



$$\begin{cases} x = v_0 \cos \alpha t \\ y = \frac{h}{2} + v_0 \sin \alpha t - \frac{1}{2} g t^2 \end{cases}$$

$$\vec{a} = -g \hat{e}_y$$

$$\vec{v} = v_0 \cos \alpha \hat{e}_x + (v_0 \sin \alpha - g t) \hat{e}_y$$

$$|\vec{v}| = \sqrt{v_0^2 + g^2 t^2 - 2 g v_0 \sin \alpha t}$$

$$\vec{v} = v \hat{T}$$

$$\hat{T} = \frac{v_0 \cos \alpha}{v} \hat{e}_x + \frac{v_0 \sin \alpha - g t}{v} \hat{e}_y$$

$$\hat{T} \cdot \hat{N} = 0$$

$$T_x N_x + T_y N_y = 0$$

$$\hat{N} = \frac{v_0 \sin \alpha - g t}{v} \hat{e}_x - \frac{v_0 \cos \alpha}{v} \hat{e}_y$$

$$\text{guess } \begin{aligned} N_x &= \pm T_y \\ N_y &= \mp T_x \end{aligned}$$

$$\boxed{\vec{a} = \dot{v} \hat{T} + \frac{v^2}{\rho} \hat{N}}$$

$$\dot{v} = \frac{1}{2v} (2gt - 2g v_0 \sin \alpha) = \frac{g}{v} (gt - v_0 \sin \alpha)$$

$$\begin{aligned} \vec{a} = -g \hat{e}_y &= \frac{g}{v} (gt - v_0 \sin \alpha) \frac{v_0 \cos \alpha}{v} \hat{e}_x + \frac{g}{v} (gt - v_0 \sin \alpha) \frac{v_0 \sin \alpha - g t}{v} \hat{e}_y \\ &+ \underbrace{\left(\frac{v_0^2 + g^2 t^2 - 2 g v_0 \sin \alpha t}{v^2} \right)}_{\rho} \frac{v_0 \sin \alpha - g t}{v} \hat{e}_x - \frac{v^2}{\rho} \frac{v_0 \cos \alpha}{v} \hat{e}_y \end{aligned}$$

$$+ \frac{v_0 \cos \alpha}{v^2} g (v_0 \sin \alpha - g t) \hat{e}_x - \frac{v}{\rho} (v_0 \sin \alpha - g t) \hat{e}_x = 0 \quad \text{comp. } \hat{x}$$

$$\rho = \frac{v^3}{v_0 \cos \alpha} = \frac{\left[v_0^2 + g^2 t^2 - 2 g v_0 \sin \alpha t \right]^{3/2}}{v_0 g \cos \alpha}$$

comp. $\hat{y}$:

$$-g = -\frac{g}{v^2} (gt - v_0 \sin \alpha)^2 - \frac{v}{\rho} v_0 \cos \alpha$$

$$-\frac{v}{\rho} v_0 \cos \alpha = +g \left[\frac{(gt - v_0 \sin \alpha)^2}{v^2} - \frac{v_0^2 \cos^2 \alpha}{v^2} \right]$$

$$+ \frac{v}{\rho} v_0 \cos \alpha = +g \frac{v_0^2 \cos^2 \alpha}{v^2}$$

$$\rho = \frac{v^3}{g v_0 \cos \alpha}$$