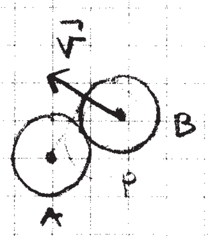
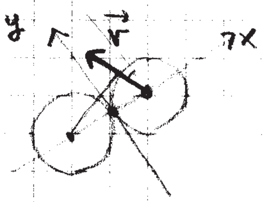
Esercizio sull'urto di 2 sfere rigide



r, m, P

Non c'è attrito di sorta $\Rightarrow$ le forze esercitate nell'urto sono lungo la congiungente dei centri

$\Downarrow$



$$\begin{cases} V_y = u_{B,y} \quad \text{e} \quad u_{Ay} = 0 \\ m_B V_x = m_B u_{B,x} + m_A u_{A,x} \\ m_B(V_x^2 + \cancel{V_y^2}) = m_B(u_{B,x}^2 + \cancel{u_{B,y}^2}) + m_A u_{A,x}^2 \end{cases}$$

$$\Rightarrow \begin{cases} m_B V_x = m_B u_{Bx} + m_A u_{Ax} \\ m_B V_x^2 = m_B u_{B,x}^2 + m_A u_{Ax}^2 \end{cases}$$

$$u_{Ax} = \frac{m_B}{m_A}(V_x - u_{Bx})$$

$$\cancel{m_B} V_x^2 = \cancel{m_B} u_{Bx}^2 + \cancel{m_A}\frac{m_B^{\cancel{2}}}{m_A^{\cancel{2}}} V_x^2 + \cancel{m_A}\frac{m_B^{\cancel{2}}}{m_A^{\cancel{2}}} u_{B,x}^2$$

$$- 2 \frac{m_B^{\cancel{2}}}{m_A^{\cancel{2}}} V_x u_{Bx} \cancel{m_A}$$

$$u_{Bx}^2 \frac{M_T}{\cancel{m_A}} - 2\frac{m_B}{\cancel{m_A}} V_x u_{Bx} + \frac{m_B - m_A}{\cancel{m_A}} V_x^2 = 0$$

$$u_{B,x} = \frac{m_B V_x \pm \sqrt{\cancel{m_B^2} V_x^2 - (\cancel{m_B^2} - m_A^2) V_x^2}}{m_A + m_B}$$

urto non c'è stato!

$$= V_x \frac{m_B \pm m_A}{m_A + m_B} = V_x \frac{m_B - m_A}{m_A + m_B}$$

$$u_{Ax} = \frac{m_B}{m_A} V_x \left(1 - \frac{m_B - m_A}{m_A + m_B}\right) = \frac{m_B}{m_A} V_x \quad \frac{2 m_A}{m_A + m_B}$$

$$= \frac{2 m_B}{m_A + m_B} \quad V_x$$

Vediamo gli angoli tra $\vec{u}_A$, $\vec{u}_B$ e $\vec{V}$:

$$\vec{u}_A \cdot \vec{u}_B = u_{Ax} u_{Bx} + u_{Ay} u_{By} = 2 m_B \frac{m_B - m_A}{(m_A + m_B)^2} V_x^2$$

$$= u_A u_B \cos\theta_{AB}$$

$$\cos\theta_{AB} = \frac{u_{Bx}}{u_B} = \frac{m_B - m_A}{m_A + m_B} \frac{V_x}{\sqrt{V_x^2 \left(\frac{m_B - m_A}{m_A + m_B}\right)^2 + V_y^2}}$$

$$\vec{u}_B \cdot \vec{V} = V_y^2 + u_{Bx} V_x = V_y^2 + V_x^2 \frac{m_B - m_A}{m_A + m_B}$$

$$\cos\theta_B = \frac{V_x^2 \frac{m_B - m_A}{m_A + m_B} + V_y^2}{V \sqrt{V_x^2 \left(\frac{m_B - m_A}{m_A + m_B}\right)^2 + V_y^2}}$$

Caso interessante è $m_A = m_B$:

$$\cos\theta_{AB} = 0 \qquad \theta_{AB} = \frac{\pi}{2}$$

$$\cos\theta_B = \frac{V_y}{V} =$$

$$= \frac{p}{2r}$$

$$u_{Bx} = 0 \qquad u_{By} = V\left(\frac{p}{2r}\right)$$

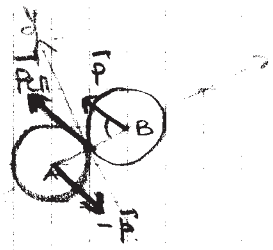
$$u_{Ax} = V_x \qquad u_{Ay} = 0$$

SR c.m.

$$\begin{cases} \vec{P}_{CM} = \vec{P}_B \\[2ex] \vec{p} = \dfrac{m_A \vec{p}_B - m_B \vec{p}_A}{m_A + m_B} \end{cases} \qquad \vec{q} = \frac{m_A \vec{q}_B - m_B \vec{q}_A}{m_A + m_B}$$

$$\Downarrow$$

$$\begin{cases} \vec{p}_A = \dfrac{m_A}{M_T} P_{CM} - \vec{p} \\[2ex] \vec{p}_B = \dfrac{m_B}{M_T} P_{CM} + \vec{p} \end{cases} \qquad \begin{matrix} \vec{q}_A = \dfrac{m_A}{M_T} P_{CM} - \vec{q} \\[2ex] \vec{q}_B = \dfrac{m_B}{m_T} P_{CM} + \vec{q} \end{matrix}$$



Non agiscono forze lungo y né su A, né su B ⇒

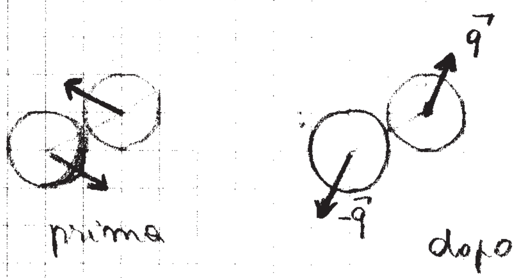
$$q_y = p_y$$

Nel S.R. CM c. energia cinetica:

$$\cancel{\frac{P_{CM}^2}{2M_T}} + \frac{p^2}{2\mu} = \cancel{\frac{P_{CM}^2}{2M_T}} + \frac{q^2}{2\mu}$$

$$p_x^2 + \cancel{p_y^2} = q_x^2 + \cancel{q_y^2} \qquad q_x = \pm p_x$$

$\Rightarrow \quad q_x = -p_x$ (altrimenti non ho urto!)



Verifichiamo con

$$\vec{q} = \vec{q}_B - \frac{m_B}{m_T}\vec{P}_B$$

$$q_x = m_B u_{B,x} - \frac{m_B}{m_T} m_B V_x$$

$$= m_B V_x \left(\frac{\cancel{m_B} - m_A}{m_T} - \cancel{\frac{m_B}{m_T}} \right) = -\frac{m_A m_B}{m_T} V_x$$

$$p_x = m_B V_x - \frac{m_B}{m_T} m_B V_x = \frac{m_B m_A V_x}{m_T}$$

$\Rightarrow$ $q_x = -p_x$ (nota che $p_x = \mu V_x$!)

$$q_y = m_B u_{B\,x} - \frac{m_B}{m_T} m_B V_y =$$

$$= m_B V_y \left(1 - \frac{m_B}{m_T}\right) = \mu V_y = p_y$$

Angolo tra $\vec{p}$ e $\vec{q}$

$$\cancel{pq}\cos\theta = p_x q_x + p_y q_y = p_x^2 - p_y^2$$

$$= \cancel{p^2}\cos^2\alpha - \cancel{p}\sin^2\alpha$$

$$\cos\theta = (1 - 2\sin^2\alpha) = 1 - \cancel{2}\frac{p^2}{\cancel{4}_{2} r^2} = \frac{2r^2 - p^2}{2r^2}.$$

$\Rightarrow$ nel C.M. l'angolo dipende da r e p, ma non dalle masse!