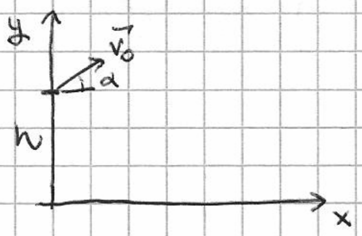
Compito del 30/01/2009

Problema 1



Per risolvere questo esercizio si può utilizzare il principio di conservazione dell'energia meccanica e osservare che $v_x(t) = v_0 \cos\alpha =$ costante

$\Rightarrow$ 1. $$\frac{1}{2} m v_0^2 + mgh = \frac{1}{2} m (v_0 \cos\alpha)^2 + mg\, h_{max}$$

$$\Rightarrow \quad h_{max} = h + \frac{(v_0 \sin\alpha)^2}{2g}$$

2. $v_x(t) = \text{cost} = v_0 \cos\alpha$

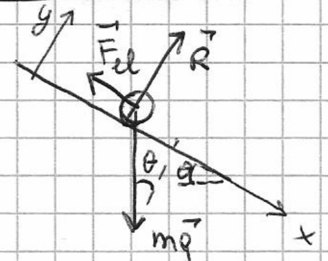
3. $$\frac{1}{2} m v_{fin}^2 = mgh + \frac{1}{2} m v_0^2 \quad \Rightarrow$$

$$v_{fin} = \sqrt{v_0^2 + 2gh}$$

L'esercizio poteva anche essere risolto partendo dalle seguenti equazioni

$$\begin{cases} x(t) = (v_0 \cos\alpha)\, t \\ y(t) = h + (v_0 \sin\alpha)\, t - \frac{1}{2} g t^2 \end{cases} \qquad \begin{cases} v_x(t) = v_0 \cos\alpha \\ v_y(t) = v_0 \sin\alpha - g t \end{cases}$$

Problema 2

4. 

$$m\vec{g} + \vec{R} + \vec{F}_{el} = 0 \qquad \begin{cases} mg \cos\theta = R \\ mg \sin\theta = F_{el} \end{cases}$$

Poiché $F_{el} = K(l_0 - l_{eq})$

$$K(l_0 - l_{eq}) = mg \sin\theta \quad \Rightarrow \quad l_{eq} = l_0 - \frac{mg}{K} \sin\theta$$

5. Dalla conservazione dell'energia meccanica, e notando che l_{min} si ha per $v = 0$ alla sommità del piano inclinato, si ottiene

$$\frac{1}{2} k \left(l_{min} - l_0\right)^2 + mg\, l_{min} \sin\theta = mg\, l \sin\theta$$

$$k\, l_{min}^2 + 2\, l_{min} \left(mg \sin\theta - k\, l_0\right) + k\, l_0^2 - mg\, l \sin\theta = 0$$

$$l_{min} = \left(k\, l_0 - mg \sin\theta \pm \sqrt{m^2 g^2 \sin^2\theta + \cancel{k^2 l_0^2} - 2k\, l_0\, mg \sin\theta - \cancel{k^2 l_0^2} + mg\, k\, l \sin\theta} \right) \frac{1}{k}$$

sol. non fisica

$$= \underbrace{l_0 - \frac{mg}{k} \sin\theta}_{l_{eq}} \; \cancel{+} \; - \; \frac{\sqrt{m^2 g^2 \sin^2\theta + mgk \sin\theta \left(l - 2 l_0\right)}}{k}$$

6. $$\frac{1}{2} k \left(\frac{1}{2} l_0 - l_0 \right)^2 + mg \frac{l_0}{5} \sin\theta = \frac{1}{2} m v^2 + mg\, l_0 \sin\theta$$

$$\cancel{\frac{1}{2}} \frac{k}{m} \frac{16}{25} l_0^2 - 2 \cancel{m} g \frac{4}{5} l_0 \sin\theta = \cancel{\frac{1}{2}} \cancel{m}\, v^2$$

$$v = \sqrt{\frac{16}{25} \frac{k}{m} l_0^2 - \frac{8}{5} g\, l_0 \sin\theta}$$

Problema 3

7. Dalla conservazione dell'energia meccanica:

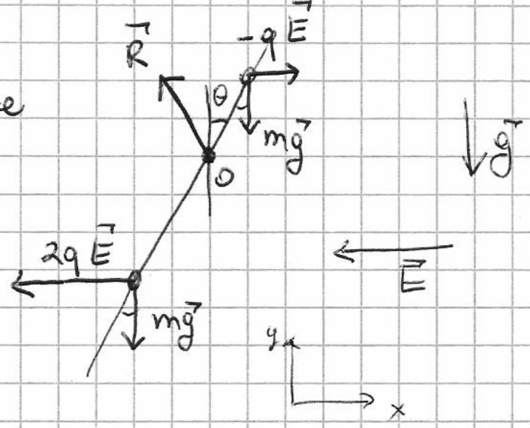
$$\frac{1}{2} \cancel{m} v_0^2 - G \frac{\cancel{m}\, m_L}{r_L} = - G \frac{\cancel{m}\, m_L}{(r_L + d)} \qquad v_0 = \sqrt{2 G m_L \left(\frac{1}{r_L} - \frac{1}{r_L + d} \right)}$$

$$\Rightarrow \quad v_0 = \sqrt{\frac{2 G m_L\, d}{r_L (r_L + d)}}$$

8. $$\cancel{m} \frac{v^2}{\cancel{(r_L + d)}} = \frac{G \cancel{m}\, m_L}{(r_L + d)^{\cancel{2}}} \quad \Rightarrow \quad v = \sqrt{\frac{G m_L}{(r_L + d)}}$$

Problema 4

Diagramma delle forze



modulo di $E = \frac{\Delta V}{d}$

9. Equilibrio $\Rightarrow$ Σ momenti delle forze rispetto a $O = 0$

$$\cancel{-mg\frac{\ell}{3}\sin\theta} \cancel{-qE\frac{\ell}{3}\cos\theta} + mg\frac{\cancel{2}\ell}{\cancel{3}}\sin\theta - \cancel{2}qE\frac{\overset{5}{\cancel{2\ell}}}{\cancel{3}}\cos\theta = 0$$

$$mg\sin\theta = 5qE\cos\theta \qquad tg\,\theta = 5\frac{q\,\Delta V}{mg\,d}$$

10. $m\vec{g} - q\vec{E} + m\vec{g} + 2q\vec{E} + \vec{R} = 0$

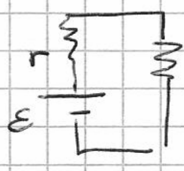
$$R_x - 2q\frac{\Delta V}{d} + q\frac{\Delta V}{d} = 0 \qquad R_x = q\frac{\Delta V}{d}$$

$$-2mg + R_y = 0 \qquad R_y = 2mg$$

$$\Rightarrow \quad R = \sqrt{\left(q\frac{\Delta V}{d}\right)^2 + 4m^2g^2}$$

Problema 5

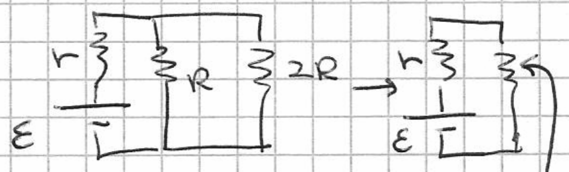
11. Il circuito a) è equivalente a



$\frac{1}{R_T} = \frac{3}{R} \Rightarrow R_T = \frac{R}{3}$

$$\Rightarrow \left(\frac{R}{3} + r\right) i_a = \varepsilon$$

Il circuito b) è equivalente a



$\frac{1}{R_T} + \frac{1}{2R} = \frac{3}{2R} \Rightarrow R_T = \frac{2R}{3}$

$$\left(\frac{2}{3} R + r\right) i_b = \varepsilon$$

$$\Rightarrow \frac{\varepsilon}{i_a} - \frac{R}{3} = r \Rightarrow \frac{2}{3} R i_b + i_b \frac{\varepsilon}{i_a} - \frac{R}{3} i_b = \varepsilon$$

$$\Rightarrow \varepsilon \left(1 - \frac{i_b}{i_a}\right) = \frac{R}{3} i_b \Rightarrow \varepsilon = \frac{R}{3} \frac{i_a i_b}{i_a - i_b}$$

12. $$r = \frac{R}{3} \frac{i_b}{i_a - i_b} - \frac{R}{3} = \frac{R}{3} \frac{2 i_b - i_a}{i_a - i_b}$$

13. $$\begin{cases} i_b = i_4 + i_6 \\ \cancel{R} i_4 = 2 \cancel{R} i_6 \end{cases} \Rightarrow i_b = 2 i_6 + i_6 \Rightarrow i_6 = \frac{i_b}{3}$$

$$\Rightarrow P = R i_6^2 = R \frac{i_b^2}{9}$$

Problema 6

14. $$\begin{cases} m_1 \frac{v_1^{\cancel{2}}}{r_0} = q \cancel{v_1} B \\ m_2 \frac{v_1^2}{r} = q v_1 B \end{cases} \Rightarrow \frac{m_1}{r_0} = \frac{m_2}{r} \Rightarrow m_2 = m_1 \frac{r}{r_0}$$

15. Poichè la forza di Lorentz non compie lavoro, si ha che

$$\cancel{\tfrac{1}{2}} m_1 v_1^2 = \cancel{\tfrac{1}{2}} m_3 v_3^2 \quad ; \quad m_3 v_3^2 = q v_3 B r = q v_1 B r_0 \Rightarrow \frac{v_1}{v_3} = \frac{r}{r_0} \Rightarrow m_3 = m_1 \left(\frac{r}{r_0}\right)^2$$