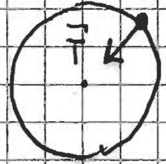
Compito del 23/07/08

Problema 1

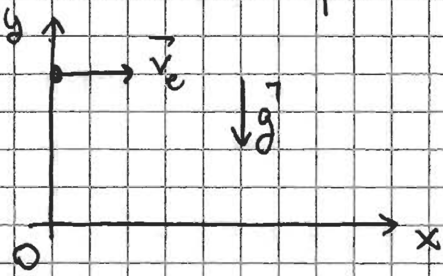
1) Moto circolare uniforme $\Rightarrow$



$$m\vec{a} = \vec{T}$$

$$m a_c = T \qquad m \frac{v_\ell^2}{\ell} = T \qquad v_\ell = \sqrt{\frac{T\ell}{m}}$$

2) La pallina viene lanciata da un'altezza h con velocità orizzontale pari a v_e $\Rightarrow$



$$\begin{cases} x(t) = v_e t \\ y(t) = h - \frac{1}{2} g t^2 \end{cases}$$

Per trovare d, imponiamo $y(t) = 0 \Rightarrow t = \sqrt{\frac{2h}{g}}$

$$d = v_e \sqrt{\frac{2h}{g}} = \sqrt{\frac{2h}{g}\frac{T\ell}{m}}$$

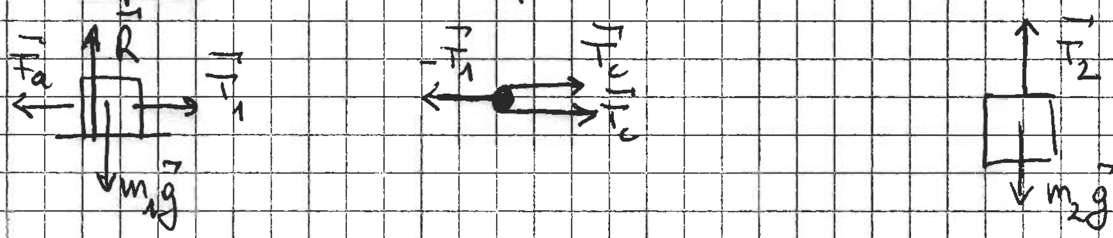
3) Si può applicare la conservazione dell'energia meccanica

$$\frac{1}{2} \cancel{m} v_e^2 + \cancel{m} g h = \frac{1}{2} \cancel{m} v_{fin}^2$$

$$\Rightarrow \quad v_{fin} = \sqrt{v_e^2 + 2gh}$$

Problema 2

4.) Diagramma delle forze



corpo 1 carrucola ($\mu \simeq 0$) corpo 2

$$\begin{cases} m_1 \vec{a}_1 = m_1 \vec{g} + \vec{F}_a + \vec{T}_1 + \vec{R} \\ \mu\, \vec{a}_c = -\vec{T}_1 + 2\vec{T}_c \simeq 0 \\ m_2 \vec{a}_2 = m_2 \vec{g} + \vec{T}_2 \end{cases}$$

Equilibrio $\Rightarrow$ $\begin{cases} m_1 \vec{g} + \vec{F}_a + \vec{T}_1 + \vec{R} = 0 \\ \vec{T}_1 - 2\vec{T}_c = 0 \\ m_2 \vec{g} + \vec{T}_2 = 0 \end{cases}$

Fili ideali $\Rightarrow$ $|T_2| = |T_c|$

Scompongo (assi x, y) $\Rightarrow$ $\begin{cases} -m_1 g + R = 0 \\ T_1 - F_a = 0 \\ -T_1 + 2T_2 = 0 \\ -m_2 g + T_2 = 0 \end{cases}$

$$\Rightarrow \begin{cases} R = m_1 g \\ F_a = T_1 \\ T_1 = 2T_2 \\ T_2 = m_2 g \end{cases} \Rightarrow F_a = 2 m_2 g$$

$$F_a \leq \mu_s R \Rightarrow \mu_s m_1 g \geq 2 m_2 g$$

$$\mu_s \geq \frac{2m_2}{m_1}.$$

5) Le formule di prima valgono con $F_a = 0$ $\Rightarrow$

$$\begin{cases} m_1 \vec{a}_1 = m_1 \vec{g} + \vec{T}_1 + \vec{R} \\ \vec{T}_1 = 2\vec{T}_c \\ m_2 \vec{a}_2 = m_2 \vec{g} + \vec{T}_2 \end{cases} \Rightarrow \begin{cases} m_1 a_1 = T_1 & a_1 \equiv a_{1x} \\ 0 = -m_1 g + R \\ T_1 = 2T_2 \\ m_2 a_2 = -m_2 g + T_2 & a_2 \equiv a_{2y} \end{cases}$$

Inoltre dalla configurazione del sistema si ha che

$$2a_{1x} = a_{2y}$$

$$\Rightarrow \quad m_1 a_1 = 2T_2$$

$$T_2 = m_2(g + a_2)$$

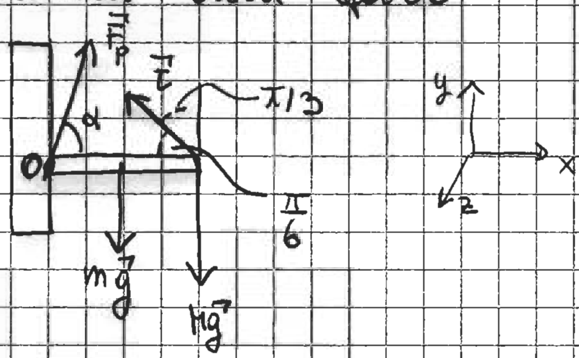
$$m_1 a_1 = 2m_2 g + 2m_2 a_2$$

$$m_1 a_1 + 4m_2 a_1 = 2m_2 g \quad \Rightarrow$$

$$a_1 = \frac{2m_2}{m_1 + 4m_2} g$$

Problema 3

6) Diagramma delle forze



Momento delle forze rispetto a O = 0

$$-\frac{\cancel{\ell}}{2} mg - \cancel{\ell} Mg + \cancel{\ell}\, \tau \underbrace{\sin\left(\frac{\pi}{6}\right)}_{1/2} = 0$$

$$\Rightarrow + \left(\frac{m}{2} + M\right) g = \frac{\tau}{2}$$

$$\Rightarrow \quad \tau = (2M + m)\, g$$

7) Risultante delle forze = 0 $\Rightarrow$

$$\vec{F}_p + m\vec{g} + M\vec{g} + \vec{\tau} = 0 \qquad \begin{cases} F_{p_x} - \tau \cos\frac{\pi}{6} = 0 \\ F_{p_y} + \tau \sin\frac{\pi}{6} - (m+M)\, g = 0 \end{cases}$$

$$F_{p_x} = (2M+m)\, g\, \frac{\sqrt{3}}{2}$$

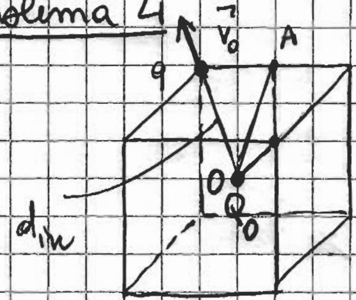
$$F_{p_y} = (m + \cancel{M})\, g - \frac{(2\cancel{M}+m)}{2}\, g = \frac{m}{2}\, g$$

$$F_p = g\sqrt{\frac{3}{4}(4M^2 + m^2 + 4Mm) + \frac{m^2}{4}} = g\sqrt{m^2 + 3M^2 + 3Mm}$$

8) $$\operatorname{tg}\alpha = \frac{F_{p_y}}{F_{p_x}} = \frac{m}{\cancel{2}} \cancel{g} \frac{\cancel{2}}{(2M+m)\, \cancel{g}\, \sqrt{3}} = \frac{m}{\sqrt{3}\,(2M+m)}$$

Problema 4

9)



Conservazione dell' energia meccanica

$$\frac{1}{2} m v_0^2 + k_e \frac{Q_0 q}{d_{in}} = k_e \frac{Q_0 q}{d_{max}}$$

d_{in} ?
$$OA = \sqrt{\left(\frac{\ell}{2}\right)^2 + \left(\frac{\ell}{2}\right)^2} = \sqrt{\frac{\ell^2}{4} \cdot 2} = \frac{\ell}{\sqrt{2}}$$

$$d_{in} = \sqrt{OA^2 + \left(\frac{\ell}{2}\right)^2} = \sqrt{\frac{\ell^2}{2} + \frac{\ell^2}{4}} = \sqrt{\frac{3}{4}\ell^2} = \frac{\ell}{2}\sqrt{3}$$

$\Rightarrow$

$$d_{max} = \frac{k_e Q_0 q}{\frac{1}{2} m v_0^2 + \frac{k_e Q_0 q}{\ell\sqrt{3}} \cdot 2}$$

Nota: $q < 0$! $\Rightarrow$ $$d_{max} = \frac{-k_e Q_0 |q|}{\frac{1}{2} m v_0^2 - \frac{2 k_e Q_0 |q|}{\ell\sqrt{3}}}.$$

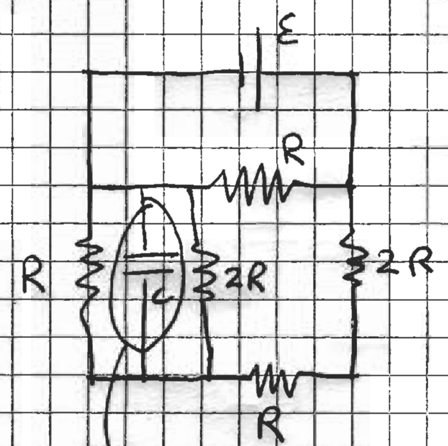
10) Flusso di E generato da Q_0 su tutto il cubo, per il teorema di Gauss, vale

$$\Phi_{cubo} = \frac{Q_0}{\varepsilon_0}$$

$$\Phi_{1\,faccia} = \frac{\Phi_{cubo}}{6} = \frac{Q_0}{6\,\varepsilon_0} = \frac{4\pi Q_0}{6} k_e$$

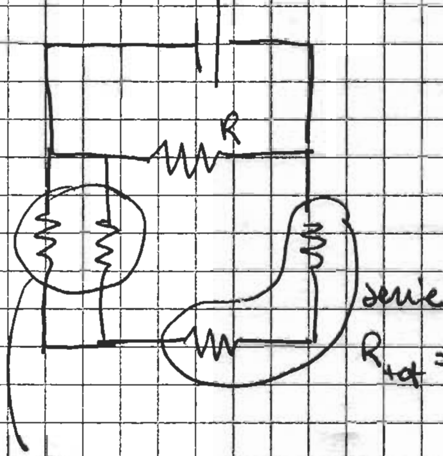
Problema 5

11)



in condizioni staz. non passa corrente

$\equiv$

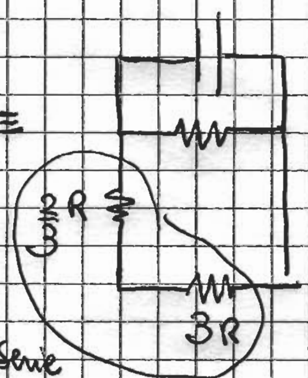


serie $R_{tot} = 3R$

Parallelo: $\frac{1}{R_{tot}} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R}$

$R_{tot} = \frac{2}{3}R$

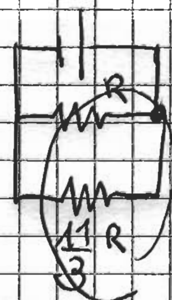
$\equiv$



serie

$R_{tot} = 3R + \frac{2}{3}R = \frac{11}{3}R$

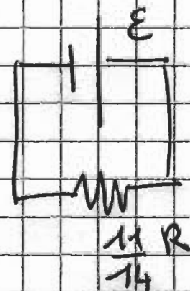
$\equiv$



Parallelo

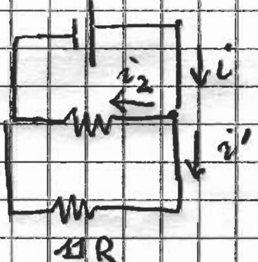
$\frac{1}{R_{tot}} = \frac{3}{11R} + \frac{1}{R} = \frac{14}{11R}$

$\equiv$



$i = \frac{\varepsilon}{R_{tot}} = \frac{14}{11}\frac{\varepsilon}{R}$

12) Da



$i_2 + i' = i$

$\cancel{R} i_2 = \frac{11}{3} i' \cancel{R} \Rightarrow i' = \frac{3}{11} i_2$

$i_2 + \frac{3}{11} i_2 = i \qquad i = \frac{14}{11} i_2 \; ; \; i_2 = \frac{11}{14} i$

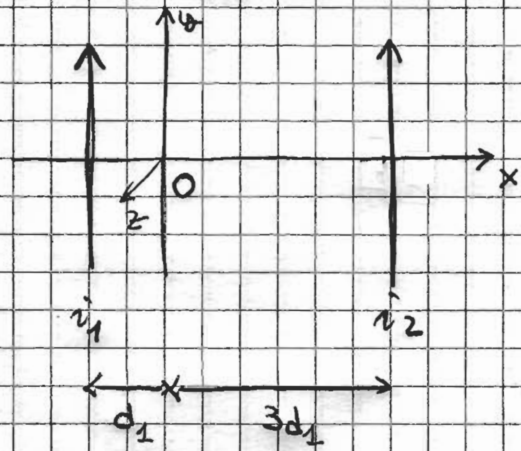
$P = i_2^2 R = R\left(\frac{\cancel{11}}{\cancel{14}} \cdot \frac{\cancel{14}}{\cancel{11}} \frac{\varepsilon}{R}\right)^2 = \frac{\varepsilon^2}{R}$

13)

$i' = \frac{3}{11} i_2 = \frac{3}{11} \cdot \frac{11}{14} i = \frac{3}{\cancel{14}} \cdot \frac{\cancel{14}}{11} \frac{\varepsilon}{R} = \frac{3}{11}\frac{\varepsilon}{R}$

$\Delta V_c = \frac{2}{3} R \cdot \frac{3}{11}\frac{\varepsilon}{R} = \frac{2}{11}\varepsilon \Rightarrow U = \frac{1}{2} C \Delta V_c^2 = \frac{1}{\cancel{2}} C \frac{\cancel{4}\varepsilon^2}{121} = \frac{2\varepsilon^2 C}{121}$

Problema 6



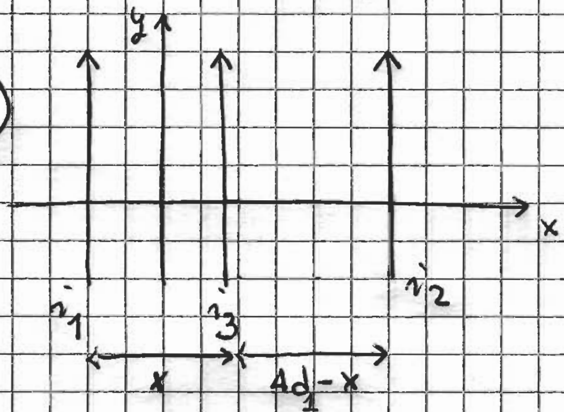
14)

$$\vec{B}_1(0) = \frac{\mu_0}{2\pi}\frac{i_1}{d_1}(-\hat{z})$$

$$\vec{B}_2(0) = \frac{\mu_0}{2\pi}\frac{i_2}{3d_1}(+\hat{z})$$

$$\vec{B}(0) = \frac{\mu_0}{2\pi}\frac{1}{d_1}\left(\frac{i_2}{3} - i_1\right)\hat{z} = \frac{\mu_0}{2\pi}\frac{1}{3d_1}(i_2 - 3i_1)\hat{z}$$

15)



$$\frac{\vec{F}_1}{l} = \frac{\mu_0}{2\pi}\frac{i_1 i_3}{x}(-\hat{x})$$

$$\frac{\vec{F}_2}{l} = \frac{\mu_0}{2\pi}\frac{i_2 i_3}{4d_1 - x}(\hat{x})$$

$$\vec{F}_1 + \vec{F}_2 = 0 \quad \Rightarrow \quad \cancel{\frac{\mu_0}{2\pi}}\frac{i_1 \cancel{i_3}}{x} = \cancel{\frac{\mu_0}{2\pi}}\frac{i_2 \cancel{i_3}}{4d_1 - x}$$

$$(4d - x)\, i_1 = i_2\, x$$

$$4d\, i_1 = (i_1 + i_2)\, x \qquad x = 4d_1 \frac{i_1}{i_1 + i_2}$$