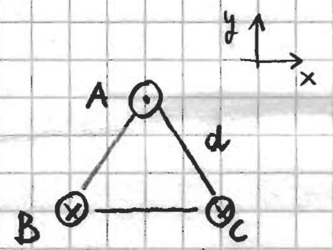
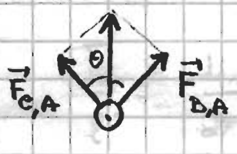
Soluzioni

1)



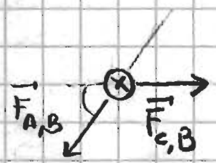
$$i_A = i_B = i_C = i$$



$$\frac{\vec{F}_A}{l} = \frac{\vec{F}_{B,A}}{l} + \frac{\vec{F}_{C,A}}{l} =$$

$$= \frac{\mu_0}{2\pi}\frac{i^2}{d}(2\cdot\cos\underset{\underset{30°}{\parallel}}{\theta})\,\hat{y} = \frac{\mu_0}{2\pi}\frac{i^2}{d}\,\cancel{2}\cdot\frac{\sqrt{3}}{\cancel{2}}\,\hat{y}$$

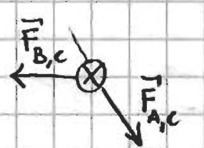
$$= \frac{\mu_0}{2\pi}\frac{i^2\sqrt{3}}{d}\,\hat{y}$$



$$\frac{\vec{F}_B}{l} = \frac{\vec{F}_{C,B}}{l} + \frac{\vec{F}_{A,B}}{l} =$$

$$= \frac{\mu_0}{2\pi}\frac{i^2}{d}\,\hat{x} + \frac{\mu_0}{2\pi}\frac{i^2}{d}\Big(-\underbrace{\cos 60°}_{1/2}\,\hat{x} - \underbrace{\sin 60°}_{\sqrt{3}/2}\,\hat{y}\Big)$$

$$= \frac{\mu_0}{2\pi}\frac{i^2}{d}\left(\frac{1}{2}\hat{x} - \frac{\sqrt{3}}{2}\hat{y}\right)$$

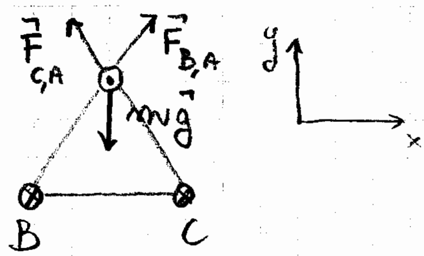


$$\frac{\vec{F}_C}{l} = \frac{\vec{F}_{B,C}}{l} + \frac{\vec{F}_{A,C}}{l} =$$

$$= -\frac{\mu_0}{2\pi}\frac{i^2}{d}\,\hat{x} + \frac{\mu_0}{2\pi}\frac{i^2}{d}\left(\cos 60°\,\hat{x} - \sin 60°\,\hat{y}\right)$$

$$= \frac{\mu_0}{2\pi}\frac{i^2}{d}\left(-\frac{1}{2}\hat{x} - \frac{\sqrt{3}}{2}\hat{y}\right)$$

2)

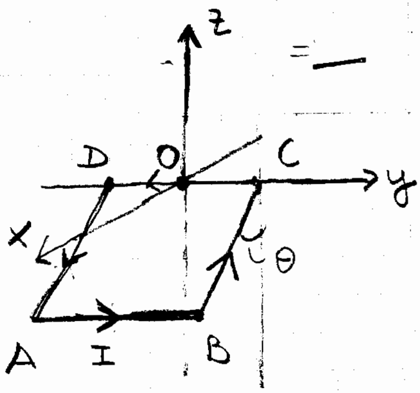


$$\underbrace{\vec F_A = 0}_{\text{equilibrio}} = m\vec g + \vec F_{B,A} + \vec F_{C,A}$$

$$(y) \quad -mg + \frac{\mu_0}{2\pi}\frac{i_A i}{d}\, l \underbrace{\cos 30^\circ}_{\frac{\sqrt3}{\cancel{2}}} \cdot \cancel{2} = 0$$

$$+\cancel{l}\cdot\pi r^2 \rho_{rame}\, g = \frac{\mu_0}{2\pi}\frac{i_A i}{d}\cancel{l}\sqrt3 \Rightarrow i_A = \frac{2\pi}{\mu_0}\frac{\pi r^2 d\, g\rho_r}{\sqrt3\, i}$$

3)



$$\vec B = B\hat z$$
$$\vec g = -g\hat z$$

$$\sum \vec F_{ext} = 0$$
$$\sum \vec M_{O,ext} = 0$$

$$\vec F_{ext} = \begin{cases} m\vec g & \text{peso} \\ \vec R & \text{reazione vincolo} \\ \vec F_{Magn} & \end{cases}$$

$$\vec F_{Magn} = \vec F_{AB} + \vec F_{BC} + \vec F_{CD} + \vec F_{DA}$$

$$\vec F_{AB} = IaB(+\hat x)$$

$$\vec F_{BC} = I\,\vec{CB}\wedge B\hat z = IBa(-\hat x\sin\theta + \hat z\cos\theta)\wedge\hat z$$
$$= +IBa\sin\theta\,\hat y$$

$$\vec F_{DC} = -IaB\,\hat x$$

$$\vec F_{DA} = -IBa\sin\theta\,\hat y$$

Che $\sum \vec{F}_{ext} = 0 \Rightarrow \quad \vec{R} + m\vec{g} = 0 \qquad \vec{R} = -m\vec{g}$

Per avere θ_{eq}, impongo

$$\sum \vec{M}_{O\,ext} = 0$$

$$\vec{M}_{O\,ext} = \begin{cases} mg\,\frac{a}{2}\sin\theta\,\hat{y} \\ \vec{M}_{O,R} = 0 \\ \vec{M}_{O,Magn} \end{cases}$$

$$\vec{M}_{O,AB} = +I\,a\,B\left(-a\cos\theta\,\hat{z} + a\sin\theta\,\hat{x}\right) \wedge \hat{x}$$

$$= -I\,a^2 B\cos\theta\,\hat{y}$$

$$\vec{M}_{O,BC} = I\,B\,a\sin\theta\left(-\frac{a}{2}\cos\theta\,\hat{z} + \frac{a}{2}\sin\theta\,\hat{x}\right) \wedge \hat{y}$$

$$\vec{M}_{O,DC} = 0$$

$$\vec{M}_{O,DA} = -I\,B\,a\sin\theta\left(-\frac{a}{2}\cos\theta\,\hat{z} + \frac{a}{2}\sin\theta\,\hat{x}\right) \wedge \hat{y}$$

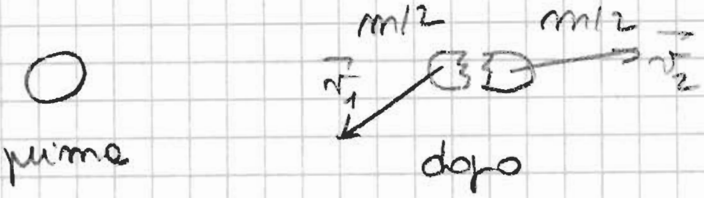
$$= -\vec{M}_{O,BC}$$

$$\Rightarrow \quad mg\,\frac{\cancel{a}}{2}\sin\theta\,\cancel{\hat{y}} - I\,a^{\cancel{2}}B\cos\theta\,\cancel{\hat{y}} = 0$$

$$\operatorname{tg}\theta_{eq} = \frac{2\,I\,a\,B}{mg}$$

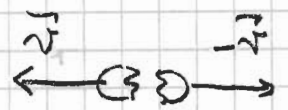
4) Sia $t=0$ l'istante in cui la particella "madre" decade. Il decadimento avviene x forze interne impulsive, ma le forze esterne sono non impulsive $\Rightarrow$ si conserva la quantità di moto

$\Rightarrow$



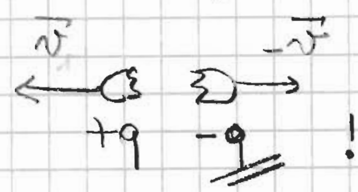
$$\frac{m}{2}\vec{v}_1 + \frac{m}{2}\vec{v}_2 = 0 \quad \Rightarrow \quad \vec{v}_1 = -\vec{v}_2 = \vec{v}$$

$\Rightarrow$

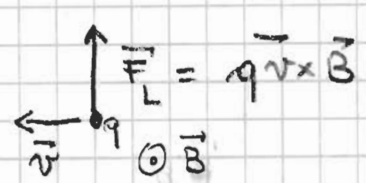


Nel decadimento pure la carica totale si conserva

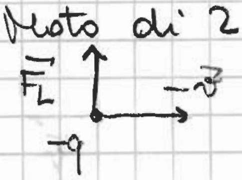
$\Rightarrow$



Moto di 1 è circolare



Moto di 2 pure circolare



$$\frac{m}{2}\frac{v^2}{r} = qBv$$

$$\omega_1 = \frac{v}{r} = \frac{2qB}{m}$$

$$r_1 = \frac{mv}{2qB}$$

$$\omega_2 = \omega_1$$

$$r_2 = r_1$$

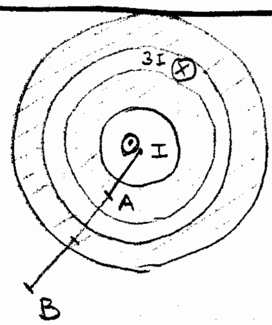
$\Rightarrow$ 1 e 2 collidono e lo fanno dopo aver percorso mezzo giro $\Rightarrow$ $T_{coll} = \frac{T}{2} = \frac{1}{\cancel{2}}\frac{\cancel{2}\pi}{\omega} = \frac{\pi m}{2qB}$

$$\Rightarrow \vec{F} = \vec{F}_{AB} + \vec{F}_{CD} = \frac{\mu_0}{2\pi} I_1 I_2 b \left(\frac{1}{c+a} - \frac{1}{c}\right)\hat{x}$$

$$= \frac{\mu_0}{2\pi} \frac{I_1 I_2 b}{(c+a)(c)} (-a\,\hat{x})$$

$$= \frac{\mu_0}{2\pi} I_1 I_2 \frac{ab}{c(c+a)} (-\hat{x})$$

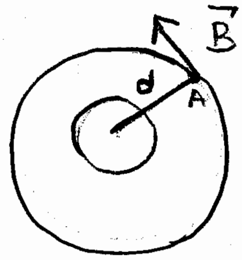
5)



$\vec{B}(A)$?

$\vec{B}(B)$?

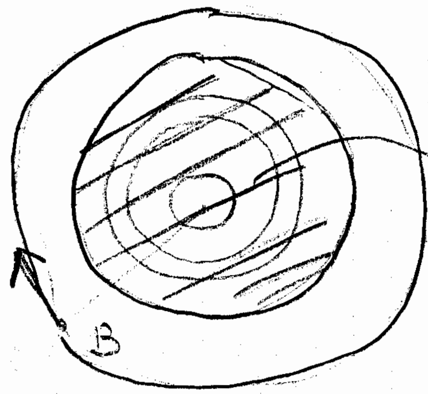
Simmetria cilindrica $\Rightarrow$



(come per il filo ∞)

$$2\pi d\, B = \mu_0 I \quad \Rightarrow \quad B(A) = \frac{\mu_0}{2\pi}\frac{I}{d}$$

Fuori dal cavo



$|I - 3I|$

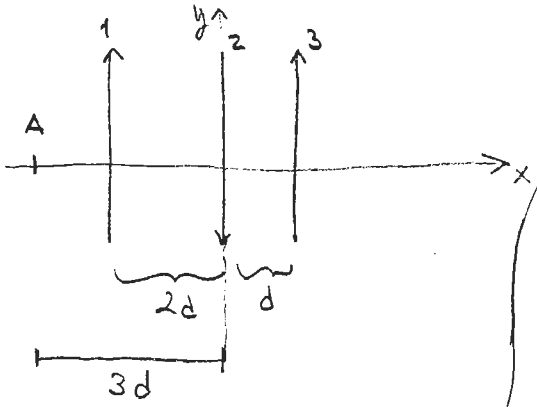
$= 2I$ ⊗

$\Rightarrow \vec{B}$ come in fig.

$$\cancel{2}\pi(3d)\, B = \mu_0 \cancel{2} I \quad \Rightarrow$$

$$B(B) = \frac{\mu_0}{\pi}\frac{I}{3d}$$

6)



$$\vec{B} = \sum_i \vec{B}_i$$

Ampère

$$2\pi d\, B_1 = \mu_0 I \Rightarrow \vec{B}_1 = \frac{\mu_0}{2\pi}\frac{I}{d}(+\hat{z})$$

Ampère

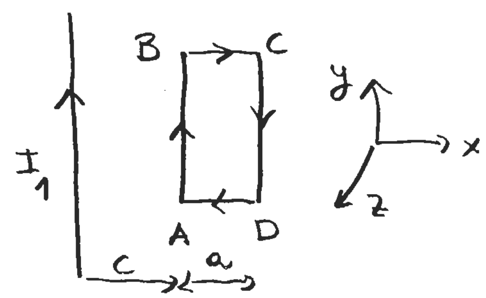
$$2\pi 3d\, B_2 = \mu_0 I \Rightarrow \vec{B}_2 = \frac{\mu_0}{2\pi}\frac{I}{3d}(-\hat{z})$$

Am

$$2\pi 4d\, B_3 = \mu_0 I \Rightarrow \vec{B}_3 = \frac{\mu_0}{2\pi}\frac{I}{4d}(+\hat{z})$$

$$\Rightarrow \vec{B} = \frac{\mu_0}{2\pi}\frac{I}{d}\hat{z}\left(1 - \frac{1}{3} + \frac{1}{4}\right) = \frac{\mu_0}{2\pi}\frac{I}{d}\frac{11}{12}(+\hat{z})$$

7)



$$\vec{B}_1(P) = \frac{\mu_0}{2\pi}\frac{I_1}{r}(-\hat{z}) \quad \forall P \in \{xy\}$$

$$\vec{F}_{AB} = \frac{\mu_0}{2\pi}\frac{I_1}{c} I_2\, b(-\hat{x})$$

$$\vec{F}_{CD} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{(c+a)}\, b(+\hat{x})$$

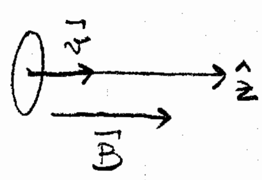
$$\underbrace{\vec{F}_{DA} = \frac{\mu_0}{2\pi} I_1 I_2 \int_c^{c+a} \frac{1}{x}\,dx\,(-\hat{y}) \qquad \vec{F}_{BC} = \frac{\mu_0}{2\pi} I_1 I_2 \int_c^{c+a} \frac{1}{x}\,dx\,(+\hat{y})}_{\vec{F}_{DA} + \vec{F}_{BC} = 0}$$

$$\vec{F} = \vec{F}_{AB} + \vec{F}_{CD} = \frac{\mu_0}{2\pi} I_1 I_2 b \left(\frac{1}{c+a} - \frac{1}{c}\right)\hat{x}$$

$$= \frac{\mu_0}{2\pi}\frac{I_1 I_2 b}{c(c+a)}(-a)\,\hat{x}$$

$$= -\frac{\mu_0}{2\pi}\frac{I_1 I_2 b a}{c(c+a)}\,\hat{x}$$

8)



$\vec{v} = v\,\hat{z}$

$\vec{B} = \beta z\ \hat{z}$

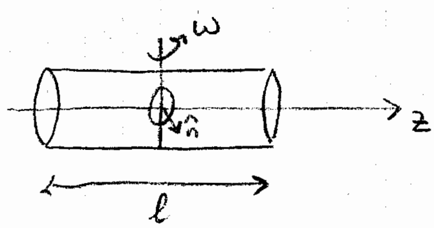
$$P = \frac{\mathcal{E}^2}{R} = \frac{1}{R}\left(\frac{d\phi_B}{dt}\right)^2$$

$$\phi_B = \pi r^2 \beta z$$

$$\frac{d\phi_B}{dt} = \pi r^2 \beta \frac{dz}{dt} = v\pi r^2 \beta$$

$$\Rightarrow \quad P = \frac{1}{R}\left(v\pi r^2\beta\right)^2$$

9)



N, i

$$\vec{B}_{in} = \mu_0 \frac{N}{\ell} i\,\hat{z}$$

angolo a $t=0$

$$\phi_B = \pi r^2 \left(\mu_0 \frac{N}{\ell} i\right) \hat{z}\cdot\hat{n} = \pi r^2 \left(\mu_0 \frac{N}{\ell} i\right) \cos(\omega t + \varphi)$$

$$\frac{d\phi_B}{dt} = -\pi r^2 \left(\mu_0 \frac{N}{\ell} i\right) \omega \sin(\omega t + \varphi)$$

$$P = \frac{\mathcal{E}^2}{R} = \frac{1}{R}\left[\pi r^2 \mu_0 \frac{N}{\ell} i\, \omega \sin(\omega t + \varphi)\right]^2$$

$$P_{max} \Rightarrow \sin(\omega t + \varphi) = \pm 1 \quad \Rightarrow$$

$$P_{max} = \frac{\left(\pi r^2 \mu_0 N/\ell\, i\omega\right)^2}{R}$$