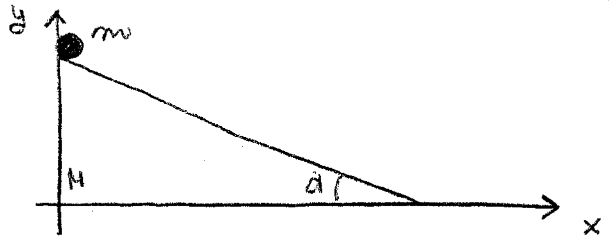
Riprendiamo il sistema in figura



Lascio il sistema dalla posizione mostrata in figura $\Rightarrow$

$\vec{v}_M(0) = 0$

$\vec{v}_m(0) = 0$

Trovare $\vec{v}_M$ e $\vec{v}_m$ quando m è arrivato in fondo. Conosco l = lunghezza del cuneo

Lavoro delle forze = variazione energia cinetica

$$= E_{c,fin} - \underset{\substack{\parallel \\ 0}}{E_{c,in}} \leftarrow \text{parte da fermo}$$

$$= \frac{1}{2} M V^2 + \frac{1}{2} m v^2$$

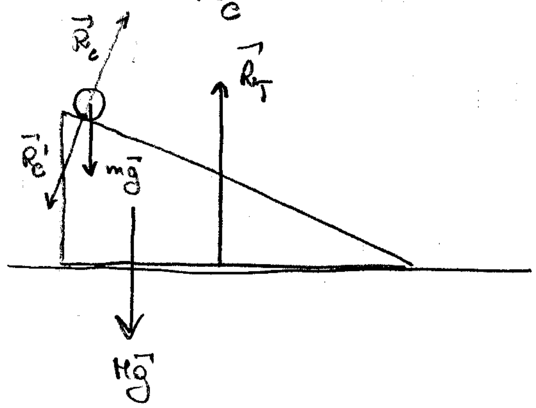
Forze: $M\vec{g}$ $\perp$ spostamento punto di applicazione $\Rightarrow \mathcal{L} = 0$

$m\vec{g}$

$\vec{R}_T$ $\perp$ spostamento punto di applic. $\Rightarrow \mathcal{L} = 0$

$\vec{R}_c$

$\vec{R}_c'$



$$\Rightarrow \mathcal{L}_{est} = \mathcal{L}_{m\vec{g}} + \mathcal{L}_{R_c} + \mathcal{L}_{R_c'}$$

$$d\mathcal{L}_{R_c} = \vec{R}_c \cdot d\vec{r}$$

$$d\mathcal{L}_{R_c'} = \vec{R}_c' \cdot d\vec{r}$$

stesso spostamento del punto di applicazione! $\Rightarrow$

Uso III principio $\Rightarrow$ $d\mathcal{L}_{R_e'} = \vec{R}_e' \cdot d\vec{r} = -\vec{R}_e \cdot d\vec{r} = -d\mathcal{L}_{R_e}$

$\Rightarrow$ $\mathcal{L}_{est} = \mathcal{L}_{m\vec{g}}$

$\mathcal{L}_{m\vec{g}} = mgl\sin\theta$

$$\Rightarrow \quad mgl\sin\theta = \frac{1}{2}mv^2 + \frac{1}{2}MV^2 = \frac{1}{2}m(v_x^2+v_y^2) + \frac{1}{2}MV^2 \quad (1)$$

Incognite: $v_x, v_y, V \Rightarrow$ 3 incognite, 1 eq $\Rightarrow$ ho bisogno di altre 2 eqns.

Forze esterne tutte $/\!/ \hat{y} \Rightarrow Q_x = \text{cost} = 0 \Rightarrow$

$$m v_x - MV = 0 \qquad v_x = \frac{M}{m}V \qquad (2)$$

Occorre un'altra eqn:

$$v_x = v_x' - V \qquad \text{(composizione velocità)}$$

$$\frac{v_y}{v_x'} = -\mathrm{tg}\,\alpha \quad \Rightarrow \quad v_y = -\mathrm{tg}\,\alpha\,(v_x + V) \qquad (3)$$

(1) – (3) : 3 eqn × 3 incognite. Risolviamo:

$$v_y^2 = \mathrm{tg}^2\alpha\,(v_x^2 + V^2 + 2v_xV) = \mathrm{tg}^2\alpha\left(\frac{M^2}{m^2}V^2 + V^2 + 2\frac{M}{m}V^2\right)$$

$$= \mathrm{tg}^2\alpha\, V^2\left(1+\frac{M}{m}\right)^2$$

$$\Rightarrow \quad mgl\sin\alpha = \frac{1}{2}\cancel{m}\frac{M^2}{m^{\cancel{2}}}V^2 + \frac{1}{2}\cancel{m}\,\mathrm{tg}^2\alpha\,\frac{(m+M)^2}{m^{\cancel{2}}}V^2 + \frac{1}{2}MV^2$$

$$= \frac{1}{2}V^2\left(\overbrace{M + \frac{M^2}{m}}^{M\left(1+\frac{M}{m}\right)} + \frac{(m+M)^2}{m}\mathrm{tg}^2\alpha\right)$$

$$V^2 = 2mgl\sin\alpha\left[(m+M)\left(\frac{M}{m} + \frac{m+M}{m}\mathrm{tg}^2\alpha\right)\right]^{-1}$$

$$V = \sqrt{2gl\sin\alpha \, \frac{m^2}{(m+M)\,(M+(m+M)\,tg^2\alpha)}}$$

$$= m\sqrt{\frac{2gl\sin\alpha}{(m+M)\,(M+(m+M)\,tg^2\alpha)}} \; ,$$

$$v_x = M\sqrt{\frac{2gl\sin\alpha}{(m+M)(M+(m+M)\,tg^2\alpha)}}$$

$$v_y = -tg\alpha\sqrt{\frac{2gl\sin\alpha}{(m+M)\,(M+(m+M)\,tg^2\alpha)}} \quad (m+M)$$

Limite $M \to \infty$ (mi aspetto M fermo e
$v = \sqrt{2gl\sin\alpha}$)

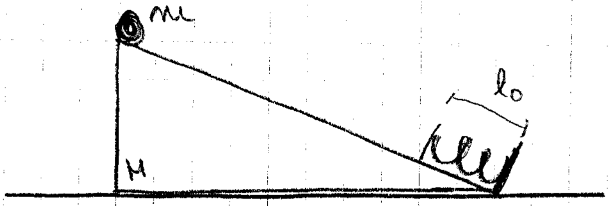
$$V \to 0$$

$$v_x \to \cancel{M}\sqrt{\frac{2gl\sin\alpha}{\cancel{M}\, M\,(1+tg^2\alpha)}} = \sqrt{\frac{2gl\sin\alpha}{\frac{1}{\cos^2\alpha}}} = \sqrt{2gl\sin\alpha}\,\cos\alpha$$

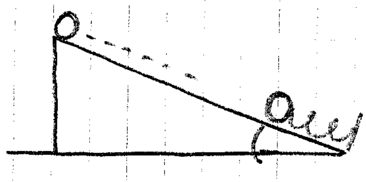
$$v_y \to -tg\alpha\sqrt{2gl\sin\alpha}\,\frac{\cancel{M}}{\cancel{M}\sqrt{1+tg^2\alpha}} = -\frac{\sin\alpha}{\cancel{\cos\alpha}}\sqrt{2gl\sin\alpha}\,\cancel{\cos\alpha}$$

$$\Rightarrow |v| = \sqrt{2gl\sin\alpha} \qquad \checkmark\checkmark$$

Supponiamo adesso che il cuneo abbia in fondo una molla (cost. elastica k, lungh. a riposo $= l_0$)
Quant'è la compressione massima:



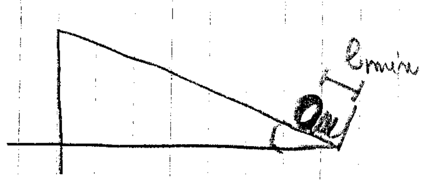
1ª parte:



m tocca la molla:

$$\frac{1}{2}MV^2 + \frac{1}{2}mv^2 = mg\,(l - l_0)\sin\alpha$$

2ª parte



la molla si comprime fino a compressione massima per cui l_{min}. Trovare l_{min}:

$$\mathcal{L}_{forze} = \Delta E_{cin} = \left(\frac{1}{2}MV_{fin}^2 + \frac{1}{2}mv_{fin}^2\right) - \left(\frac{1}{2}MV^2 + \frac{1}{2}mv^2\right)$$

$$= \left(\frac{1}{2}MV_{fin}^2 + \frac{1}{2}mv_{fin}^2\right) - mg\,(l - l_0)\sin\alpha$$

Troviamo v_{fin} e V_{fin}:

Forze lungo // $\Rightarrow$ $Q_x = \text{cost} = 0 \Rightarrow$

$$m\,v_{fin_x} - M\,V_{fin} = 0$$

$$\begin{cases} v_{fin,x} = v'_{fin,x} - V \\ v_{fin\,y} = v'_{fin\,y} \end{cases}$$

compressione max $\Rightarrow$ $v'_{fin,x} = v'_{fin,y} = 0$

$$\Rightarrow \quad v_{fin,x} = -V \Rightarrow \quad V = v_{fin,x} = v_{fin,y} = 0$$

$$\Rightarrow \mathcal{L}_{forze} = -mg(l-l_0)\sin\alpha$$

Forze che fanno lavoro:

$$\mathcal{L}_{peso\ m\vec{g}} \quad , \quad \mathcal{L}_{F elastica}$$

$$\mathcal{L}_{m\vec{g}} = mg(l_0 - l_{min})\sin\alpha$$

$$\mathcal{L}_{Fel} = \int_{l_0}^{l_{min}} -k(x-l_0)\,dx = \int_0^{l_{min}-l_0} -ky\,dy$$

$$= -\frac{1}{2}k(l_{min}-l_0)^2$$

$$-\frac{1}{2}k(l_{min}-l_0)^2 + \cancel{mg l_0 \sin\alpha} - mg\, l_{min}\sin\alpha$$

$$= -mg\, l\sin\alpha + \cancel{mg l_0 \sin\alpha}$$

$$\frac{1}{2}k(l_{min}-l_0)^2 + mg(l_{min}-l_0)\sin\alpha - mg(l-l_0)\sin\alpha = 0$$

$$l_{min} - l_0 = x$$

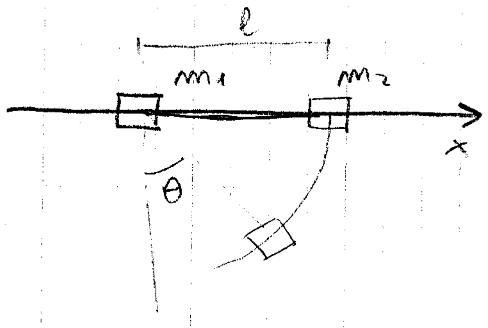
$$kx^2 + 2mg\sin\alpha\, x - 2mg(l-l_0)\sin\alpha = 0$$

$$x = \frac{-mg\sin\alpha \pm \sqrt{m^2g^2\sin^2\alpha + 2mgk(l-l_0)\sin\alpha}}{k}$$

soluzione giusta (x deve essere <0)

$$l_{min} = l_0 - \frac{mg\sin\alpha}{k}\left(1 + \sqrt{1 + 2\frac{k(l-l_0)}{mg\sin\alpha}}\right)$$

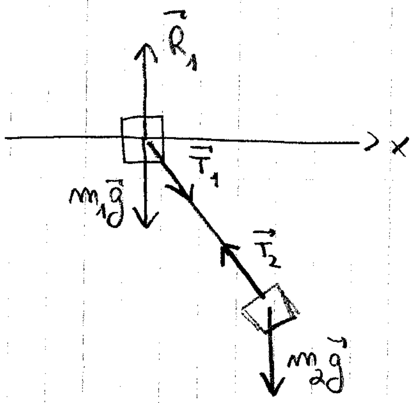
Esercizio del pendolo doppio



m_1 libero di scorrere lungo x senza attrito.

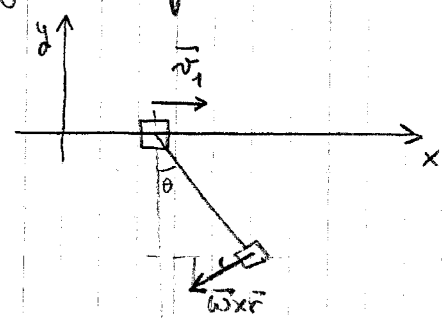
Dato m_1, m_2 e l (lunghezza del filo), trovare $v_1(\theta)$ e $v_2(\theta)$

Forze



Forze esterne // $\vec{y}$ $\Rightarrow$ $m_1 v_1 + m_2 v_{2x} = 0$ (1)

In generale posso scrivere $\vec{v}_2 = \vec{v}_1 + \vec{\omega} \times \vec{r}$



$\Rightarrow$ $v_{2,x} = v_1 - \omega l \cos\theta$

$v_{2y} = - \omega l \sin\theta$

$\Rightarrow$ $m_1 v_1 + m_2 v_1 - m_2 \omega l \cos\theta = 0$

$\Rightarrow$ $v_1 = \frac{m_2}{m_{12}} \omega l \cos\theta$

Adesso devo trovare ω:

$\mathcal{L}_{Forze} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$ $\leftarrow$ teorema delle forze vive

$\mathcal{L}_{R_1} = 0$ $\quad (\vec{R}_1 \perp d\vec{r}_1)$

$\mathcal{L}_{m_1 g} = 0$ $\quad (m_1 \vec{g} \perp d\vec{r}_1)$

$\mathcal{L}_{T_1} = \int \vec{T}_1 \cdot d\vec{r}_1 = T_1 \cdot \Delta x_1 \sin\theta$

$$\mathcal{L}_{T_2} = \int \vec{T}_2 \cdot d\vec{r}_2 = \int \vec{T}_2 \cdot \left(d\vec{r}_1 + d(\vec{\omega}\times\vec{r}_2) \right) \underset{\uparrow}{=} \int \vec{T}_2 \cdot d\vec{r}_1 = - \mathcal{L}_{T_1}$$

$d(\vec{\omega}\times\vec{r}_2) \perp$ filo

$$\Rightarrow \mathcal{L}_{Forze} = \mathcal{L}_{m_2 g} = m_2 g\, l \cos\theta$$

$$m_2 g\, l \cos\theta = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 \left(v_1^2 + \omega^2 l^2 - 2 v_1 \omega l \cos\theta \right)$$

$$= \cancel{\frac{1}{2} m_{12} \frac{m_2^2}{m_{12}^2} \omega^2 l^2 \cos^2\theta} + \frac{1}{2} m_2 \omega^2 l^2$$

$$- m_2 \frac{m_2}{m_{12}} \omega^2 l^2 \cos^2\theta \quad \frac{1}{2}$$

$$= \frac{1}{2} m_2 \omega^2 l^2 \left(1 - \frac{m_2}{m_{12}} \cos^2\theta \right)$$

$$= \frac{1}{2} m_2 \omega^2 l^2 \left(\frac{m_1 + m_2 \sin^2\theta}{m_{12}} \right)$$

$$\Rightarrow \omega = \sqrt{2 \frac{g}{l} \cos\theta \frac{m_1 + m_2}{m_1 + m_2 \sin^2\theta}} .$$

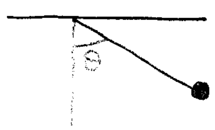
— Consideriamo $m_1 \longrightarrow \infty$:

$$\omega \longrightarrow \sqrt{2\frac{g}{l} \cos\theta}$$

$$|\vec{v}_1| \longrightarrow 0$$

$$|\vec{v}_2| \longrightarrow \sqrt{2g\, l \cos\theta}$$

Questo caso corrisponde al pendolo semplice!



$$\mathcal{L} = mg\, l \cos\theta = \frac{1}{2} m v^2$$

$$v = \sqrt{2g\, l \cos\theta} = |\vec{v}_2| \quad !$$