Reazione nucleari di interesse astrofisico

Laura E. Marcucci

Univ. of Pisa e INFN-Pisa

Riassunto del corso:

- teoria del momento angolare: fondamentale per la costruzione della funzioni d'onda
- teoria perturbativa dipendente dal tempo: calcolo della sezione d'urto (e fattore astrofisico)
- funzioni d'onda: stati legati (d, ^{3}He, ^{3}H) e stati di scattering (pp, np, pd, nd)
- reazioni considerate: $p+p \rightarrow d+e^+ + \nu_e$ e $n+p \rightarrow d+\gamma$

Campi di applicazione:

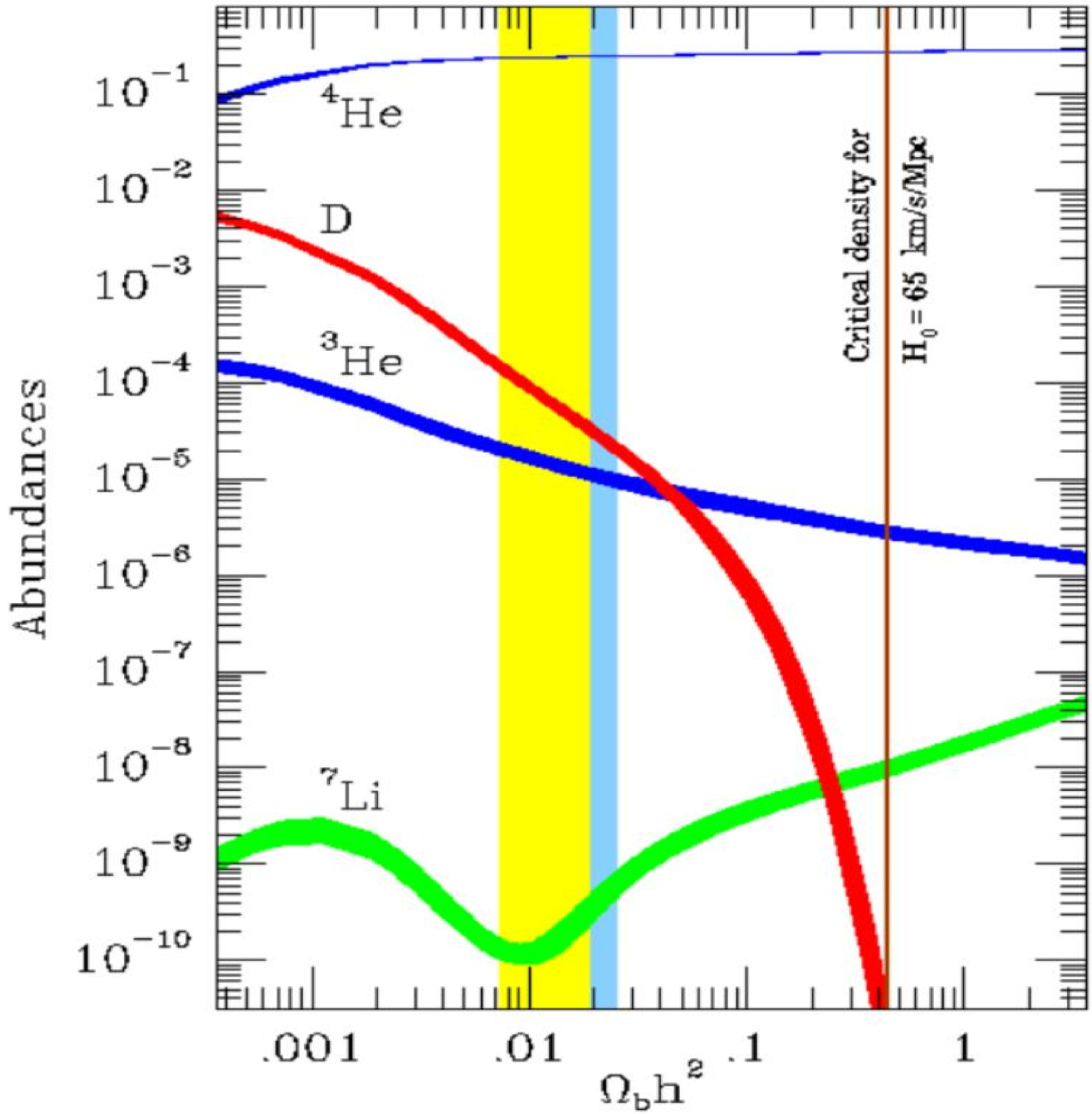
- Big Bang nucleosynthesis (BBN)
- Produzione solare di energia attraverso una catena di reazioni nucleari

Big Bang nucleosynthesis (BBN)
(naïve picture)

- Step 1 (kT<few MeV): thermal eq.; $n/p \sim 1$
- Step 2 (kT~0.7 MeV): weak int. freeze-out, $n/p \sim 1/6 \rightarrow 1/7$
- Step 3 (kT~0.5 MeV): nuclear reactions, which form d, ^{3}He, ^{4}He, ^{7}Li

11 key nuclear reactions, among which:

- $n + p \rightarrow d + \gamma$
- $p + d \rightarrow {}^3\text{He} + \gamma$
- ${}^3\text{H} + \alpha \rightarrow {}^7\text{Li} + \gamma$
- ${}^3\text{He} + \alpha \rightarrow {}^7\text{Be} + \gamma$
- ${}^7\text{Be} + n \rightarrow {}^7\text{Li} + p$



Schramm&Turner, RMP **70**, 303 (1998)

Solar neutrinos: *pp* chain

$$p+p \rightarrow d+e^{+}+\nu_e$$

$$p+p+e^{-} \rightarrow d+\nu_e$$

$$p+d \rightarrow {}^{3}\mathrm{He}+\gamma$$

$${}^{3}\mathrm{He}+{}^{3}\mathrm{He} \rightarrow {}^{4}\mathrm{He}+p+p$$

$$p+{}^{3}\mathrm{He} \rightarrow {}^{4}\mathrm{He}+e^{+}+\nu_e$$

$${}^{3}\mathrm{He}+{}^{4}\mathrm{He} \rightarrow {}^{7}\mathrm{Be}+\gamma$$

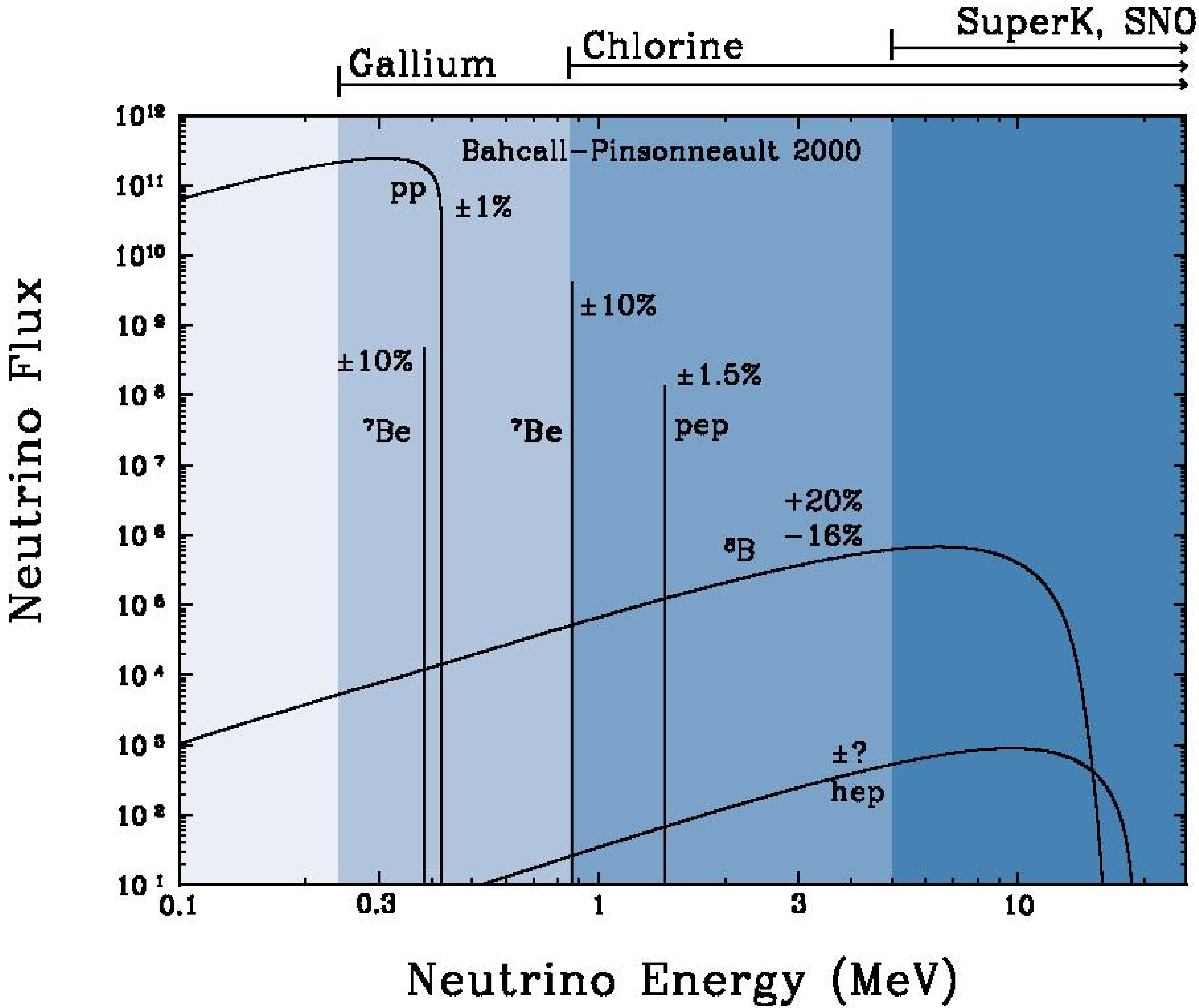
$${}^{7}\mathrm{Be}+e^{-} \rightarrow {}^{7}\mathrm{Li}+\nu_e$$

$${}^{7}\mathrm{Be}+p \rightarrow {}^{8}\mathrm{B}+\gamma$$

$${}^{7}\mathrm{Li}+p \rightarrow {}^{4}\mathrm{He}+{}^{4}\mathrm{He}$$

$${}^{8}\mathrm{B} \rightarrow {}^{4}\mathrm{He}+{}^{4}\mathrm{He}+e^{+}+\nu_e$$

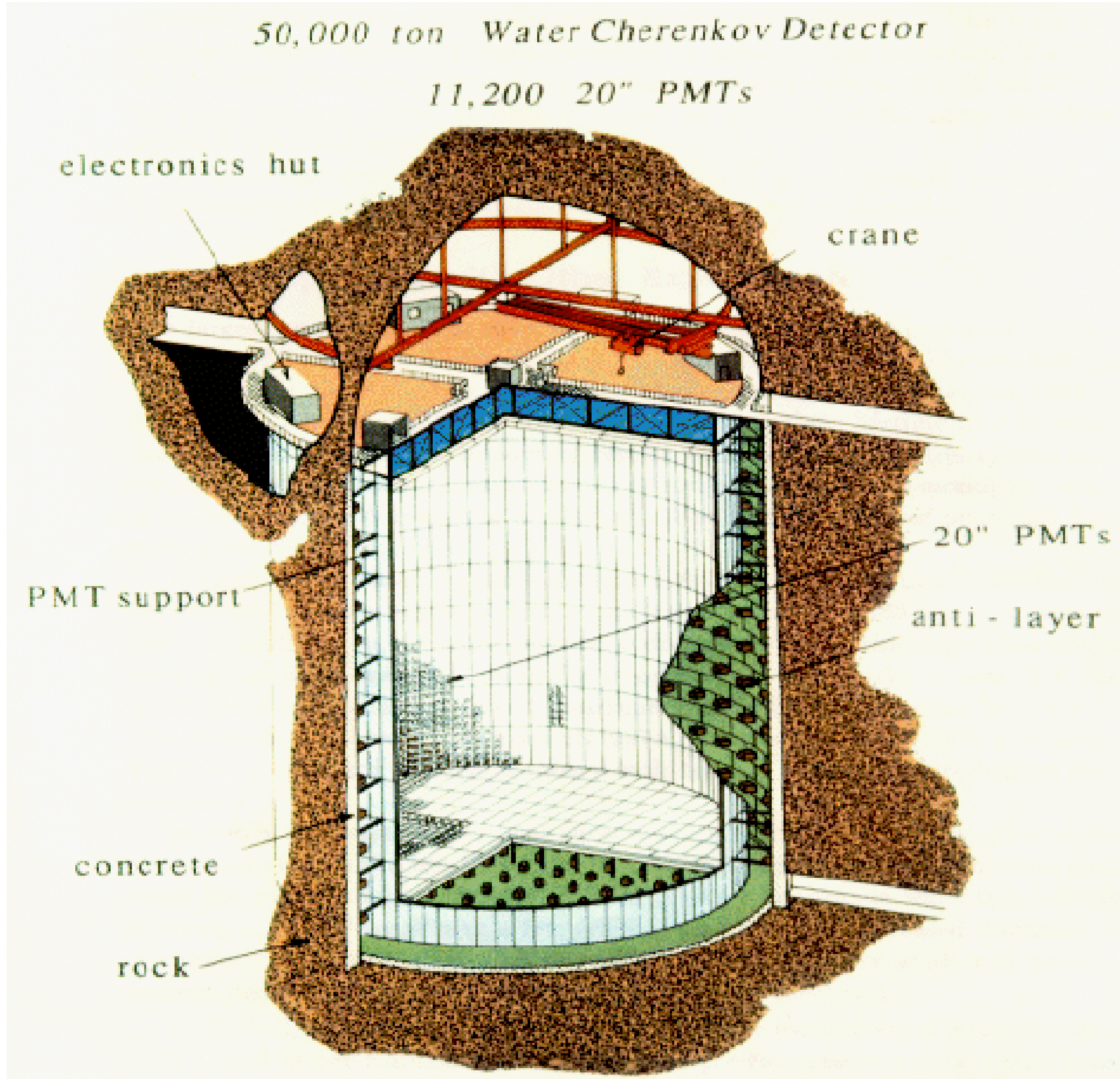
www.sns.ias.edu/~jnb

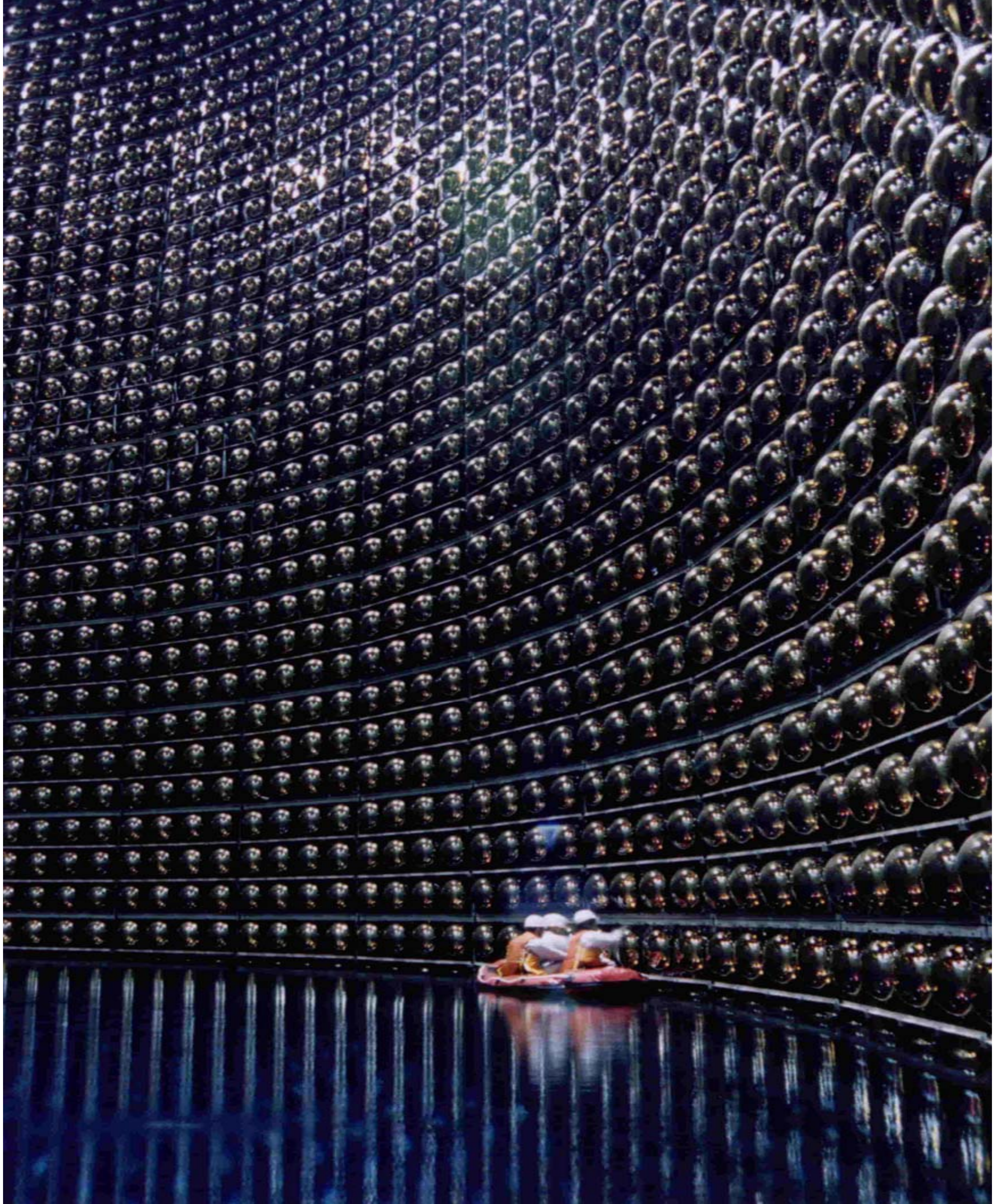


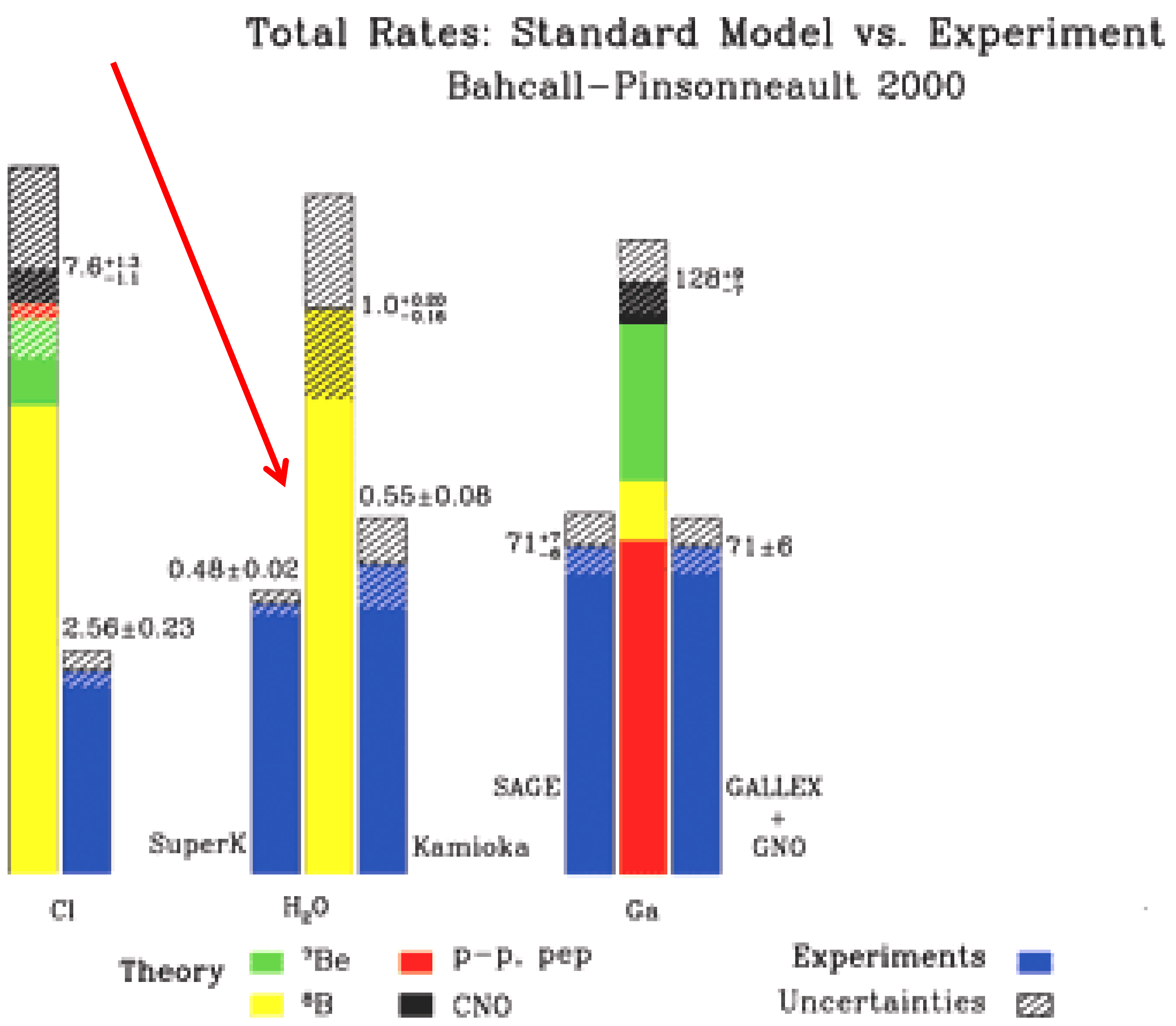
The SuperKamiokande (SK) experiment

Elastic scattering
$\nu + e^- \rightarrow \nu + e^-$

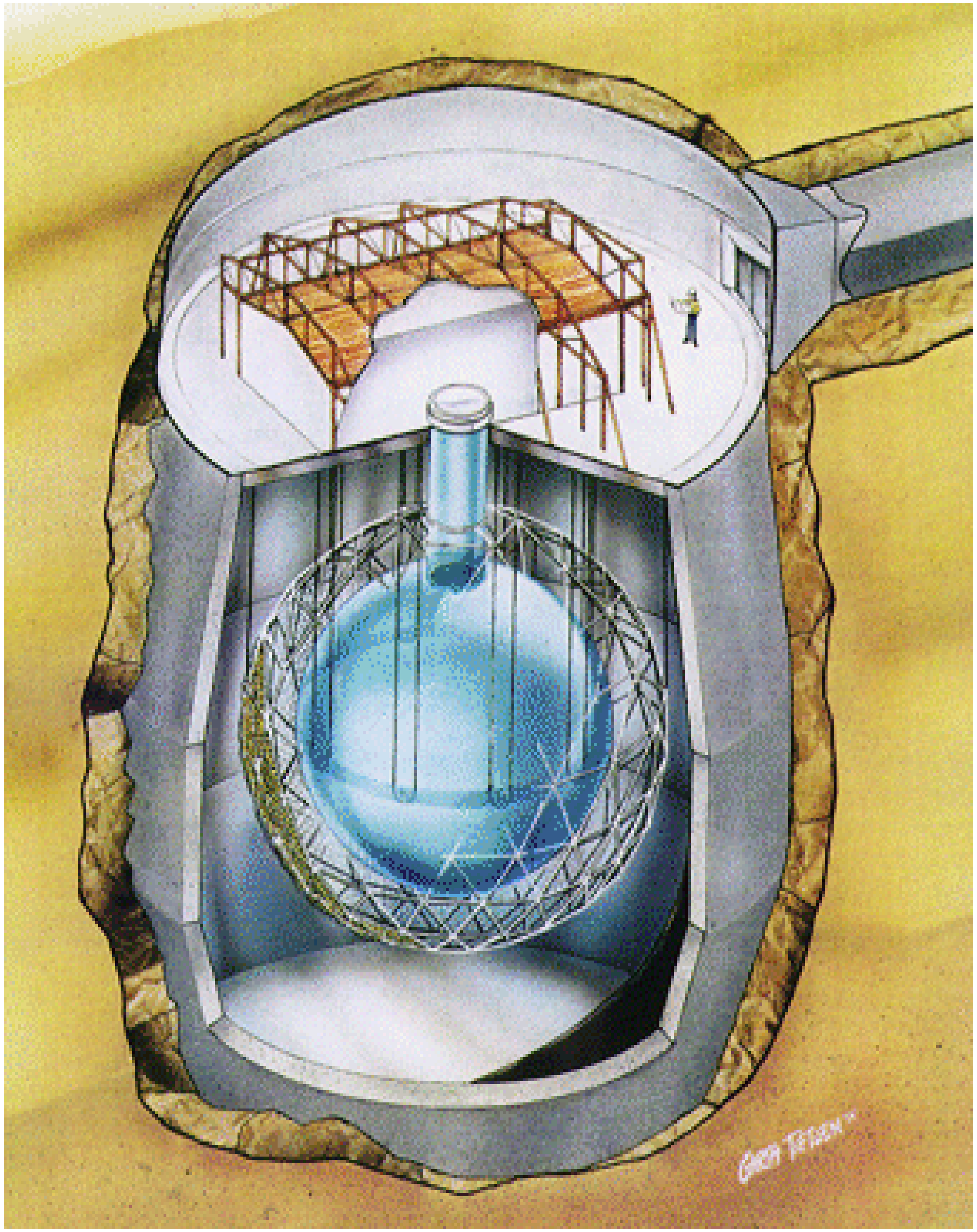
Sensitive to all the ν, but in fact particularly to ν_e

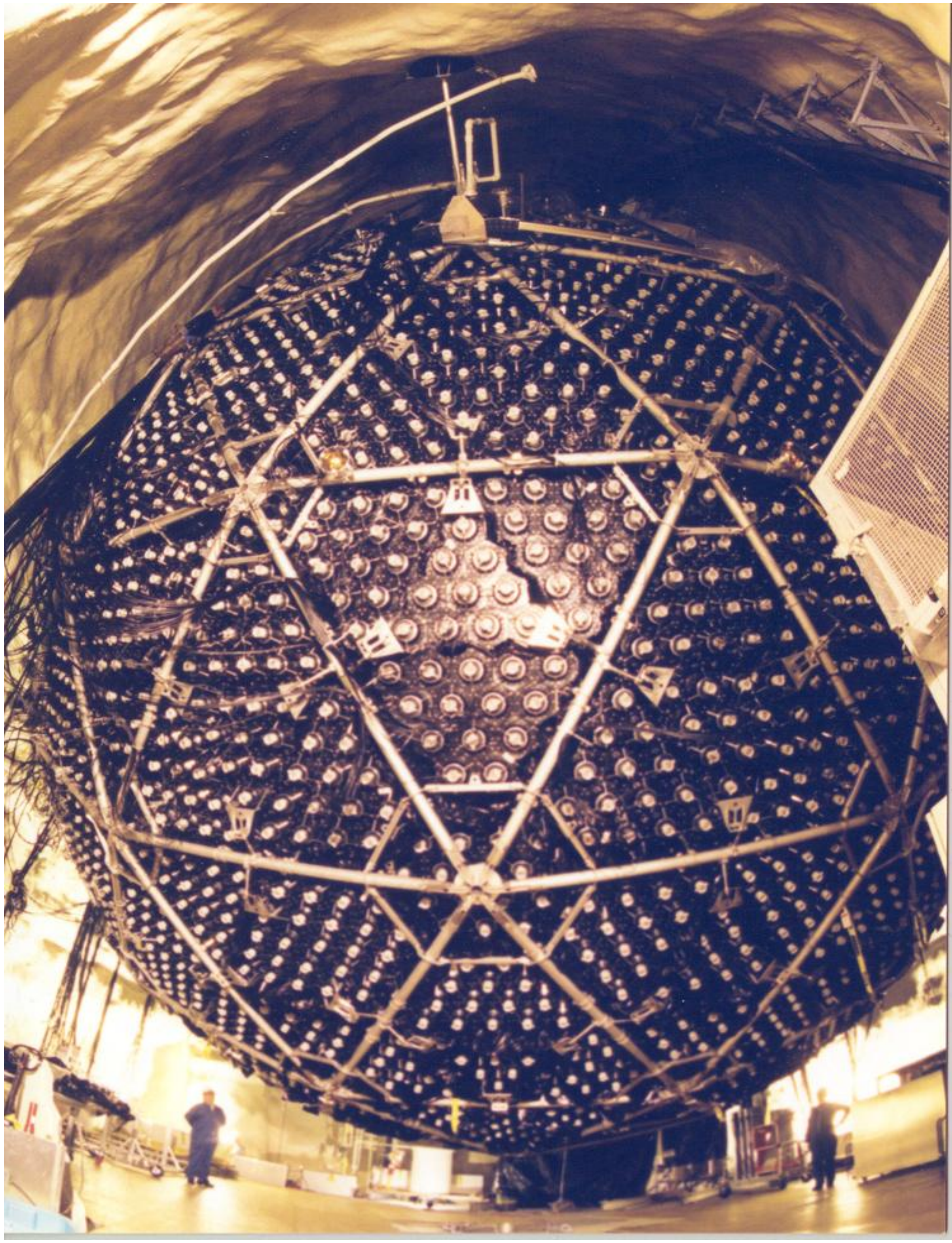





Total Rates: Standard Model vs. Experiment
Bahcall–Pinsonneault 2000
$7.6^{+1.3}_{-1.1}$
$1.0^{+0.20}_{-0.16}$
128^{+9}_{-7}
0.55±0.08
0.48±0.02
71^{+7}_{-6}
71±6
2.56±0.23
SAGE
GALLEX
+
GNO
SuperK
Kamioka
Cl
H_2O
Ga
Theory
^7Be
^8B
p–p, pep
CNO
Experiments
Uncertainties

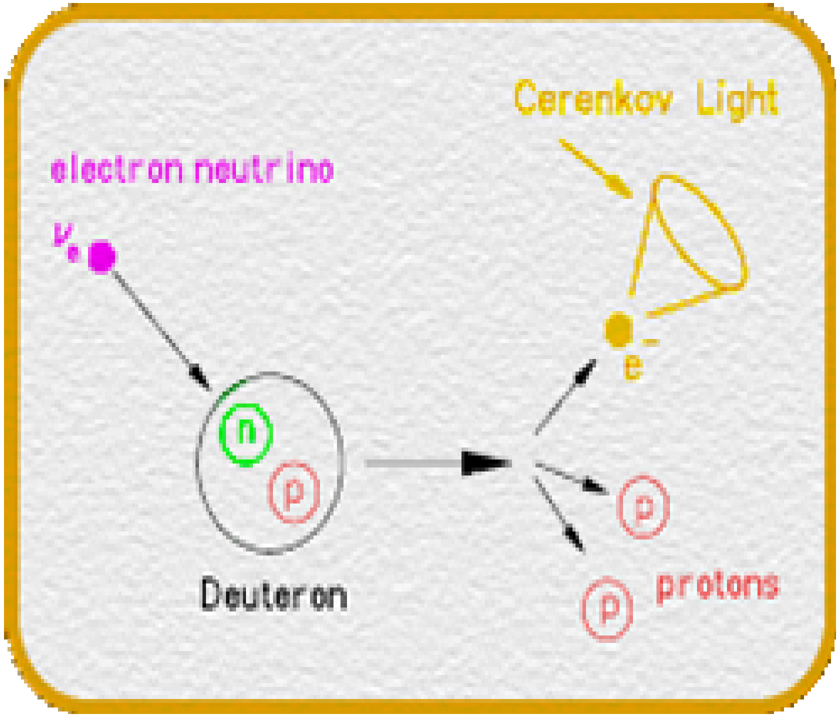
The Sudbury Neutrino Observatory (SNO)



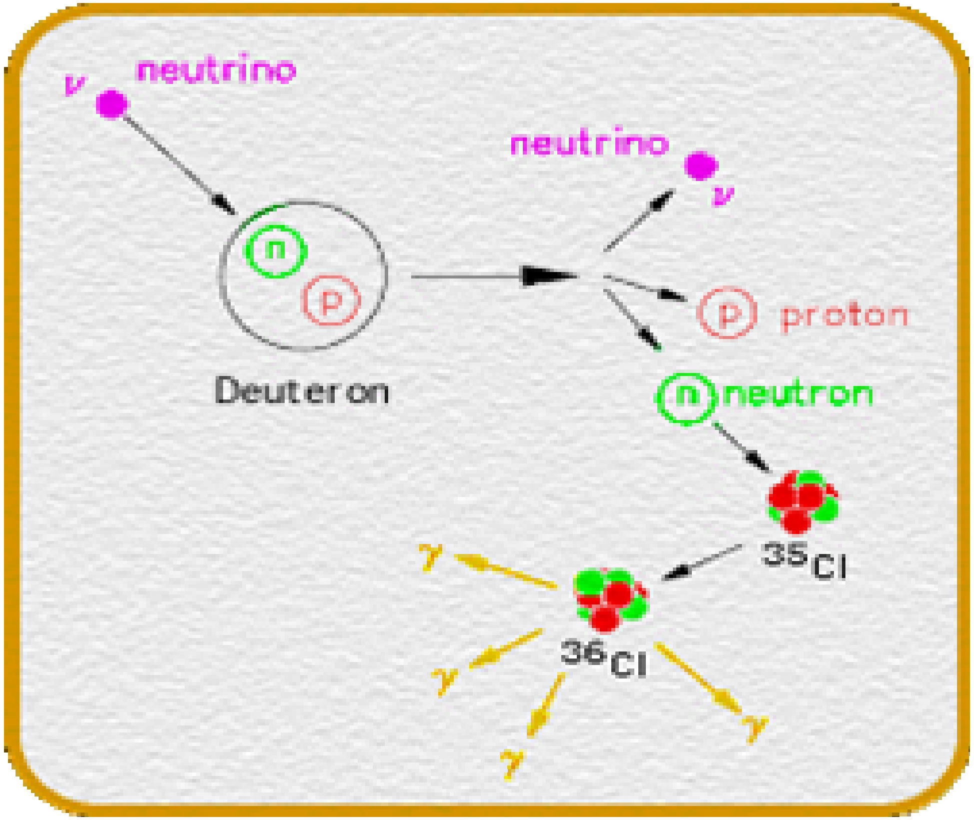


Neutrino interactions in D_2O

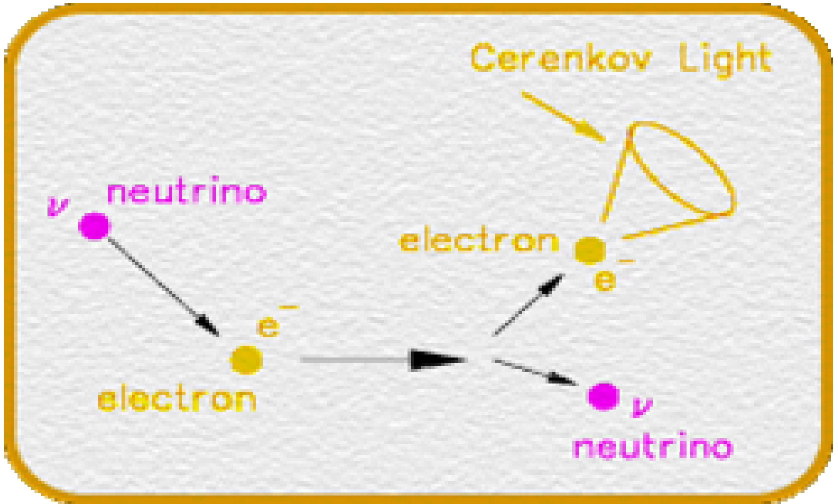
Charged Current Reaction (CC):

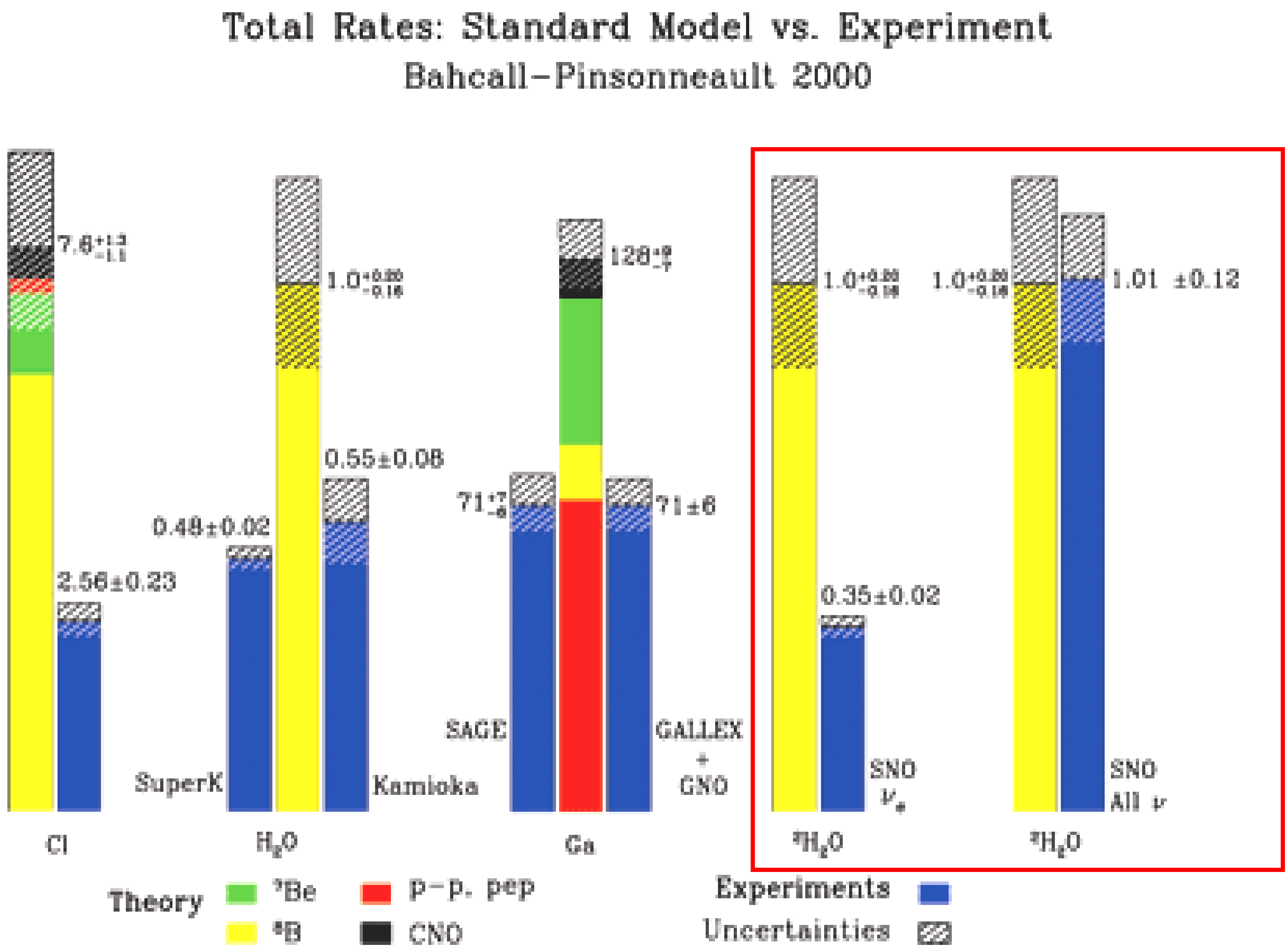


Neutral Current Reaction (NC):



Elastic scattering (ES):




Total Rates: Standard Model vs. Experiment
Bahcall–Pinsonneault 2000
7.6
2.56±0.23
0.48±0.02
1.0
0.55±0.08
128
71
71±6
SAGE
GALLEX
+
GNO
SuperK
Kamioka
1.0
0.35±0.02
1.0
1.01 ±0.12
SNO
ν_e
SNO
All ν
Cl
H_2O
Ga
^2H_2O
^2H_2O
Theory
^7Be
p-p, pep
^8B
CNO
Experiments
Uncertainties

Key ingredients of the ab initio "microscopic" calculation:

$$\sigma \approx |V_{fi}|^2 = |\langle f \,|\, V \,|\, i \rangle|^2$$

known

$$V \approx j_\mu \cdot J^\mu \rightarrow \langle f \,|\, V \,|\, i \rangle \approx \langle f\text{-lep}|\, j_\mu \,|i\text{-lep}\rangle$$

$$\cdot \langle f\text{-hadr}|\, J^\mu \,|i\text{-hadr}\rangle$$

1. Realistic description of |i-hadr> and |f-hadr>: *realistic Hamiltonians + accurate techniques* (HH method)
2. Realistic description of J^μ : realistic model for the *nuclear electroweak transition operator*

Realistic Hamiltonian

$$H = T + \sum_{ij} v_{ij} + \sum_{i<j<k} V_{ijk}$$

non-relativistic kinetic energy

NN potential:
AV18

three-nucleon interaction:
Urbana IX

A=3-7 binding energies

$^{A}Z(J^{\pi};T)$	CHH	VMC	GFMC	Expt
$^{3}H(\frac{1}{2}^{+};\frac{1}{2})$	8.479	8.227(6)	8.461(6)	8.482
$^{3}He(\frac{1}{2}^{+};\frac{1}{2})$	7.750	7.476(6)	7.708(6)	7.718
$^{4}He(0^{+};0)$	27.89	27.40(3)	28.31(2)	28.30
$^{6}Li(1^{+};0)$		28.05(5)	31.25(8)	31.99
$^{7}Li(\frac{3}{2}^{-};\frac{1}{2})$		33.07(7)	37.5(1)	39.24
$^{7}Li(\frac{1}{2}^{-};\frac{1}{2})$		33.13(7)	37.6(1)	38.76
$^{7}Be(\frac{3}{2}^{-};\frac{1}{2})$		31.49(7)	35.9(1)	37.60
$^{7}Be(\frac{1}{2}^{-};\frac{1}{2})$		31.58(8)	35.9(2)	37.17

Pieper&Wiringa, Ann. Rev. Part. Sci. **51**, 53 (2001)

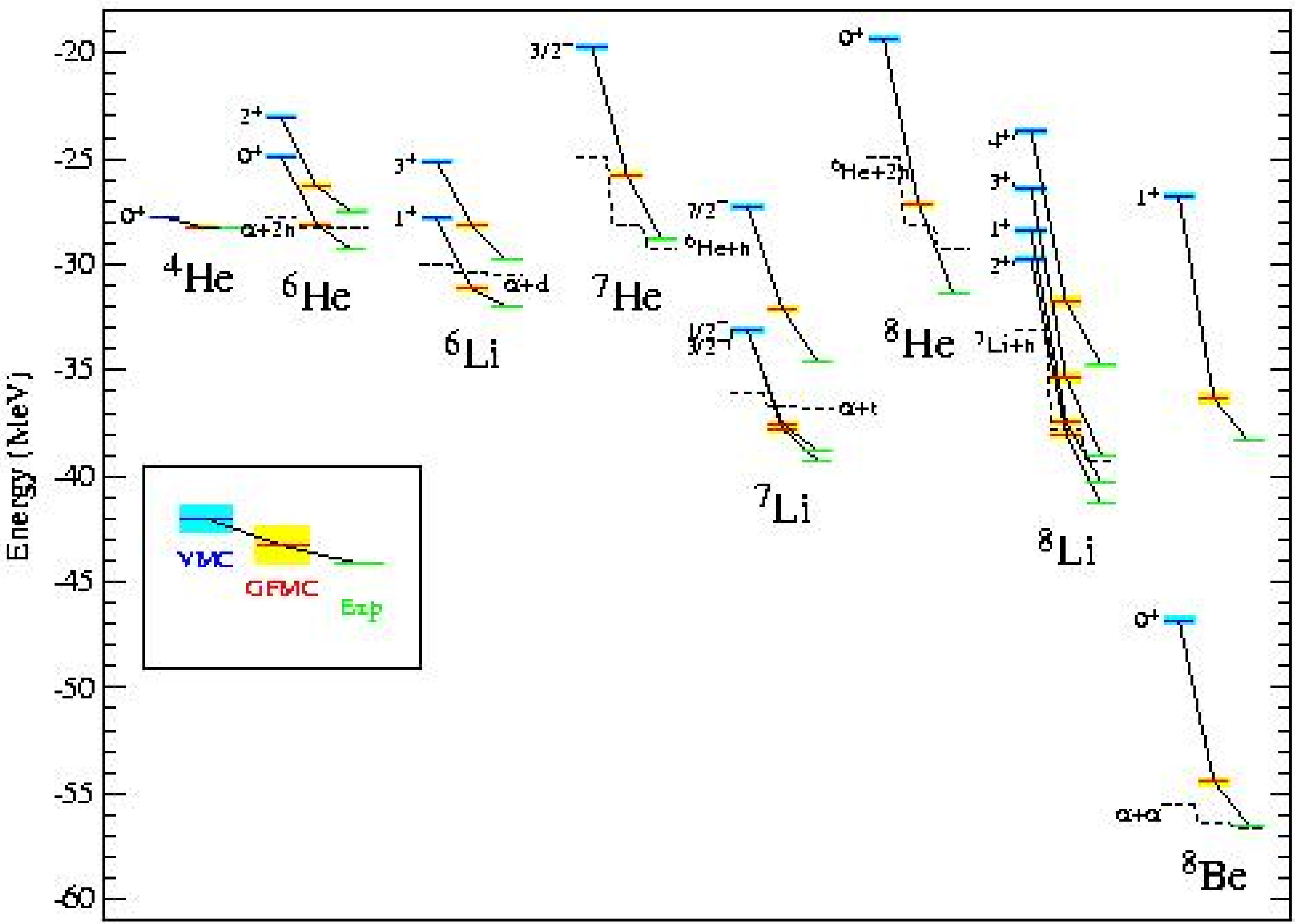
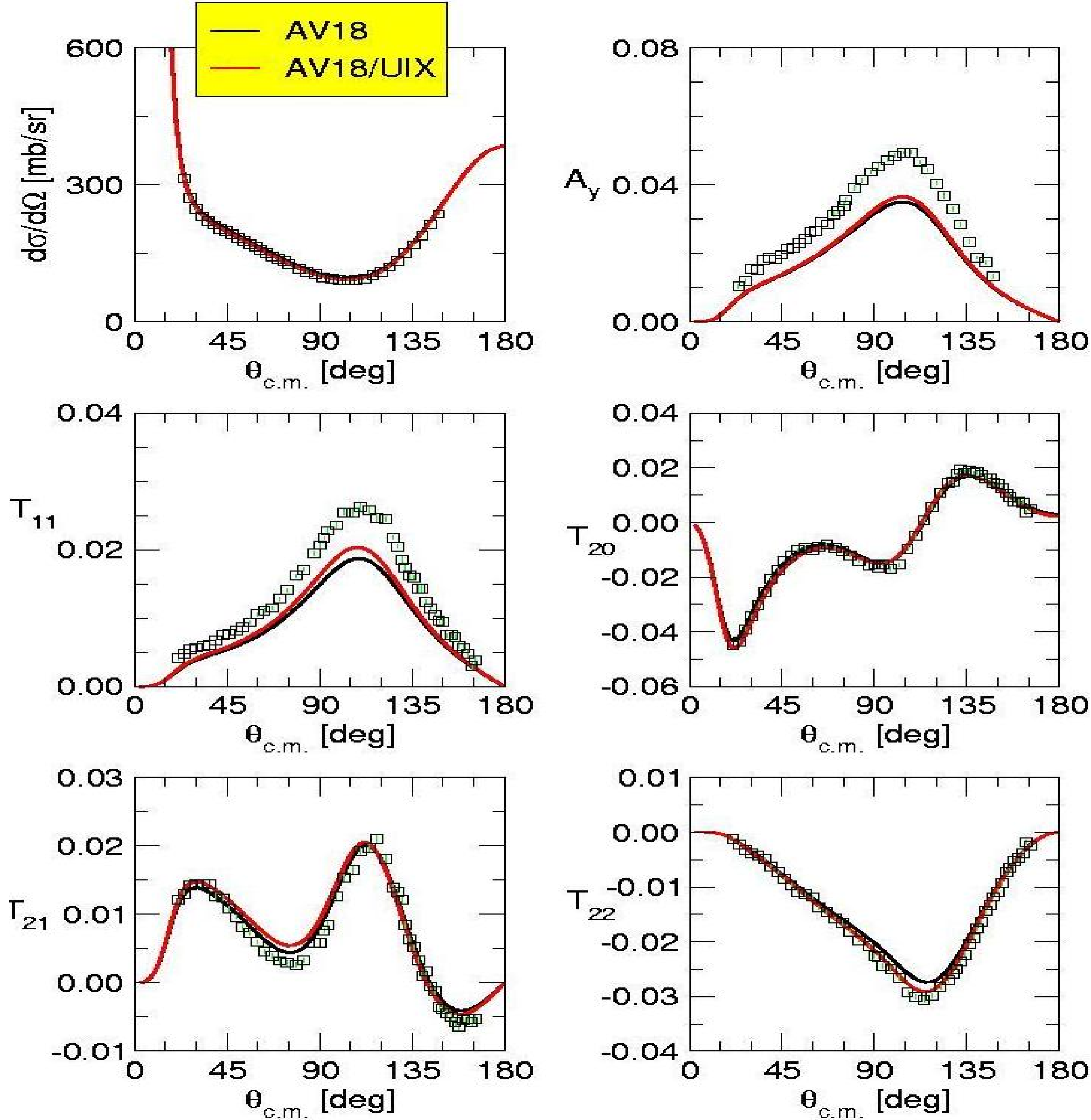
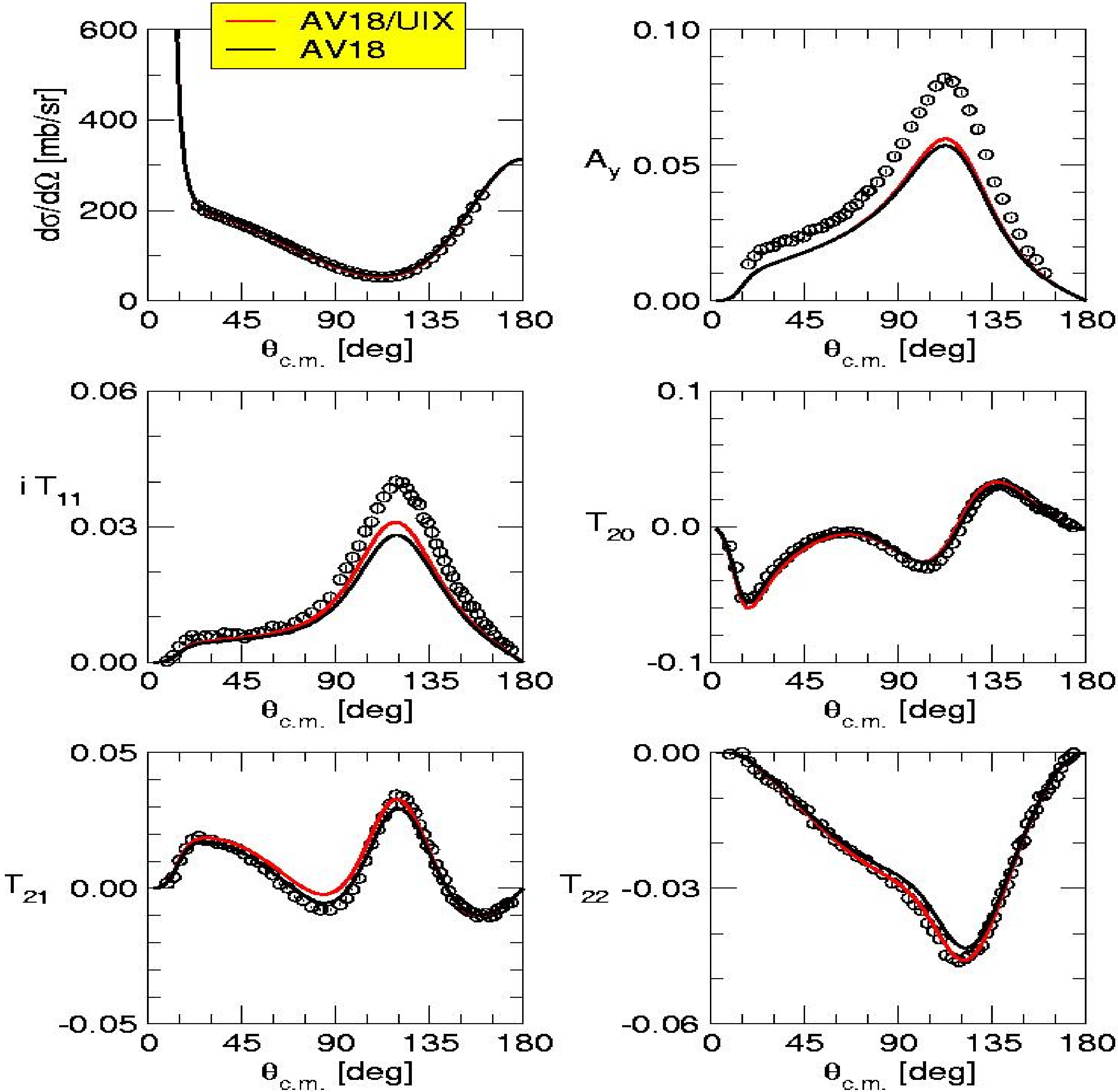


Figure 4: VMC and GFMC energies using AV18/UIX compared to experiment. Black dashed lines show the indicated breakup thresholds for each method. The Monte Carlo statistical errors are shown by the light blue and yellow bands.



pd elastic scatt. at 2.00 MeV



pd elastic scatt. at 3.33 MeV

Electroweak nuclear transition operator

Electromagnetic sector:

charge and current operators

Weak sector:

vector charge and current operators

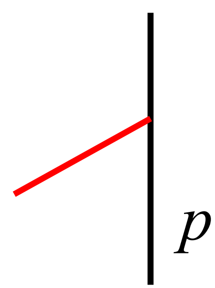
axial charge and current operators

Conserved vector current hypothesis →
weak vector operators from the *EM* ones
with a rotation in the isospin space

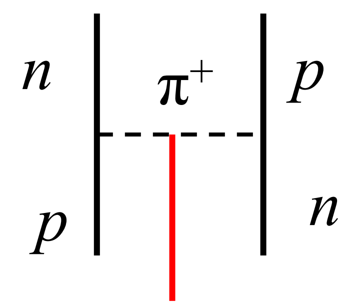
Electromagnetic current operator

Realistic model: the e.m. current <u>must satisfy</u> the current conservation relation (CCR) with the Hamiltonian
$H=T+\sum_{ij} v_{ij}+\sum_{i<j<k}V_{ijk}$

one-body current



two-body current



three-body current •••

Similar situation for the axial current operator

Gauge invariance and CCR

$$\frac{\partial \rho(\boldsymbol{r},t)}{\partial t} + \nabla \cdot \boldsymbol{J}(\boldsymbol{r},t) = 0$$

$$\boldsymbol{J}(\boldsymbol{q}) = \int d\boldsymbol{x}\, exp(i\ \boldsymbol{q} \cdot \boldsymbol{x})\, \boldsymbol{J}(\boldsymbol{x}) \Rightarrow \nabla \cdot \boldsymbol{J} \rightarrow i\, \boldsymbol{q} \cdot \boldsymbol{J}(\boldsymbol{q})$$

$$\frac{\partial \rho}{\partial t} \rightarrow -i\,\hbar[H, \rho]$$

$$\Rightarrow \quad \boldsymbol{q} \cdot \boldsymbol{J} = [H, \rho]$$

$$H = T + V$$

$$\Rightarrow \quad \boldsymbol{q} \cdot \boldsymbol{J} = [T, \rho] + [V, \rho]$$

$$\boldsymbol{J} = \boldsymbol{J}_1 + \boldsymbol{J}_2 \rightarrow$$

$$\boldsymbol{q} \cdot \boldsymbol{J}_1 = [T, \rho] \quad \boldsymbol{q} \cdot \boldsymbol{J}_2 = [V, \rho]$$

$n+p \rightarrow d+\gamma$ *radiative capture at thermal energies*

Total cross section [mb] for *np* radiative capture (AV18):

One-body	**304.6**
Full	**332.7**
Expt.	**332.6(7)**

$$p+p \rightarrow d+e^{+}+\nu_e$$

$$S(E) = E\,\sigma(E)\, e^{2\pi Z_1 Z_2\, \alpha/v}$$

$$S(E) \approx \Lambda(E)^2$$

Square of the overlap integral $\Lambda(E=0)$ for two modern NN interaction models.

NN model	AV18	CD-Bonn
Λ^2 (one-body)	6.965	6.985
Λ^2 (full)	7.076	7.060

Schiavilla *et al.*, PRC **58**, 1263 (1998)

H. A. Bethe and C. L. Critchfield: Phys. Rev. **54**, 248 (1938)

TABLE I. *Numerical results for two values of the radius.*

	$r_0=e^2/mc^2$	$r_0=e^2/2\,mc^2$
x_0	0.645	0.322
V_0 (Mev)	20.9	66.5
D (Mev)	10.3	47.0
μ	2.94	5.45
ν	2.18	4.65
$(rd \log w/dr)$	0.236	0.110
$\Phi(r_0)$	1.050	1.025
$\Theta(r_0)$	0.769	0.854
$(rd \log \Phi/dr)$	0.050	0.025
ζ	0.814	0.915
λ	2.63	4.80
Λ_1	0.689	0.277
Λ_2	1.949	1.547
Λ_3	1.205	1.030
$(1+x_0)(1+\mu^{-2})$	1.835	1.367
Λ	2.84	2.44
Λ^2	8.08	5.93

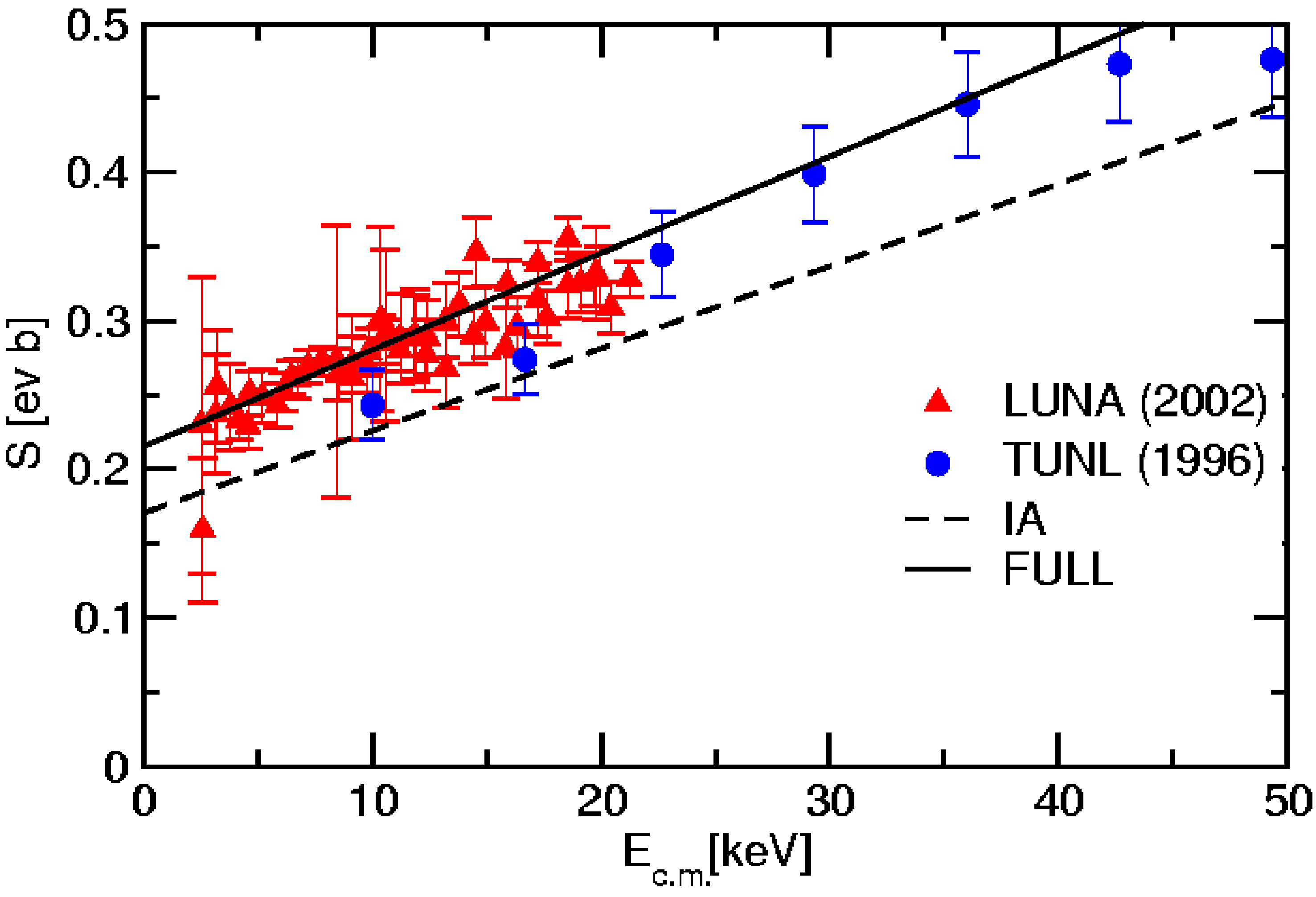
[13] Breit, Condon and Present, Phys. Rev. 50, 825 (1936).

$$n + d \longrightarrow {}^3\mathrm{H} + \gamma$$

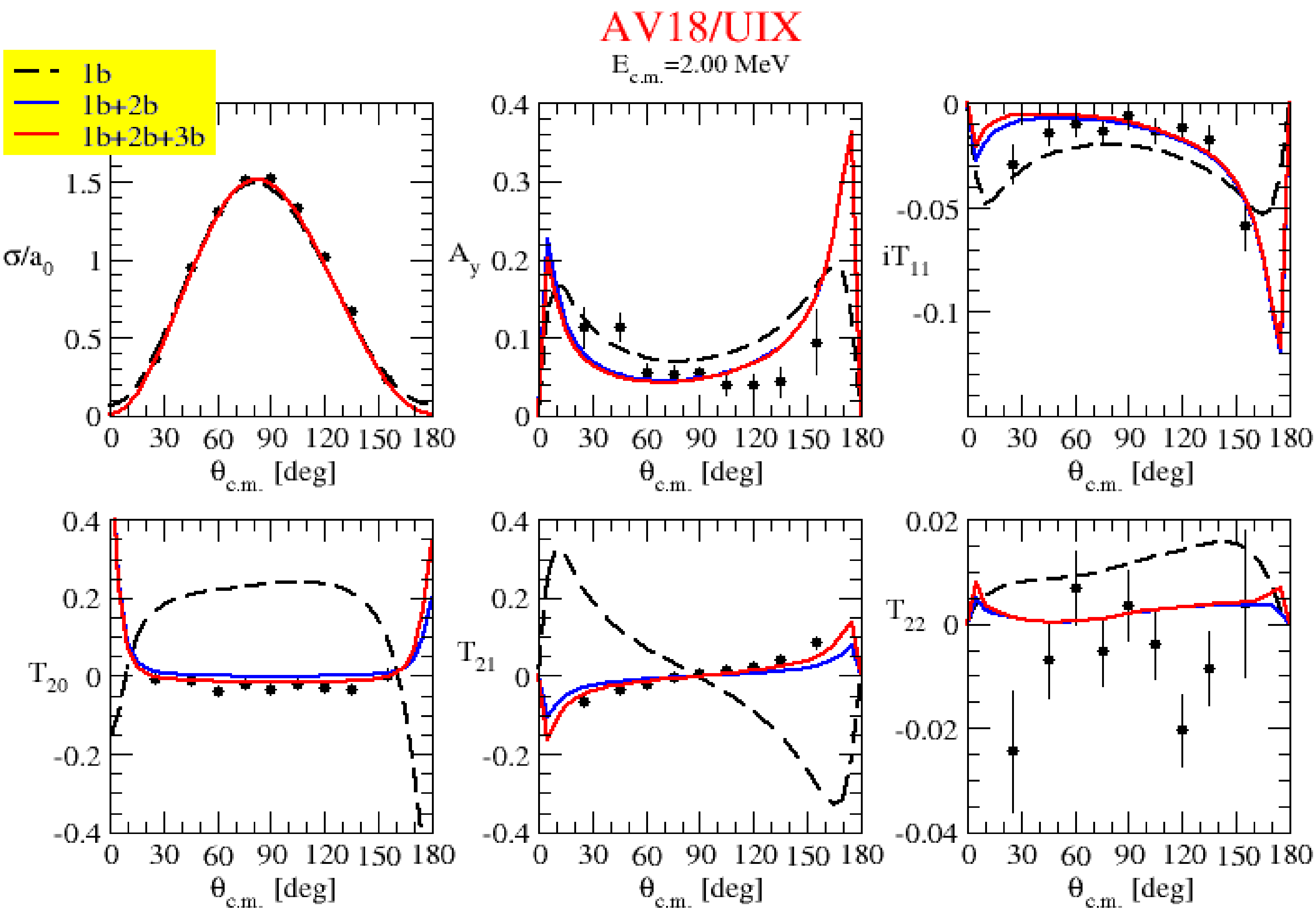
Total cross section [mb] for *nd* radiative capture (AV18/UIX):

One-body	**0.227**
Full	**0.556**
Expt.	**0.508(15)**

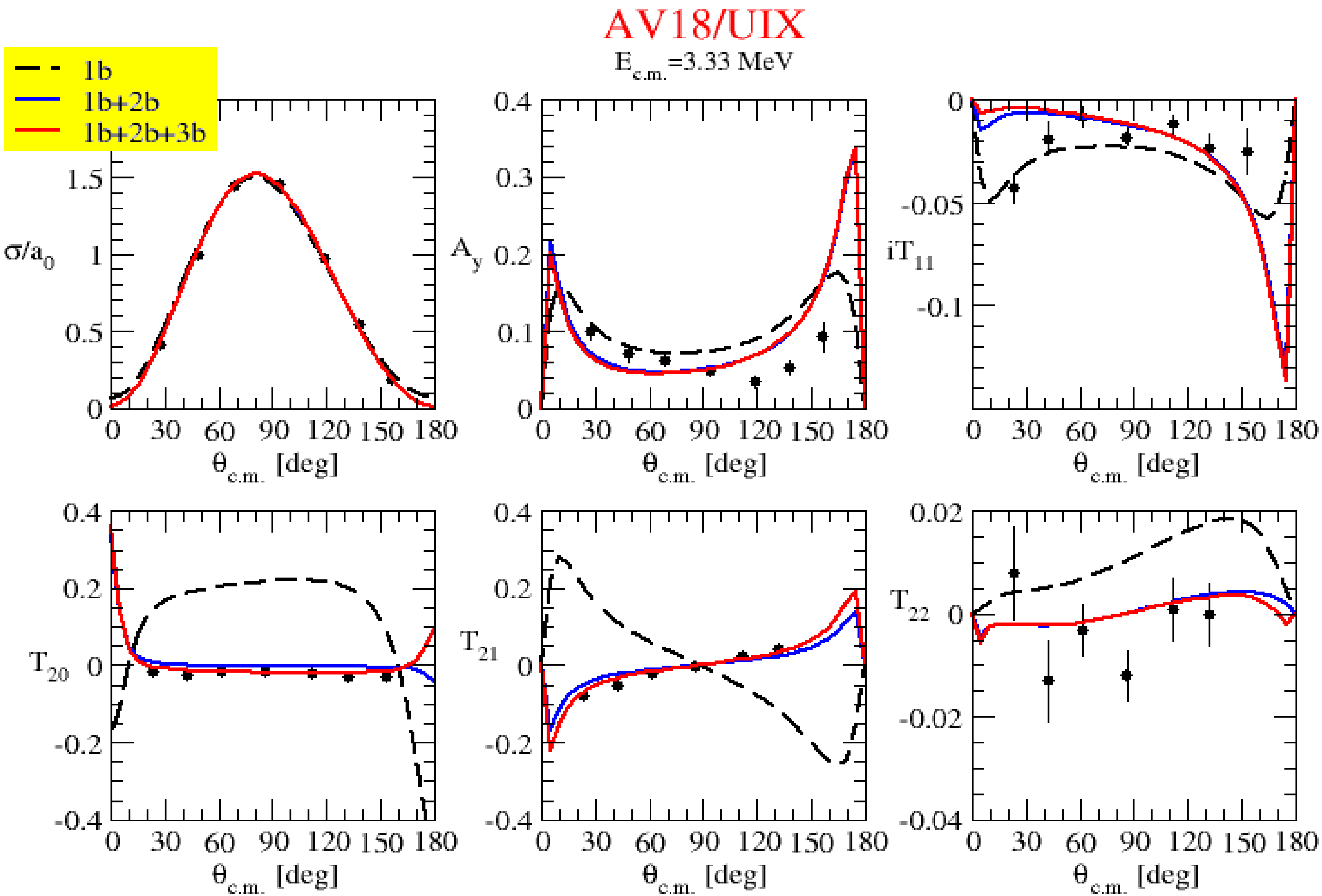
$p + d \longrightarrow {}^3\mathrm{He} + \gamma$



$p+d \longrightarrow {}^3\mathrm{He} + \gamma$



$p+d \longrightarrow {}^3\mathrm{He} + \gamma$



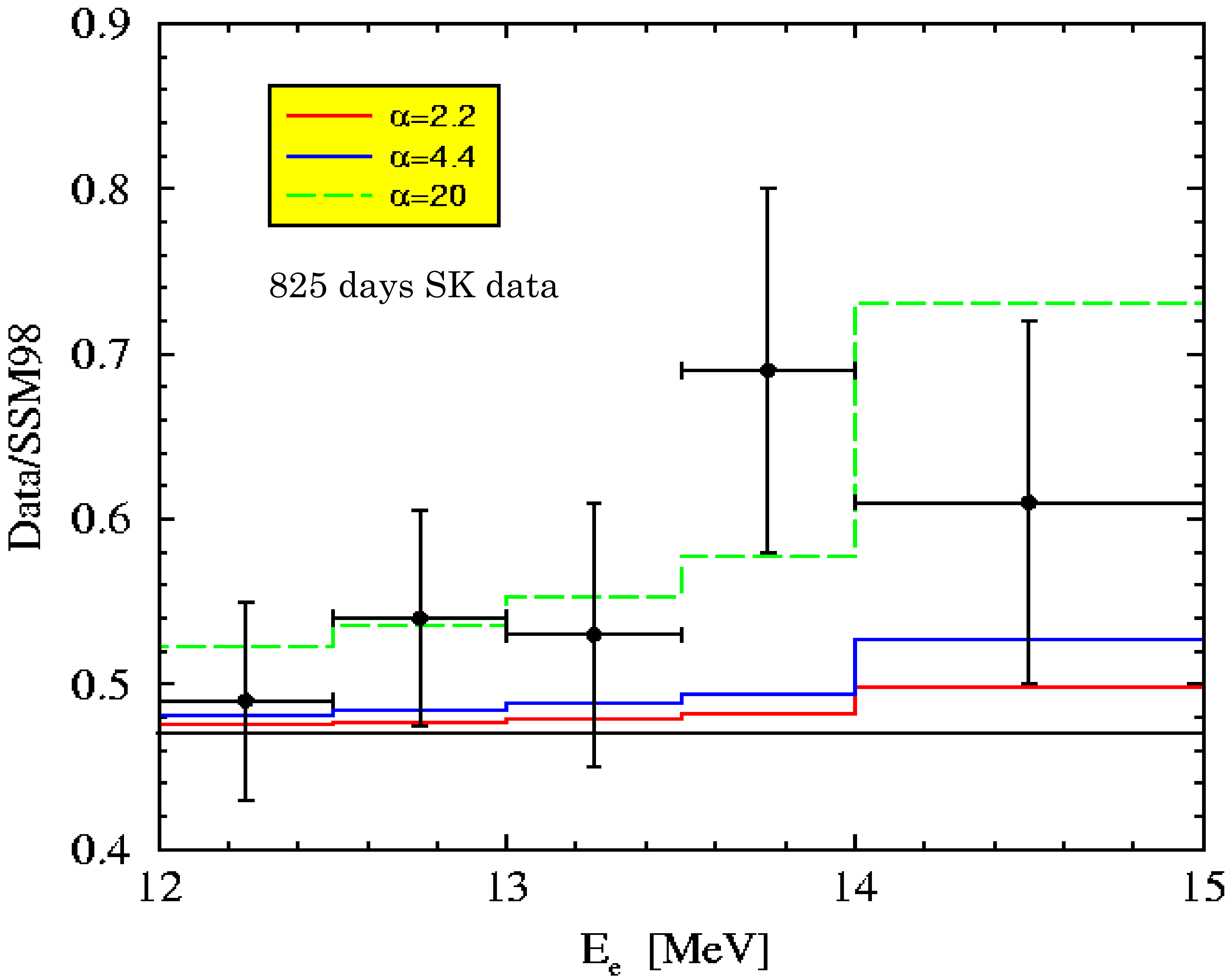
$p+{}^3He \longrightarrow {}^4He+e^+ +\nu_e$: *the hep reaction*

- $S_{SSM98} = 2.3 \times 10^{-20}$ keV b

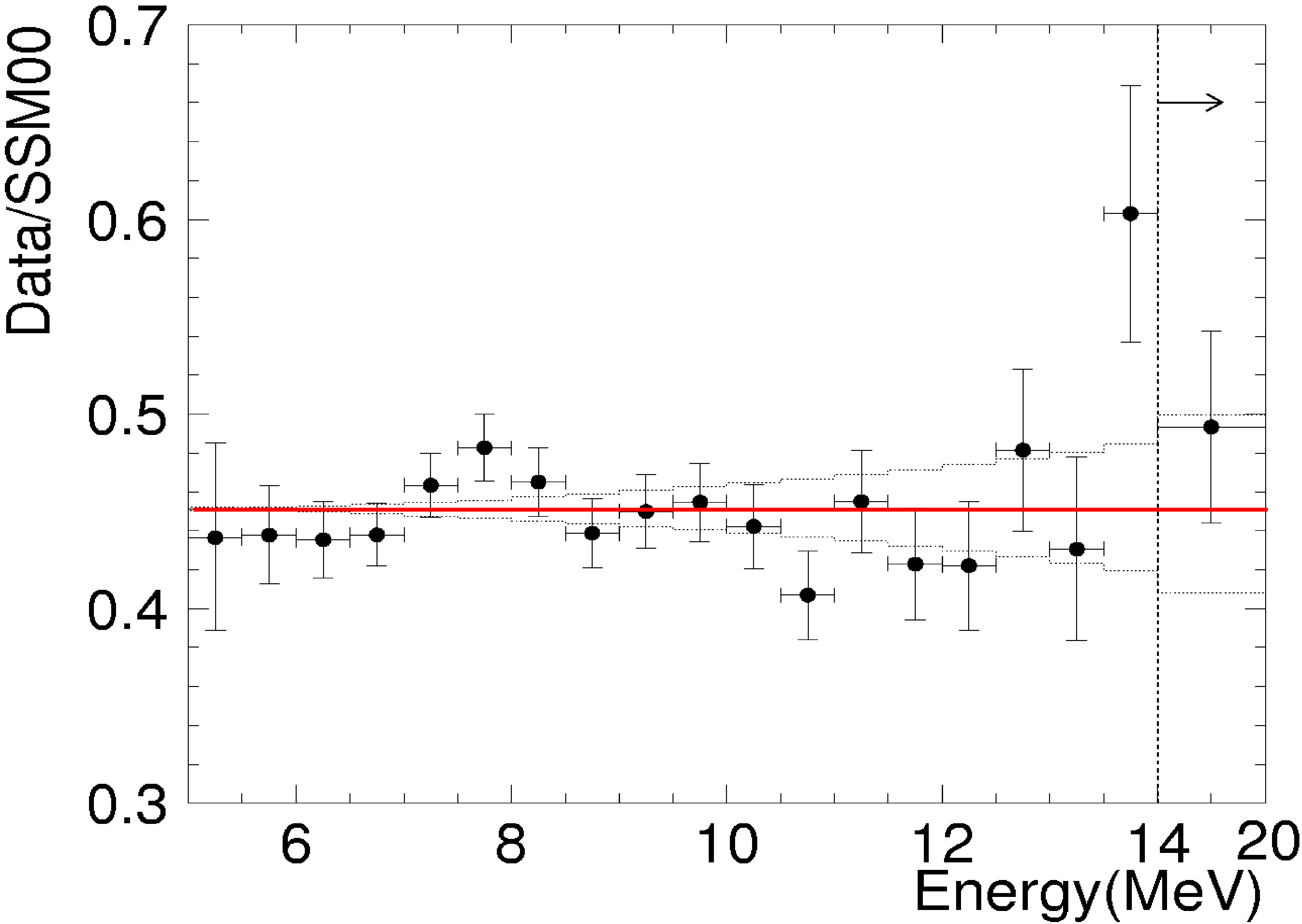
- New calculation (Marcucci *et al.*, PRL **84**, 5959 (2000); PRC **63**, 015801 (2001)):

  1. S (10 keV) $= 10.1 \times 10^{-20}$ keV b
  2. no c.m. energy dependence
  3. no Hamiltonian model dependence (if H reproduces accurately the initial and final state w.f.'s)
  4. importance of the P-waves (40%)
  5. importance of MEC (Δ)

- SSM revisited (Bahcall *et al.*, Astr. J. **555**, 990 (2001)): $S_{SSM00} = 10.1 \times 10^{-20}$ keV b

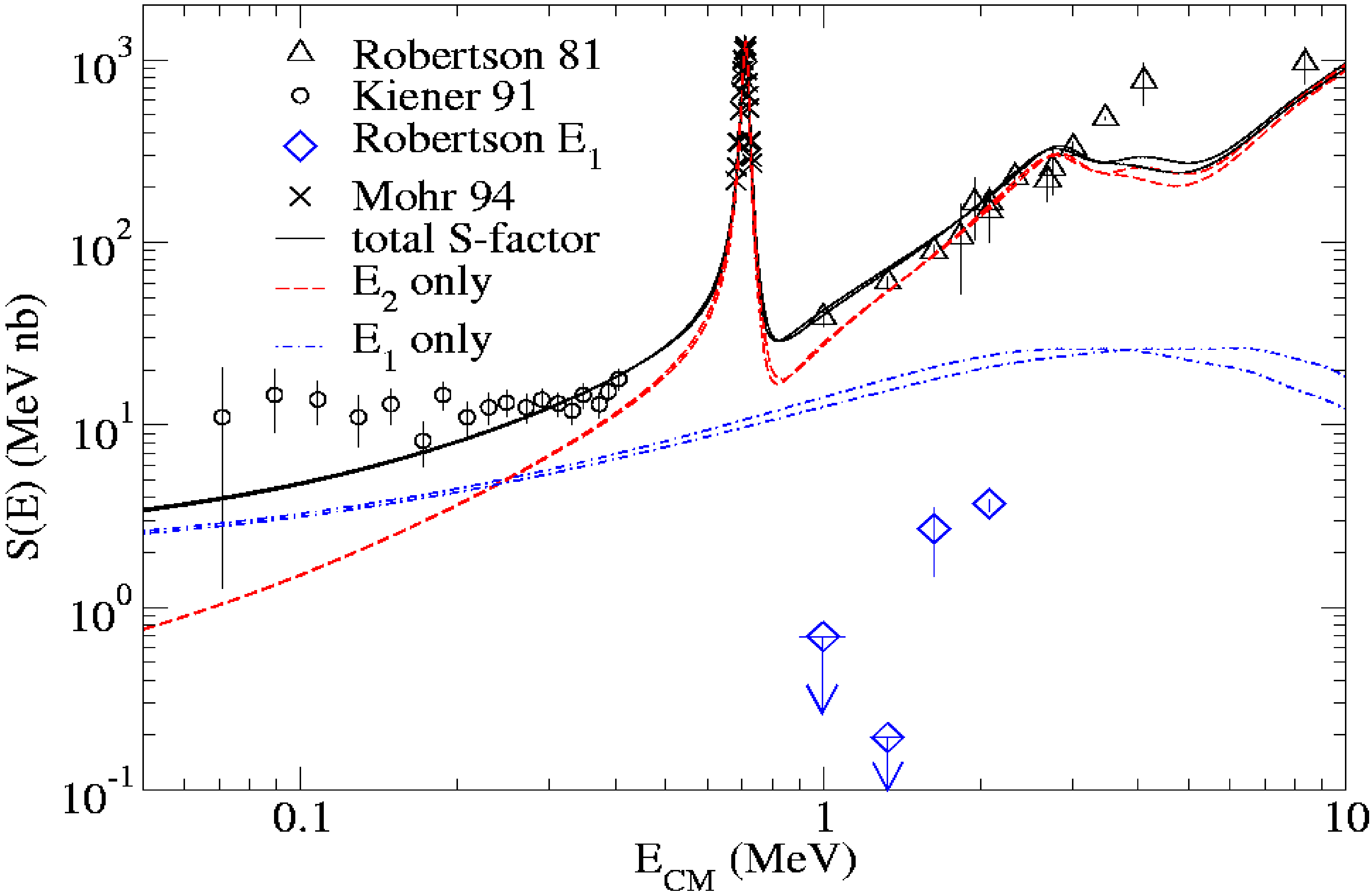


0.9
0.8
0.7
0.6
0.5
0.4
12
13
14
15
Data/SSM98
E_e [MeV]
α=2.2
α=4.4
α=20
825 days SK data



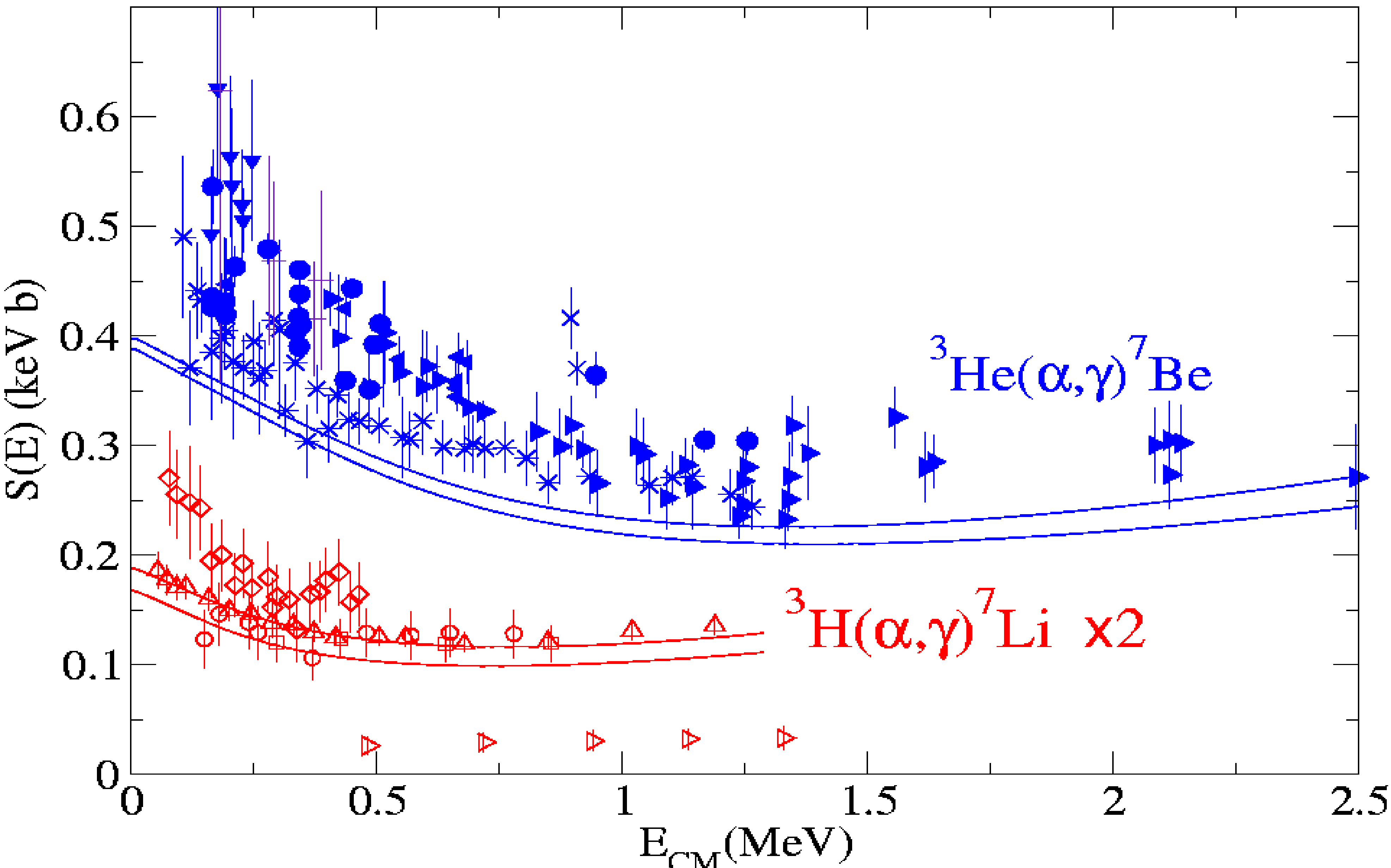
The SK collaboration, PRL **86**, 5651 (2001)

$\alpha+d \longrightarrow {}^6\text{Li}+\gamma$



Nollett *et al.*, PRC**63**, 024003 (2001)

$^3H + \alpha \rightarrow {}^7Li + \gamma$ and $^3He + \alpha \rightarrow {}^7Be + \gamma$



Nollett, PRC**63**, 054002 (2001)

$^7Be + e^- \longrightarrow {}^7Li + \nu_e$ and $^7Be + e^- \longrightarrow {}^7Li^* + \nu_e$

Gamov-Teller matrix elements for A=3,7 nuclei.

	CHH		VMC		
	IA	Full	IA	Full	Expt.
$^3H \rightarrow {}^3He$	1.597	1.658	1.602		1.658
$^7Be \rightarrow {}^7Li$			2.345(3)	2.419(5)	2.599
$^7Be \rightarrow {}^7Li^*$			2.142(2)	2.200(3)	2.323

Marcucci *et al.*, Nucl. Phys. **A777** (2006)

Conclusions and outlook

- Nuclear reactions of astrophysical interest are an important field of research for nuclear theory

can be tested　　　　provides inputs

- Further investigation and comparison with experimental results (electron scattering)