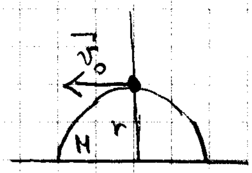
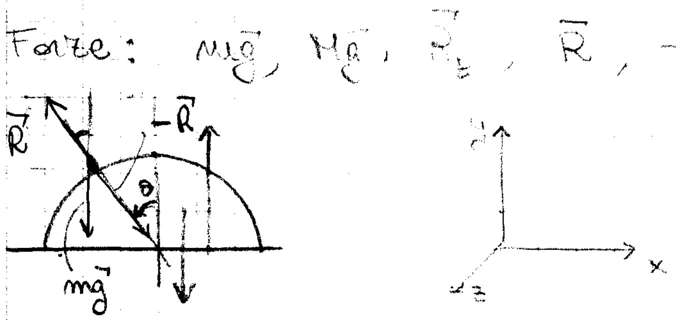
Esercizio della pallina su semisfera mobile



1) A che angolo si stacca?

Conservazione dell'energia si può applicare?

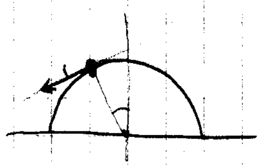
Forze: $m\vec{g}$, $M\vec{g}$, $\vec{R}_z$, $\vec{R}$, $-\vec{R}$



$m\vec{g}$ e $M\vec{g}$ conservative, le altre non fanno lavoro $\Rightarrow$

$$\frac{1}{2} m v_0^2 + mgr = \frac{1}{2} M V^2 + \frac{1}{2} m v^2 + mgr\cos\theta \qquad (1)$$

So che $\vec{v} = \vec{V} + \vec{\omega} \times \vec{r}$ $\qquad \vec{\omega} = +\dot{\theta}\hat{z}$



Inoltre lungo l'asse $\hat{x}$ ho conservazione della q.d.m. totale (forze esterne sono lungo y) $\Rightarrow$

$$\begin{cases} MV + m\dot{x} = -mv_0 \\ \dot{x} = V - r\dot{\theta}\cos\theta \\ \dot{y} = -r\dot{\theta}\sin\theta \end{cases} \Rightarrow \quad MV + m(+V - \dot{\theta}r\cos\theta) = -mv_0$$

$$V = -\frac{m}{m+M}(v_0 - r\dot{\theta}\cos\theta)$$

$\Rightarrow$ (1) diventa:

$$\frac{1}{2} m v_0^2 + mgr = \frac{1}{2} M V^2 + \frac{1}{2} m \left(V^2 + \dot{\theta}^2 r^2 - 2V\dot{\theta}r\cos\theta\right)$$

$$+ mgr\cos\theta$$

$$\frac{1}{2} m v_0^2 + mgr = \frac{1}{2}\cancel{(m+M)}\frac{m^2}{(m+M)^2}(v_0 - r\dot\theta\cos\theta)^2$$

$$+\frac{1}{2} m r^2\dot\theta^2 - m r\dot\theta\cos\theta\,\frac{-m}{m+M}(v_0 - r\dot\theta\cos\theta) + mgr\cos\theta$$

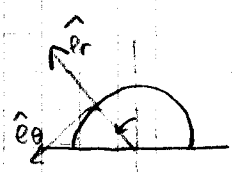
$$\frac{1}{2}\cancel{m} v_0^2 + \cancel{m}gr = \frac{1}{2}\frac{m^{\cancel{2}}}{M_T}v_0^2 + \cancel{\frac{1}{2}\frac{m^2}{M_T}r^2\dot\theta^2\cos^2\theta} - \cancel{\frac{m^2}{M_T}v_0 r\dot\theta\cos\theta}$$

$$+\frac{1}{2}\cancel{m} r^2\dot\theta^2 + \cancel{\frac{m^2}{M_T}v_0 r\dot\theta\cos\theta} - \overset{1/2}{\cancel{1}}\frac{m^{\cancel{2}}}{M_T}r^2\dot\theta^2\cos^2\theta + \cancel{m}gr\cos\theta$$

$$\frac{1}{2} v_0^2\left(\frac{M+\cancel{m}-\cancel{m}}{M_T}\right) + gr(1-\cos\theta) + \frac{1}{2}r^2\dot\theta^2\left[\frac{m}{M_T}\cos^2\theta - 1\right] = 0$$

$$\frac{1}{2}\frac{M}{M_T}v_0^2 + gr(1-\cos\theta) + \frac{1}{2}r^2\dot\theta^2\left[\frac{m}{M_T}\cos^2\theta - 1\right] = 0 \qquad (2)$$

Condizione perchè si stacchi: $R=0 \Rightarrow$



$$m a_r = R - mg\cos\theta \qquad \text{SR semisfera}$$

$$a_r = -\omega^2 r = -r\dot\theta^2$$

$$\Rightarrow R=0 \Rightarrow \cancel{m} r\dot\theta^2 = \cancel{m} g\cos\theta \Rightarrow$$

$$\frac{1}{2}\frac{M}{M_T}v_0^2 + gr - \cancel{gr\cos\theta} + \frac{1}{2}\frac{m}{M_T}gr\cos^3\theta - \overset{3/2}{\cancel{\tfrac{1}{2}}}rg\cos\theta = 0$$

$$\boxed{\frac{m}{M_T}gr\cos^3\theta - 3gr\cos\theta + \frac{M}{M_T}v_0^2 + 2gr = 0} \qquad (3)$$

Eq. difficile. Più semplice per $M \gg m \Rightarrow$

$$-3\cancel{gr}\cos\theta + 2\cancel{gr} + \frac{v_0^2}{gr} = 0$$

$$3\cos\theta = 2 + \frac{v_0^2}{gr} \Rightarrow \cos\theta = \left(\frac{2gr + v_0^2}{3gr}\right)$$

2) Quanto deve essere v_0 perchè si stacchi subito?

Da (3) con $\theta = 0$

$$\underbrace{\frac{m}{M_T} gr - \cancel{1} gr} + \frac{M}{M_T} v_0^2 \cancel{+ 2gr} = 0$$

$$\left(\frac{\cancel{m} - \cancel{m} - M}{M_T} \right) gr + \frac{M}{\cancel{M_T}} v_0^2 = 0$$

$$v_0 = \sqrt{gr}$$

3) Trovare le eqn. del moto finchè m resta attaccato a M.

a) I eqn. cardinale:

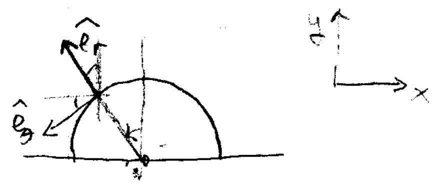
$$\begin{cases} m\ddot{x} = -R \sin\theta \\ m\ddot{y} = R\cos\theta - mg \\ M\ddot{x}_M = +R\sin\theta \\ M\ddot{y}_M = 0 = -Mg + R_t - R\cos\theta \end{cases}$$

$$m\ddot{x} + M\ddot{x}_M = 0 \qquad (\text{ f. esterne solo } /\!/ \hat{x})$$

$$\vec{a} = \vec{a}_{tr} + \vec{a}'$$

Nota che $\vec{v}' = r\dot{\theta}\,\hat{e}_\theta$

$$\vec{a}' = -r\dot{\theta}^2\hat{e}_r + r\ddot{\theta}\,\hat{e}_\theta$$



$$\Rightarrow \begin{cases} \ddot{x} = \ddot{x}_M + r\dot{\theta}^2\sin\theta - r\ddot{\theta}\cos\theta \\ \ddot{y} = \quad -r\dot{\theta}^2\cos\theta - r\ddot{\theta}\sin\theta \end{cases}$$

$$\Rightarrow \begin{cases} m\ddot{x}_M + m r\dot{\theta}^2\sin\theta - m r\ddot{\theta}\cos\theta = -R\sin\theta \\ \quad - m r\dot{\theta}^2\cos\theta - m r\ddot{\theta}\sin\theta = R\cos\theta - mg \\ M\ddot{x}_M = R\sin\theta \end{cases}$$

Risolvo

$$+\frac{m}{M}R\sin\theta + m r\dot{\theta}^2\sin\theta - m r\ddot{\theta}\cos\theta = -R\sin\theta$$

$$\cos\theta\left(R\sin\theta\left(1+\frac{m}{M}\right)\right) = \left[-m r\dot{\theta}^2\sin\theta + m r\ddot{\theta}\cos\theta\right]\cos\theta$$

$$-\cancel{m} r\dot{\theta}^2\sin\theta\cos\theta + \cancel{m} r\ddot{\theta}\sin^2\theta =$$

$$\sin\theta\left(1+\frac{m}{M}\right)\left(\cancel{m}g - \cancel{m}r\dot{\theta}^2\cos\theta - \cancel{m}r\ddot{\theta}\sin\theta\right)$$

$$r\ddot{\theta}\left(\underbrace{\sin^2\theta + \cos^2\theta}_{1} + \sin^2\theta\frac{m}{M}\right)$$

$$+ r\dot{\theta}^2\sin\theta\cos\theta\left[\cancel{1} - \cancel{1} + \frac{m}{M}\right] - g\sin\theta\left(1+\frac{m}{M}\right) = 0$$

$$r\ddot{\theta}\left(\underbrace{1+\frac{m}{M}}_{\frac{m+M}{M}} - \frac{m}{M}\cos^2\theta\right)\frac{M}{m+M} + r\dot{\theta}^2\sin\theta\cos\theta\,\frac{m}{\cancel{M}}\,\frac{\cancel{M}}{m+M}$$

$$- g\sin\theta = 0$$

$$r\ddot{\theta}\left(1 - \frac{m}{m+M}\cos^2\theta\right) + r\dot{\theta}^2\sin\theta\cos\theta\,\frac{m}{m+M} - g\sin\theta = 0$$

$\dot{\theta}^2$ si ottiene dall'Eq. (2)

b) dall' Eq. (2) : $E = \text{cost} \Rightarrow \frac{dE}{dt} = 0$

$$+ g\cancel{r}\sin\theta\,\cancel{\dot{\theta}} + \cancel{\tfrac{1}{2}}r^2\,\cancel{2}\cancel{\dot{\theta}}\,\ddot{\theta}\left[\frac{m}{m+M}\cos^2\theta - 1\right]$$

$$+ \cancel{\tfrac{1}{2}}r^2\dot{\theta}^2\left[-\frac{m}{m+M}\cancel{2}\sin\theta\cos\theta\,\cancel{\dot{\theta}}\right] = 0$$

$$r\ddot{\theta}\left[1-\frac{m}{m+M}\cos^2\theta\right]+r\dot{\theta}^2\sin\theta\cos\theta\,\frac{m}{m+M}-g\sin\theta=0$$

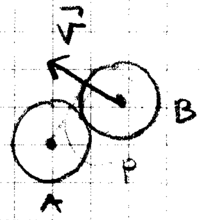
Assai più veloce che con 2) !!

$$r\dot{\theta}^2=\frac{\cancel{\frac{1}{2}}\frac{M}{M_T}\frac{v_0^2}{r}+\cancel{2}g\cancel{r}(1-\cos\theta)}{\cancel{\frac{1}{2}}\cancel{r}\left(\frac{m}{M_T}\cos^2\theta-1\right)}$$

$$\Rightarrow\quad r\ddot{\theta}\left[1-\frac{m}{m+M}\cos^2\theta\right]+\frac{m}{m+M}\sin\theta\cos\theta\,\frac{\frac{M}{M+m}\frac{v_0^2}{r}+2g(1-\cos\theta)}{\left(\frac{m}{m+M}\cos^2\theta-1\right)}$$

$$-g\sin\theta=0$$

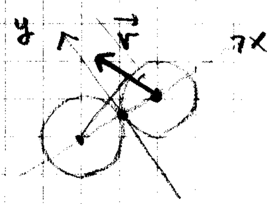
Esercizio sull'urto di 2 sfere rigide



r, m, P

Non c'è attrito di sorta $\Rightarrow$ le forze esercitate nell'urto sono lungo la congiungente dei centri

$\Downarrow$



$$\begin{cases} V_y = u_{B,y} \quad \text{e} \quad u_{Ay} = 0 \\ m_B V_x = m_B u_{B,x} + m_A u_{A,x} \\ m_B(V_x^2 + \cancel{V_y^2}) = m_B(u_{B,x}^2 + \cancel{u_{B,y}^2}) + m_A u_{A,x}^2 \end{cases}$$

$$\Rightarrow \begin{cases} m_B V_x = m_B u_{Bx} + m_A u_{Ax} \\ m_B V_x^2 = m_B u_{B,x}^2 + m_A u_{Ax}^2 \end{cases}$$

$$u_{Ax} = \frac{m_B}{m_A}(V_x - u_{Bx})$$

$$\cancel{m_B} V_x^2 = \cancel{m_B} u_{Bx}^2 + \cancel{m_A} \frac{m_B^{\cancel{2}}}{m_A^{\cancel{2}}} V_x^2 + \cancel{m_A} \frac{m_B^{\cancel{2}}}{m_A^{\cancel{2}}} u_{B,x}^2 - 2 \frac{m_B^{\cancel{2}}}{m_A^{\cancel{2}}} V_x u_{Bx} \cancel{m_A}$$

$$u_{Bx}^2 \frac{M_T}{\cancel{m_A}} - 2 \frac{m_B}{\cancel{m_A}} V_x u_{Bx} + \frac{m_B - m_A}{\cancel{m_A}} V_x^2 = 0$$

$$u_{B,x} = \frac{m_B V_x \pm \sqrt{\cancel{m_B^2} V_x^2 - (\cancel{m_B^2} - m_A^2) V_x^2}}{m_A + m_B}$$

urto non c'è stato!

$$= V_x \frac{m_B \pm m_A}{m_A + m_B} = V_x \frac{m_B - m_A}{m_A + m_B}$$

$$u_{Ax} = \frac{m_B}{m_A} V_x \left(1 - \frac{m_B - m_A}{m_A + m_B}\right) = \frac{m_B}{m_A} V_x \frac{2 m_A}{m_A + m_B}$$
$$= \frac{2 m_B}{m_A + m_B} V_x$$

Vediamo gli angoli tra $\vec{u}_A$, $\vec{u}_B$ e $\vec{V}$:

$$\vec{u}_A \cdot \vec{u}_B = u_{Ax} u_{Bx} + u_{Ay} u_{By} = 2 m_B \frac{m_B - m_A}{(m_A + m_B)^2} V_x^2$$

$$= u_A u_B \cos\theta_{AB}$$

$$\cos\theta_{AB} = \frac{u_{Bx}}{u_B} = \frac{m_B - m_A}{m_A + m_B} \frac{V_x}{\sqrt{V_x^2 \left(\frac{m_B - m_A}{m_A + m_B}\right)^2 + V_y^2}}$$

$$\vec{u}_B \cdot \vec{V} = V_y^2 + u_{Bx} V_x = V_y^2 + V_x^2 \frac{m_B - m_A}{m_A + m_B}$$

$$\cos\theta_B = \frac{V_x^2 \frac{m_B - m_A}{m_A + m_B} + V_y^2}{V \sqrt{V_x^2 \left(\frac{m_B - m_A}{m_A + m_B}\right)^2 + V_y^2}}$$

Caso interessante è $m_A = m_B$:

$$\cos\theta_{AB} = 0 \qquad \theta_{AB} \quad \frac{\pi}{2}$$

$$\cos\theta_B = \frac{V_y}{V} =$$

$$= \frac{p}{2r}$$

$$u_{Bx} = 0 \qquad u_{By} = V \left(\frac{p}{2r}\right)$$

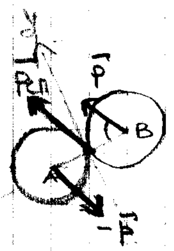
$$u_{Ax} = V_x \qquad u_{Ay} = 0$$

SR c.m.

$$\begin{cases} \vec{p}_{CM} = \vec{p}_B \\ \vec{p} = \dfrac{m_A \vec{p}_B - m_B \vec{p}_A}{m_A + m_B} \end{cases} \qquad \vec{q} = \frac{m_A \vec{q}_B - m_B \vec{q}_A}{m_A + m_B}$$

$$\Downarrow$$

$$\begin{cases} \vec{p}_A = \dfrac{m_A}{M_T} p_{CM} - \vec{p} \\ \vec{p}_B = \dfrac{m_B}{M_T} p_{CM} + \vec{p} \end{cases} \qquad \begin{aligned} \vec{q}_A &= \frac{m_A}{M_T} p_{CM} - \vec{q} \\ \vec{q}_B &= \frac{m_B}{m_T} p_{CM} + \vec{q} \end{aligned}$$



Non agiscono forze lungo y né su A, né su B $\Rightarrow$

$$q_y = p_y$$

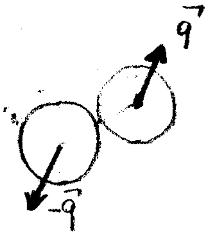
Nel S.R. CM c. energia cinetica:

$$\cancel{\frac{p_{CM}^2}{2M_T}} + \frac{p^2}{2\mu} = \cancel{\frac{p_{CM}^2}{2M_T}} + \frac{q^2}{2\mu}$$

$$p_x^2 + \cancel{p_y^2} = q_x^2 + \cancel{q_y^2} \qquad q_x = \pm p_x$$

$\Rightarrow$ $q_x = -p_x$ (altrimenti non ho urto!)

prima



Verifichiamo con

$$\vec q = \vec q_B - \frac{m_B}{m_T}\vec P_B$$

$$q_x = m_B u_{B,x} - \frac{m_B}{m_T} m_B V_x$$

$$= m_B V_x\left(\frac{\cancel{m_B}-m_A}{m_T} - \frac{\cancel{m_B}}{m_T}\right) = -\frac{m_A m_B}{m_T} V_x$$

$$p_x = m_B V_x - \frac{m_B}{m_T} m_B V_x = \frac{m_B m_A V_x}{m_T}$$

$\Rightarrow$ $q_x = -p_x$ (nota che $p_x = \mu V_x$!)

$$q_y = m_B u_{B x} - \frac{m_B}{m_T} m_B V_y =$$

$$= m_B V_y \left(1 - \frac{m_B}{m_T}\right) = \mu V_y = p_y$$

Angolo tra $\vec p$ e $\vec q$

$$\cancel{pq}\cos\theta = p_x q_x + p_y q_y = p_x^2 - p_y^2$$

$$= \cancel{p^2}\cos^2\alpha - \cancel{p^2}\sin^2\alpha$$

$$\cos\theta = (1 - 2\sin^2\alpha) = 1 - \cancel{2}\frac{p^2}{\cancel{4}_2 r^2} = \frac{2r^2 - p^2}{2r^2}.$$

$\Rightarrow$ nel c.m. l'angolo dipende da r e p, ma non dalle masse!