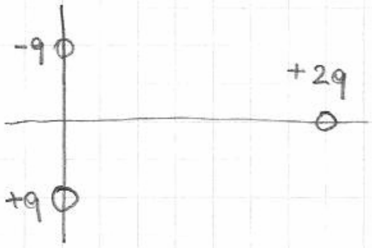
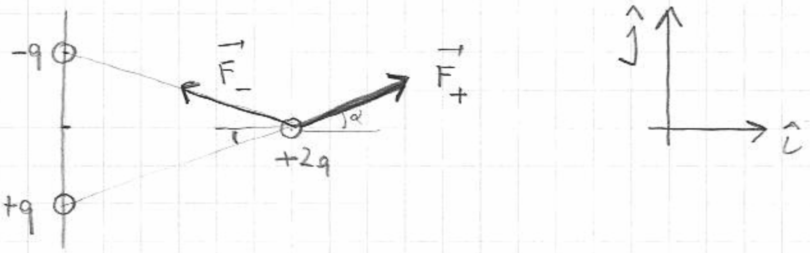
Soluzioni

1)



(a)



$$\vec{F}_+ = k_e \frac{2q^2}{(a^2+d^2)} (\cos\alpha \ \hat{i} + \sin\alpha \ \hat{j})$$

$$= k_e \frac{2q^2}{(a^2+d^2)} \left(\frac{d}{\sqrt{a^2+d^2}} \hat{i} + \frac{a}{\sqrt{a^2+d^2}} \hat{j} \right)$$

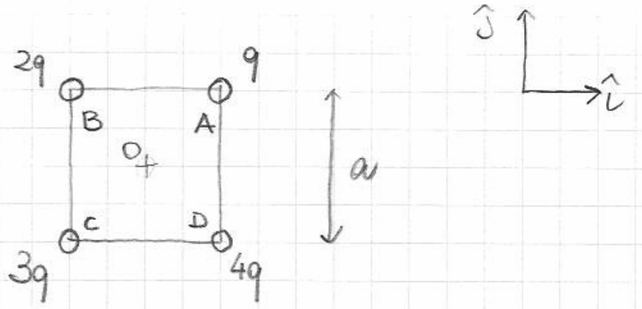
$$\vec{F}_- = k_e \frac{2q^2}{(a^2+d^2)} (-\cos\alpha \ \hat{i} + \sin\alpha \ \hat{j})$$

$$= k_e \frac{2q^2}{(a^2+d^2)} \left(\frac{-d}{\sqrt{a^2+d^2}} \hat{i} + \frac{a}{\sqrt{a^2+d^2}} \hat{j} \right)$$

$$\vec{F} = \vec{F}_+ + \vec{F}_- = k_e \frac{2q^2}{(a^2+d^2)} \frac{2a}{\sqrt{a^2+d^2}} \hat{j}$$

(b) $\vec{F} = m\vec{a} \Rightarrow \vec{a} = k_e \dfrac{2q^2}{m} \dfrac{2a}{(a^2+d^2)^{3/2}} \hat{j}$

2)



(a) $\vec{E}(A) = \vec{E}_{2q} + \vec{E}_{3q} + \vec{E}_{4q}$

$$= k_e \frac{2q}{a^2}\,\hat{\imath} + k_e \frac{3q}{2a^2}\left(\frac{\sqrt{2}}{2}\hat{\imath} + \frac{\sqrt{2}}{2}\hat{\jmath}\right)$$

$$+ k_e \frac{4q}{a^2}\,\hat{\jmath}$$

$$= k_e \frac{q}{a^2}\left[\left(2 + \frac{3\sqrt{2}}{4}\right)\hat{\imath} + \left(\frac{3\sqrt{2}}{4} + 4\right)\hat{\jmath}\right]$$

$$= k_e \frac{q}{a^2}\left(\frac{8+3\sqrt{2}}{4}\,\hat{\imath} + \frac{16+3\sqrt{2}}{4}\,\hat{\jmath}\right)$$

(b) $\vec{F}_q = q\,\vec{E}(A) = k_e \frac{q^2}{a^2}\left(\frac{8+3\sqrt{2}}{4}\,\hat{\imath} + \frac{16+3\sqrt{2}}{4}\,\hat{\jmath}\right)$

(c) $\vec{E}(O) = \vec{E}_q + \vec{E}_{2q} + \vec{E}_{3q} + \vec{E}_{4q} =$

$$= k_e \frac{q}{\frac{a^2}{2}}\left(-\frac{\sqrt{2}}{2}\hat{\imath} - \frac{\sqrt{2}}{2}\hat{\jmath}\right)$$

$$+ k_e \frac{2q}{\frac{a^2}{2}}\left(\frac{\sqrt{2}}{2}\hat{\imath} - \frac{\sqrt{2}}{2}\hat{\jmath}\right)$$

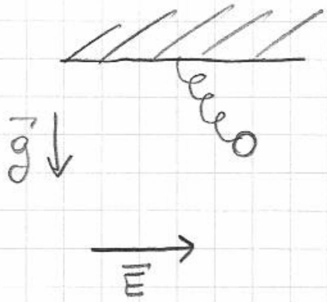
$$+ k_e \frac{3q}{\frac{a^2}{2}}\left(\frac{\sqrt{2}}{2}\hat{\imath} + \frac{\sqrt{2}}{2}\hat{\jmath}\right)$$

$$+ k_e \frac{4q}{\frac{a^2}{2}}\left(-\frac{\sqrt{2}}{2}\hat{\imath} + \frac{\sqrt{2}}{2}\hat{\jmath}\right) =$$

$$= k_e \frac{2q}{a^2} \left\{ \hat{i} \left[-\frac{\sqrt{2}}{2} + 2\frac{\sqrt{2}}{2} + \frac{3}{2}\sqrt{2} - 4\frac{\sqrt{2}}{2} \right] \right.$$

$$\left. + \hat{j} \left[-\frac{\sqrt{2}}{2} - 2\frac{\sqrt{2}}{2} + \frac{3}{2}\sqrt{2} + 4\frac{\sqrt{2}}{2} \right] \right\}$$

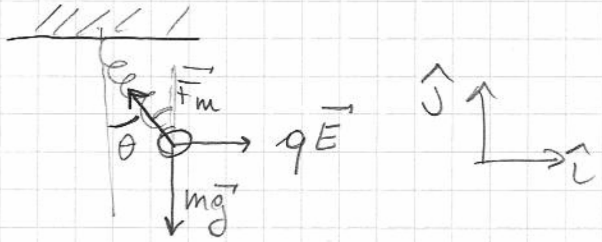
$$= k_e \frac{2q}{a^2} \cancel{4}^{2} \frac{\sqrt{2}}{\cancel{2}} \hat{j} = k_e \, 4\sqrt{2} \, \frac{q}{a^2} \hat{j}$$

3) 

k, ℓ_0

m

Diagramma delle forze su m



Equilibrio $\Rightarrow$ $\vec{F}_m + m\vec{g} + q\vec{E} = 0$

$$\Rightarrow \begin{cases} -F_m \sin\theta + qE = 0 \\ F_m \cos\theta - mg = 0 \end{cases}$$

$$F_m = k(\ell_{eq} - \ell_0) \Rightarrow$$

$$k(\ell_{eq} - \ell_0)\sin\theta = qE$$

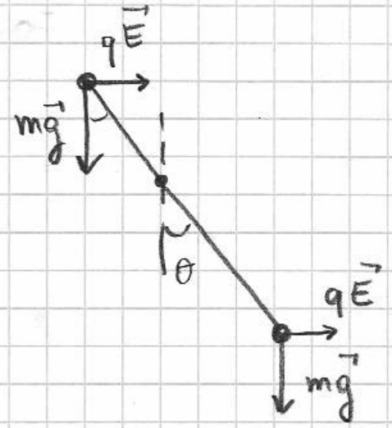
$$k(\ell_{eq} - \ell_0)\cos\theta = mg$$

$$\mathrm{tg}\,\theta_{eq} = \frac{qE}{mg}$$

$$k(\ell_{eq} - \ell_0) = \frac{mg}{\cos\theta_{eq}} = mg\sqrt{1 + \mathrm{tg}^2\theta_{eq}}$$

$$= mg\sqrt{1 + \frac{q^2E^2}{m^2g^2}} \Rightarrow \ell_{eq} = \ell_0 + \frac{mg}{k}\sqrt{1 + \frac{q^2E^2}{m^2g^2}}$$

4)



$\theta = 30°$

(a) Equilibrio $\Rightarrow$ Σ momenti delle forze ext $= 0$ $\Rightarrow$

$$mg\,\cancel{\frac{l}{3}}\sin\theta - qE\,\cancel{\frac{l}{3}}\cos\theta - mg\,2\cancel{\frac{l}{3}}\sin\theta + qE\,2\cancel{\frac{l}{3}}\cos\theta = 0$$

$$-mg\sin\theta + qE\cos\theta = 0$$

$$\Rightarrow \quad E = \frac{mg}{q}\,tg\,\theta =$$

$$-Ed = \Delta V \quad \Rightarrow \quad |\Delta V| = \frac{mg}{q}\,d\,tg\,\theta$$

(b) $\Delta V = 0 \Rightarrow E = 0 \Rightarrow F_{elettrica} = 0$

Conserv. dell'energia:

$$\frac{1}{2}\cancel{m}v_1^2 + \frac{1}{2}\cancel{m}v_2^2 + \cancel{m}g\frac{l}{3} - \cancel{m}g\frac{2l}{3} = \cancel{m}g\frac{l}{3}\cos\theta - \cancel{m}g\frac{2l}{3}\cos\theta$$

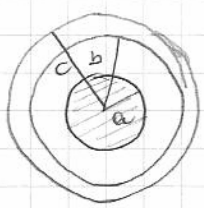
1 e 2 ruotano intorno ad O $\Rightarrow$

$$v_1 = \omega\frac{l}{3}, \quad v_2 = \omega\frac{2l}{3} \quad \Rightarrow \quad v_2 = 2v_1$$

$$\Rightarrow \quad \frac{1}{2}v_1^2 + \frac{\cancel{1}}{\cancel{2}}\,\overset{2}{\cancel{4}}v_1^2 - g\frac{l}{3} = -g\frac{l}{3}\cos\theta$$

$$\Rightarrow \quad \frac{5}{2}v_1^2 = g\frac{l}{3}(1-\cos\theta) \quad \Rightarrow \quad v_1 = \sqrt{\frac{2}{15}gl(1-\cos\theta)}$$

5)



$a, \ Q > 0$

b, c

a) Applichiamo ripetutamente il teorema di Gauss, sapendo che $\vec{E}$ deve essere radiale, perché il problema ha simmetria sferica $\Rightarrow$

$r < a \qquad 4\pi r^2 E = \frac{1}{\varepsilon_0} q_{in} = \frac{1}{\varepsilon_0} \frac{Q}{\frac{4}{3}\pi a^3} \frac{4}{3}\pi r^3$

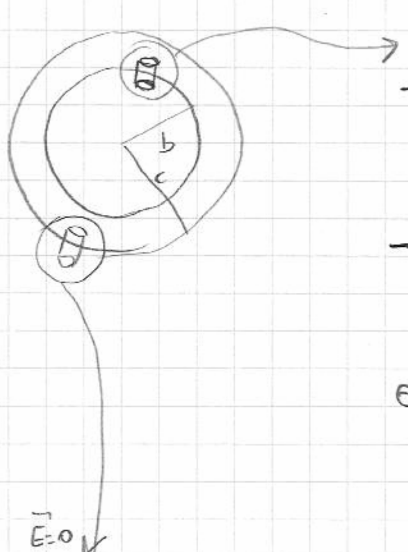
$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0} Q \frac{r}{a^3}$$

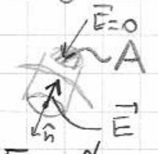
$a < r < b \qquad 4\pi r^2 E = \frac{1}{\varepsilon_0} Q \Rightarrow E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$

$b < r < c \qquad$ ho un conduttore $\Rightarrow E = 0$

$r > c \qquad 4\pi r^2 E = \frac{1}{\varepsilon_0} Q \Rightarrow E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$

b)

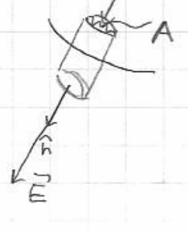




$$-E_{r=b^-} \cancel{A} = \frac{1}{\varepsilon_0} \sigma_{in} \cancel{A} \Rightarrow$$

$$-\frac{1}{4\pi\varepsilon_0} \frac{Q}{b^2} = \frac{1}{\varepsilon_0} \sigma_{in} \Rightarrow$$

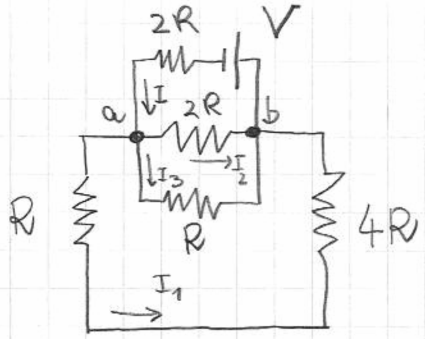
$$\sigma_{in} = \frac{-Q}{4\pi b^2}$$



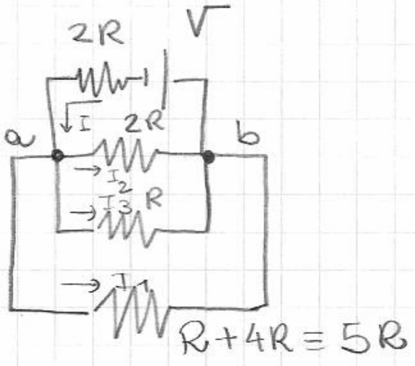
$$+E_{r=c^+} \cancel{A} = \frac{1}{\varepsilon_0} \sigma_{out} \cancel{A} \Rightarrow$$

$$\sigma_{out} = \frac{1}{4\pi\varepsilon_0} \varepsilon_0 \frac{Q}{c^2} = \frac{Q}{4\pi c^2}$$

5)



$\equiv$



a)

$I = I_1 + I_2 + I_3$ (nodo a o b)

$\Delta V_{ab} = 2R\, I_2 = R\, I_3 \Rightarrow I_3 = 2\, I_2$

$\Delta V_{ab} = R\, I_3 = 5R\, I_1 \Rightarrow I_3 = 5\, I_1$ $\Rightarrow I_2 = \frac{5}{2} I_1$

Calcolo I : il circuito è equivalente a:

$$\left(\frac{1}{2R} + \frac{1}{R} + \frac{1}{5R}\right)^{-1} = \left(\frac{5 + 10 + 2}{10R}\right)^{-1} = \frac{10}{17} R$$

$$\Rightarrow I \left(2R + \frac{10}{17} R\right) = V$$

$$\Rightarrow I = \frac{V}{\frac{44R}{17}} = \frac{17}{44} \frac{V}{R} = 1.932 \text{ A}$$

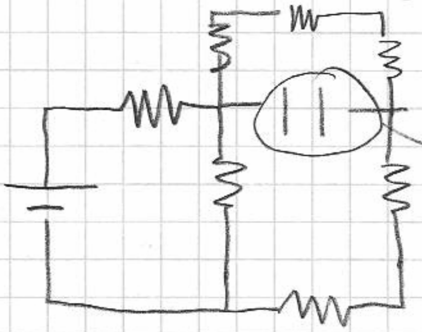
$$I = I_1 + \frac{5}{2} I_1 + 5 I_1 = \frac{2 + 5 + 10}{2} I_1 = \frac{17}{2} I_1$$

$$\Rightarrow I_1 = \frac{17}{44} \frac{V}{R} \frac{2}{17} = \frac{1}{22} \frac{V}{R} = 0.227 \text{ A}$$

$$I_2 = \frac{5}{44} \frac{V}{R} = 0.568 \text{ A}$$

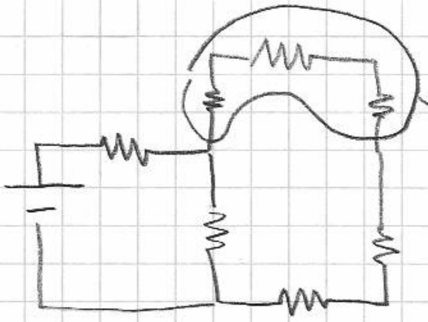
$$I_3 = 5/22\ V/R = 1.136 \text{ A}$$

b) $\Delta V_{ab} = 5 R I_1 = \frac{5}{22} V = 5.68 V$

7) 

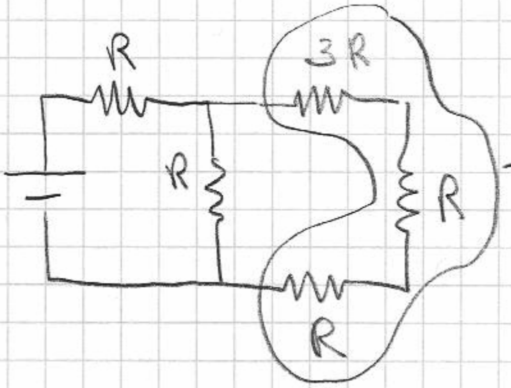
In condizioni stazionarie il ramo con C è come se fosse aperto

$\Downarrow$



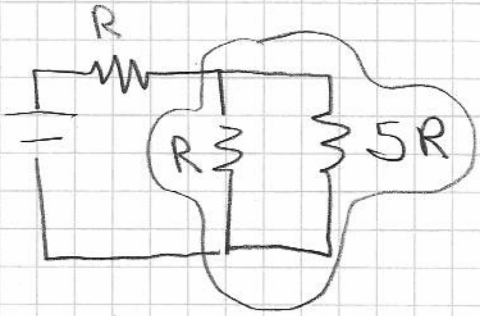
3 resistenze in serie

$\Downarrow$



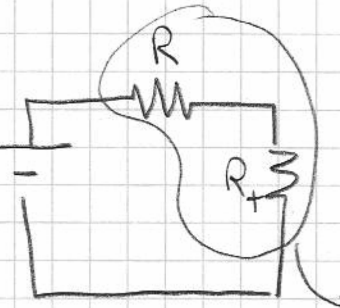
3 resistenze in serie

$\Downarrow$



2 resistenze in parallelo

$\Downarrow$



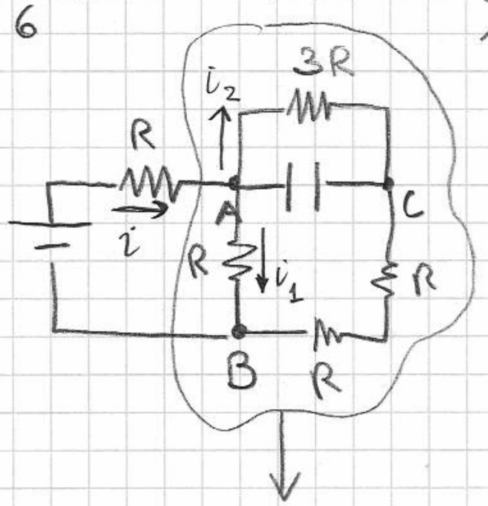
$$\frac{1}{R_+} = \frac{1}{R} + \frac{1}{5R} = \frac{6}{5R} \Rightarrow R_+ = \frac{5}{6} R$$

2 resistenze in serie

$$\frac{5}{6} R + R = \frac{11}{6} R \Rightarrow$$

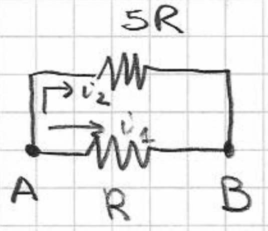
$$\varepsilon = \frac{11}{6} R\, i \Rightarrow i = \frac{6}{11}\frac{\varepsilon}{R}$$

(b)



$i = i_1 + i_2$ (1ª legge di Kirchoff)

equiv. a



$$5\cancel{R}\, i_2 = \cancel{R}\, i_1 = \Delta V_{AB}$$

$$\Rightarrow \quad i_2 + 5 i_2 = i \Rightarrow i_2 = \frac{i}{6}$$

$$\Delta V_{AC} = 3R\, i_2 = \cancel{3}R \frac{i}{\cancel{6}2} = R\frac{i}{2}$$

$$U = \frac{1}{2} C\, \Delta V_{AC}^2 = \frac{1}{2} C \left(R\frac{i}{2}\right)^2 = \frac{1}{2} C \left(\frac{\cancel{R}}{\cancel{2}} \frac{\overset{3}{\cancel{6}}\varepsilon}{11\cancel{R}}\right)^2$$

$$= \frac{1}{2} C \left(\frac{3\varepsilon}{11}\right)^2$$