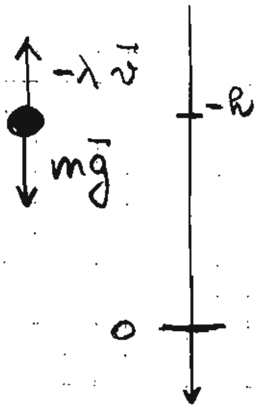
Esercizio del "paracadutista"

Forze



$$m\vec{a} = m\vec{g} - \lambda\vec{v} \qquad [\lambda] = \frac{[F]}{[L][T]^{-1}} = \frac{[M]\cancel{[L]}[T]^{-2}}{\cancel{[L]}\cancel{[T]^{-1}}}$$

$$m\ddot{y} = mg - \lambda\dot{y}$$

$$\ddot{y} = g - \underbrace{\frac{\lambda}{m}}_{k}\dot{y} = g - k\dot{y} \qquad [k] = [T]^{-1}$$

1) velocità limite v_L t.c. $\dot{v} = 0 \Rightarrow 0 = -g + k v_L$

$$\Rightarrow v_L = \frac{g}{k} = \frac{mg}{\lambda}$$

2) soluzione generale $\dot{v} = g - kv$

$$v(t) = v_{om}(t) + v_{part}(t)$$

$$\dot{v}_{om} = -k v_{om} \qquad v_{om}(t) = A e^{-\alpha t}$$

$$-\cancel{A}\alpha \cancel{e^{-\alpha t}} = -k\cancel{A}\cancel{e^{-\alpha t}} \qquad \alpha = k$$

$$\Rightarrow v(t) = A e^{-kt} + \frac{g}{k}$$

A dalle cond. al contorno

$$v(t) = \dot{y}(t) = A e^{-kt} + \frac{g}{k} \Rightarrow$$

$$y(t) = \int A e^{-kt}\,dt + \int \frac{g}{k}\,dt + B$$

$$= A\frac{e^{-kt}}{-k} + \frac{g}{k}t + B \Rightarrow$$

$$y(t) = -\frac{A}{k}e^{-kt} + \frac{g}{k}t + B$$

B dalle cond. al contorno

Per es.
$$\begin{cases} y(t=0) = -h \\ v(t=0) = -v_0 \end{cases} \qquad \begin{cases} -\frac{A}{k} + B = -h \\ A + \frac{g}{k} = -v_0 \end{cases} \qquad \begin{matrix} B = -h - \frac{g}{k^2} - \frac{v_0}{k} \\ A = -\frac{g}{k} - v_0 \end{matrix}$$

v verso l'alto

$$y(t) = \left(\frac{g}{k^2}+\frac{v_0}{k}\right)e^{-kt} + \frac{g}{k}t - h - \frac{g}{k^2} - \frac{v_0}{k}$$

$$= -h + \left(\frac{g}{k^2}+\frac{v_0}{k}\right)\left(e^{-kt}-1\right) + \frac{g}{k}t$$

3) $\bullet \xrightarrow{\vec{v}_0}$ $\quad \vec{v}_0 \equiv$ velocità dell'aereo da cui si lancia il paracadutista

Eq. del moto $\begin{cases} m\ddot{x} = -\lambda \dot{x} \\ m\ddot{y} = mg - \lambda \dot{y} \longrightarrow \text{uguale a prima} \end{cases}$

$$y(t) = -\frac{A}{k}e^{-kt} + \frac{g}{k}t + B$$

$$\dot{v}_x = -k v_x \qquad \dot{x}(t) = C e^{-kt}$$

$$x(t) = -\frac{C}{k}e^{-kt} + D$$

Cond. al contorno $\begin{cases} x(0) = 0 \\ \dot{x}(0) = v_0 \end{cases}$ $\begin{cases} y(0) = -h \\ \dot{y}(0) = 0 \end{cases}$

$\begin{cases} -\frac{C}{k} + D = 0 \\ C = v_0 \end{cases}$ $\quad D = \frac{v_0}{k} \quad C = v_0$ $\qquad x(t) = \frac{v_0}{k}\left(1 - e^{-kt}\right)$

lo trovavo subito dalla $y(t)$ del punto 2) con $g=0$, $h=0$, e $v_0 \rightleftarrows -v_0$

$\begin{cases} -\frac{A}{k} + B = -h \\ A + \frac{g}{k} = 0 \end{cases}$ $\quad B = -h - \frac{g}{k^2} \quad A = -\frac{g}{k}$ $\qquad y(t) = -h + \frac{g}{k^2}\left(e^{-kt}-1\right) + \frac{g}{k}t$

stessa del punto 2) con $v_0 = 0$