

H. A. Bethe and C. L. Critchfield: Phys. Rev. **54**, 248 (1938)

TABLE I. *Numerical results for two values of the radius.*

	$r_0 = e^2/mc^2$	$r_0 = e^2/2\ mc^2$
x_0	0.645	0.322
V_0 (Mev)	20.9	66.5
D (Mev)	10.3	47.0
μ	2.94	5.45
ν	2.18	4.65
$(rd \log w/dr)$	0.236	0.110
$\Phi(r_0)$	1.050	1.025
$\Theta(r_0)$	0.769	0.854
$(rd \log \Phi/dr)$	0.050	0.025
ξ	0.814	0.915
λ	2.63	4.80
A_1	0.689	0.277
A_2	1.949	1.547
A_3	1.205	1.030
$(1+x_0)(1+\mu^{-2})$	1.835	1.367
A	2.84	2.44
A^2	8.08	5.93

¹² Breit, Condon and Present, Phys. Rev. **50**, 825 (1936).

Reazione nucleare di interesse astrofisico

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Riassunto del corso:

- teoria del momento angolare: fondamentale per la costruzione della funzioni d'onda
- teoria perturbativa dipendente dal tempo: calcolo della sezione d'urto (e fattore astrofisico)
- funzioni d'onda: stati legati (d , ^3He , ^3H) e stati di scattering (pp , pd , nd)

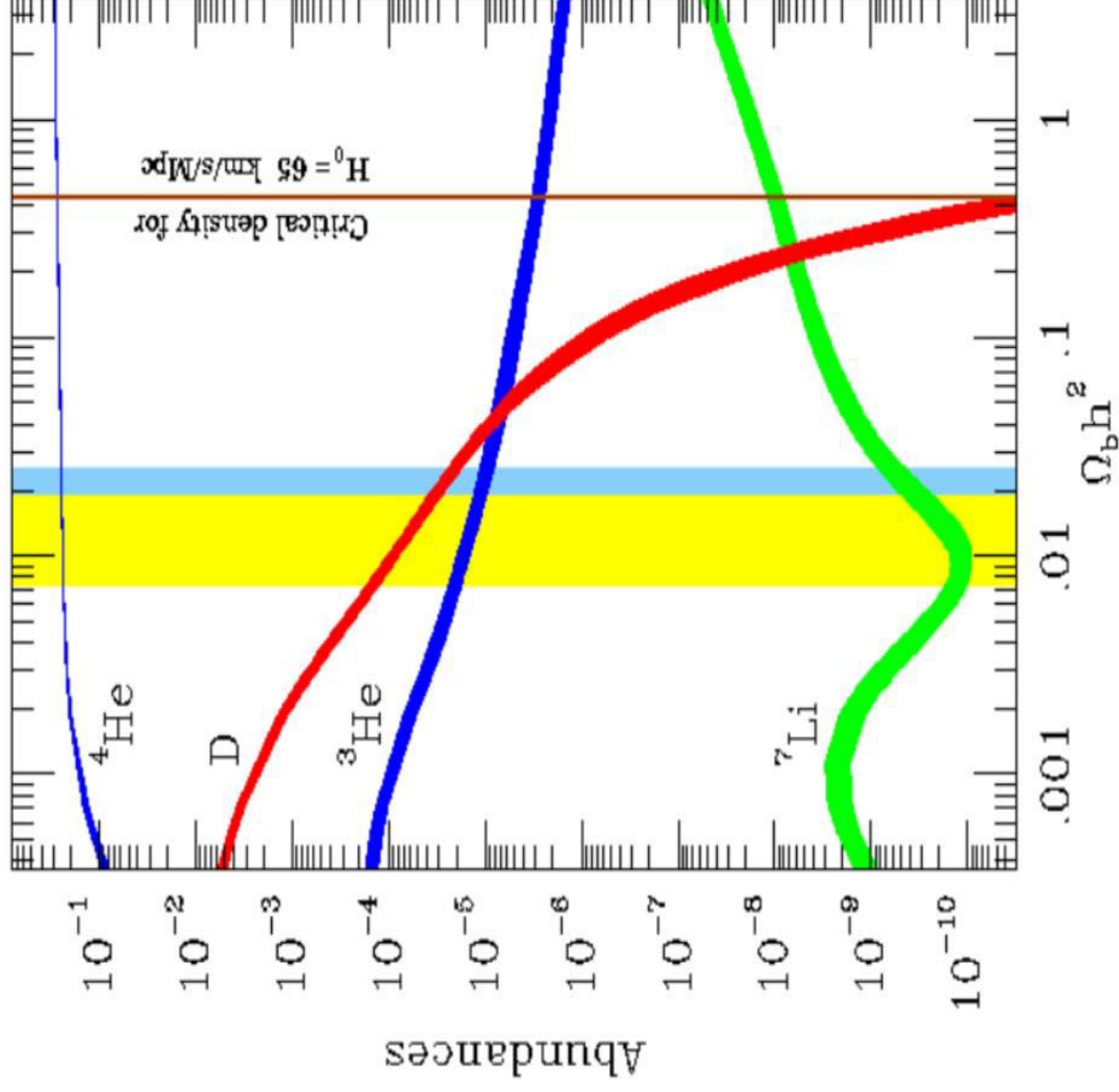
Campi di applicazione:

- Big Bang nucleosynthesis (BBN)
- Produzione solare di energia attraverso una catena di reazioni nucleari

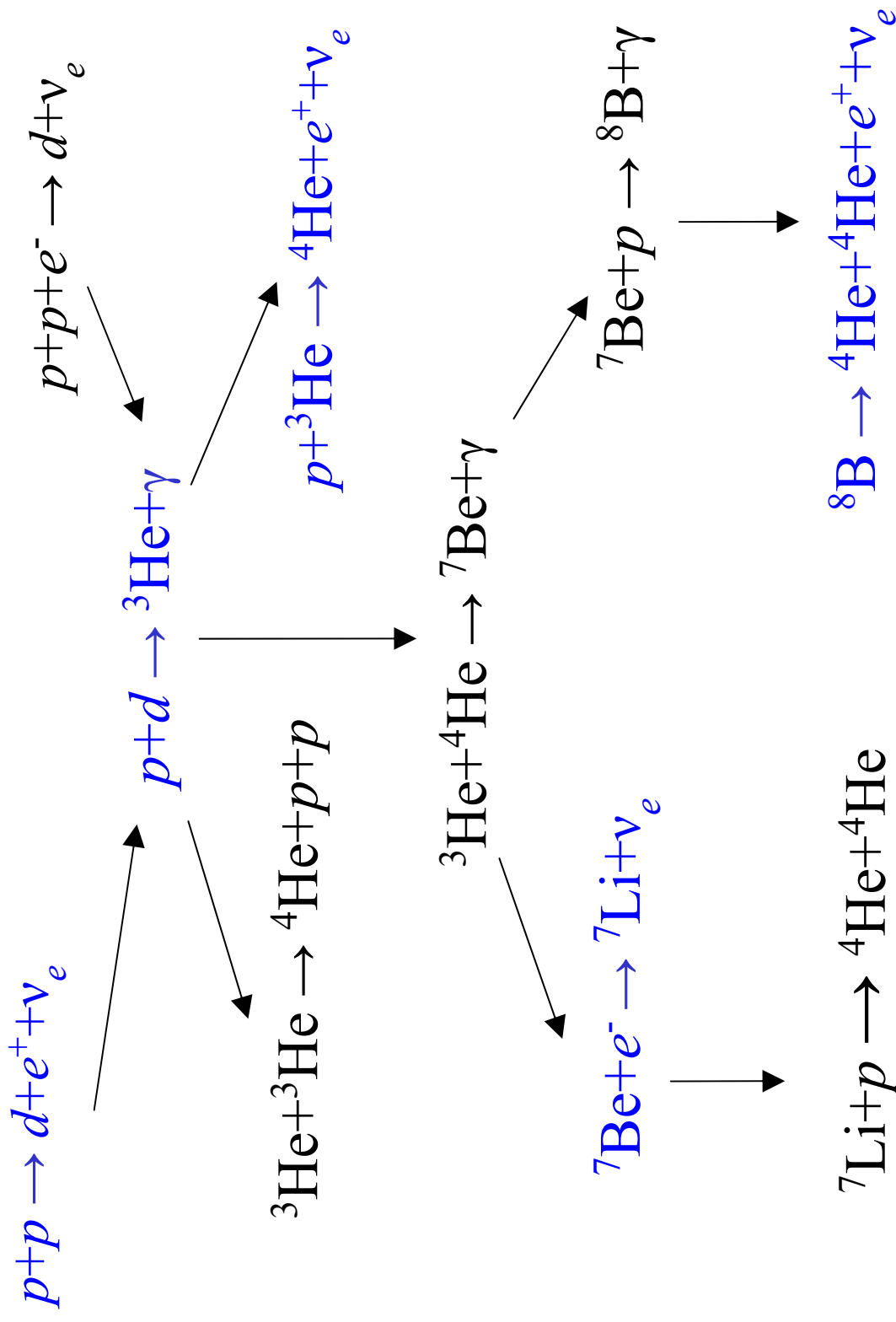
Big Bang nucleosynthesis (BBN)

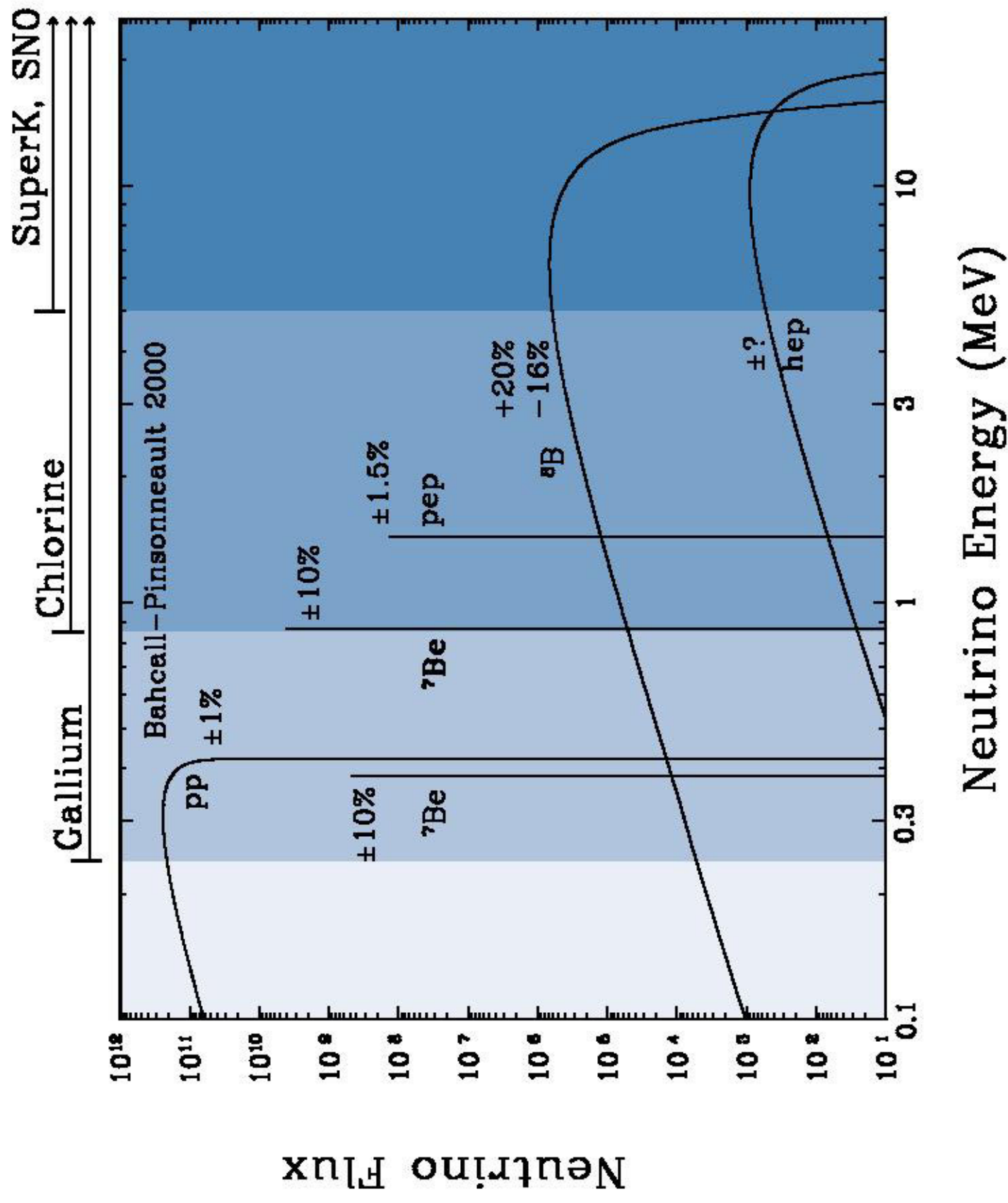
(naïve picture)

- Step 1 ($kT < \text{few MeV}$):
thermal eq.; $n/p \sim 1$
 - Step 2 ($kT \sim 0.7 \text{ MeV}$): weak
int. freeze-out, $n/p \sim 1/6$
 $\rightarrow 1/7$
 - Step 3 ($kT \sim 0.5 \text{ MeV}$):
nuclear reactions, which
form d , ${}^3\text{He}$, ${}^4\text{He}$, ${}^7\text{Li}$
- 11 key nuclear reactions,
among which:
- $n + p \rightarrow d + \gamma$
 - $p + d \rightarrow {}^3\text{He} + \gamma$
 - ${}^3\text{H} + \alpha \rightarrow {}^7\text{Li} + \gamma$
 - ${}^3\text{He} + \alpha \rightarrow {}^7\text{Be} + \gamma$
 - ${}^7\text{Be} + n \rightarrow {}^7\text{Li} + p$



Solar neutrinos: pp chain



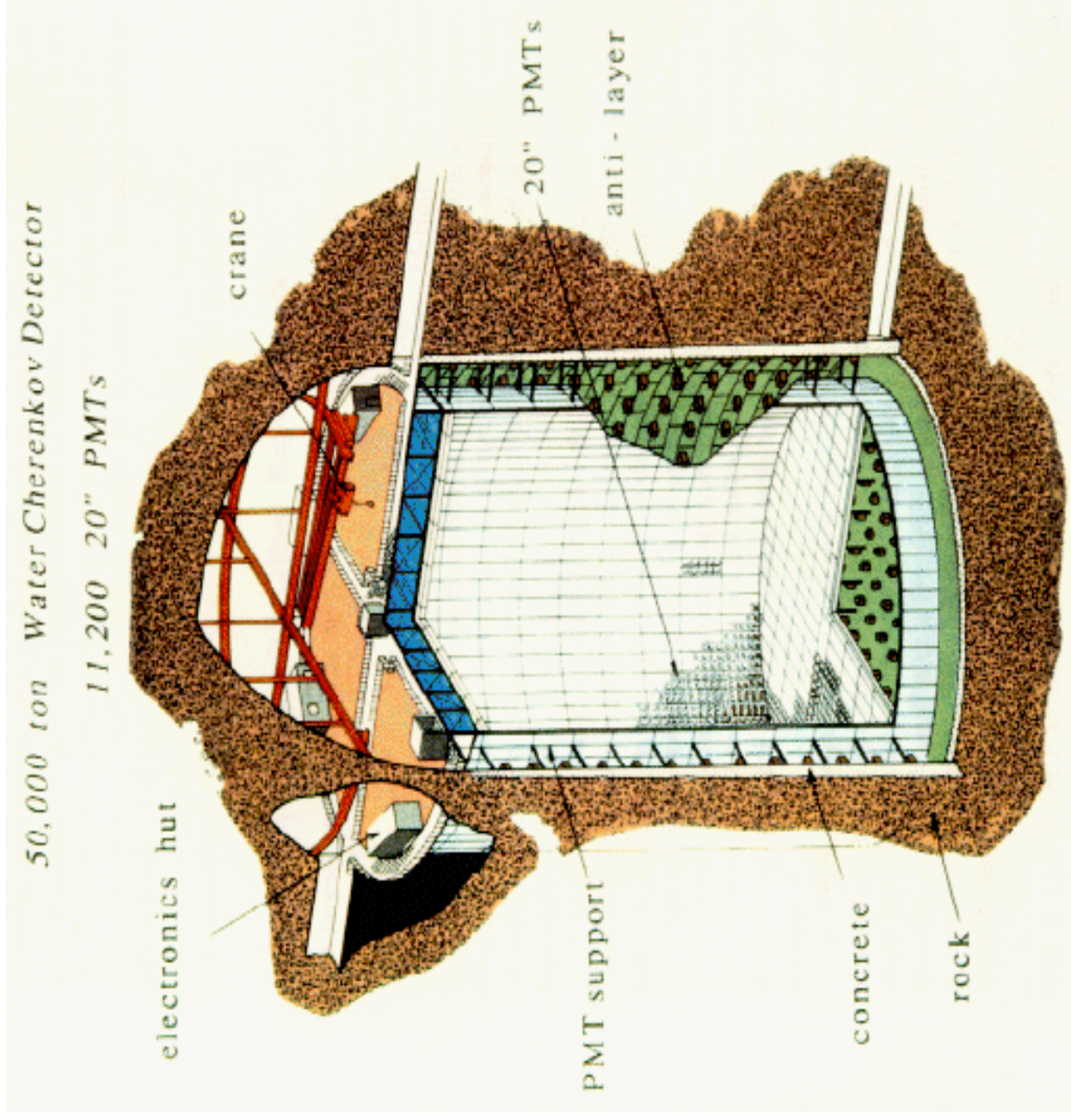


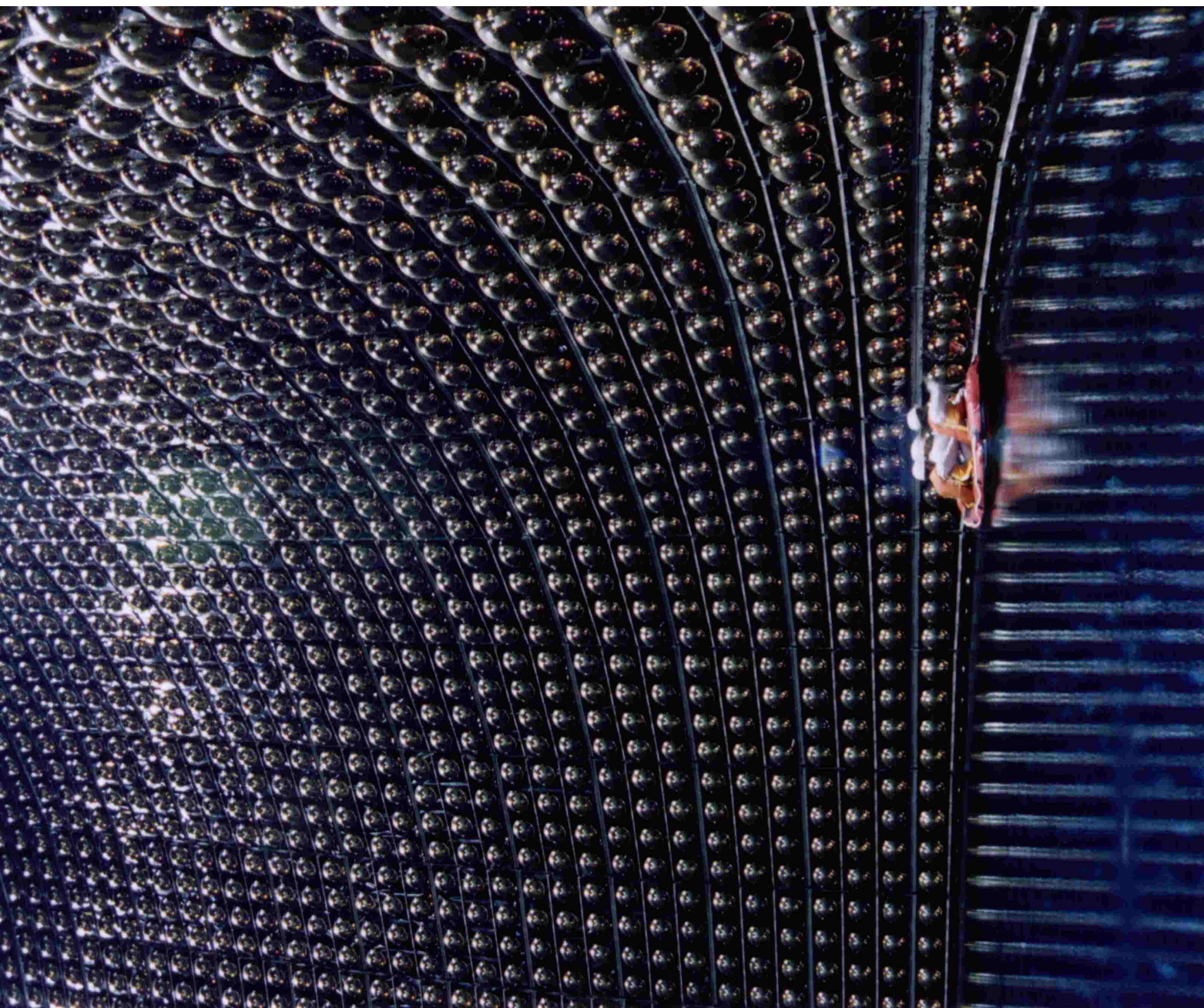
The SuperKamiokande (SK) experiment

Elastic scattering

$$\nu + e^- \rightarrow \nu + e^-$$

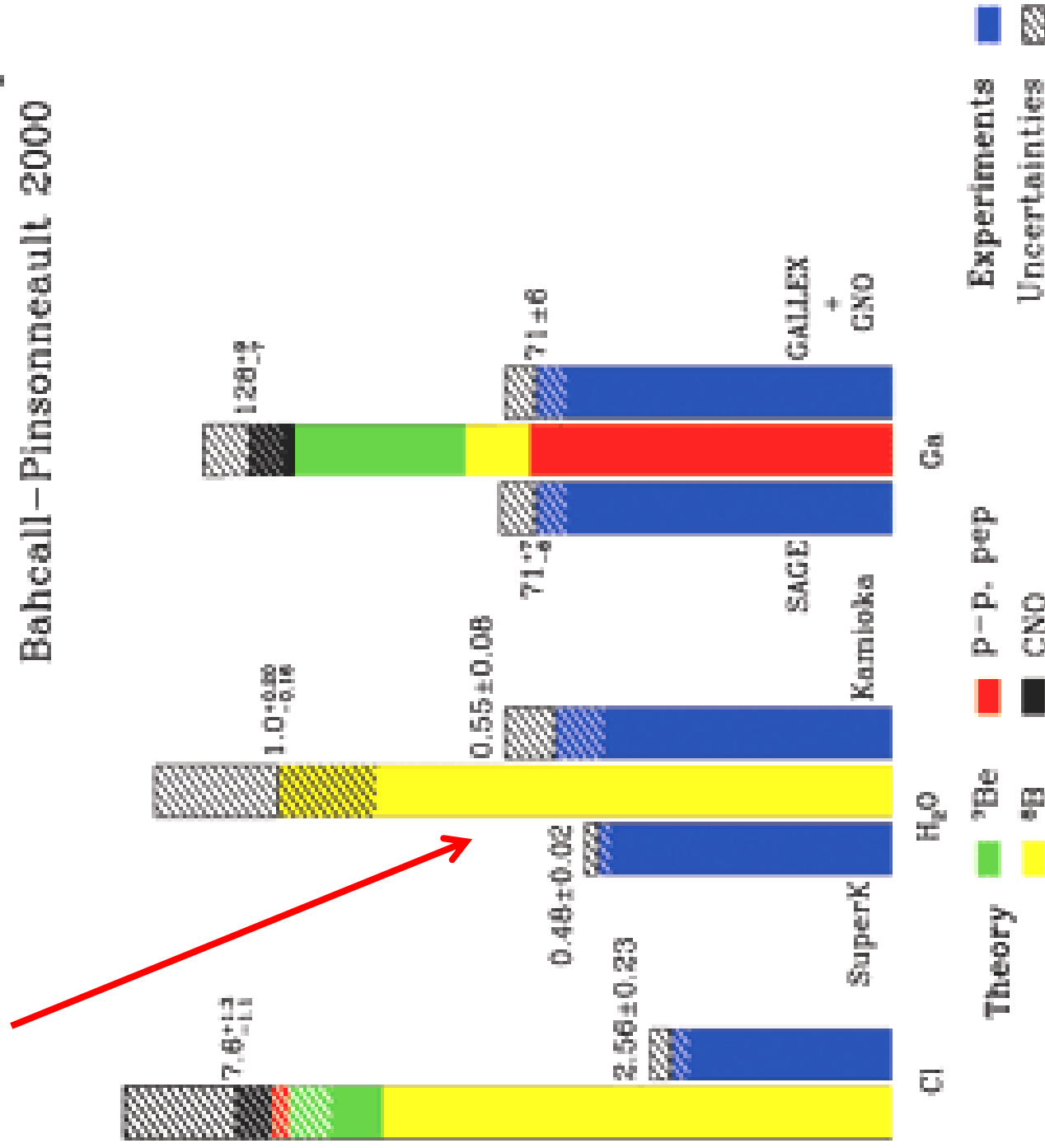
Sensitive to all the ν , but
in fact particularly to ν_e



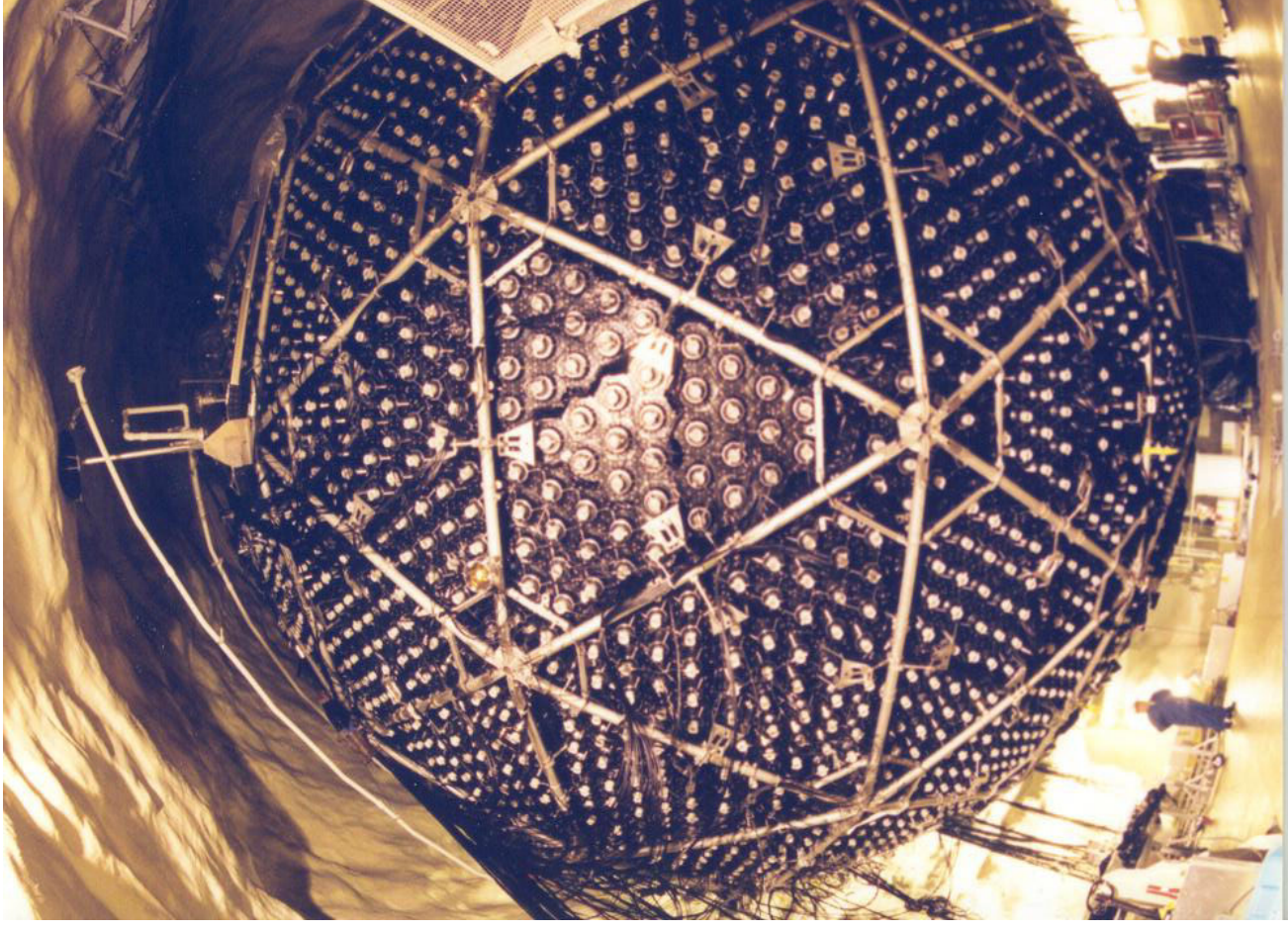
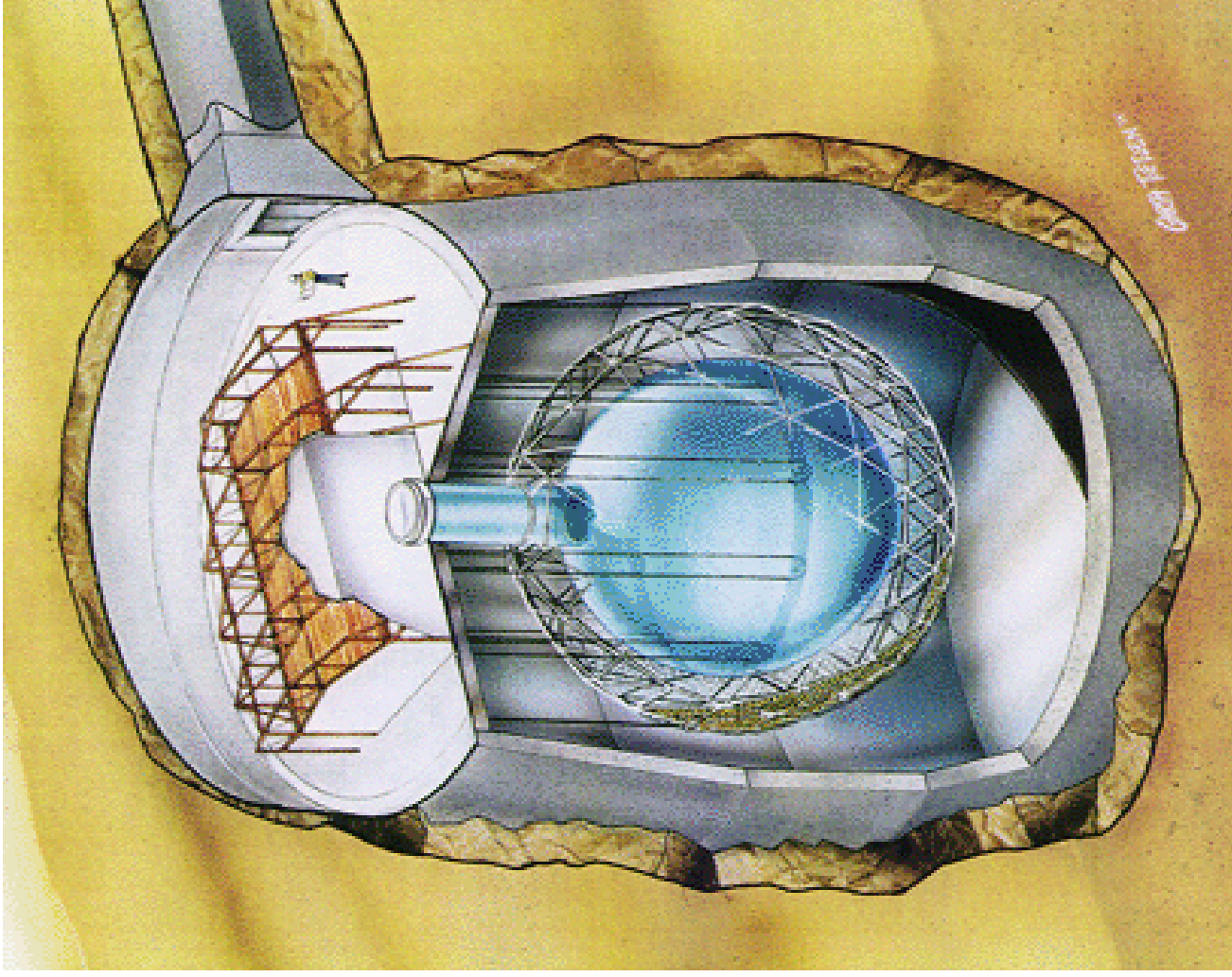


Total Rates: Standard Model vs. Experiment

Bahcall-Pinsonneault 2000

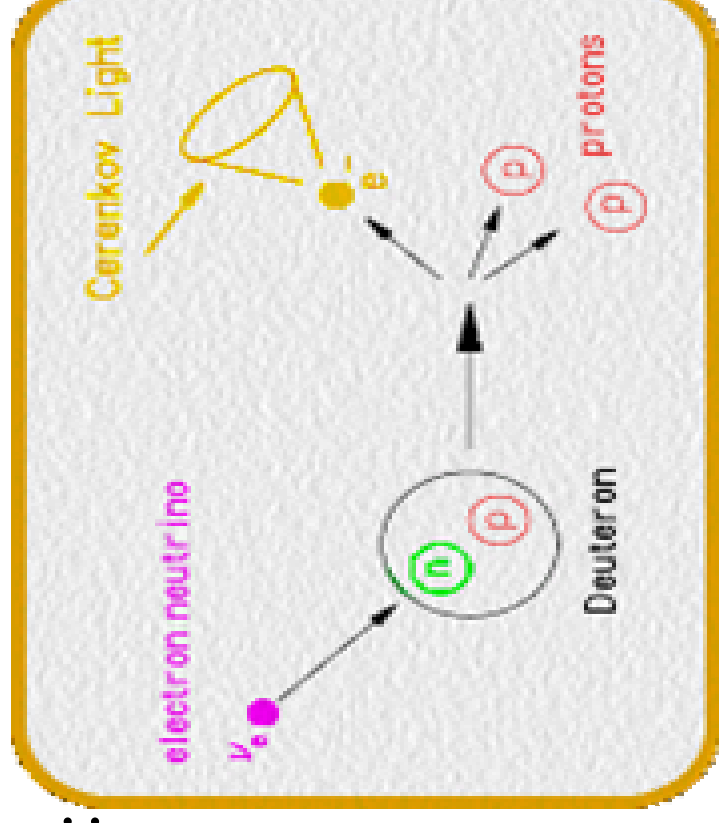


The Sudbury Neutrino Observatory (SNO)

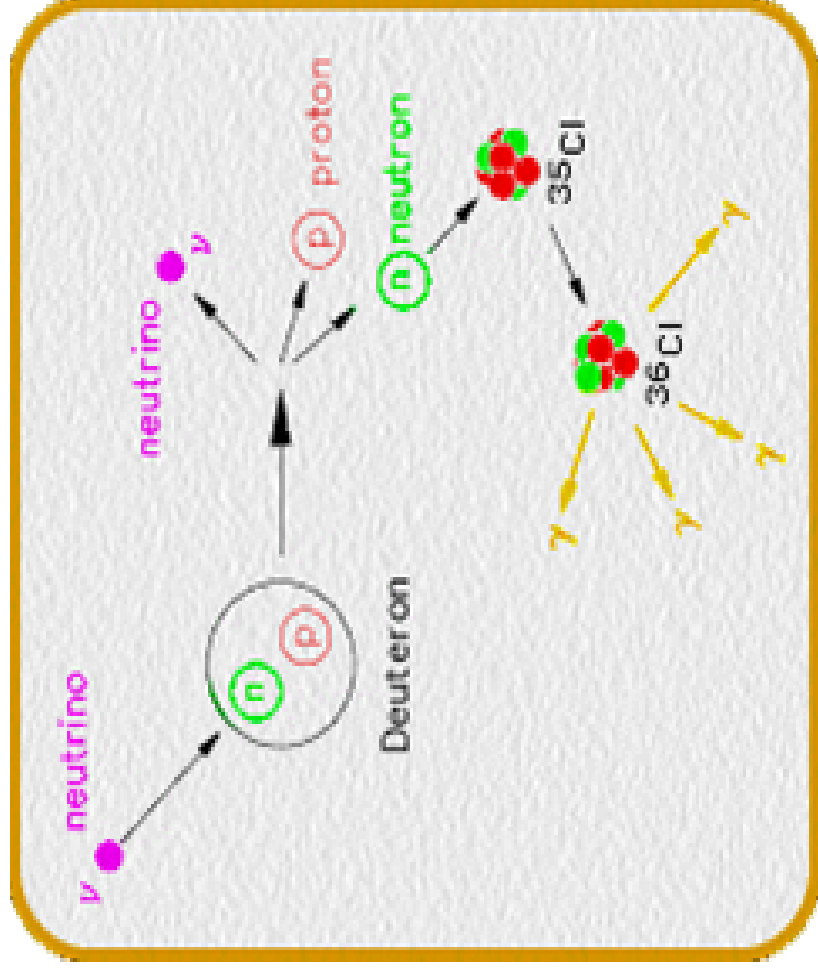


Neutrino interactions in D₂O

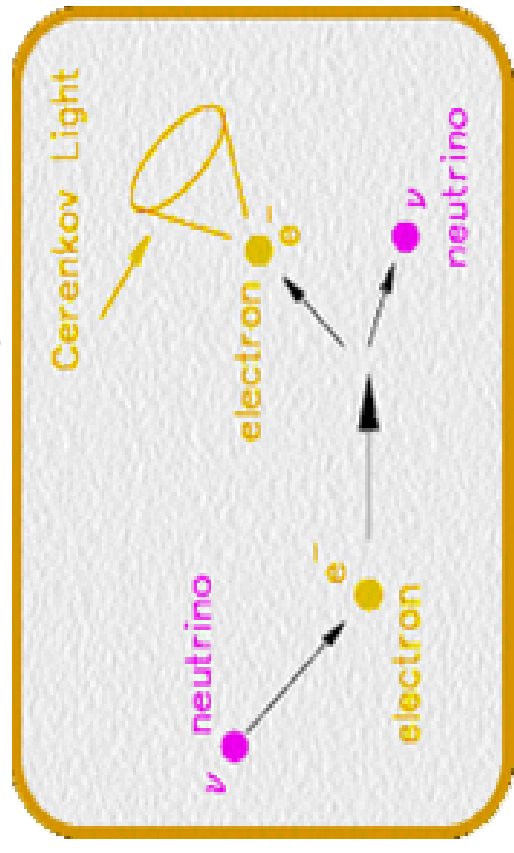
Charged Current Reaction (CC):



Neutral Current Reaction (NC):

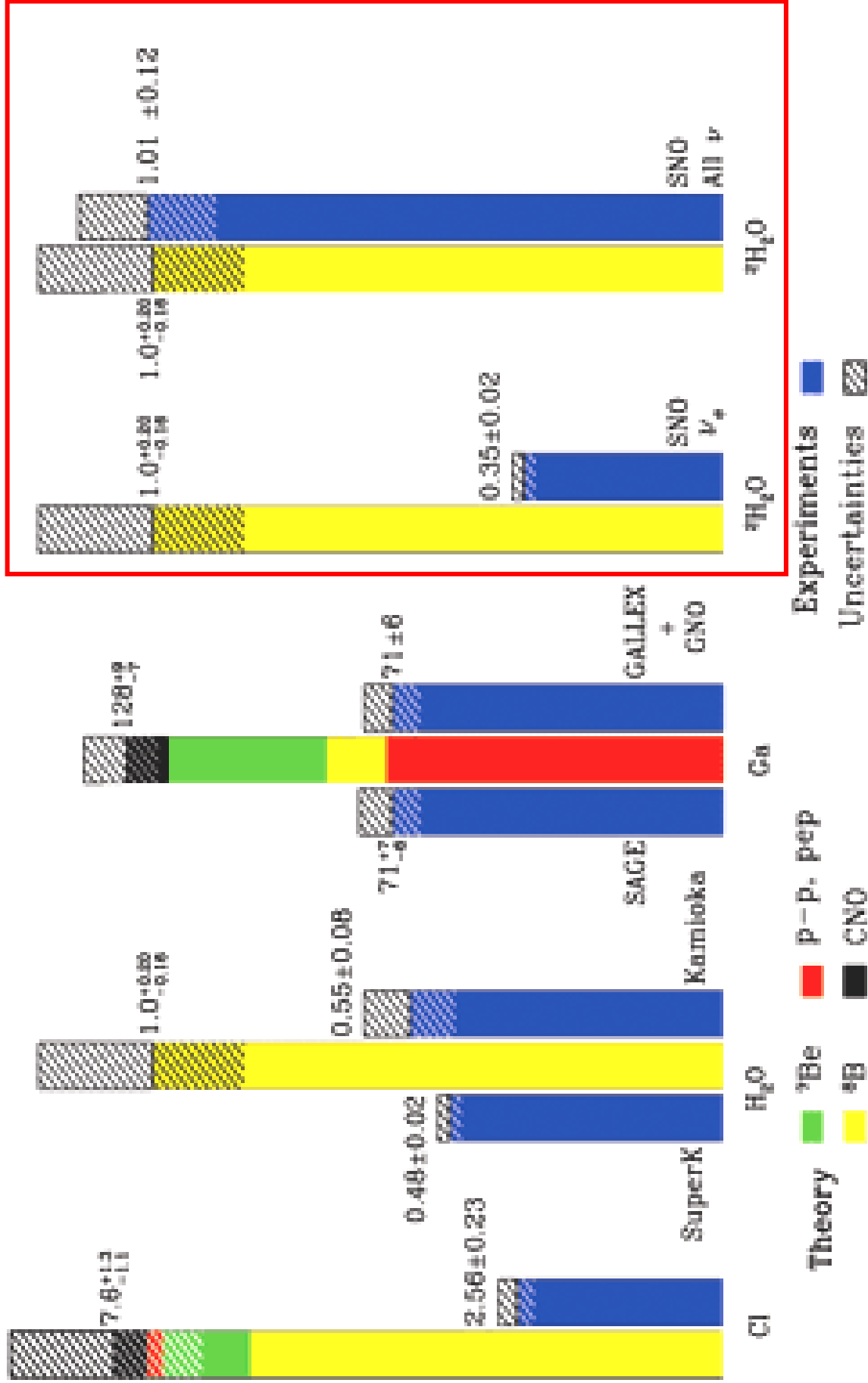


Elastic scattering (ES):



Total Rates: Standard Model vs. Experiment

Bahcall-Pinsonneault 2000



Key ingredients of the ab initio “microscopic” calculation:

$$\sigma \approx |V_{fi}|^2 = |\langle f | V | i \rangle|^2$$

$$V \approx \mathbf{j}_\mu \cdot \mathbf{J}^\mu \rightarrow \langle f | V | i \rangle \approx \langle f | \mathbf{j}_\mu | i \rangle \cdot \langle f | \mathbf{J}^\mu | i \rangle$$

known 

1. Realistic description of $|i\text{-hadr}\rangle$ and $|f\text{-hadr}\rangle$:
realistic Hamiltonians + accurate techniques
(HH method)
2. Realistic description of $\mathbf{J}^\mu$: realistic model for the
nuclear electroweak transition operator

Realistic Hamiltonian

$$H = T + \sum_{ij} v_{ij} + \sum_{i < j < k} V_{ijk}$$

non-relativistic
kinetic energy

NN potential:
AV18

three-nucleon interaction:
Urbana IX

A=3-7 binding energies

${}^AZ(J^\pi; T)$	CHH	VMC	GFMC	Expt
${}^3\text{H}(\frac{1}{2}^+; \frac{1}{2})$	8.479	8.227(6)	8.461(6)	8.482
${}^3\text{He}(\frac{1}{2}^+; \frac{1}{2})$	7.750	7.476(6)	7.708(6)	7.718
${}^4\text{He}(0^+; 0)$	27.89	27.40(3)	28.31(2)	28.30
${}^6\text{Li}(1^+; 0)$		28.05(5)	31.25(8)	31.99
${}^7\text{Li}(\frac{3}{2}^-; \frac{1}{2})$		33.07(7)	37.5(1)	39.24
${}^7\text{Li}(\frac{1}{2}^-; \frac{1}{2})$		33.13(7)	37.6(1)	38.76
${}^7\text{Be}(\frac{3}{2}^-; \frac{1}{2})$		31.49(7)	35.9(1)	37.60
${}^7\text{Be}(\frac{1}{2}^-; \frac{1}{2})$		31.58(8)	35.9(2)	37.17

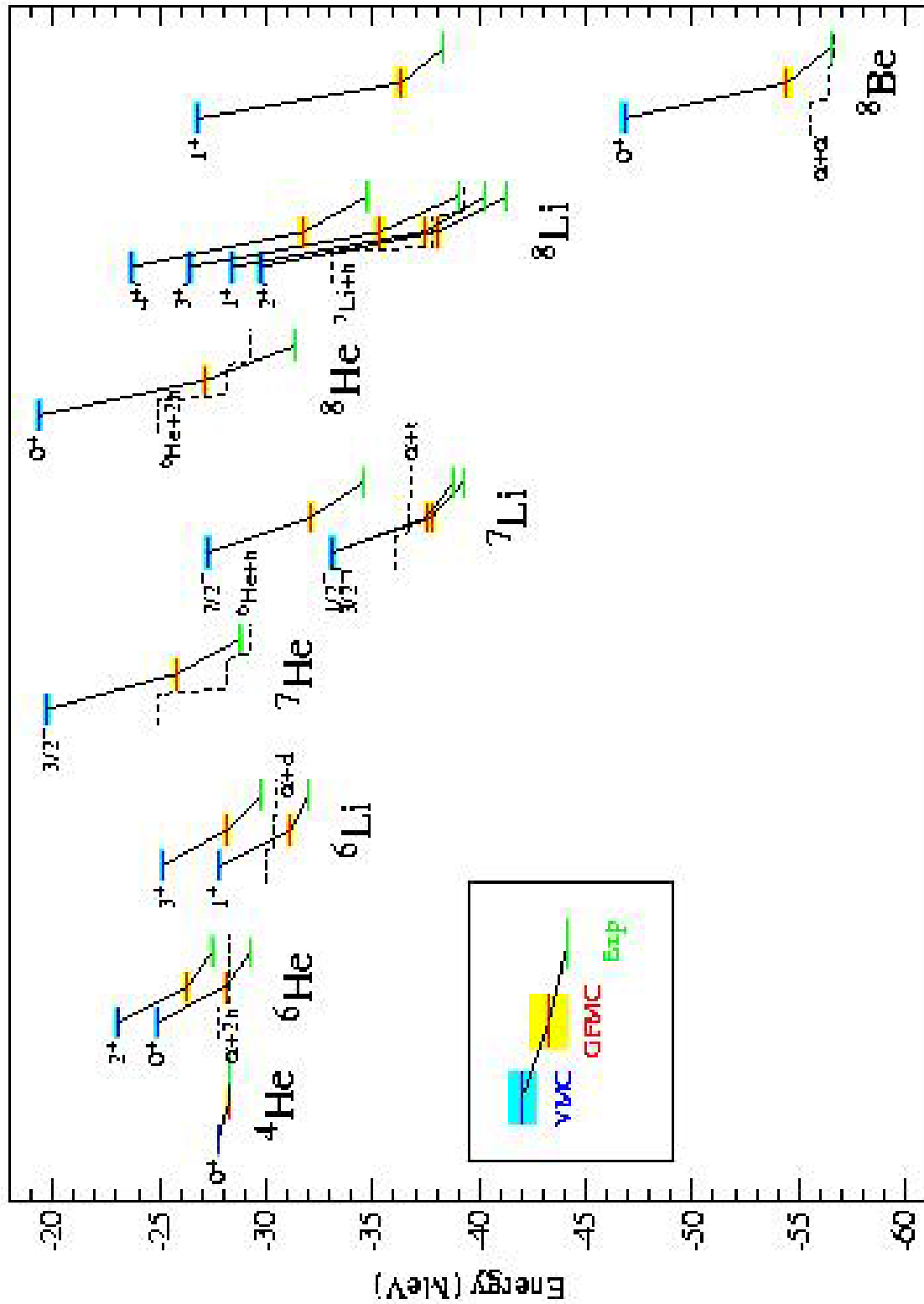
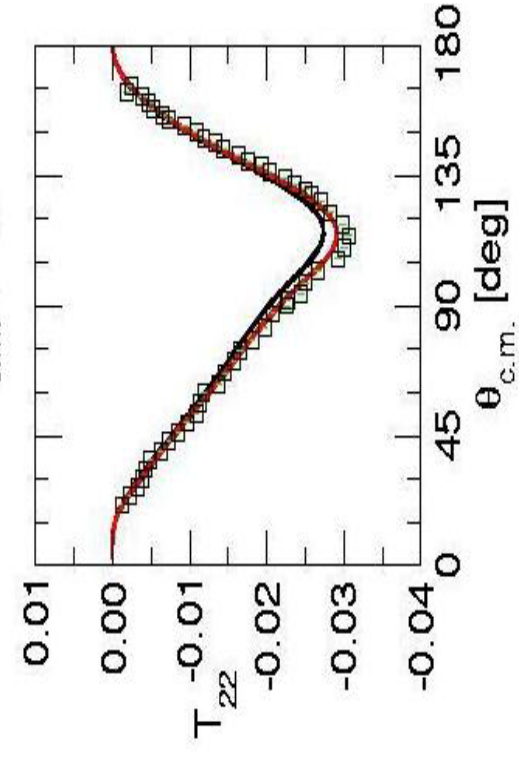
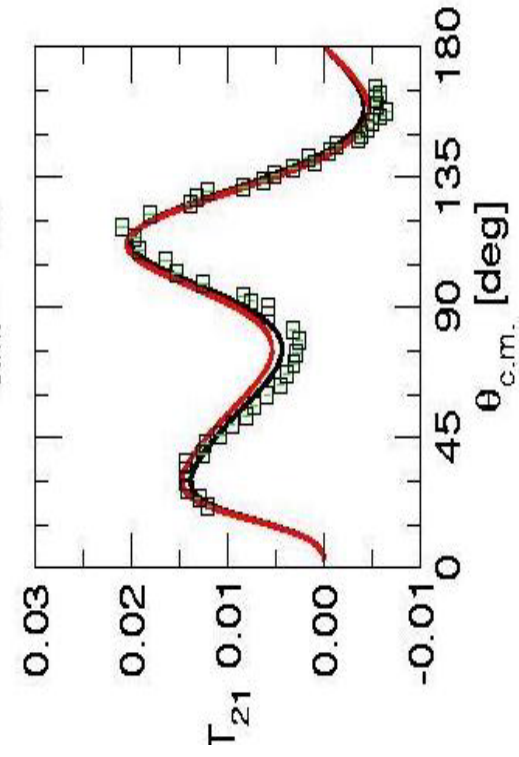
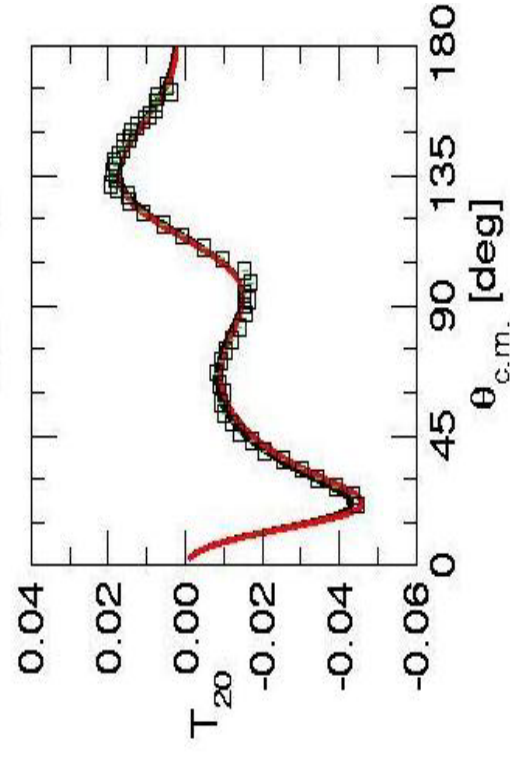
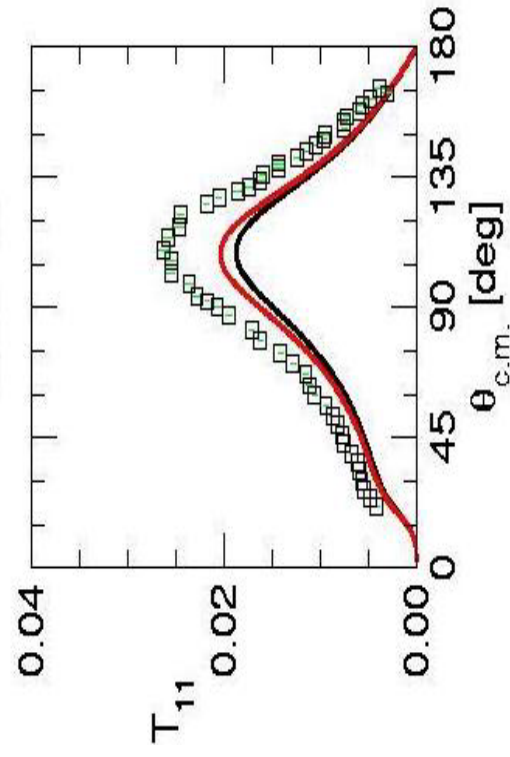
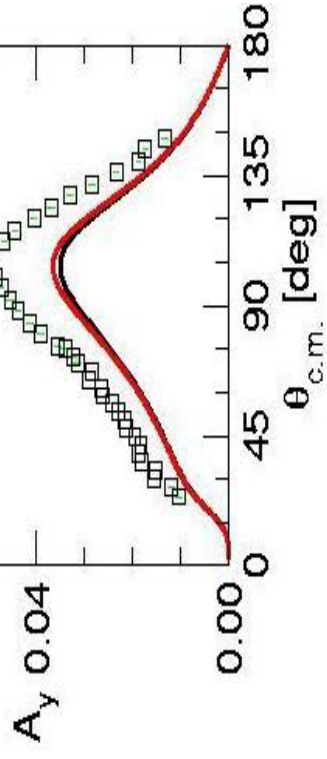
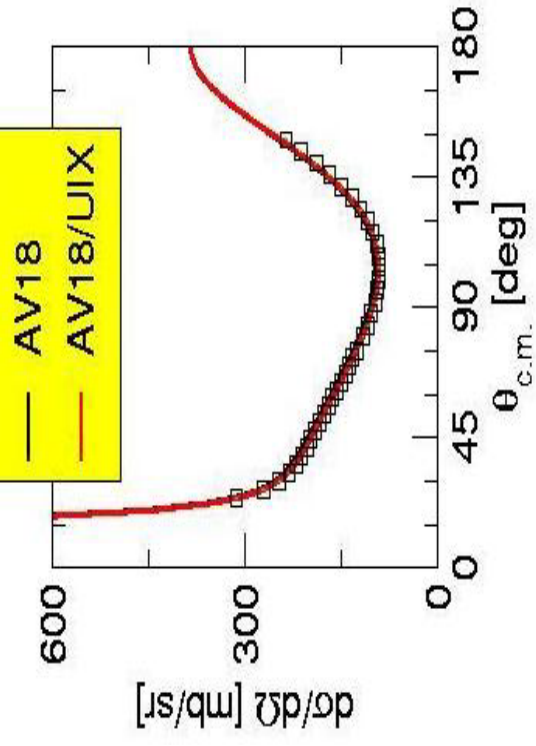
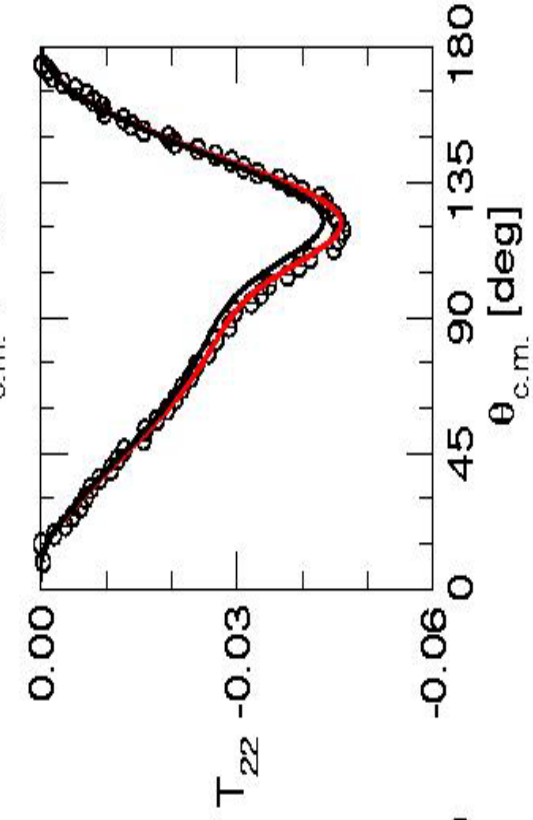
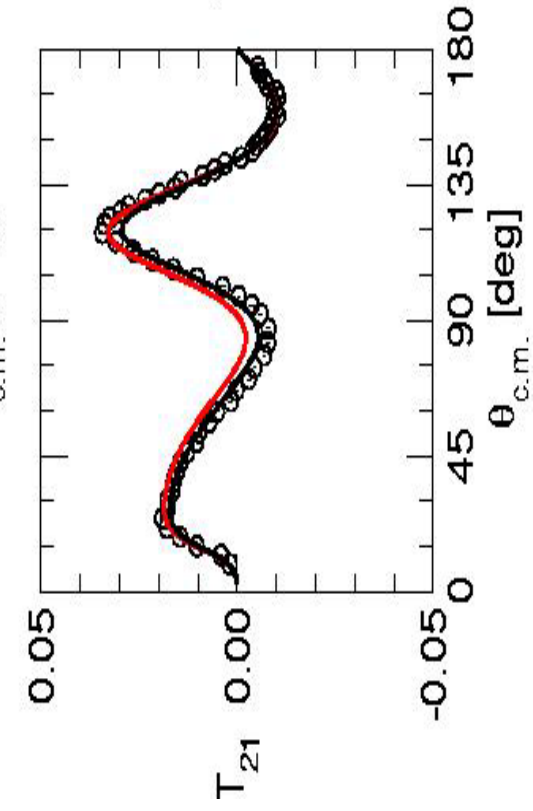
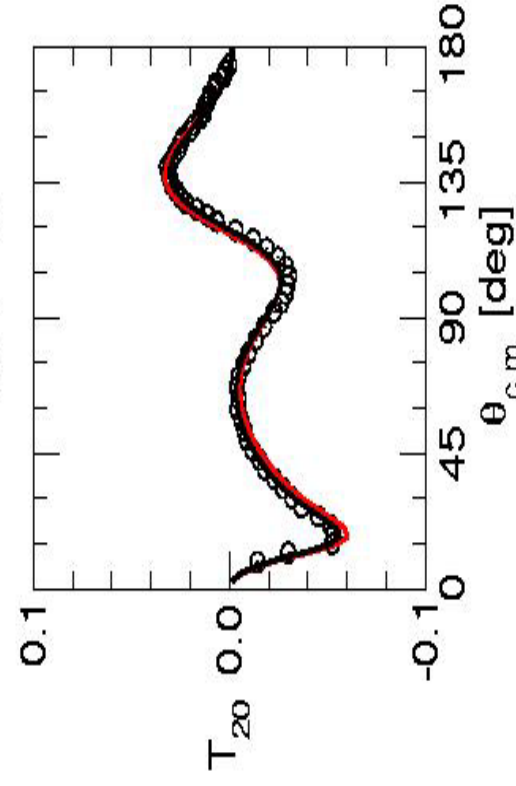
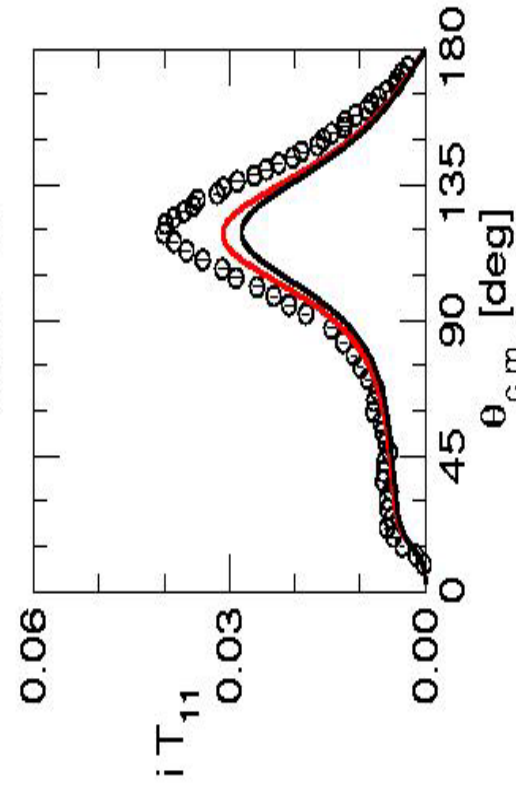
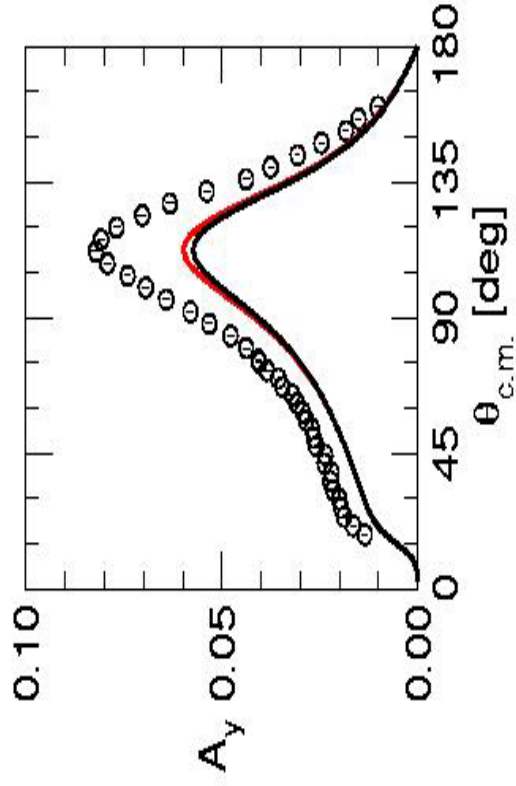
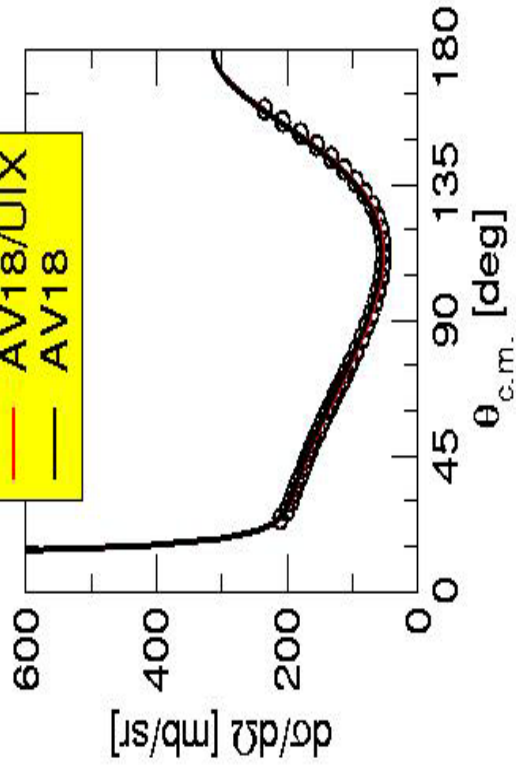


Figure 4: VMC and GFMC energies using AV18/UIX compared to experiment. Black dashed lines show the indicated breakup thresholds for each method. The Monte Carlo statistical errors are shown by the light blue and yellow bands.



pd elastic
scatt. at
2.00 MeV



pd elastic
scatt. at
3.33 MeV

Electroweak nuclear transition operator

Electromagnetic sector:

charge and current operators

Weak sector:

*vector charge and current
operators*

*axial charge and current
operators*

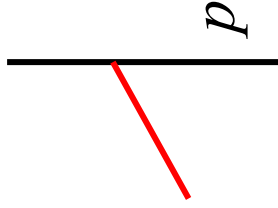
Conserved vector current hypothesis $\rightarrow$
weak vector operators from the *EM* ones
with a rotation in the isospin space

Electromagnetic current operator

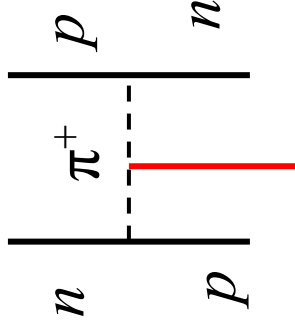
Realistic model: the e.m. current must satisfy the current conservation relation (CCR) with the Hamiltonian

$$H = T + \sum_{ij} v_{ij} + \sum_{i < j < k} V_{ijk}$$

one-body current



two-body current



three-body current

...

Similar situation for the axial current operator

Gauge invariance and CCR

$$\frac{\partial \rho(\mathbf{r}, t)}{\partial t} + \nabla \cdot \mathbf{J}(\mathbf{r}, t) = 0$$

$$\mathbf{J}(\mathbf{q}) = \int d\mathbf{x} \exp(i \mathbf{q} \cdot \mathbf{x}) \mathbf{J}(\mathbf{x}) \Rightarrow \nabla \cdot \mathbf{J} \rightarrow i \mathbf{q} \cdot \mathbf{J}(\mathbf{q})$$

$$\frac{\partial \rho}{\partial t} \rightarrow -i \hbar [H, \rho]$$

$$\Rightarrow \mathbf{q} \cdot \mathbf{J} = [H, \rho]$$

$$H = T + V$$

$$\Rightarrow \mathbf{q} \cdot \mathbf{J} = [T, \rho] + [V, \rho]$$

$$\mathbf{J} = \mathbf{J}_1 + \mathbf{J}_2 \rightarrow$$

$$\mathbf{q} \cdot \mathbf{J}_1 = [T, \rho] \quad \mathbf{q} \cdot \mathbf{J}_2 = [V, \rho]$$

$n+p \rightarrow d + \gamma$ radiative capture at thermal energies

Total cross section [mb] for np radiative capture (AV18):

One-body

304.6

Full

332.7

Expt.

332.6(7)

$$p+p \rightarrow d+e^+ + \nu_e$$

$$S(E) = E \sigma(E) \, e^{2\pi Z_1 Z_2 \alpha/\nu}$$

$$S(E) \approx \Lambda(E)^2$$

Square of the overlap integral $\Lambda(E=0)$ for two modern NN interaction models

NN model	AV18	CD-Bonn
Λ^2 (one-body)	6.985	6.985
Λ^2 (full)	7.076	7.060

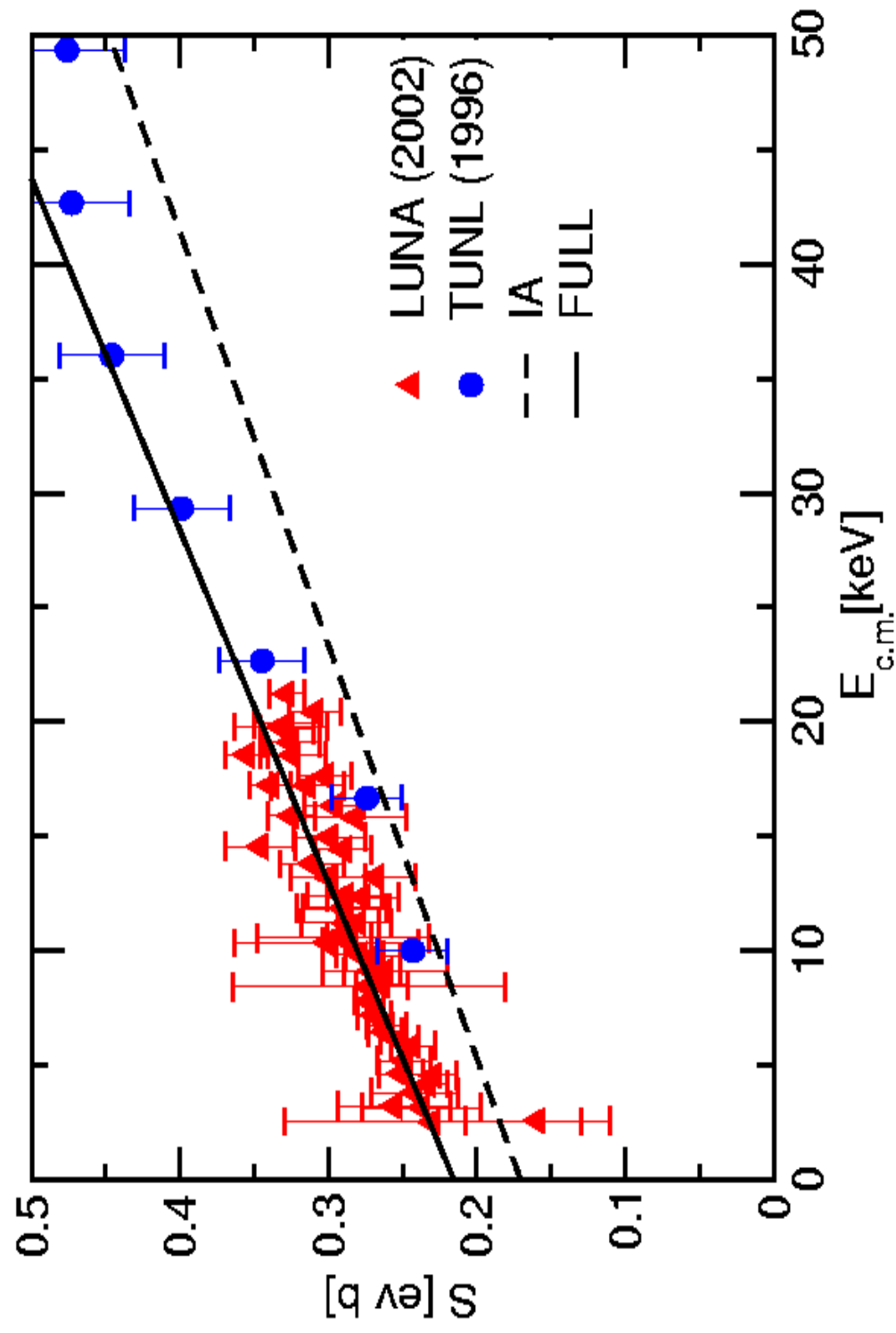


Total cross section [mb] for nd radiative capture
(AV18/UIX):

One-body **0.227**

Full **0.556**

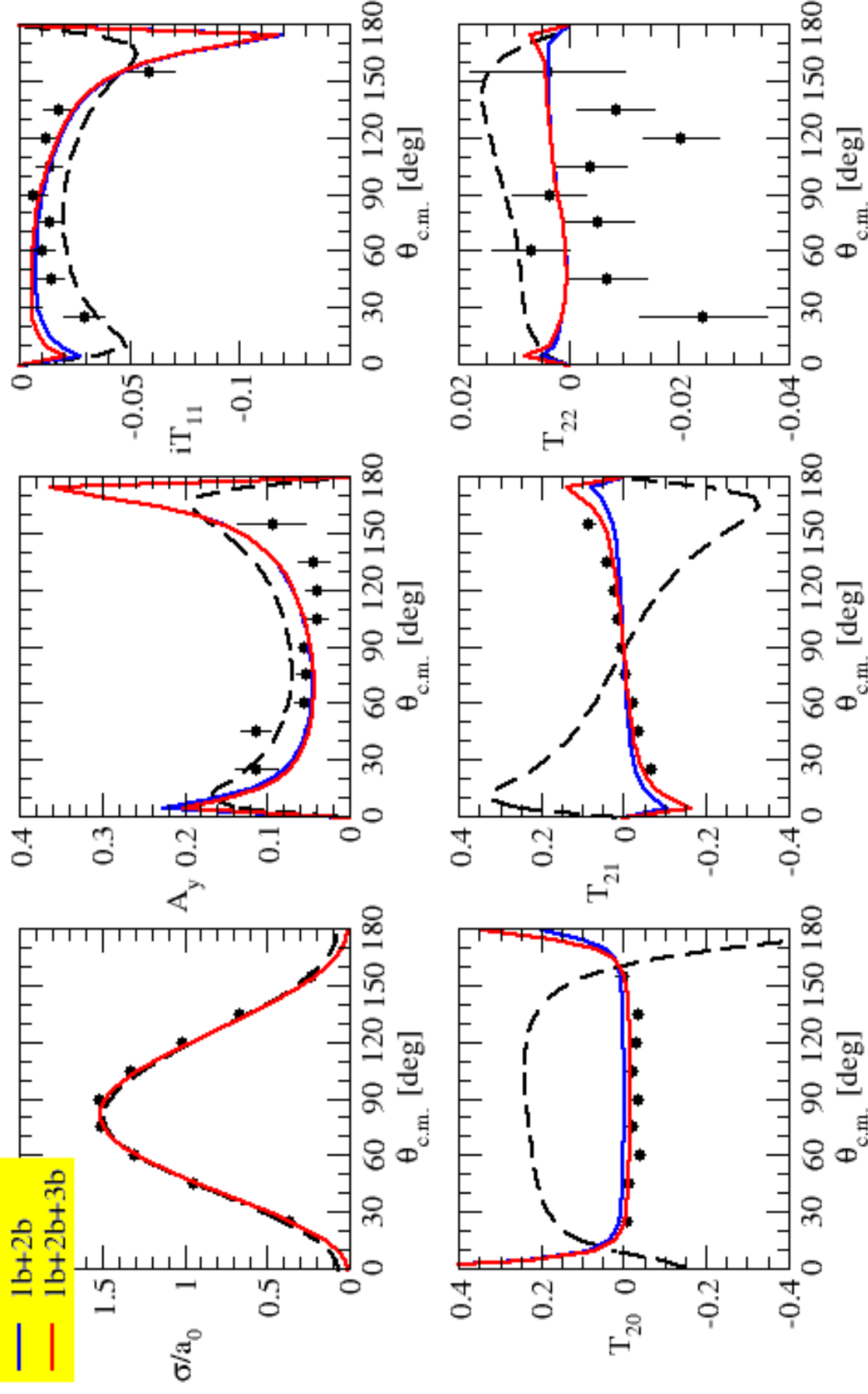
Expt. **0.508(15)**





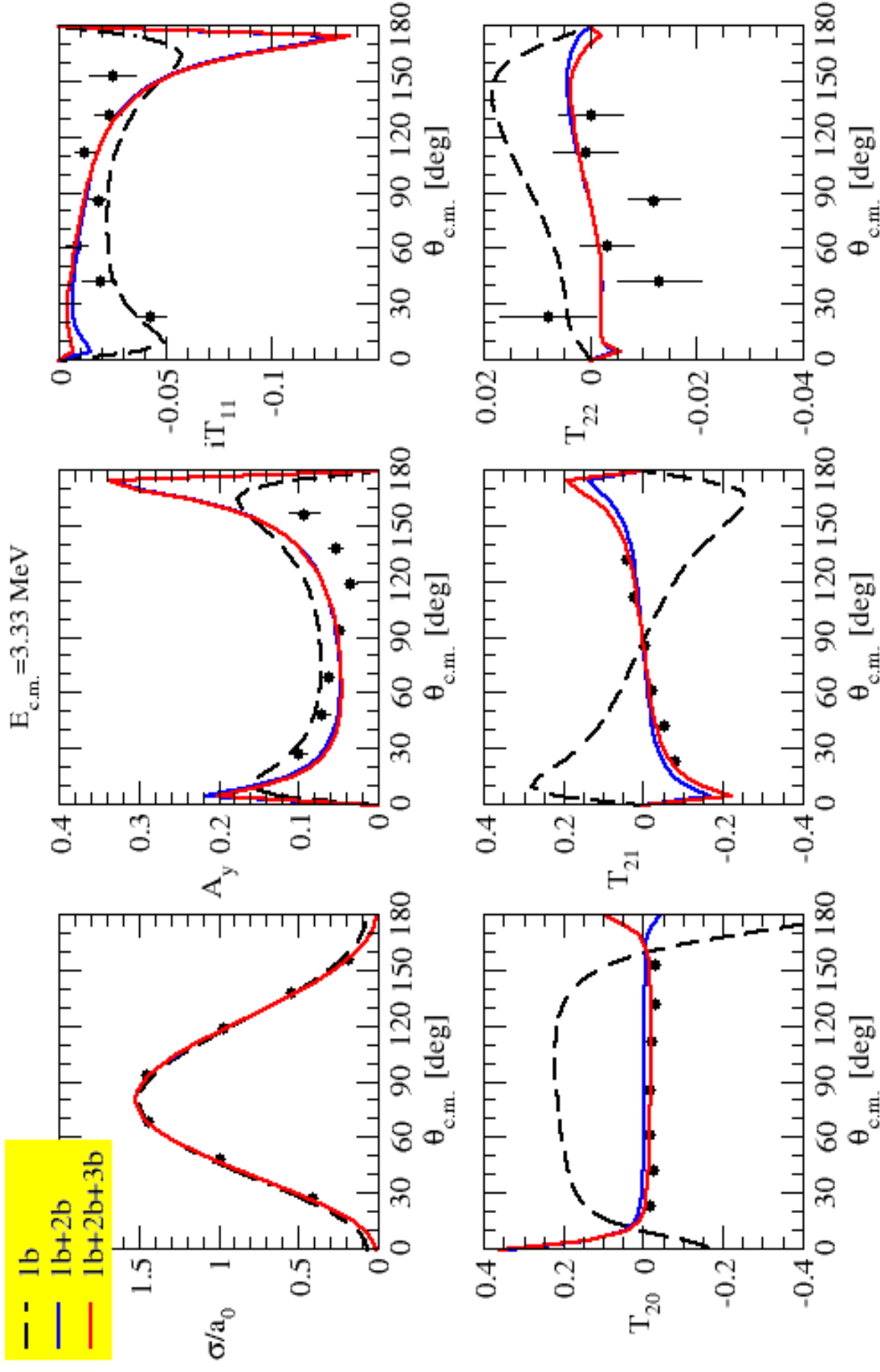
AV18/UIX

$E_{\text{c.m.}} = 2.00 \text{ MeV}$





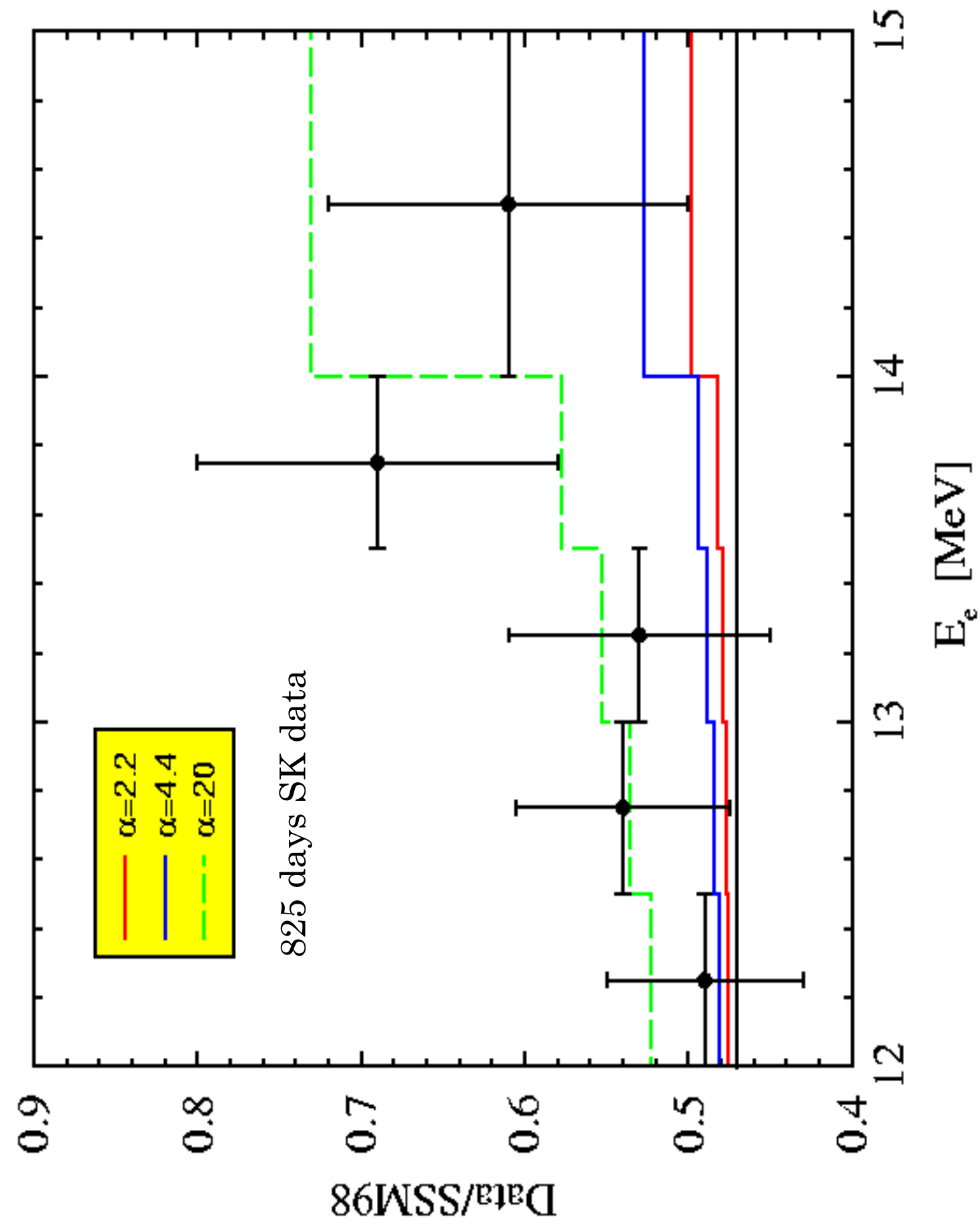
AV18/UIX

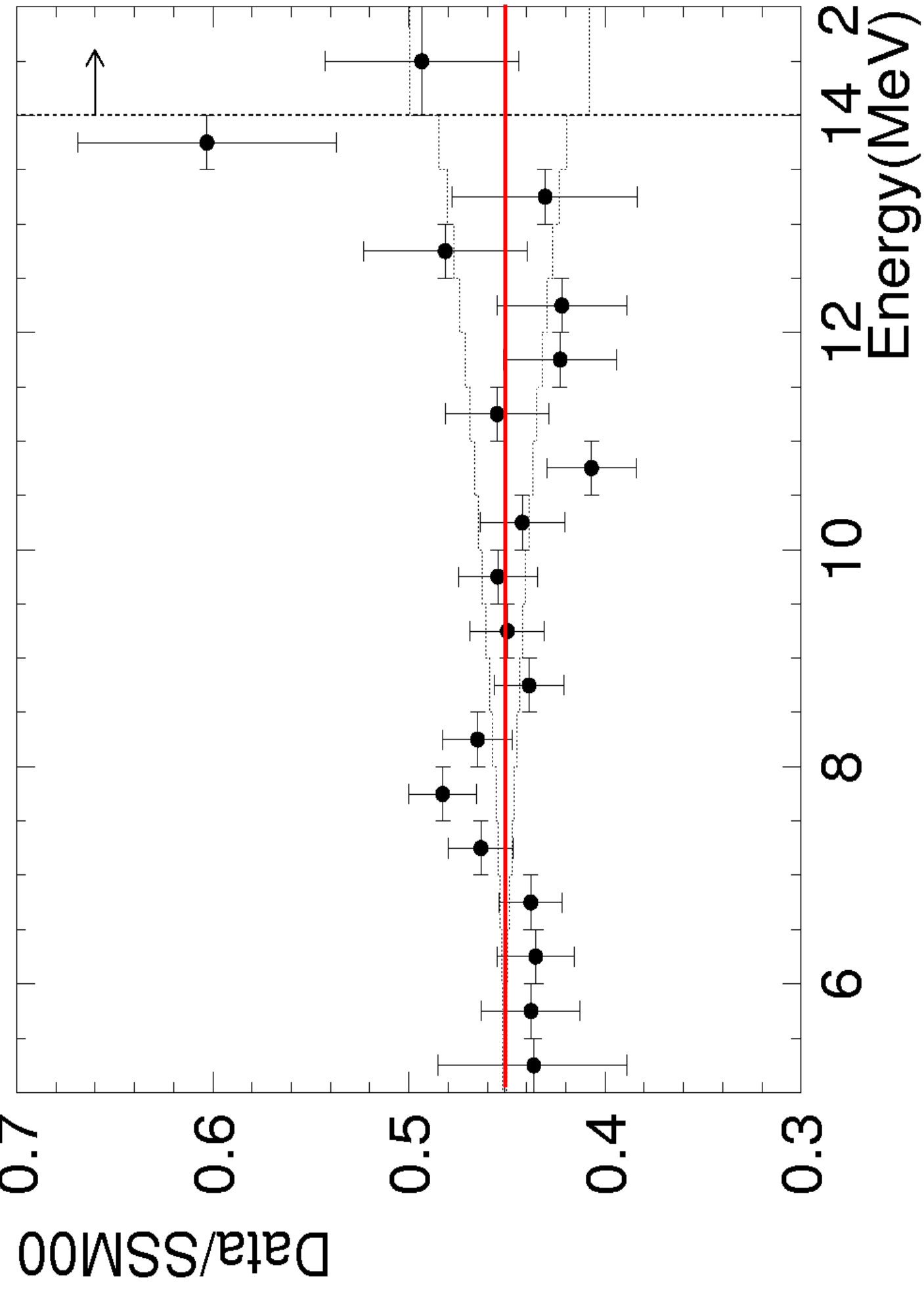




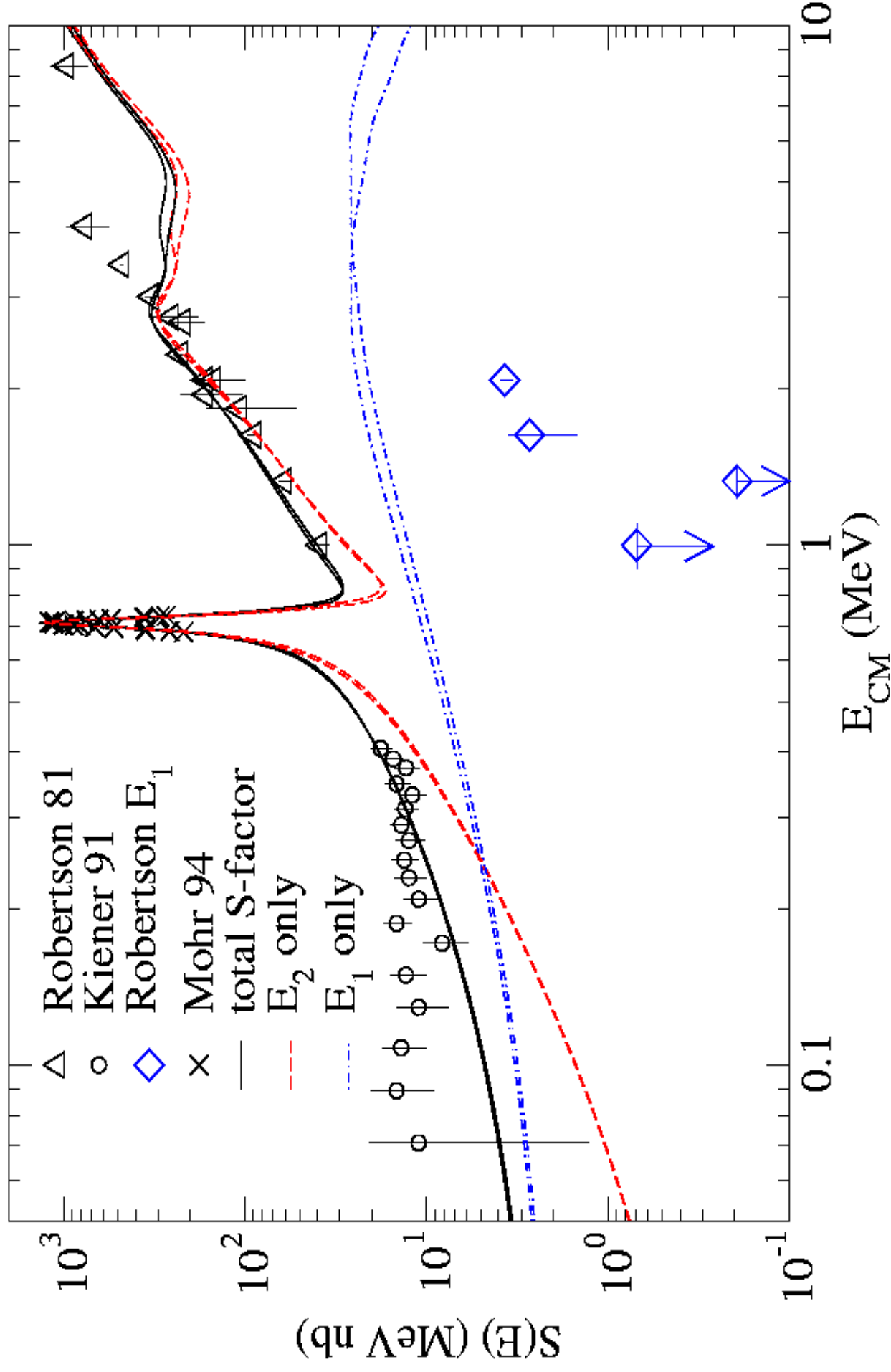
- $S_{\text{SSM98}} = 2.3 \times 10^{-20} \text{ keV b}$
 - New calculation (Marcucci et al., PRL **84**, 5959 (2000); PRC **63**, 015801 (2001)):
1. $S(10 \text{ keV}) = 10.1 \times 10^{-20} \text{ keV b}$
 2. no c.m. energy dependence
 3. no Hamiltonian model dependence (if H reproduces accurately the initial and final state w.f.'s)
 4. importance of the P-waves (40%)
 5. importance of MEC (Δ)

- SSM revisited (Bahcall et al., Astr. J. **555**, 990 (2001)):
 $S_{\text{SSM00}} = 10.1 \times 10^{-20} \text{ keV b}$

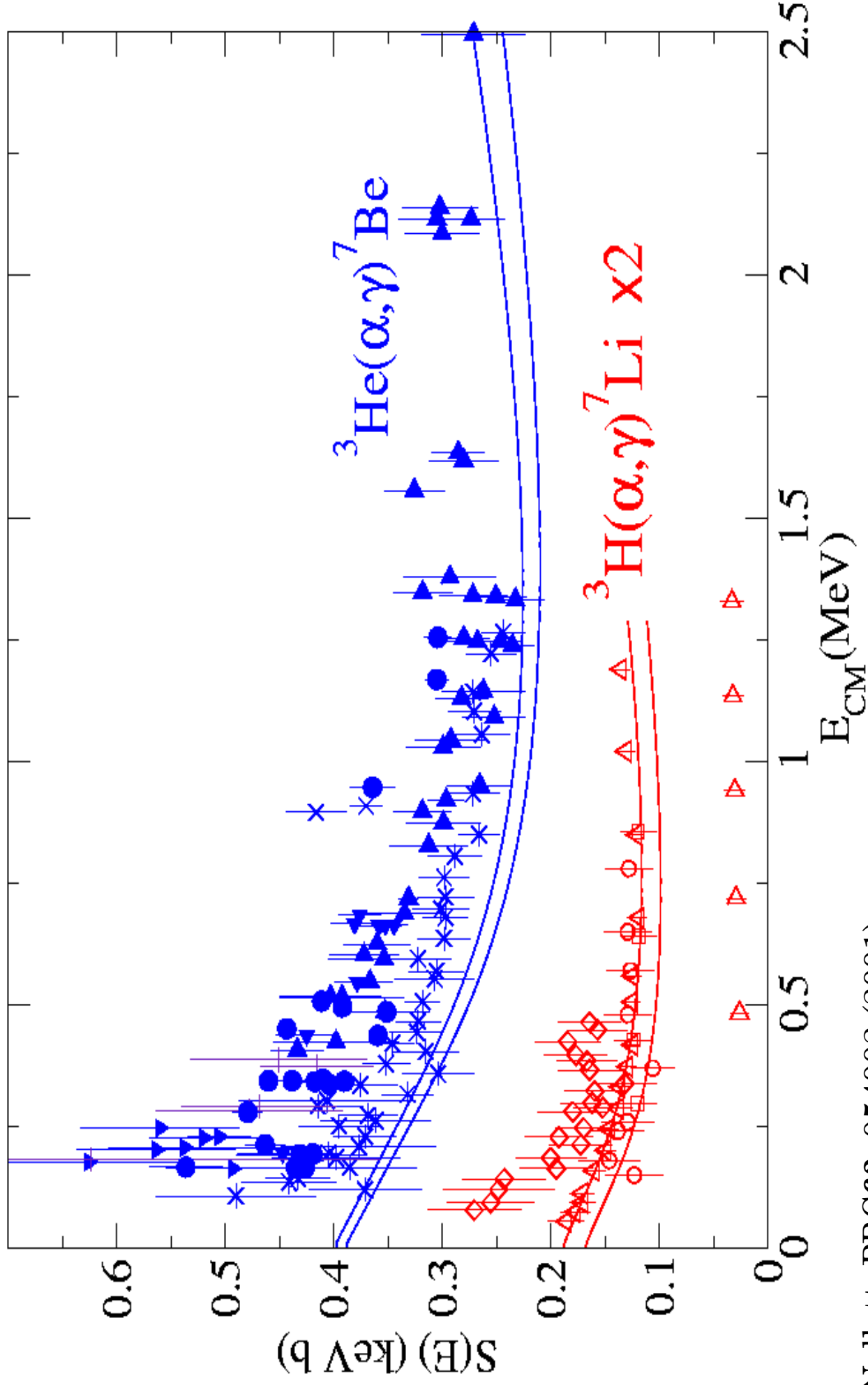




The SK collaboration, PRL 86, 5651 (2001)



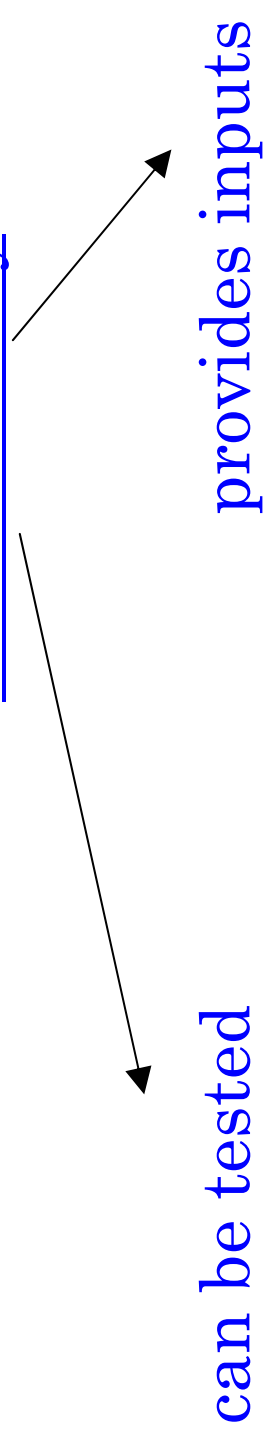
${}^3\text{H} + \alpha \rightarrow {}^7\text{Li} + \gamma$ and ${}^3\text{He} + \alpha \rightarrow {}^7\text{Be} + \gamma$



$${}^7\text{Be} + e^- \longrightarrow {}^7\text{Li} + \nu_e \quad \text{and} \quad {}^7\text{Be} + e^- \longrightarrow {}^7\text{Li}^* + \nu_e$$

Gamov-Teller matrix elements for A=3,7 nuclei.					
	CHH		VMC		
	IA	Full	IA	Full	Expt
${}^3\text{H} \rightarrow {}^3\text{He}$	1.597	1.658	1.602		1.658
${}^7\text{Be} \rightarrow {}^7\text{Li}$			2.345(3)	2.419(5)	2.599
${}^7\text{Be} \rightarrow {}^7\text{Li}^*$			2.142(2)	2.200(3)	2.323

Conclusions and outlook

- Nuclear reactions of astrophysical interest are an important field of research for nuclear theory

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graph TD; A["Nuclear theory"] --> B["can be tested"]; A --> C["provides inputs"]
```

can be tested

provides inputs
- Further investigation and comparison with experimental results (electron scattering)