y_3}{m_1+m_2+m_3}$$

Per calcolare y_3:



$$\vec F_{el} + m_3 \vec g = 0$$

$$\Downarrow$$

$$k|y_3| - m_3 g = 0$$

$$\Rightarrow |y_3| = \frac{m_3 g}{k}$$

$$\Rightarrow y_{CM}(0) = \frac{2m}{4m}\left(-\frac{m_3 g}{k}\right) = -\frac{m_3 g}{2k} = -\frac{mg}{k}$$

$$y_{CM}(t) = -\frac{mg}{k} - \frac{1}{2} g \frac{(2x_{CM})^2}{v_{12}^2} = -\frac{mg}{k} - 8g\frac{x_{CM}^2}{v_1^2}$$

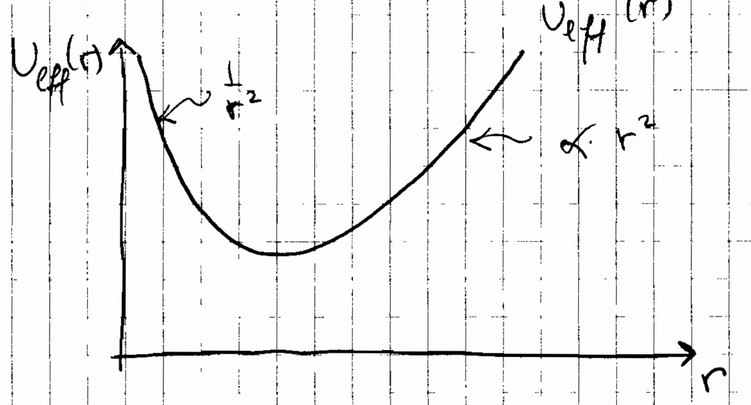
3) $$E_{rel} = \frac{1}{2}\mu v_{rel}^2 + \frac{1}{2}kr^2 = \frac{1}{2}\mu \dot r^2 + \frac{1}{2}\mu r^2 \dot\theta^2 + \frac{1}{2}kr^2$$

$\vec L_{rel} = \text{cost}$ ($\vec L$ è cost. se il polo è il CM)

Poiché $L_{rel} = \mu r^2 \dot\theta \underset{t=0}{=} \mu\, 2|y_{CM}(0)|\, v_{12} = m\, |y_{CM}(0)|\, v_1$

$$\mu = \frac{(m_1+m_2)m_3}{m_1+m_2+m_3} = m$$

$$\Rightarrow E_{rel} = \frac{1}{2}\mu\dot r^2 + \underbrace{\frac{L_{rel}^2}{2\mu r^2} + \frac{1}{2}kr^2}_{U_{eff}(r)}$$



4) Per trovare r_{max} e r_{min}, bisogna imporre:

$$E_{rel} = U_{eff}(r)$$

$$E_{rel} = \frac{1}{2}\mu v_{12}^2 + \frac{1}{2} k \left(2 y_{cm}(0)\right)^2$$

$$E_{rel} = \frac{1}{2} k r^2 + \frac{L_{rel}^2}{2\mu r^2}$$

$$2\mu E_{rel} r^2 - k\mu r^4 - L_{rel}^2 = 0$$

$$k\mu r^4 - 2\mu E_{rel} r^2 + L_{rel}^2 = 0$$

$$r^2 = \frac{E_{rel}\mu \pm \sqrt{E_{rel}^2 \mu^2 - k\mu L_{rel}^2}}{k\mu}$$

$$= \frac{E_{rel}}{k}\left(1 \underset{r_{min}^2}{\overset{r_{max}^2}{\pm}} \sqrt{1 - \frac{L_{rel}^2 k}{E_{rel}^2 \mu}}\right)$$

$$\Rightarrow r_{max} = \sqrt{\frac{E_{rel}}{k}\left(1 + \sqrt{1 - \frac{L_{rel}^2 k}{\mu E_{rel}^2}}\right)} = 0.543\ m$$

$$r_{min} = \sqrt{\frac{E_{rel}}{k}\left(1 - \sqrt{1 - \frac{L_{rel}^2 k}{\mu E_{rel}^2}}\right)} = 0.392\ m$$

Da notare che r_{min} è proprio la lunghezza della molla a $t=0$, cioè $r_{min} = 2|y_{cm}(0)|$